Basic Electrical Technology
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Lecture - 38
Induction Motor – 3

Hello everybody, in the last class we were discussing about the induction motor and then we had concluded with the manner in which the active power flows through the machine. We saw that the active power which enters the machine through the stator and then there is some portion of the power that gets lost in the stator as I square R loss due to the winding resistance and then it goes into the magnetic domain and in the magnetic domain you have the core losses, the hysteresis currents like in a transformer and the remaining power goes into the rotor as the effective rotor power and in the rotor again there are rotor conductors, there will be I square losses in the rotor conductors and minusing that I square R losses the remaining power goes to the mechanical shaft and the mechanical shaft if there is bearing frictions associated some portion of that power goes in and ultimately you will have the shaft power.

So let us work out an example to get an idea of the power flow in the induction motor. So let us take a 3 phase induction motor. Let me take an example from the book. So we have a 3 phase induction motor. It has a synchronous speed n s as 1200 rpm and it draws 80 kilowatts from the source from the 3 phase source. The copper loss the copper loss plus the core loss or the iron loss in the stator amounts to 5 kilowatts. So the motor runs runs at 1152 rpm revolutions per minute then let us try to find the power flow for such amount.

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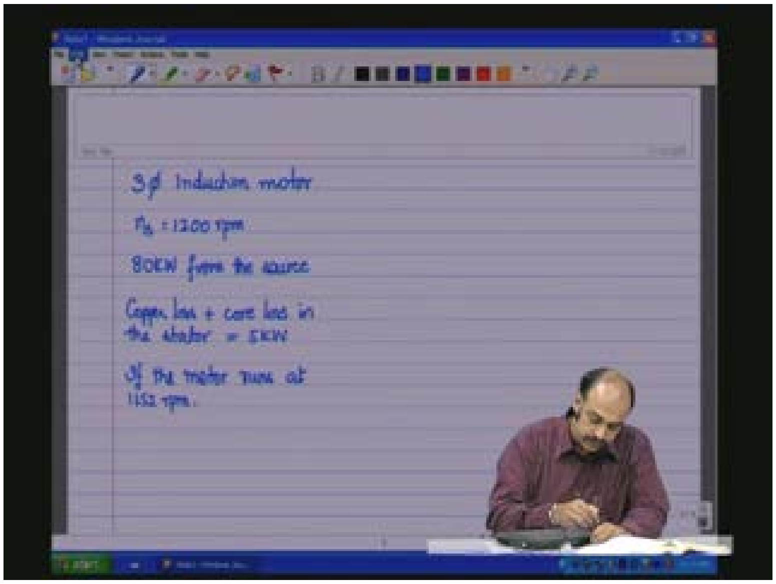

3φ Induction motor

$n_s = 1200$ rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at 1152 rpm.

So let us first make a calculation. Let us have this so that we have an idea of what we are doing. Copy; so we have the stator and to the stator there is an input power P s and there is some loss P Ls I square R loss, there is also some loss this is the iron losses P i in the stator and the remaining power goes to the rotor which is P r. Now in the rotor in the rotor there is some loss due to the finite resistance of the rotor conductors so I square R loss there, the remaining power goes as the mechanical power which is developed. And in the bearings you can have some friction losses and there is some loss which goes off as friction so we will say P friction and the remaining power goes as the shaft power P shaft. now And we also know here that if we take this portion here the rotor portion P Lr is equal to P r minus P m and we saw that P r is nothing but torque into omega s minus P m torque into omega m so it is nothing but T omega s you take it you take out as common and you have 1 minus omega m by omega s and that is equal to slip times P r.

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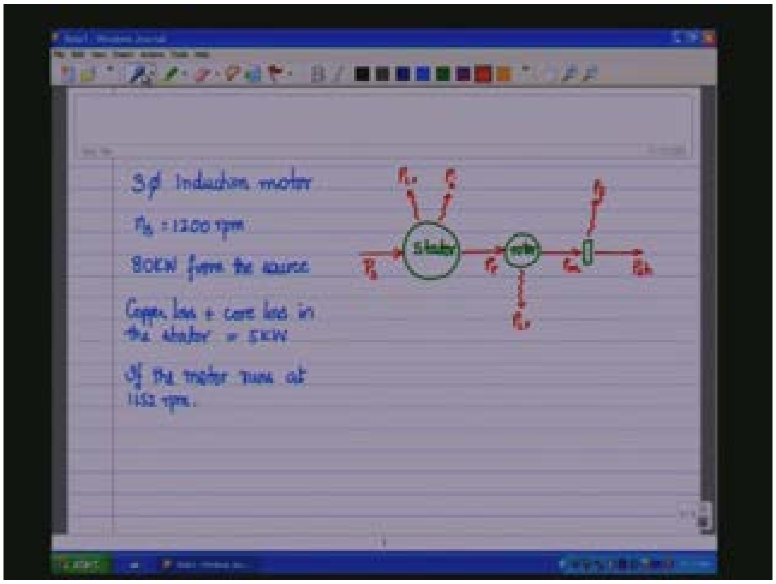

3ϕ Induction motor

$n_s = 1200$ rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at 1152 rpm.

So therefore, therefore P Lr the loss in the rotor is dependent on the slip and therefore we can replace that (Refer Slide Time: 7:40) by S times P r. So this is how power flow occurs.

Now let us calculate the various powers that will come here. Now if you look at the input what is the input? For this 3 phase induction motor which has a synchronous speed of 1200 rpm the input is 80 kilowatts okay so we have 80 kilowatts as the input.

Now out of this copper loss plus core loss in the stator is 5 kilowatts so this together is 5 kilowatts. So therefore P r which is equal to P s minus P Ls minus P i which is equal to 80 minus 5 which is equal to 75 kilowatts, so this is 75 kilowatts.

Now what is S?

The slip S is given by n s minus n m by n s which is equal to 1200 rpm minus 1152 rpm which is the motor speed. Look at these two (p........9:30) by 1200 and that is equal to 0.04.

So now what is sP r?

sP r is equal to 0.04 into 75 which is equal to 4 into 7.5 which is 3 kilowatts so this is 3 kilowatts. So this 3 kilowatts what goes to the power as the mechanical power will be 72 kilowatts.

