Basic Electrical Technology
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Lecture - 6
Modelling of Circuit

In the last session we discussed about the Kirchoff's laws. There were two laws the Kirchoff's voltage law and the Kirchoff's current law. We also stated that there are two categories of junctions in electric circuits: one is the voltage junction and other is the current junction. Now these two laws along with these junctions the voltage junction being called the loop and the current junction being called the node will be essential for doing any analysis of the circuit.

So today we shall discuss about how we go about modelling circuits such that it will result in the mathematical representation of the circuit. So this mathematical representation of the circuit will then be used for analysing the circuit and knowing more about the circuit. So today's topic is on modelling of circuits.

Consider a circuit a simple circuit with a source a resistance that is a capacitance all of them connected in series in this manner (Refer Slide Time: 2:44). The source we have V i, there is a resistance R and capacitance C. The source has the voltage the arrow direction in that as shown here, the voltage across the resistance the voltage across the capacitance shown like this. So like the Kirchoff's voltage law because this is a junction and what junction is it this is a voltage junction or a loop and what does Kirchoff's voltage law state? The Kirchoff's voltage law or the KVL states that the algebraic sum of all the voltages in this loop must be zero. Therefore, V i is equal to V R which is the voltage across the resistance plus V C which is the voltage across the capacitance. This is the equation of the circuit but however this equation does not fully describe the circuit which is having the components like the resistance and the capacitance. We need to have much more detail to understand the circuit and analyse the circuit.

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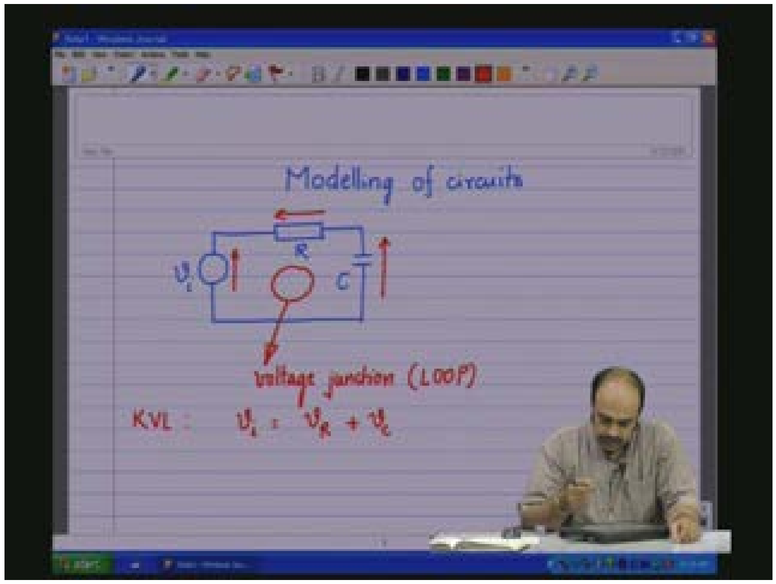

So the question arises; how do we dynamically describe the circuit. So how do we describe the dynamic behaviour of circuits?

In this session we are going to take one approach, there are many approaches of course; but we will take one approach which is a kind of universal in its approach whether it be electric circuits or whether it be the magnetic domain or the mechanical domain however we are restricting ourselves to the electric domain the state space approach that is called the state space approach. So throughout the session and also in the future classes we are going to use only the state space approach to mathematically represent the circuits and then thereby afterwards use it for analysis.

In the state space approach the behaviour of the system at any given instant is fully defined by two main variables: one is the input. You have two keys, one is the input and the other the state variables. The input is the source voltage or the source current as the case may be in any particular given circuit and the state variables; you may not have come across the state variables, let me give you an idea about the state variables before we proceed further in the state space approach.

So if we have the knowledge about the input variable value and the state variable values then the whole system can be defined. So let x be the state variable and let u be the input variable then the state equation can be written as x dot equals Ax plus Bu. And there is one more equation the output equation which is written as y is equal to Cx plus Du; this is the standard form of representing all systems in the state space domain and this is called the state equation.

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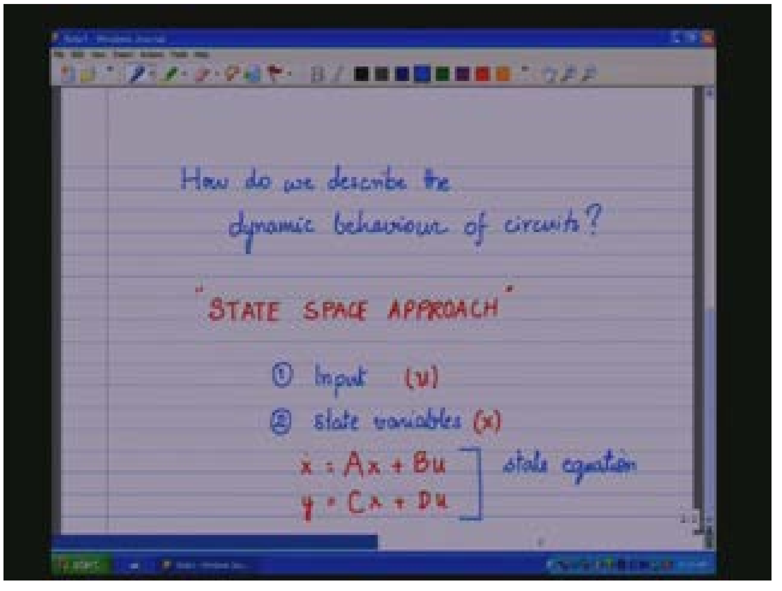

I will come back to uh state equation again. Let us first have a look at what a state variable is, the state variable. What can become a state variable?

We mentioned earlier that the inductance, capacitance and probably in other domains like the inertias they are all energy storing elements. The variable associated with the energy storing elements, for example; in the case of the inductor the current is the variable that is associated with the energy storage because it is kinetic energy that is getting stored in the inductor. So the current becomes a candidate for being considered as a state variable and in the case of the capacitance which is a potential energy storing element the voltage can be a candidate for being considered as a state variable. So, in short we can say the variable associated with energy storing components can be considered as state variable. But if you see in the case of the signal processing circuits where you use lot of memories, RAMs and in microcontroller circuits where there are lot of memories involved there also there is some storage but in that storage there is information. Thus, the variable associated with the information storage also can be considered as state variables. So it is not only energy storing but even variables associated with information storage.

Now we can state from these two that any variable that is associated with a component having memory can be considered as state variable and that would be the most general statement. Any variable that is associated with a component having memory then that variable is a candidate for being considered as a state variable.

