Basic Electrical Technology

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Lecture - 7
Modelling of Circuit – 2

Hello everybody, in the last session we discussed at length how we go about obtaining the mathematical representation of circuits. We saw that there were three essential steps to be followed in the process of obtaining the mathematical representation: the first being identifying the energy storage elements, second we identify the state variables associated with energy storing elements and then one energy storage element by element we go about obtaining the dynamic equations around the particular energy storing element making use of the Kirchhoff's voltage law and the Kirchhoff's current law that is the KVL and the KCL. That is the general procedure for obtaining the state space equation.

In this session I would like to introduce a degree of complexity in the circuit. The process is essentially the same but just to show that you should be able to do with any given circuit you should be able to obtain the mathematical representation for a circuit which is of any degree of complexity. I will introduce now a complexity which will bring in some intermediate variables which you need to address and use that to obtain the state equations.

However, the intermediate variables will be used only to obtain some intermediate equations otherwise the state equation representation the state equation the standard form will not be altered.

Now, starting with the circuit now let me take a circuit which is of the following nature. Let us say we have a resistance, let us have an L, then a capacitance. This is one part of the circuit, this is the series; all are in series, this is one loop.

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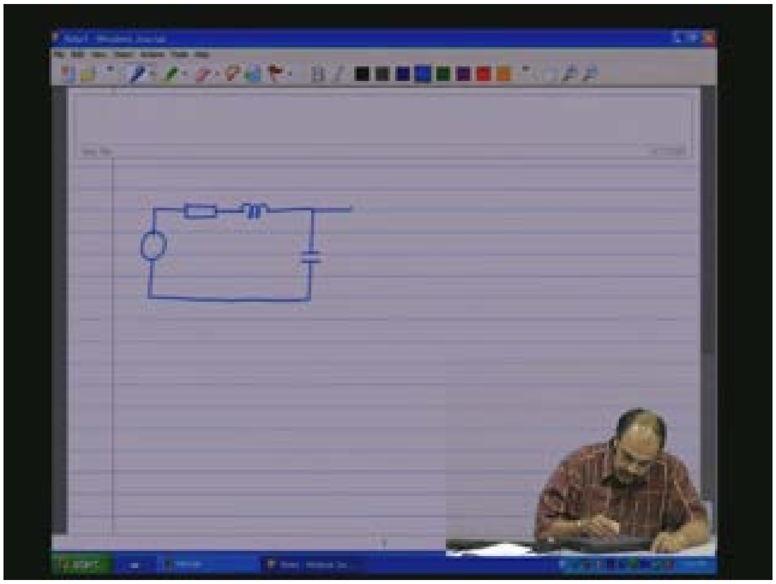

Now I am going to connect few more components, another resistance and a resistance connected to ground. So we have one more loop there and sorry…. We have let us say one more inductance and a resistance and probably a source. So we could consider a circuit which is something like this. We have a source V i 1 and another source V i 2 two sources. let us have the parameters; we have the parameter here let us say R 1, this is L 1, this is C, R 2, R 3, L 2, R 4 and the source V i2, so this is the circuit.

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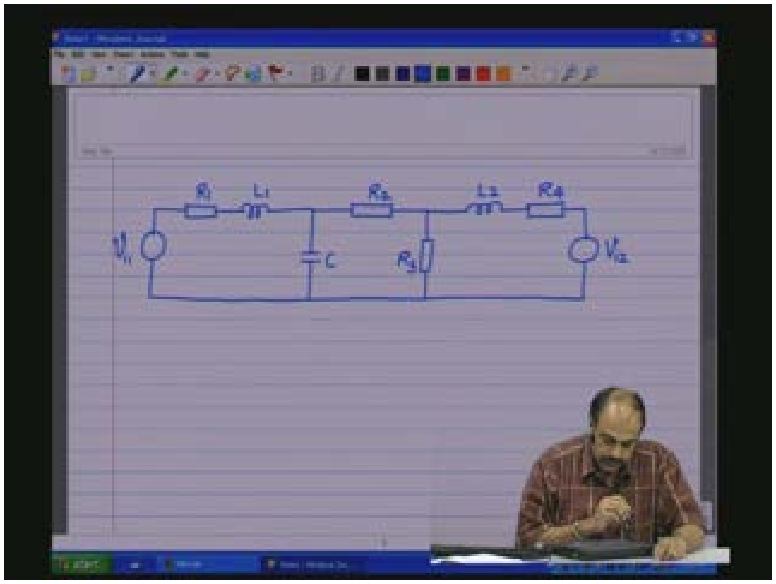