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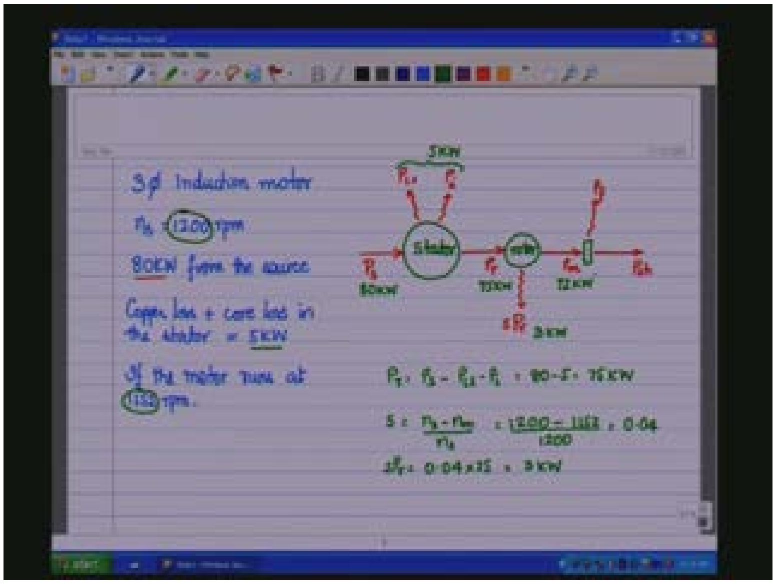

3ϕ Induction motor

$n_s = 1200$ rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at 1152 rpm.

$P_r = P_s - P_{cu} - P_c = 80 - 5 = 75KW$

$s = \frac{n_s - n_m}{n_s} = \frac{1200 - 1152}{1200} = 0.04$

$sP_r = 0.04 \times 75 = 3KW$

Now let us say that the friction loss friction loss which is given by let us say like I square R it is omega square into B and we know omega square which is 1152 rpm into radians which is 2 pi by 60 is equal to 2 kilowatts so which means 2 kilowatts goes off here so what is available at the shaft is 70 kilowatts 70 kilowatts.

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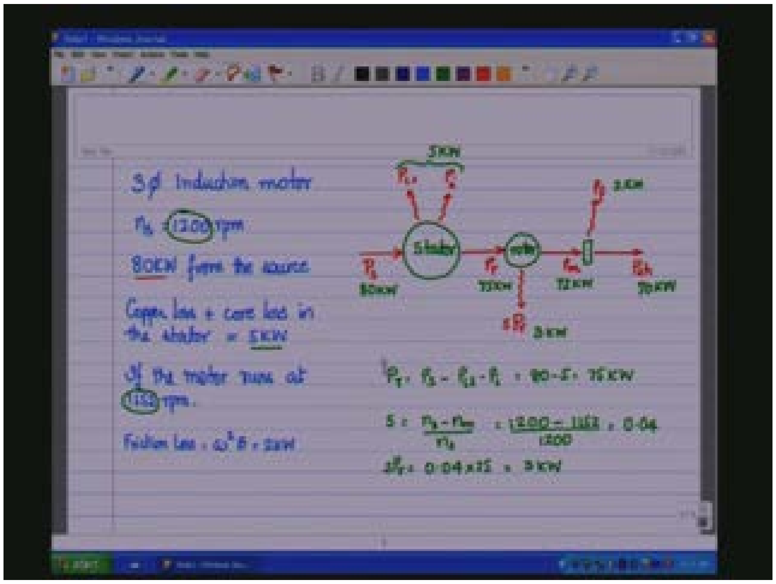

3ϕ Induction motor

n_s = 1200 rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at 1152 rpm.

Friction loss = $\omega^2 B$ = 2KW

$P_r = P_s - P_{c1} - P_c = 80 - 5 = 75$ KW

$s = \frac{n_s - n_m}{n_s} = \frac{1200 - 1152}{1200} = 0.04$

$sP_r = 0.04 \times 75 = 3$ KW

Let us say the efficiency. Efficiency is given by the power that is available at the shaft that can be delivered to the load: P shaft by the power that is input at the stator P s which is 70 by 80 which is 7 by 8 this is the efficiency of this particular motor. This is close to something like 80 percent efficiency which is equal to 87.5 percent or let me put it this way this is equal to 0.875 and then in terms of percentage if you multiply it by 100 so you will get efficiency equal to 87.5 percent.

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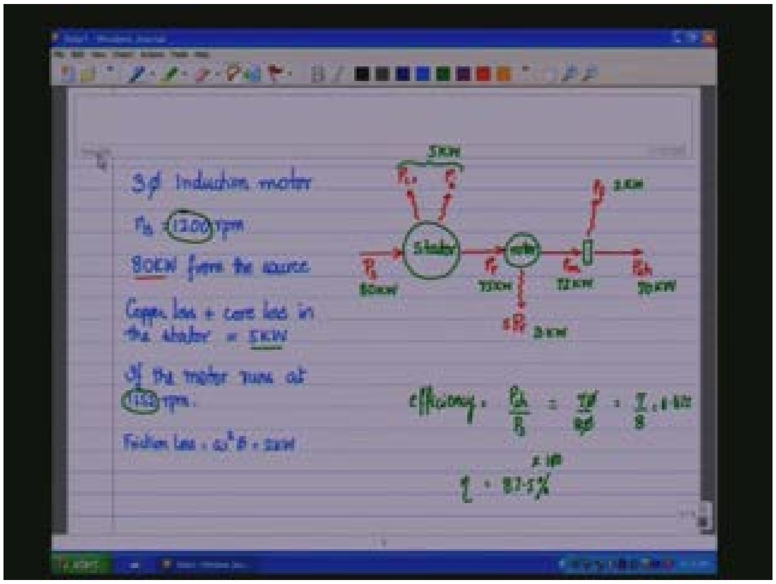

3φ Induction motor

n_s = 1200 rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at [illegible] rpm.

Friction loss = $\omega^2 B$ = 2KW

efficiency = $\frac{P_{sh}}{P_s} = \frac{70}{80} = \frac{7}{8} = 0.875$

$\times 100$

$\eta = 87.5\%$

Now let us see what is the torque. See; the torque that is generated by the rotor can be found out by the power. See; P r is the power that goes into the rotor and there is of course some amount of power that goes off due to the I square R losses so what is the mechanical power is P m. Now if P m is 1 minus s into P r; P r minus sP r; P r minus sP r that is 72 kilowatts now that is equal to torque that is generated by the motor let me call that one as T g into omega into omega m.

Or, on the other hand, if we want to find the torque which is independent of omega m you could use this equation also. But anyway we have this and therefore T g is equal to P m by omega m. T g is also equal to P r by omega s or equal to P r by omegas s if it is a synchronous frequency that you want to...........