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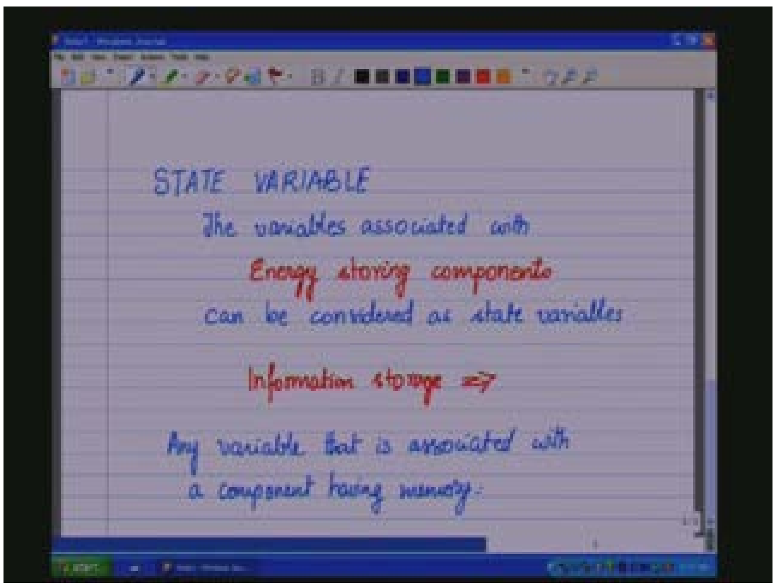

For example, let me take the case of the inductor, the inductor L. In the inductor the energy is stored by virtue of the kinetic energy by virtue of the flow of the current and it is given by half Li square which we saw in one of the earlier sessions.

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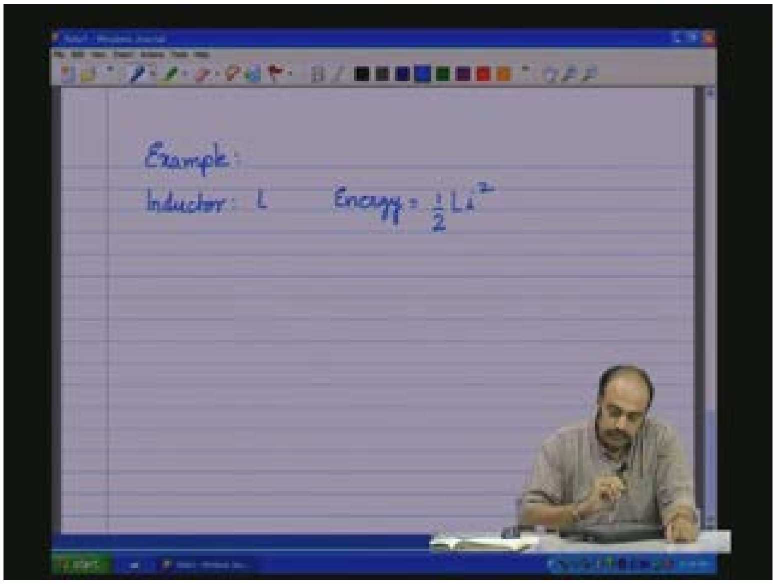

This current here (Refer Slide Time: 13:06) is the means by which the energy is getting stored and this current is a candidate for state variable and that can be a state variable. Or, if you consider the capacitance C the energy here is stored by virtue of the potential across the capacitance which is half Cv square and this voltage across the capacitance can be considered as state variable. Or in any other domain for that matter if you take an inertia an inertial element J which is being rotated by the shaft of a motor, J also stores the kinetic energy of the angular rotation and the energy is given by half J omega square and therefore the angular speed of rotation is a candidate for being considered as state variable. And also in the information domain the variable stored in the memories in the RAM memories can also be considered as state variables in the signalling information domain.

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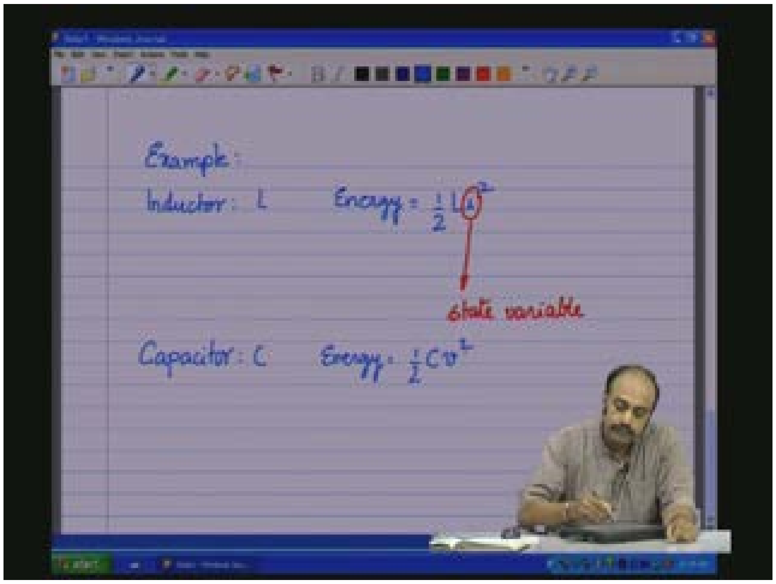

So now that being the case that we consider the variables associated with components to store energy or store information or in other words components that have memory or history then such variables can well be considered for state variable to be applied in the analysis of the circuits or to develop the mathematical representation.

So now coming to the circuit consider the circuit that we wrote quite some time ago that we have a source, we have a resistance and a capacitance. All these are connected in series, all these share the same current and they are all in a loop and therefore this is a voltage junction; and if it is a voltage junction what should agree? The KVL should be obeyed; the Kirchoff's Voltage law should be obeyed for this loop. So here you have the R (Refer Slide Time: 16:20), you have the C and you have the source which is V i. Now there is a voltage which is considered in this direction, there is a voltage across R which is this direction and there is a voltage across C which is in this direction.

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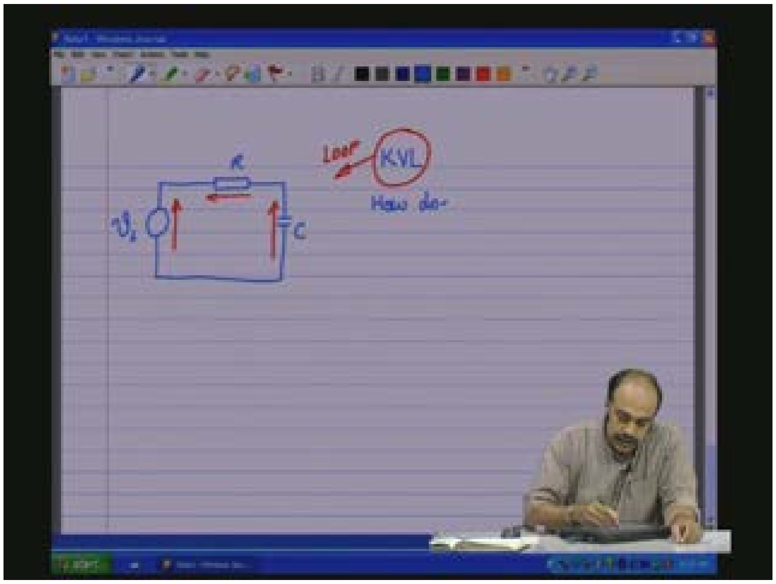

Now the question; how does one get the mathematical representation of this circuit? How does one obtain the mathematical model of the circuit?