So you see that there is a loop here, (Refer Slide Time: 5:29) there is another loop here and another loop here. So there are three voltage junctions and there are two nodes here; one node is at this point, another node is at this point. So the first step; you know the process of obtaining the state equation is, identify the energy storing elements. So here you have L 1 C L 2 three energy storing elements and therefore there are there has to be three differential equations to describe the circuit.

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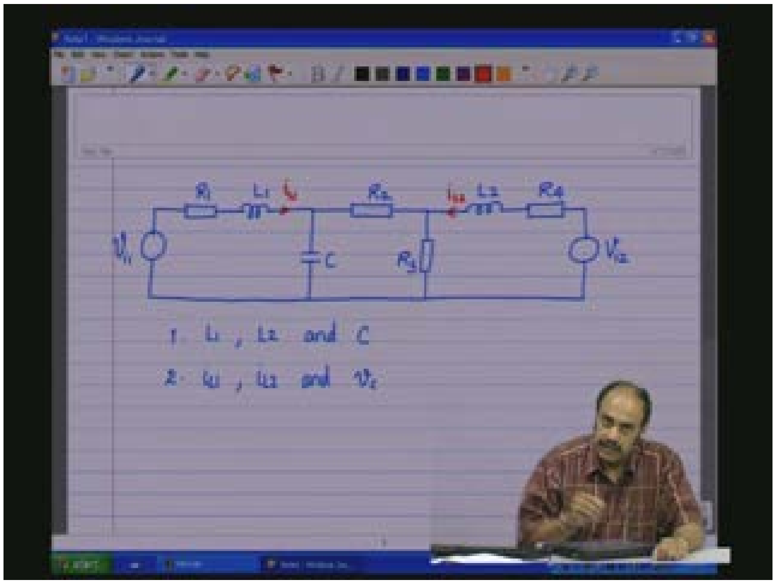

So L 1 L 2 and C these are the energy storing elements that we need to be bothered about for the dynamics and then identify the state variables associated with this energy storing elements. So coming to L 1; so it is a current through L 1 which is let us say let us name it as i L 1. So i L 1 and then the current through L 2 and let us name it as i L 2 and then the voltage across C we will name it as V c voltage across C and let us also indicate these state variables in the circuit. So we have i L 1 here, let me indicate this as i L 2 here; notice that I have put an arrow direction for the current flow here, it could as well be the other direction no issues there and the sign will automatically get changed in the equation and you will not face any problems as such.

Now the third state variable is across the C the voltage across the C that is V c. So now we have the state variables, the input variables and the parameters all listed on the circuit and therefore we should be able to make or obtain the state equations. However, here there are two nodes here; this node voltage is defined as V c (Refer Slide Time: 8:16) but this node voltage is not defined with a variable so this could give rise to a problem so that is the reason where we need to introduce a new intermediate variable. We will come to that as we formulate the equation.

Now the third point or the third step in our process of obtaining the mathematical representation of this circuit is we take the dynamic elements or the energy storing elements one by one and try to solve the equation around them. So first we take the equation around L 1 d i L 1 by dt and then we take the equation around C dV c by dt and when we take the equation L 2 d i L 2 by dt. So these are the three dynamic equations that we need to formulate which will finally describe your system in this case this particular circuit completely.