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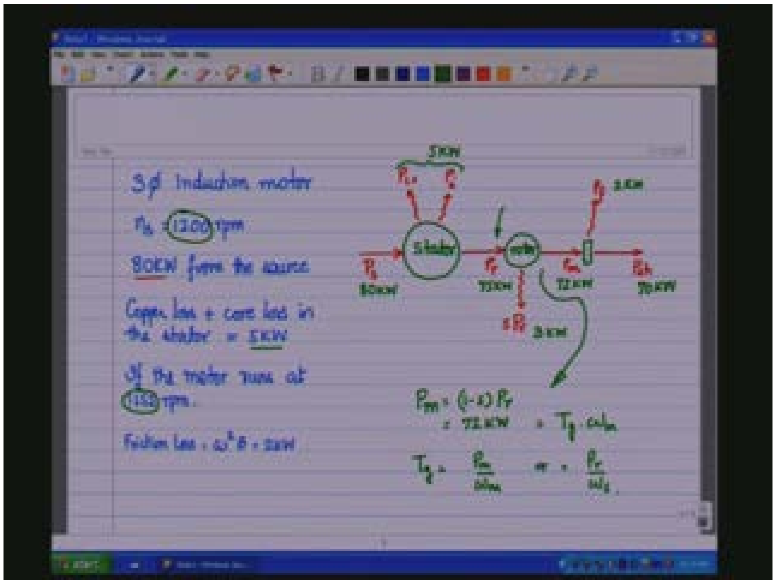

3ϕ Induction motor

n_s = 1200 rpm

80KW from the source

Copper loss + core loss in the stator = 5KW

If the motor runs at 1152 rpm.

Friction loss = $\omega^2 B$ = 2KW

$$P_m = (1-s)P_r$$
$$= 72KW = T_g \cdot \omega_m$$
$$T_g = \frac{P_m}{\omega_m} \quad \text{or} = \frac{P_r}{\omega_s}$$

Now we know those speeds of course in rpm. So, to convert it into radians per second what do you do; to convert it to radians per second we have T g which is equal to P m divided by omega m which is equal to P m divided by n m in rpm to convert it into radians per second we have into 60 by 2 pi. So in this case we have P ms 72, n ms 1152 into 60 by 2 so this is the generated torque the mechanical at the mechanical shaft.

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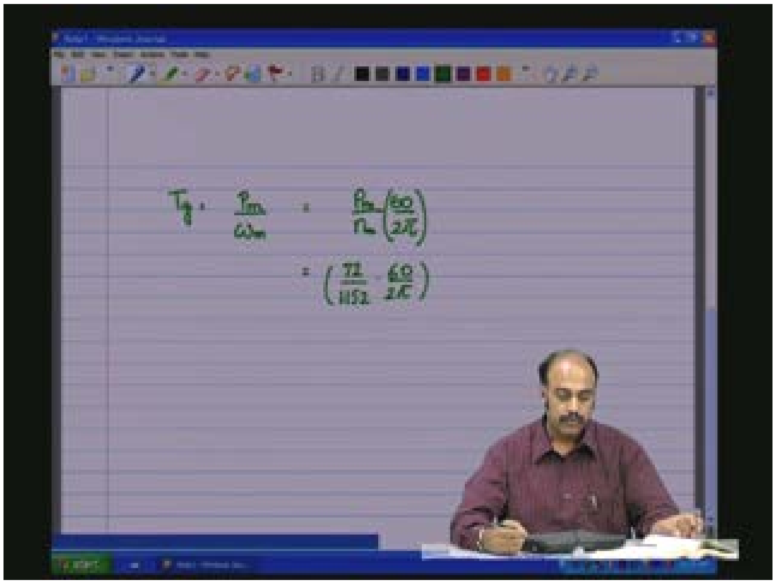

$$T_g = \frac{P_m}{\omega_m} = \frac{P_m}{n_m}\left(\frac{60}{2\pi}\right)$$
$$= \left(\frac{72}{1152}\cdot\frac{60}{2\pi}\right)$$

So now let us look at one important aspect of the induction motor and that is the equivalent circuit. How do we represent the induction motor induction motor as induction motor as an equivalent circuit equivalent circuit.

You see that the operation of the induction motor is very similar to that of the 3 phase transformer. So therefore, the equivalent circuit of the 3 phase induction motor will be very similar to that of the 3 phase transformer. Let us also consider that because of the symmetry in the induction motor that is all the 3 phases are balanced you could consider the single phase equivalent circuit per phase equivalent circuit of the induction motor because it is the equivalent circuit is going to be the duplicate for every other phase.

So if we take equivalent circuit of a transformer we have........ let us say we start with the primary terminals okay. Now, starting from the primary terminals we have an R primary and here it would be...... I will call it is a R 1 so that we keep it general between the transformer and the motor, and then we have an X sigma 1 the primary side leakage, we have the magnetizing reactance and R c which is the core loss component. Now this is followed by an ideal transformer which has some ratio 1 is to n which has a ratio 1 is to n dot polarities okay and on the secondary side you have the leakage reactance x sigma 2 and the R 2 which is the winding resistance of the secondary side; and this gets connected to the load, but we are just indicating up to the terminals so let us say we take only up to the terminals the external circuits need not come into the picture. So this is the way the transformer is having the equivalent circuit.

The same equivalent circuit of the transformer can also be applied to the induction motor. So let us go step by step to achieve the equivalent circuit of the transformer starting from the standstill condition. So under standstill condition what happens is there is no motion of the shaft which means there is no power delivered to the shaft. The frequency of the rotor f r is S times the frequency of the stator, so under standstill S is equal to 1. So as S is equal to 1 the frequency of the rotor is also the same as that of the stator. let us further make this ratio 1 is to 1, will make this 1 is to 1 and let us short the rotor; the rotor is now shorted because in the case of the induction motor the rotor is shorted or in the case of the wound rotor induction motor you could probably add a resistance there and then that resistance could gradually be cut off. So effectively we have the resistance externally put or a short plus R 2 which is the winding resistance. Now all these things are put together in this form and this looks exactly like that of a transformer.

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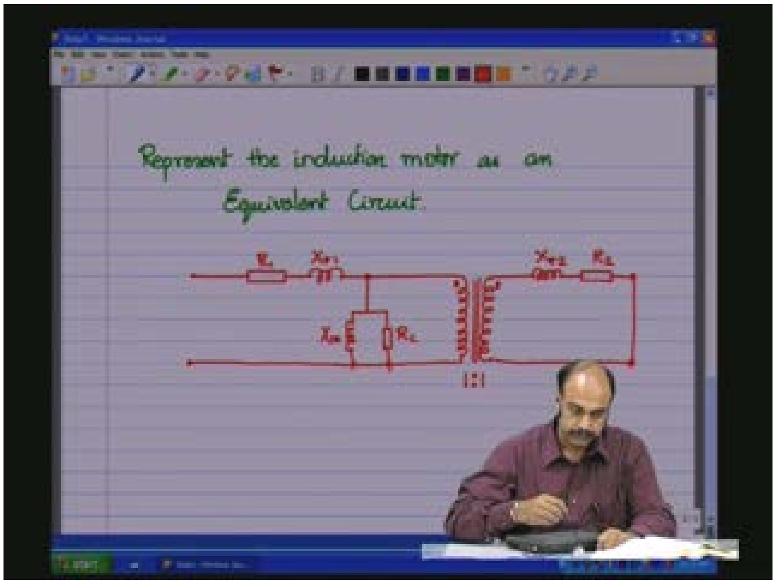

Represent the induction motor as an Equivalent Circuit.