Let us go step by step. Step 1: First after having the circuit, this one, identify the energy storing or the memory components, identify the energy storing components. After having identified the energy storing components, in this case for this circuit there is only one energy storing component which is the capacitor here.

The step 2 would be to identify the state variable. So in this circuit there is only one energy storing component therefore there has to be only one state variable and as it is it is a capacitor, the state variable for the capacitor is we see because the energy is stored by virtue of the potential across the capacitance and therefore we write down the state variable for the capacitance which is the voltage across the capacitor we see.

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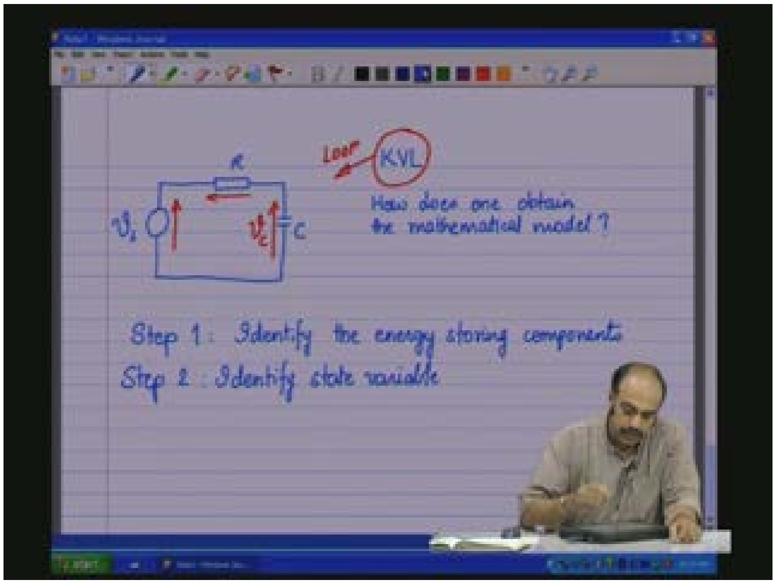

Now having done this that is having identified the energy storing component and having identified the state variable, put down or write down the state variable at the corresponding or appropriate place in the circuit. The circuit should contain only the input variables, the state variables and the parameter variables like the RNCs; and the whole equation should be constructed only with these variables. Now in this case the whole equation should be constructed with V i, the V C the state variable or the parameter C, the parameter of the circuit.

Step 3: Consider the dynamic equation at the energy storing components one by one, order immaterial. What it means is we consider the dynamic equation at the energy storing component one by one and if a particular circuit contains more than one energy storing component you can do it in any order. So in this case there is only one energy storing component which is the capacitance so we need to look at the equation of the capacitors alone. So the equation of the capacitance is CdV c by dt is equal the current through the capacitance.

So, if you look at the circuit here we see CdV c by dt equal to the current through the capacitance and what is the current through the capacitance it is the same as the current through the resistance which is the same as the current through V i because this is a loop. So what is the current through the resistance? V i minus V c divided by R and that is the current that is flowing through the capacitance therefore which is equal to V i minus V c whole by R. Note that in this equation, only the input and the state variables are used apart from the parameters.

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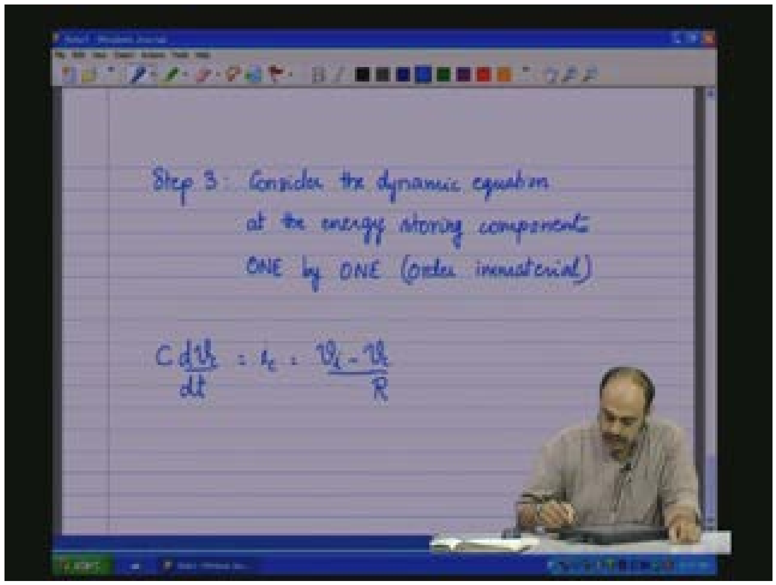

Now this equation can be rewritten in the following form: dV c by dt equals minus V c by RC plus V i by RC.

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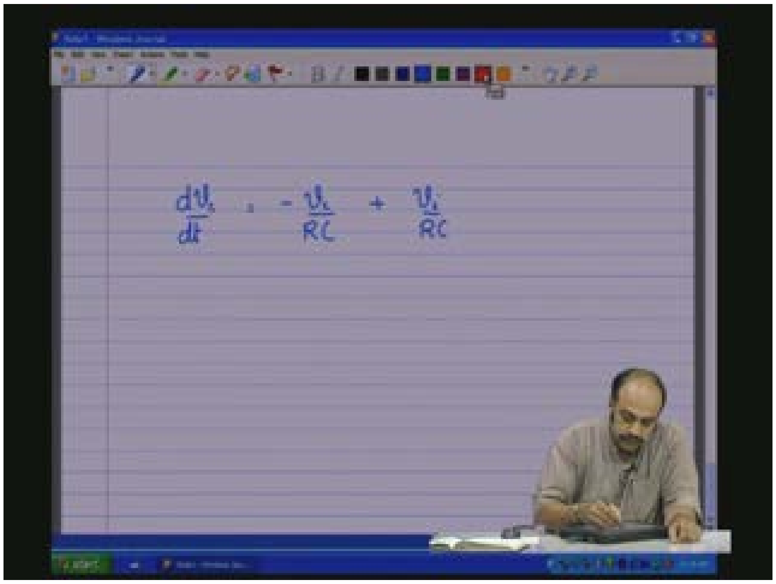

This is now in the standard form. If you look at this; this is dV c by dt which is V c dot equals…. now this can be written as minus 1 by RC into V c plus 1 by RC into V i. Now see that compare this with the standard form which is X dot which is equal to A x plus B u there is one to one correspondence between these two. V c dot V c is the state variable which is x, A is the parameter matrix the system parameter matrix, this is the system parameter matrix (Refer Slide Time: 24:04), B is the input matrix and if there are more than one state variable this is a vector, x is the vector, if there is more than one input variable then this is also a vector U.