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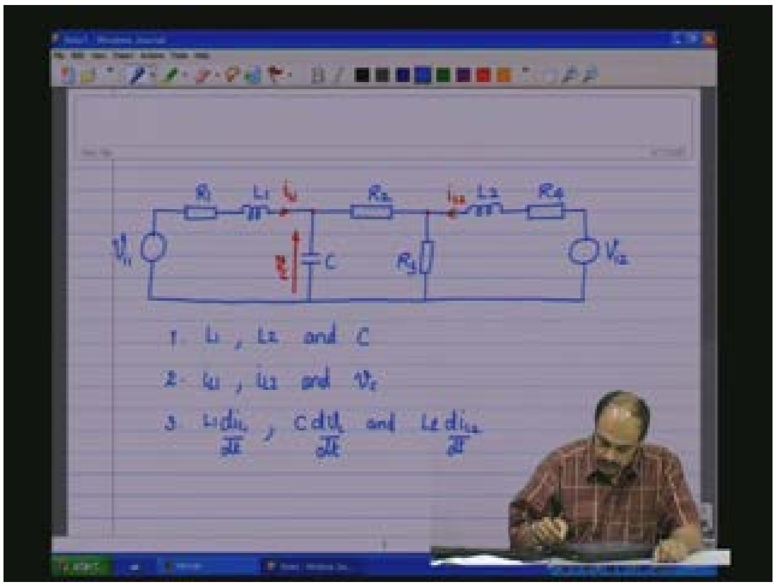

So now we start formulating the equations. Let us go back to the circuit here (Refer Slide Time: 00:09:54) let me delete these portions so that I can start writing the equation right here. So first we take L 1. L 1 d i L 1 by dt equals should equal should be equal to the voltage across L. Now the voltage across L by Kirchhoff's voltage law should be equal to the algebraic sum of all the other voltages. So let us see the other voltages which are there in this loop; there is a source voltage in this direction (Refer Slide Time: 10:55), so if the current is assumed to be flowing this direction we have the voltage across the resistor in this direction and of course the inductance voltage in this direction. In fact, we need to find the voltage across the inductance.

Now what is this voltage here?

This is in terms of the state variables i L 1 into R 1. So now we have the equation which is very simple; we have V i 1 minus i L 1 R 1 minus V c should add up to the voltage across the inductor such that the sum of the voltages algebraic sum of voltages in this loop in this voltage junction is zero by Kirchhoff's voltage law. So this is our first equation and now the second equation.

Now the equation for C is C dV c by dt which should be equal to the current through C. Now the current through C is i L 1 minus the current through R 2 but what is the current through R 2. This node potential minus this node potential divided by R 2 will be the current through R 2 but we have not given a variable to this node potential, therefore we shall now introduce an intermediate variable at this point in a different colour and let me call it as V x (Refer Slide Time: 12:56). So with this now the intermediate variable is being put there. we have the current through C which is i L 1 minus current through R 2 which is V c minus V x by R 2 this is the current through R 2 and by Kirchhoff's current law at this node the sum of i L 1 the current through R 2 and the current through C should algebraically add up to zero.

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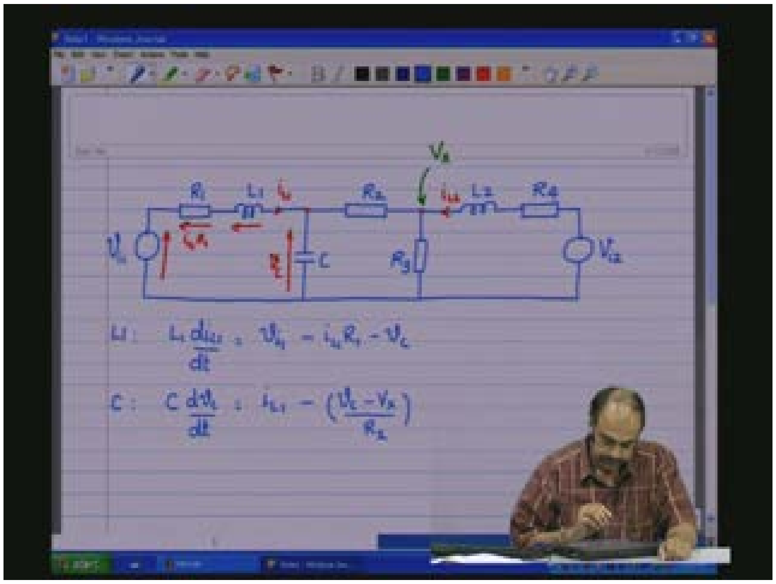

This is the second equation but of course here this equation is in terms of the state variable V c, state variable i L 1 and a variable V x. We should later on express V x in terms of state variables. Of course we will come to that one.