So now here if you look at the currents flowing in this the frequency of the currents and the voltages here is the same as f r is equal to f s whatever the frequent source frequency we are applying here.

Now, if the motor starts rotating if the motor starts rotating you have f r which is equal to sf s meaning the frequency of all this portion of the circuit is s times f s which is different from...... the s is smaller and therefore it is going to be lower than the frequency of this portion of the circuit.

Now as this is 1 is to 1 so whatever we have the back emf here which is let us say we call it E 1 we have E 2 equal to E 1 and the induced emf onto the rotor side E 2 is equal to E 1 if it had been standstill but would be equal to s times e one because of the rotation that we saw earlier so this is s times E 1.

Now because the frequency is sf s the reactance here also will change. what was The reactance is j 2 pi f s into L sigma 2; this was equal to j X sigma 2 during standstill. So as it starts rotating the frequency is changing and therefore it is no longer sf s it is f r which is no longer f s but it is f r which is sf s so there is one more term which is added here and that is s and that is s into standstill reactance. So what gets changed is the reactance js X sigma 2; of course all these are also j. R 2 the resistance is not dependent on the frequency so they are not going to get changed okay. So these are the changes that occur when the motor is in motion, which is one is the induced emf is s times E 1, the frequency is sf s and because of that the reactance is changed and that is this okay.

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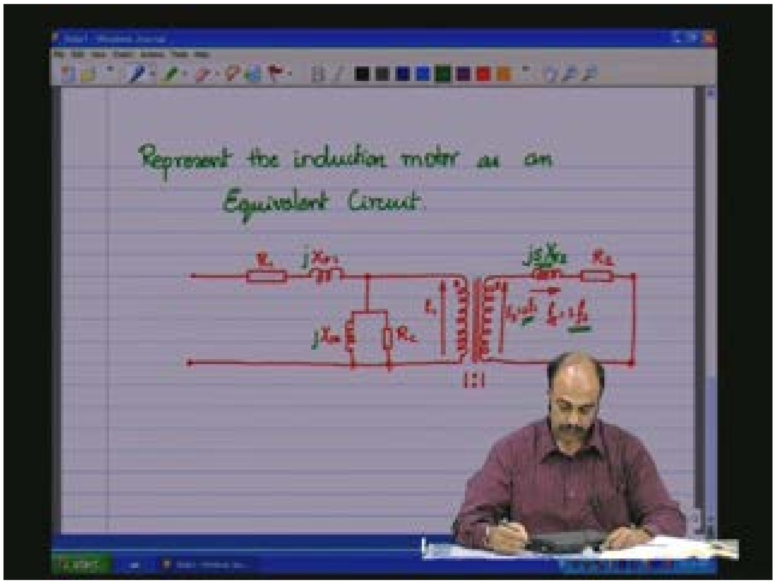

Represent the induction motor as an Equivalent Circuit.

Now this is the actual equivalent circuit of the induction motor but we could we we could make a slight approximation that is for high power motors this current the current which is going to flow through the magnetizing portion i 0 that we used to call is going to be a smaller fraction of the total load current and therefore this can be shifted on this side that is this portion (Refer Slide Time: 26:50) can be shifted to this portion. The errors will be a very small insignificant fraction without loss of generality but it will make the analysis much more easier. Therefore, we call this one as R sorry this is jX m and this is R c the core loss component. So this would be the modified equivalent circuit for higher powers that is for powers greater than let us say 2 to 3 kilowatts this can be safely used without much error. But if it is fractional horse power motors then we have to use the one which is the more accurate model which is which means that the magnetizing component will come there in between. okay

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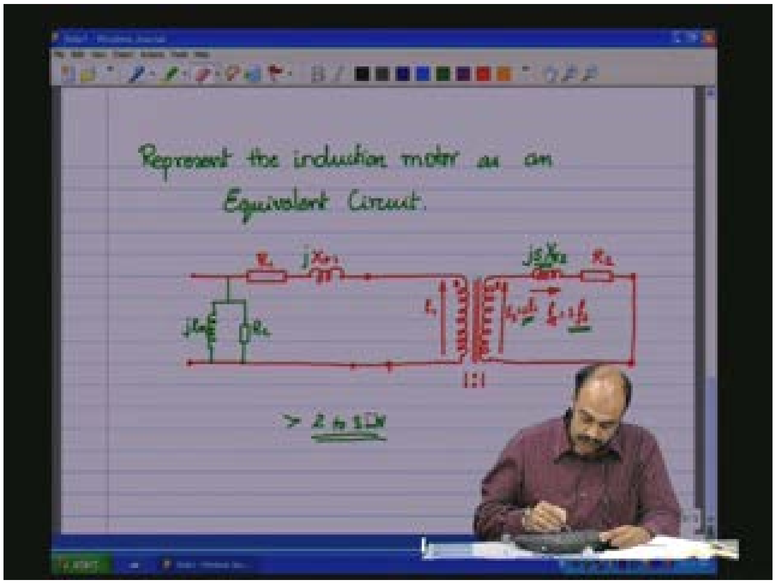

Represent the induction motor as an Equivalent Circuit.

Now we have the situation, let us copy this figure and take it to the next page so that we have more space, yeah. Now let us see the current. Now if you take the current which is flowing here (Refer Slide Time: 28:42) let us say this is I 1 it is the same current that is also flowing here which is I 2. I 2 and I 1 will be the same; effective values will be the same because this is the effective values that I am talking of because of the 1 is to 1 turns ratio. So, even if the frequencies are different the effective values are the same. So as a result I 1 is equal to I 2 and let us look at this portion of the circuit. So I 1 I 2 is given by s E 1 that is the source divided by the impedance which is R 2 plus js X sigma 2 js X sigma 2 and this is equal to E 1 by R 2 by S plus J x sigma 2, this is equal to E 1 by Z. Now this is the Z 2 as seen from this side because Z 2 is E 1 by I 1. So E 1 by I 1 is here. So therefore, as seen from here Z 2 is this which is equal to R 2 by S plus jX sigma 2.