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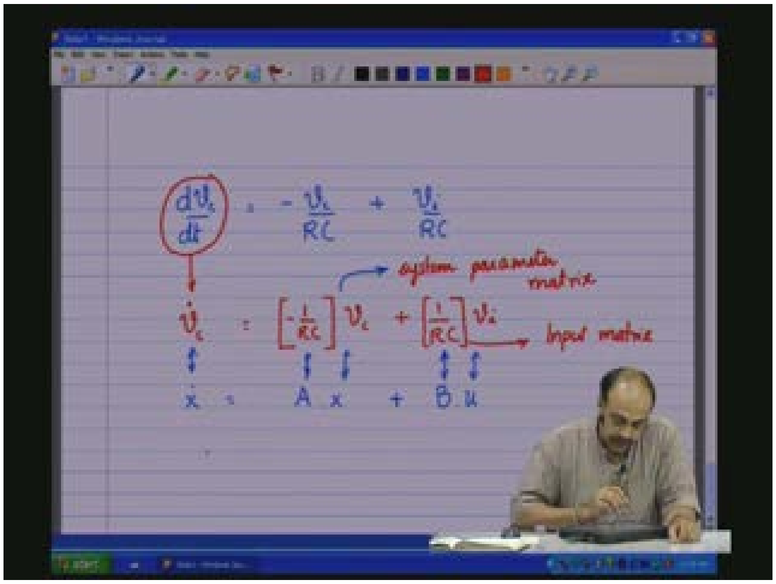

Now there is one more equation that you have to write to complete the state equation which is Y the output is given by 1 into V c plus 0 into V i the general form which comes in the form of y. Compare this with the general form of the output equation y equals C x plus D u; again the equivalence between these two; this is the state vector and this is the output matrix, C is the output matrix, D is the feed forward matrix the input that comes directly to the output without going through the dynamics of the system.

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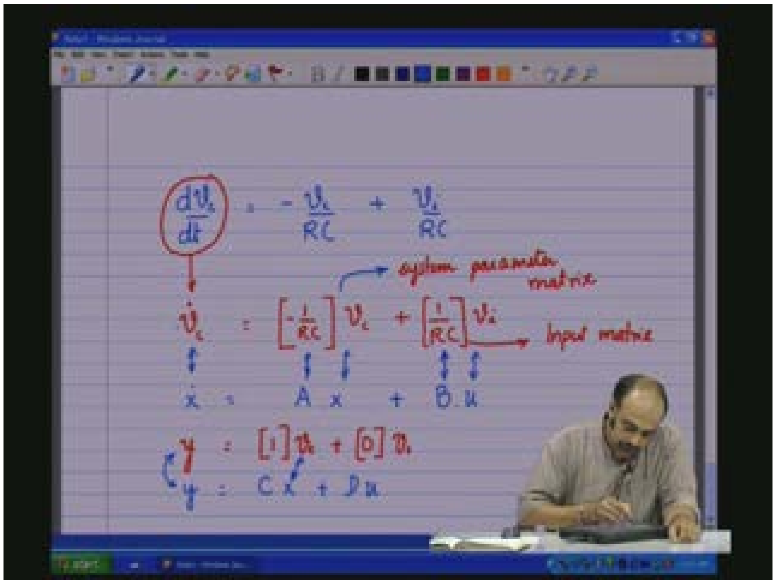

Now these two equations together these two equations together form the state equations. therefore V c dot equals minus 1 by RC into V c plus 1 by RC into V i that is one equation and here this gives you the dynamic information and Y which is 1 into V c in this case because the output is same as V c plus 0 into V i so these two equations together are called the state equation for that circuit. This is the dynamic equation which is a first-order differential equation and this is the output equation here.

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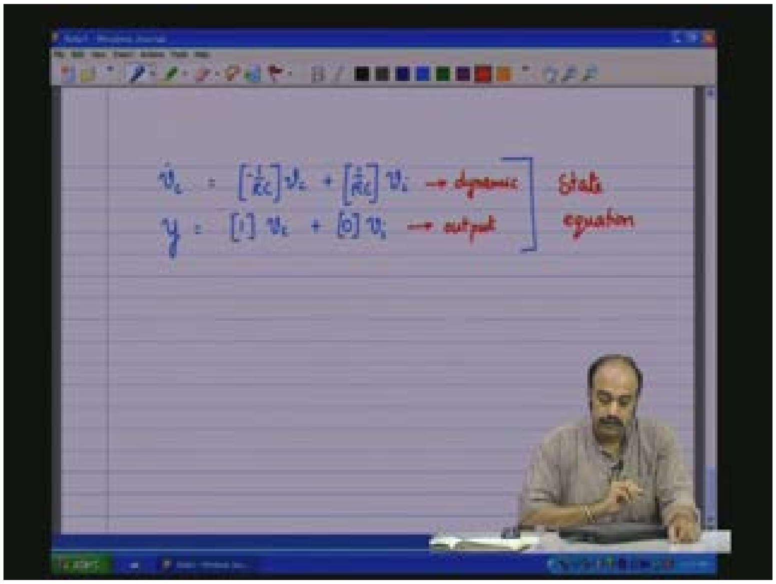

So the dynamic equation is a first-order differential equation and the output equation together describes the system completely. And we see that this equation to describe it completely needs just the input variable and the state variable and the state variable is nothing but the variable associated with the energy storing component of the particular circuit.

Now take another example with slight modification. Let me put the inductor instead of a capacitance. So we have a resistance and an inductor, it is an RL circuit. The previous circuit was a RC circuit so we have an RL circuit V i and we have the source having the direction like this (Refer Slide Time: 28:12), we have a passive component and another passive component.

So in this case again going step by step what is the first step? The first step is to identify the component which stores energy and in this case it is the inductor L. The second step is to identify the state variable for the energy storing component and for L the energy is stored as half L i square by virtue of the flow of current and therefore i L therefore i L is the state variable i L is the state variable, this is the state variable. And in the circuit we just have those variables only that are essential for making the equation which is the input variable V i, the state variable i L and the parameters of the circuit R and L, you need not mention any other variable other than this, these variables are sufficient to completely describe the circuit.