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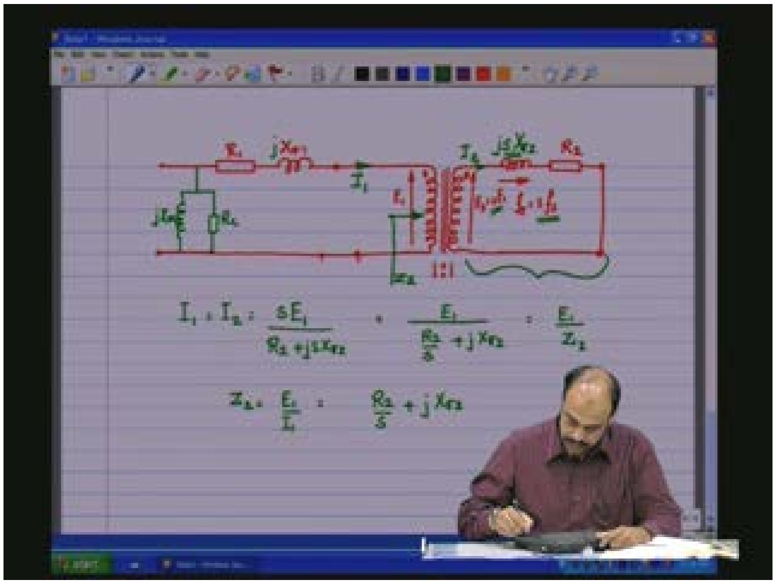

$I_1 = I_2 = \frac{sE_1}{R_2 + jsX_{\sigma 2}} = \frac{E_1}{\frac{R_2}{s} + jX_{\sigma 2}} = \frac{E_1}{Z_2}$

$Z_2 = \frac{E_1}{I_1} = \frac{R_2}{s} + jX_{\sigma 2}$

So let us shift everything on to this side so that we can eliminate the transformer portion so which means we have a reactance magnetizing reactance, core loss component; this is followed by R 1 and that is followed by jX sigma 1 the primary side. Now here let us put Z 2 which is which is jX sigma 2 plus R 2 by S and then you have the ideal transformer and as it is shifted on to this side there is of course a short here and this is so it is as good as that 1 is to 1 transformer not being there so we can effectively remove all these things (Refer Slide Time: 32:48) and we have it like that.

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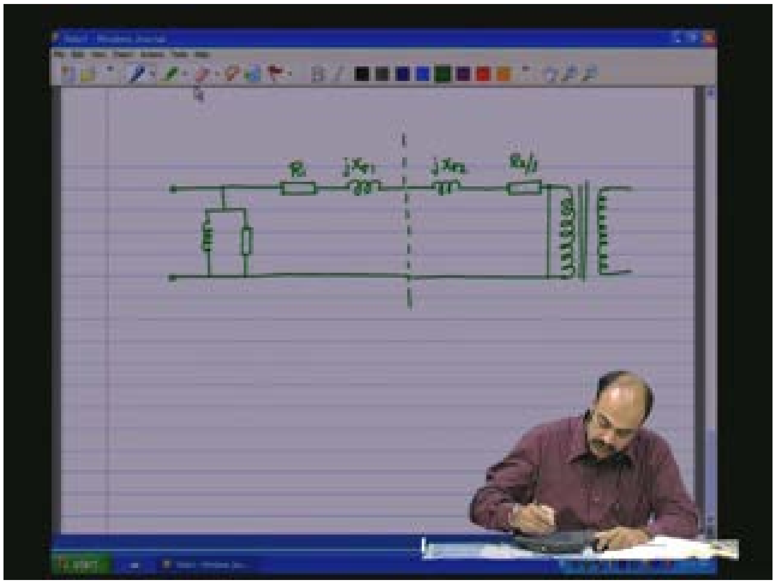

So this becomes the effective equivalent circuit of the induction motor now and to this we add the following things.

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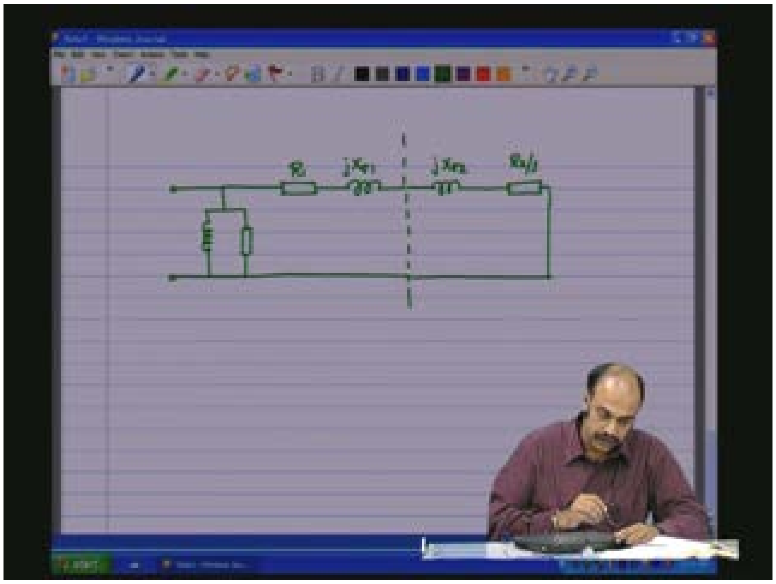

Let me slightly modify this in placing. Let me make it like that so that you will see this type in the textbooks R 2 by s. So you will see that the S is figuring in the rotor effective rotor resistance so this is like the effective load; at standstill R 2 is..... so if I say the load this is the effective load effective load at standstill s is equal to 1, the load resistor is equal to R 2. As s increases as s increases you will see that sorry as the s decreases you will see that this portion starts increasing and for the same current that will flow you have more power which comes into effect that is I square R. But if you look at the actual operation you will see that in the induction motor characteristic we have something like that, is it not? This is the torque and let me say that this is the speed shaft speed and we are here, so this is the operating point that we are going to be most of the time.

This is n s, this is where slip is equal to 0, this is at slip at maximum torque slip at or we will call it as S m slip at maximum torque. So this is the range of the variation of slip because the operating point can be somewhere here or here or here or here along these (Refer Slide Time: 35:52). So when s is 0 it is infinite, the resistance is infinite. That is at s equal to 0 load resistor is infinite which means at this point there is no power transfer to the load like open load like open circuit. Now as s starts decreasing sorry here increasing this is s is equal to 1, so as s starts increasing somewhere here the effective load resistor starts coming down. So the load resistor is equal to R 2 by S. So, as S starts increasing the effective load resistor starts coming down and therefore it can draw more current and I square R that goes to the load.

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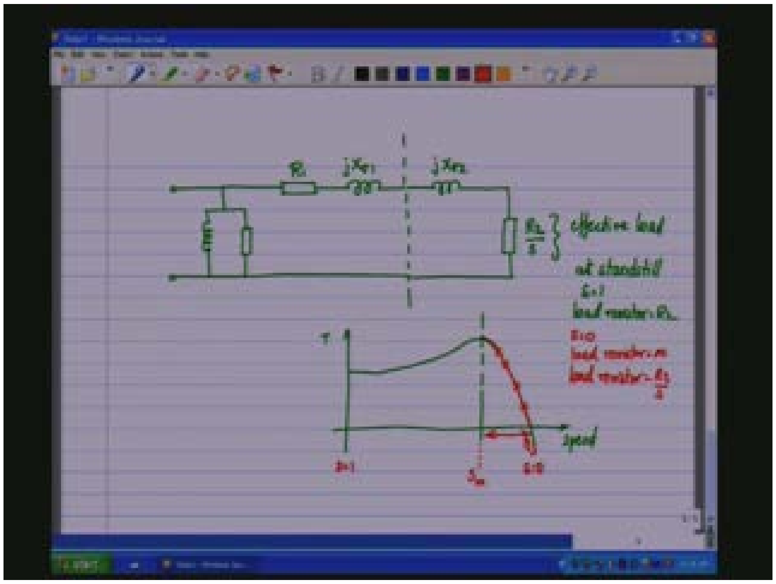

So we have the final equivalent circuit which is like this. We have X m we have jX m; I will write all those things later. We have R 1 and..... so this is the equivalent circuit of the induction motor. This is jX m, this is R c, this is R 1 the stator resistance, this is jX sigma 1 standstill stator leakage reactance stator side leakage reactance, rotor side leakage reactance and this is R 2 rotor effective rotor resistance which is dependent on the slip and we connect the source effective value let us say this is E g which is from the generator is connected like that. So this current is the stator current and we call that one as I s and out of this I s the stator current some portion goes for the magnetizing component I 0. So in I 0 we have I m I c and in this you have I 1 which flows through. So I 1 is what is flowing through and is the major part of the equivalent circuit which is the stator on the rotor equivalent brought on towards the primary side.

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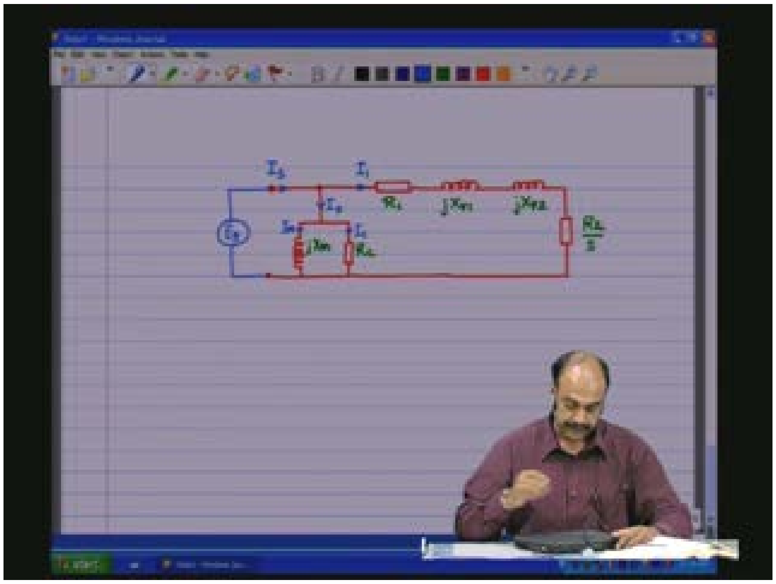

Now let us draw the vector diagram for this with respect to let us say E g let us have this axis. Now let us say E g is my reference, we will draw E g, this is E g so we start with the phasor E g. Now, lagging E g is all the currents because they are all inductive in reactance and this I s is also going to lag E g, so let us say this is I s. Now I s is the sum of I 0 plus I 1.

Now if you take I 0 to compute I 0 the drop I c I c is going to be in phase with E g so we have I c and we have I m and that is going to give you I 0, resultant of that is I 0. Now I 0 and I 1; I 1 if you look at the voltage at this potential is I 1 and I 1 is going to lag E g but we have here I s so I s minus I 0 is I 1 so I s minus I 0 let me draw this a bit clearly in black......... so we have this I s, we take parallel of I 0 minus so I 0 minus I 0 will be I 1 so that is I 1. So this is how the vector diagram would look like in the case of the induction motor.

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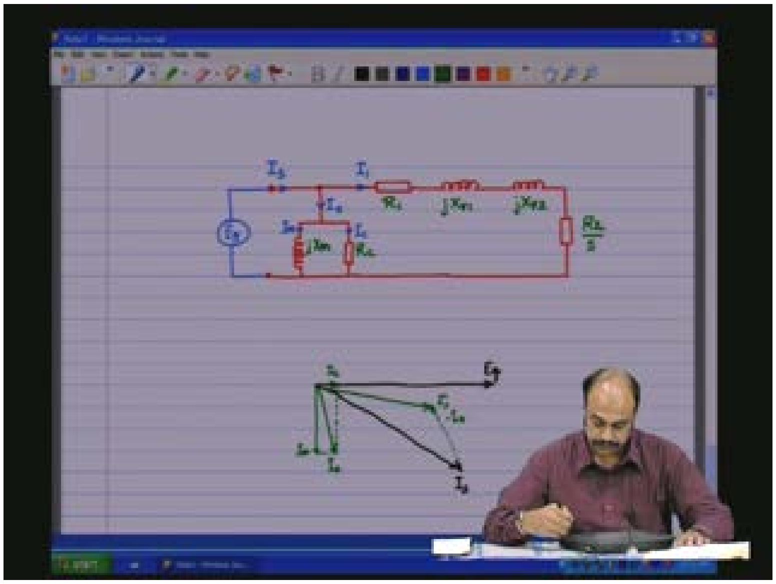

Let us have this diagram that is the equivalent circuit, let me copy that and paste. Let us see how the let us list down the power flow equations okay.

What is the active power absorbed by the motor the active power absorbed by the motor?
So the active power absorbed by the motor is E g square by R m R c that is the core loss component plus I 1 square R 1 that is the loss in the this resistance plus I 1 square R 2 by S that is the one which goes to the load. So this is the active power absorbed by the motor, we will call this one as P.

And the reactive power reactive power absorbed by the motor: so the reactive power absorbed by the motor we will call that one as Q is the one which stored in jX m, so this is E g square by X m magnetizing in the leakages X sigma 1 and X sigma 2 the leakage reactances plus I 1 square X sigma 1 plus X sigma 2 so this is the reactive power which is absorbed by the motor.