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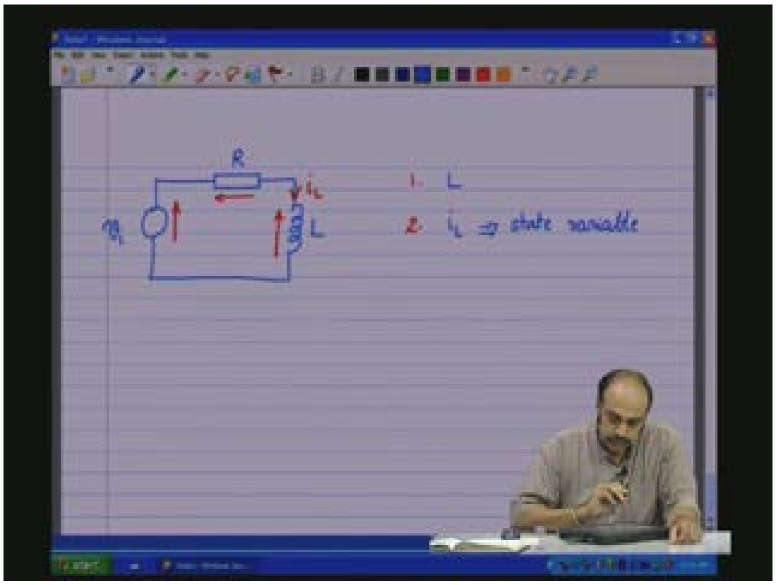

Then we consider the equation third step of the dynamic element and in this case the dynamic element of the energy storing element is the inductor so you consider the equation L di by dt is equal to V L or the voltage across the inductor. I will mention it as voltage across L.

Thus, applying this third condition we have L di L by dt equals and what is the voltage across the inductor; one end is ground here the circuit ground, the other end here is V i minus the voltage across R and we know the state variable i L is flowing through whole of the circuit and therefore the drop across this is nothing but i L into R. So V i minus i L into R is the voltage across the inductance V i minus i L into R is the voltage across the inductance.

Hence, you see again the whole dynamic equation is in terms of only the state variable and the input variable. So putting it in the standard form: diL by dt is equal to where i L is the state variable minus R by L into i L the state variable plus 1 by L into the input variable and then the output variable for us let us say is i L. So if it is i L or if it is across the resistance let us say, so let us say the output is V R just for a change this is what we want V R, so the output V R is equal to R into i L plus 0 into V i. So the same matrix is R in this case. So these two together fully describe the circuit and that is the state equation for this circuit. You see, it is so simple.

Actually whatever be the complexity of the circuit you just have to follow these three steps. that is step 1: identify the energy storing elements, step 2: identify the state variables corresponding or associated with these energy storing elements, step 3: with no particular order starting at any particular energy storing element the equation describing this dynamic behaviour can be put down and then followed by the equation describing the dynamic behaviour of any other all other energy storing elements in the circuit. Hence, this is the method that we will be following.

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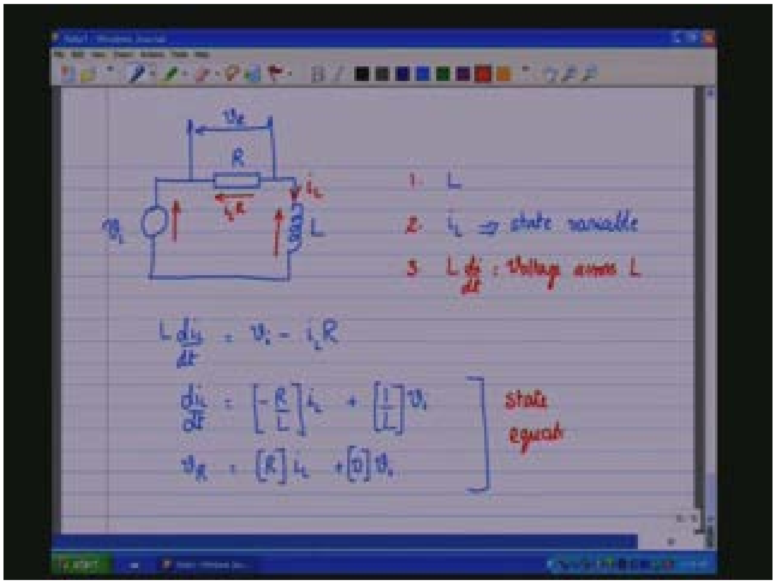

Now let us just take a bit more complex, actually simple but just a bit more complex compared to the earlier RL and RC circuit that we have been seeing. The third example is like this. Let us have an R, let us have an L, let us have a node with the capacitance C and let us have another resistance R across C as shown here (Refer Slide Time: 34:19). So this is the input V i, this is one resistor R 1 which is in series with an inductor L, there is a capacitance and there is another resistance R 2 which is across C. Now let us say this is the circuit that you want to analyse which means that you have to have a mathematical representation of this circuit, the complete dynamic behaviour of this circuit.

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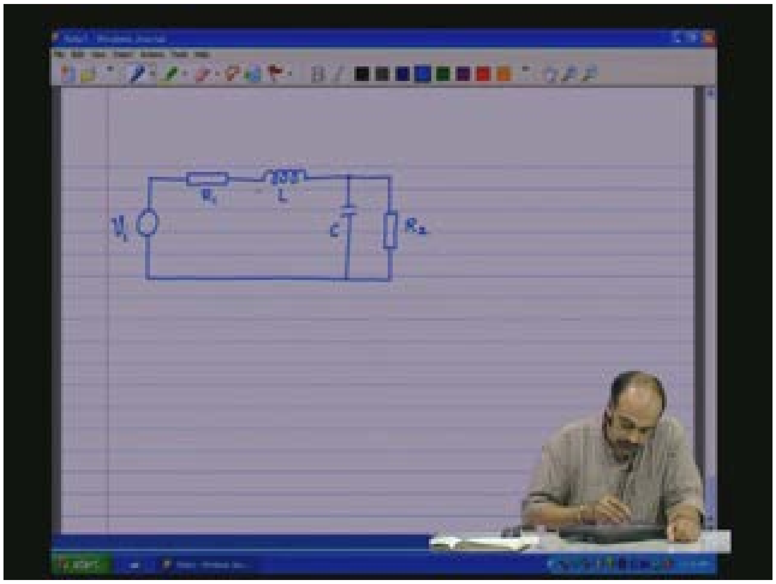

Notice that there is a loop here that is there is a voltage of the source represented like that, we have the voltage across R represented like this, the voltage across the inductor, the voltage across the capacitance. There is a loop here and there is a node here (Refer Slide Time: 35:33). So what does it state? The node is a current junction, it states that all the currents the algebraic sum of all the currents of these branches should adopt to zero, the loop states that the algebraic sum of all the voltages in this loop should adopt to zero. This is the KVL and this is the KCL. We make use of these two laws of course.

Now step 1: identify the energy storage elements. in this circuit there are two energy storage elements: there is an L and a C. We have L and C. Second step: identify the state variables associated with these energy storage elements. So, for L we shall call it as i L, the current through the inductor is a state variable and for C we shall call it as V c, the voltage across C is a state variable. Now put these state variables or mark these state variables in the circuit. So in the case of the inductor state variable this is i L and in the case of the capacitance state variable there is V c the voltage across this capacitance. Now let me erase these things so that it will not disturb the overall……. Yeah.