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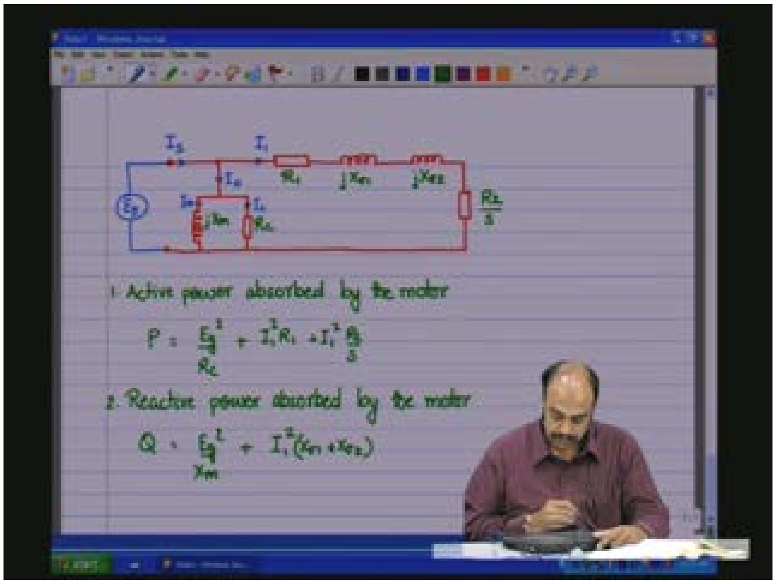

1\. Active power absorbed by the motor

$$P = \frac{E_g^2}{R_c} + I_1^2 R_1 + I_1^2 \frac{R_2}{s}$$

2\. Reactive power absorbed by the motor

$$Q = \frac{E_g^2}{X_m} + I_1^2 (X_m + X_{e2})$$

Let us shift this, yeah, then 3). The apparent power the apparent power absorbed by motor is that is the VA is given by square root of P square that is the active power plus Q square.

Then 4) what is the power factor power factor?

Now the power factor of the motor is actually as seen at the stator as seen at the stator here. Now we have the active and the reactive power and therefore we know that this is the active power orthogonal is the reactive power and then this is the VA. This is the power factor angle. So VA cos theta should be equal to P. So power factor Pf is equal to cos theta is equal to P by VA.

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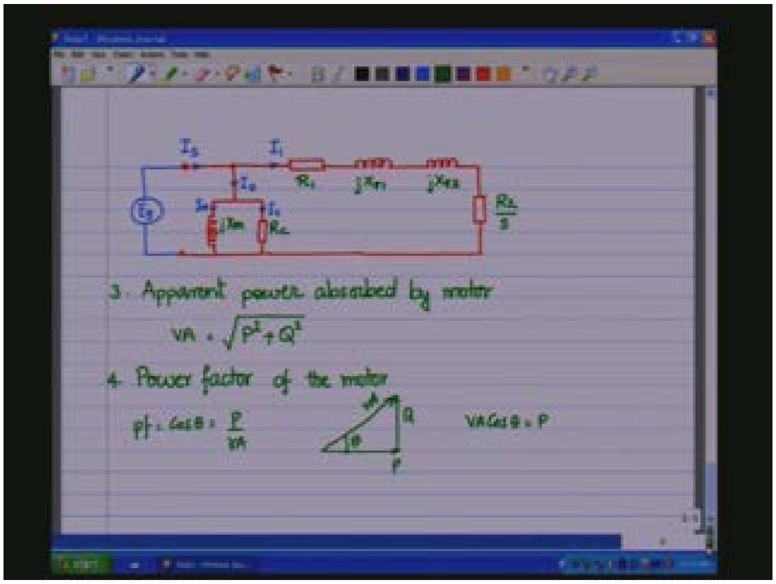

3. Apparent power absorbed by motor

$$VA = \sqrt{P^2 + Q^2}$$

4. Power factor of the motor

$$pf = \cos\theta = \frac{P}{VA}$$

$$VA \cos\theta = P$$

Then what else can we obtain from the circuit, then we have the line current. The line current I s I stator is........ now we know the voltage, we know the apparent power; so from the apparent power in the voltage we know the apparent power VA divided by E g will be the effective value of the stator current, this is also called the stator current. So let us have also the active power supplied to rotor. So what gets supplied to the rotor; the power that enters the active power that enters the stator minus the stator I square R loss, the stator resistance minus the core loss that is due to R c. So active power supply to the rotor which is P r is nothing but the active power of the input P minus I c square R c minus I 1 square R 1 or we could just take this portion of the circuit it is also I 1 square R 2 by S I 1 square R 2 by S is it not? If we look at the active power here you will see that this is the core loss component, this is the stator winding resistance loss component and this is the P r which goes to the rotor and this is the portion which is the active power that is supplied to the rotor and that is this.

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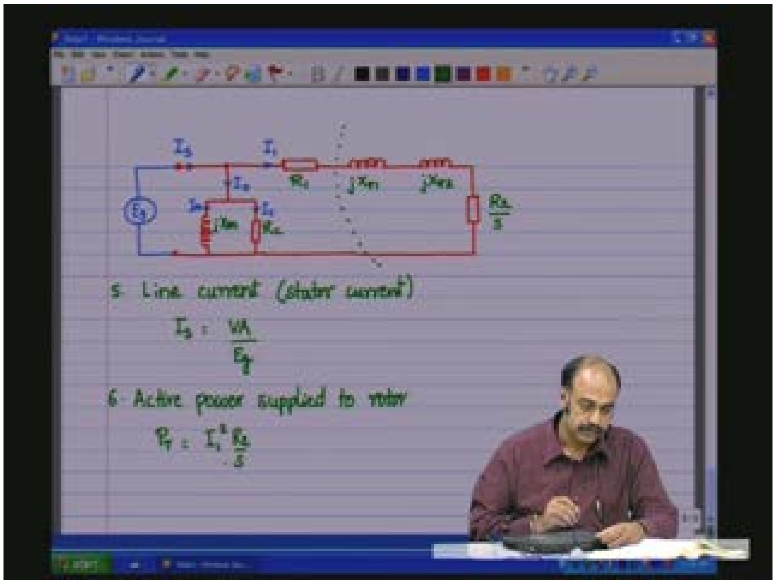

5. Line current (stator current)

$$I_s = \frac{VA}{E_g}$$

6. Active power supplied to rotor

$$P_r = I_1^2 \frac{R_2}{S}$$

Next paste. so we have a 0.7 here now the power dissipated in the rotor, power lost in the rotor conductors is....... power lost is given by slip times P r; P Lr actually will be basically I 1 square R 2 which is nothing but slip times P r is it not? If we look at slip times P r which is equal to slip times (I 1 square R 2 by S) which is equal to I 1 square R 2 this is the power lost in the rotor. Then we have to find out the mechanical power P m the mechanical power developed by the motor.

The mechanical power developed by the motor P m is P r minus what is lost in the what is lost in the rotor and that is nothing but this so you have I 1 square R 2 by S minus I 1 square R 2 which is I 1 square R 2 (1 by S minus 1) or which is equal to I 1 square R 2 (1 minus S by S) which is equal to P r (1 minus S).

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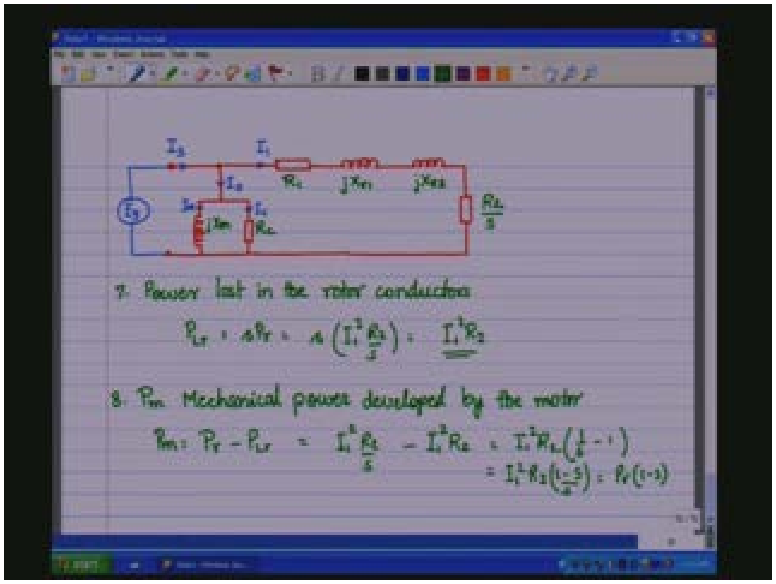

7. Power lost in the rotor conductors

$$P_{Lr} = sP_r = s\left(I_1^2 \frac{R_2}{s}\right) = \underline{\underline{I_1^2 R_2}}$$

8. P_m Mechanical power developed by the motor

$$P_m = P_r - P_{Lr} = I_1^2 \frac{R_2}{s} - I_1^2 R_2 = I_1^2 R_2\left(\frac{1}{s} - 1\right)$$
$$= I_1^2 R_2\left(\frac{1-s}{s}\right) = P_r(1-s)$$

Then we have 0.9 the torque developed by the motor. So T which is equal to P m into P m by n m into 60 by 2pi which is equal to 60 by 2pi....... P m is P r into (1 minus S) and n m is n s into (1 minus s) so you can so say 60 by 2pi P r by n s. So either you can use either you can use this equation you know P m and n m or you can use this equation P r and n s. Then of course the last point let us put down that is the efficiency efficiency.

So efficiency is given by P shaft that is if you minus the friction so P m minus P friction by the P in this case P s so the total input power P which is calculated; if friction is not known then you can say P m by P so all the information can be put to an end also from the directly from the equivalent circuit of the motor.

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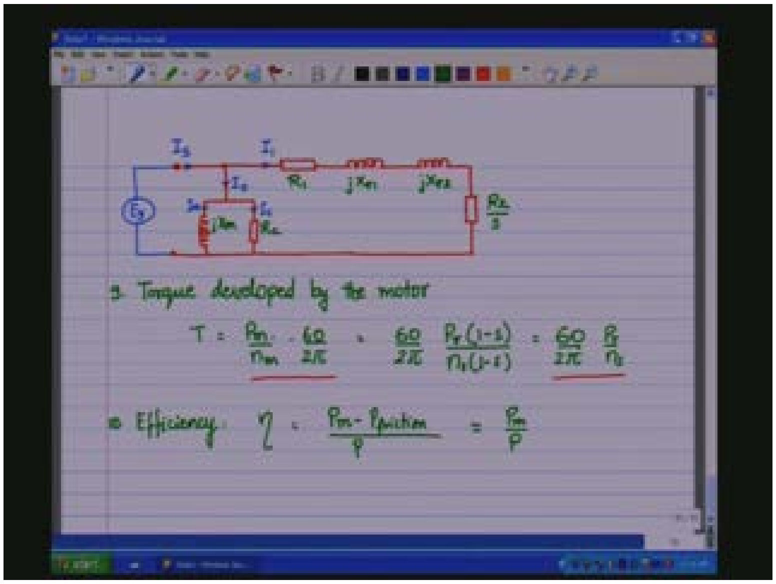

3. Torque developed by the motor

$$T = \underline{\frac{P_m}{n_m} \cdot \frac{60}{2\pi}} = \frac{60}{2\pi} \frac{P_r(1-s)}{n_s(1-s)} = \underline{\frac{60}{2\pi} \frac{P_r}{n_s}}$$

- Efficiency: $\eta = \frac{P_m - P_{friction}}{P} = \frac{P_m}{P}$

So we saw here that the induction motor is not actually a very complex system, it can be represented like a transformer. You have the equivalent circuit almost like a transformer except two minor variations. in fact under the standstill condition it is just like the transformer and in the case of the motor in motion there is a slip, the load is R 2 by S which is a function of the slip and the leakage reactances when you take it into the equivalent stator side equivalence they do not have a variation with respect to frequency because the stator frequency is constant the variation the variation the speed is taken care of by the R 2 by S component. Otherwise the equivalent circuit is just the same as in the case of the transformer.

We saw the phasor diagram and also from the phasor diagram from the phasor diagram and from the equivalent circuit we were able to also list down the various equations for the active power flow and the reactive power flow components of the induction motor.

Now, if the slip was negative we said that the induction motor becomes a generator. In the case of the induction motor acting as the generator the equivalent circuit is still the same; the only difference is that the slip S is negative. So R 2 by S becomes negative which means the polarity is reverse which means now that it is pumping energy back into the mains so it will look like that because the slip becomes negative this becomes the source (Refer Slide Time: 58:23) this portion would become a source in the case of induction generator, the polarity would reverse and then starts pumping energy back into the mains.

So, by reversal of polarity it could be the reversal in the voltage polarity or reverse in the current because it is now acting against I 1 and then it will pump more current than I 1 and therefore the current reverses and therefore the power flow is from the mechanical side to the electrical side.
So the same equivalent circuit can be used even for the induction generator.

We will work out a very simple example of the induction generator in the next class and then conclude the topic of the induction motor.

Thank you for now.