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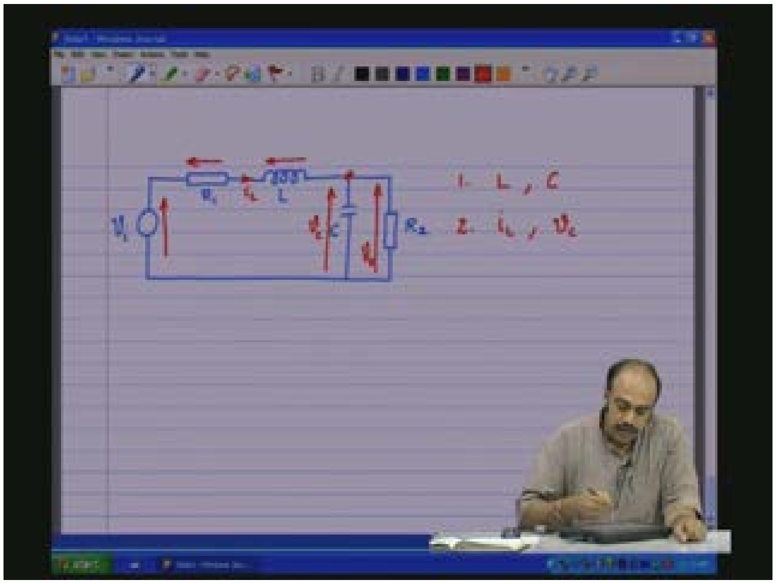

Now it is required to have a look at the output voltage V 0. Note that the V 0 can be expressed in terms of the state variables. This is the output the Y that we used to describe which is in this case same as V c. But on the circuit we just keep only those variables which we need to form the equation which is the input variable V i, the state variable V c, state variable i L, parameters of the circuits R 1, L, C and R 2. These are the only variables that need to be written down on the circuit and the whole dynamic behaviour of the circuit should be formulated with just these variables nothing more nothing less.

So what is the step 3?

One by one we take the equation describing the dynamic behaviour of the energy storing elements. We have two energy storing elements L and C first we consider the dynamic behaviour of L and then we consider the equation defining the dynamic behaviour of C and then write down the expressions only in terms of these variables that you see on the circuit. So first the inductance equation given by L di L by dt should be equal to the voltage across the inductance. Now the voltage across the inductance should be expressed only in terms of these variables.

Now what is the voltage across R 1 here?

The voltage across R 1 can be expressed in terms of the state variable, the current i L is flowing through R 1 and therefore attached to be i L R 1. Now looking at the graph circuit here and applying KVL L di by dt is equal to V i minus i L R 1 will be the voltage of this node with respect to ground and the voltage of this node with respect to ground minus V c. Looking at the circuit we see that this is the dynamic behaviour that we have obtained by considering the equation revolving around the inductor element and you see that the whole equation is formulated only with the input, the state variables are alone.

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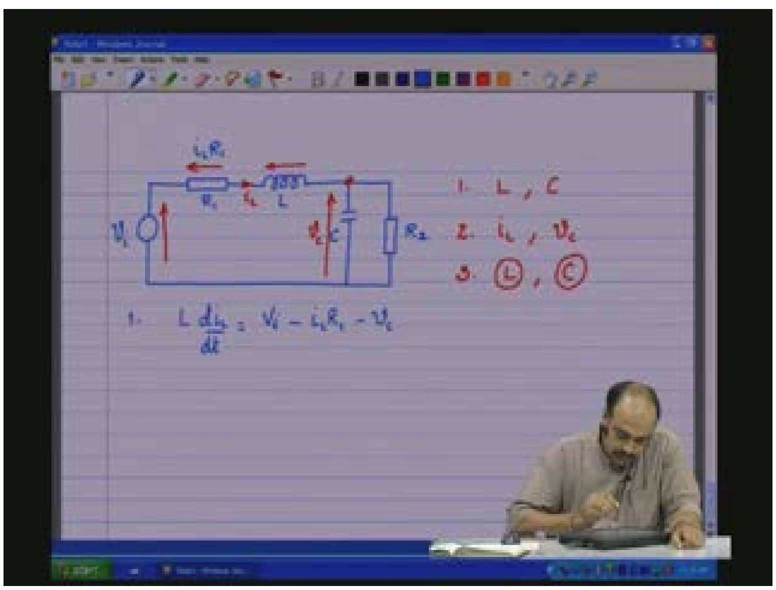

So let us consider the second element and the second element in this case is the capacitor C. C dV c by dt equals the current through the capacitance. Now what is the current through the capacitance? There is a current flowing from a inductor i L here, there is a current through the capacitance here and there is a current which flows through R 2 and by KCL it is known that the algebraic sum of this current and this current and this current should add up to zero. So here we have i L and what is this current here (Refer Slide Time: 41:54) the one which goes through R 2. Can it be expressed in terms of the state variable? We know that the node voltage here is V c. so V c by R 2 is the current which is flowing through R 2 here. Therefore, the current flowing through C is i L minus V c by R 2 as simple as that. Therefore, in these two equations there are two dynamic components or the energy storing components and therefore there are two equations and these two equations totally describe the dynamic behaviour of the circuit. However, we need to put it in the standard form, express it in the standard form.

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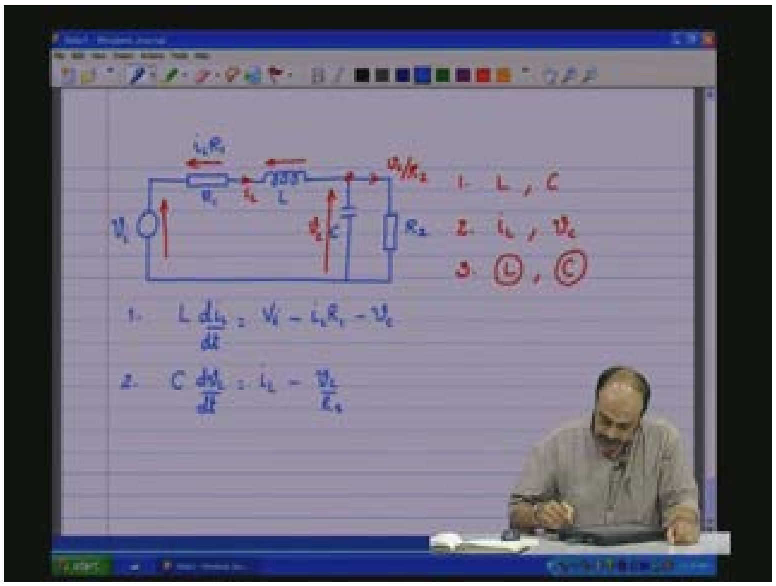

So let us define X, let me go to the next page. Let me rewrite these two equations: L di L by dt equals V i minus i L R 1 minus V c and the other equation is C dV c by dt equals i L minus V c by R 2. These are the two equations. Now we need to remove L from here (Refer Slide Time: 43:37) so I remove L from here and divide everything throughout by L and then I remove C from here and divide everything throughout by C so that now we have the state vector and we can now rewrite this as i L dot V c dot enclosed in matrix. So let us put it in the matrix form, it is very very elegant and let us write the A matrix also in the matrix form V c this is the state vector X plus we have to write the input scaling matrix and there is of course only one input which is V i.

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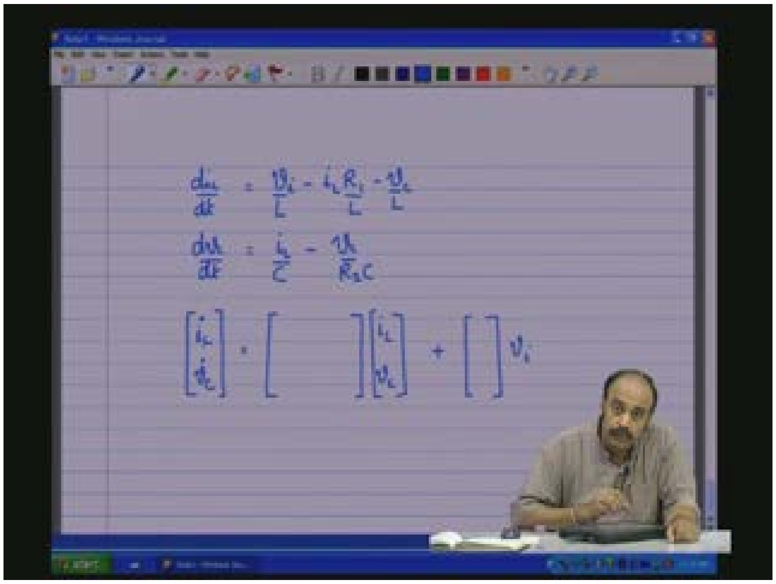

So i L dot equals……. i L is multiplied by minus R 1 by L R 1 by L and minus 1 by L into V c minus one by L into V c. then for a second equation dV c by dt V c dot equals i L into 1 by C i L into 1 by C and V c into 1 by R 2 minus 1 by R 2 C into V c plus here you have 1 by L into V i there is no V i term here and that is zero. So this is one equation which describes the dynamic behaviour of the circuit and then we need to get the output equation which is V 0 which is same as V c and therefore we have [0 1] you have i L V c plus 0 into V i. Now this is your state equation which completely describes the circuit as shown in this figure (Refer Slide Time: 46:45).

Therefore, you see here this is the derivative of the state vector X dot and this is the state vector X and this is of course the input vector, of course in this case this is a single element and therefore it is a one by one matrix and here is the input matrix and this is the matrix which describes the circuit in terms of the circuit parameters Rs and Cs and all those things and here you see this is the output Y, the C matrix here because in this case V 0 is same as V c so it is the C matrix and this is of course the X matrix, the D matrix is zero because V i is not directly passed on to the output and this is U.

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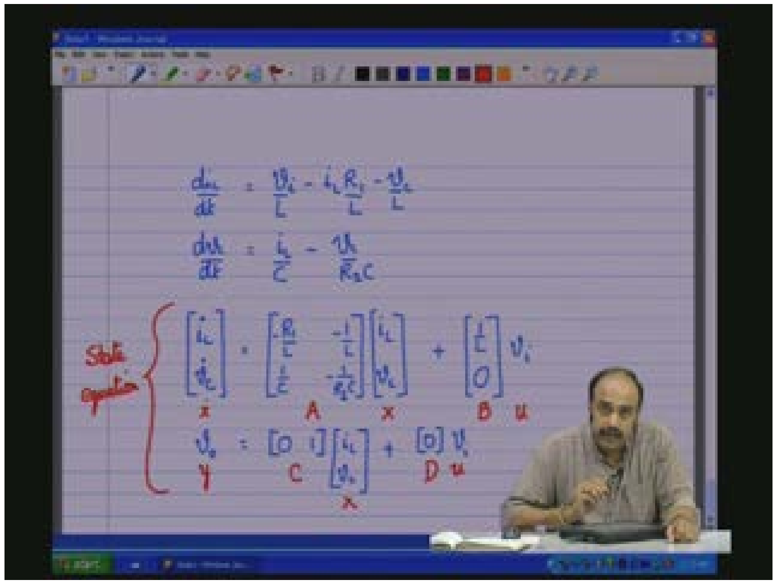

Now you see that this is a one to one correspondence between the general standard representation of the state equation. So this is a complete mathematical representation of that particular circuit. So all circuits can be represented in this form.

Well, at this point I would like to state that we have two important laws which is the Kirchhoff's Voltage law which we have been applying for all the loops for all the loops here and the second is the Kirchoff's Current law which has been which is being applied for all the nodes. All the circuits all electric circuits are composed of alternately these loops and nodes so you have the voltage junction which alternates with the current junction and so on. This happens in even the most complex circuits. And to obtain the mathematical representation we just have a very simple three step process.

first step is to identify all the energy storing components. After having identified the energy storing components, identify the state variables, it is very simple. In the case of electric circuits when you restrict electric circuits which is if it is an inductor the current the inductor current is a state variable and if it is a capacitance the voltage across the capacitance is a state variable. So after having identified all the inductor currents and the state variables, mark them on the circuit.

And if it is a loop like you see here let us say just check if all the voltages across the components have been identified.

Like for example; in the case of the resistance here it is not identified but express it it will always be expressible in terms of a state variable. Therefore try to identify that state variable which will decide the voltage and so on. And as R is in series with L the state variable i L is used for finding the voltage across R so i L R 1 is the voltage across that one. So all the elements the voltages across them have been identified. Now apply KVL for this loop, you will get the dynamic equation one dynamic equation corresponding to that particular energy storing element. Then choose the other energy storing element in this case it is a C and here as it is connected to the node you apply the KCL because C dV c by dt is going is going to be equal to the currents through the capacitance.

Hence, for the inductor you are applying the KVL to get the voltage across it and for the capacitance you are applying the KCL to get the current through the inductor sorry to get the current through the capacitance. With these two you have defined the dynamic behaviour of the entire circuit just only in terms of these energy storing elements and the input elements and the parameters of the circuit. And this process is valid for whatever may be the complicity of the circuit and whatever may be the domain; whether it be in the electrical domain or it be in mechanical domain or it be in the hydraulic domain or it be in the magnetic domain and so on and so forth.

Just one point of caution that you need to be careful about is what happens if I have two inductors in series or two capacitors in series. For example; in a case like this let us say we had a resistance, an inductor an inductor followed by a capacitance and a resistance. so here you have let us say R 1 (Refer Slide Time: 52:49), L 1 L 2 C R 2 and an input source V i. You will see that there are three energy storing elements; apparently it looks like there are three energy storing elements L 1 L 2 and C and therefore there should be three state variables. But it is not actually so; this L 1 and L 2 are actually one single inductance, it can be considered as one single inductance L equivalent or either one of two, either the L 1 inductor or the L 2 inductor can be considered as the energy storing element and the corresponding variable can be taken as the state variable which means in this case as they are all coming in a loop the current through L 1 and the current through L 2 are same so only one of them can act as state variable.

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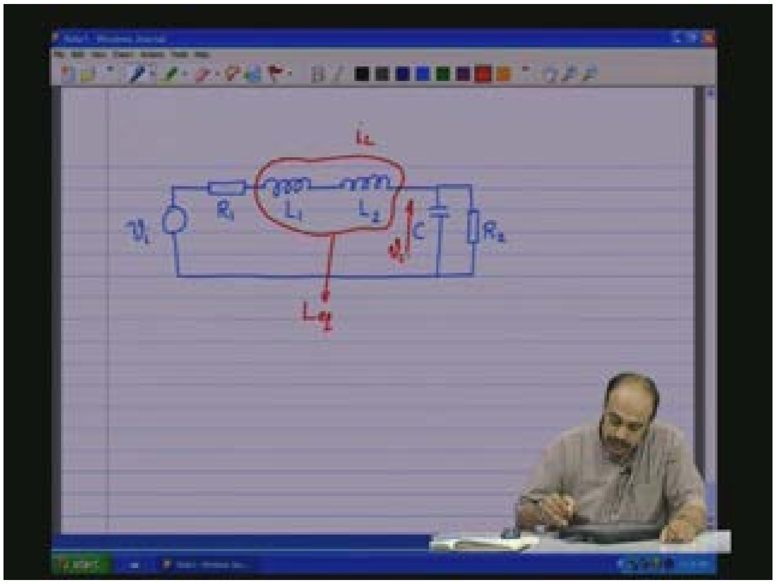

Likewise you can have a similar problem which means that there will be one single i L either by taking it as equivalent inductance or taking it as the state variable corresponding to L 2 or taking this as the state variable corresponding to L 1 and other state variable will be V c.

The other problem could be something like this. I have an R, I have an L, and I have a C, another C and let us say a resistance. This also could lead to a conflict in making a decision of what should be the number of state variables. So I have a state variable here iL, we have V i, now either we combine C 1 and C 2 and make it C equivalent and have one common state variable V c for that or associate V c with either C 1 or associate V c with C 2 any one of them so there will be only two state variables V c and i L.

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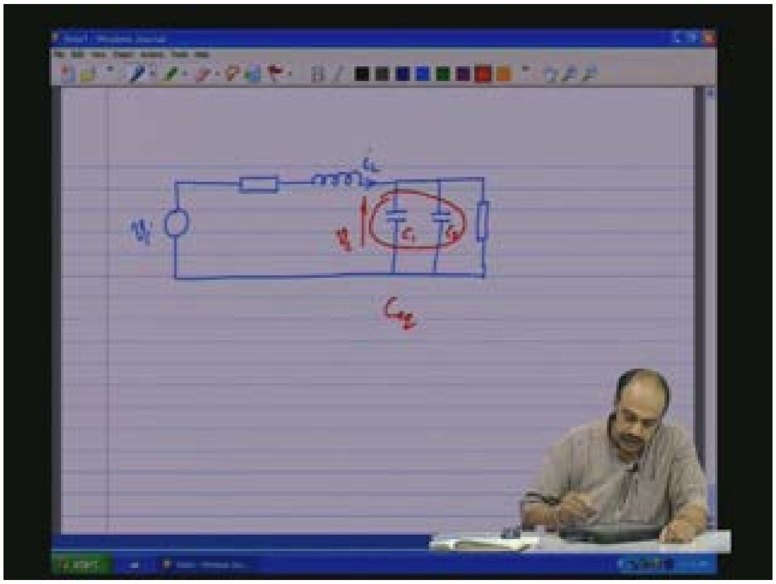

There is one more case which you may land up with and that is a network something like this. Let us say you have a R, you have a L and you have another inductance there, you have a R and something like this. So here we would be considering let us stay this is i L1 and let us say this is considered as i L2 so once this becomes the state variable this current here through the third inductor is automatically determined by the Kirchoff's current law here and this is the most fundamental, there should not be any accumulation of current here. So, by Kirchoff's current law this is i L1 plus i L2 and therefore this is not a state variable, there only two state variables even though it apparently seems that there are three energy storing elements. Only any two of them here would have the memory and any two of them……….; either you can say i L1 and i L2 are state variables or i L1 and the current through the middle R is a state variable the other one being determined by KCL or it could be i L2 and the middle R being the state variables the current through the inductor one L1 being determined by KCL.

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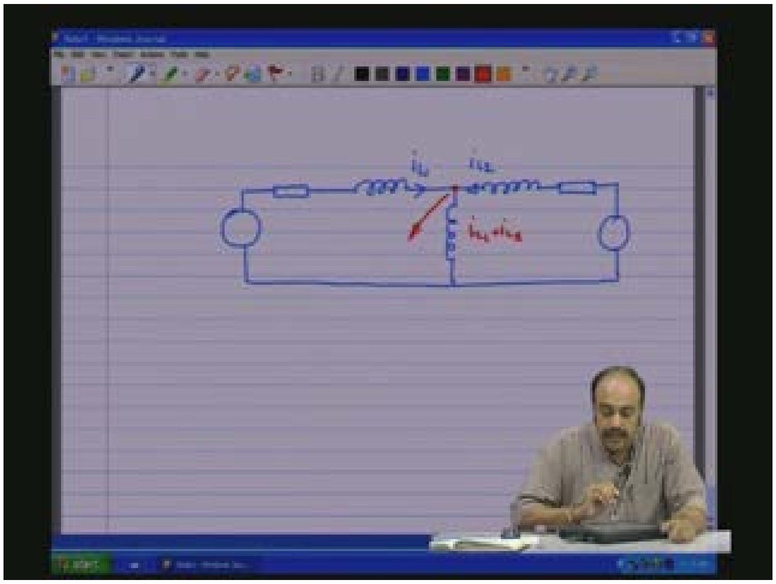

These are some of the special cases that you should be aware of and be careful about when you are doing the decision on the state variables. Otherwise the process remains the same and for the majority of the cases except for these special cases you will find the process that we stated or that we discussed today that is the step 1, step 2, and step 3 is valid for whatever may be the domains. With this we stop the discussion here about obtaining the mathematical representation of the circuits. Thank you.