Now if you look at this potential V x here, now the potential V x here is nothing but V c minus the potential across R 2. Or in other words, if we assume that there is a current flowing in this direction then we have a potential here which is in this direction as shown (Refer Slide Time: 14:57) and this is the potential V x. So now V c plus V c minus V x by R 2 being the current which flows through which flows through this into R 2 is of course V x. Of course this is a trivial equation which we cannot do much about.

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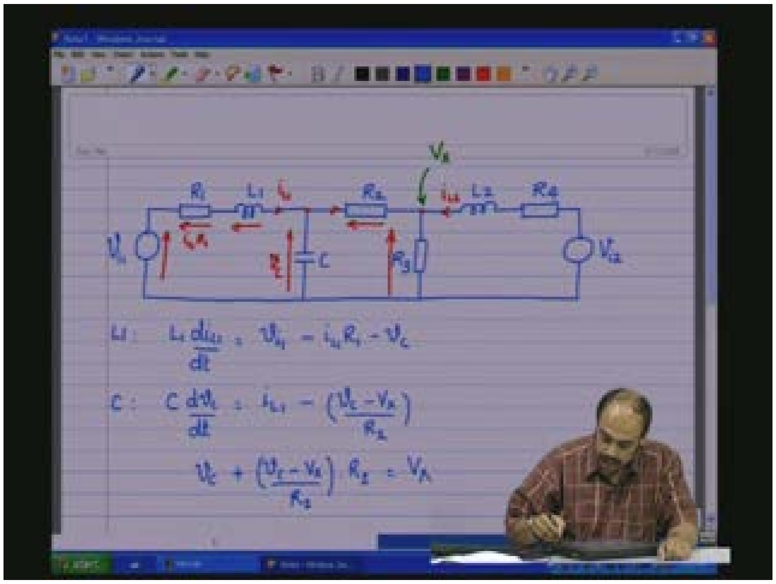

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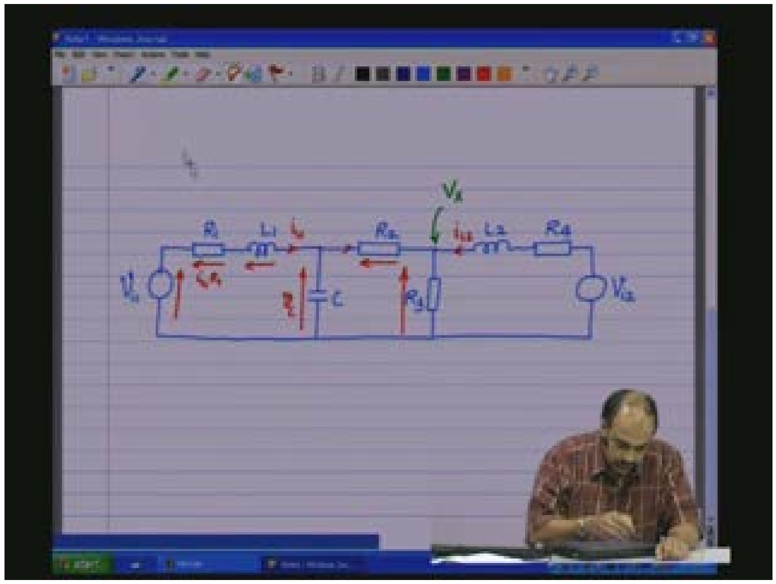

So we have the circuit here so that we again can proceed with our operations. Now, coming to this node here what is the current that goes through R 3?

It is this current plus this current which meaning the current through R 2 and the current through i L 2 which is going through R 3 and therefore that current into R 3 is going to the x. So V x equals i R 2 that is the current through R 2 plus i L 2 which is the state variable here into R 3. Or what is V R 2? V R 2 is nothing but the potential difference across this by R 2. So V c minus V x by R 2 is the current through R 2 plus i L 2 is the current which is the state variable which is coming as the inductor current i L 2 of L 2 this whole thing into R 3 is V x.

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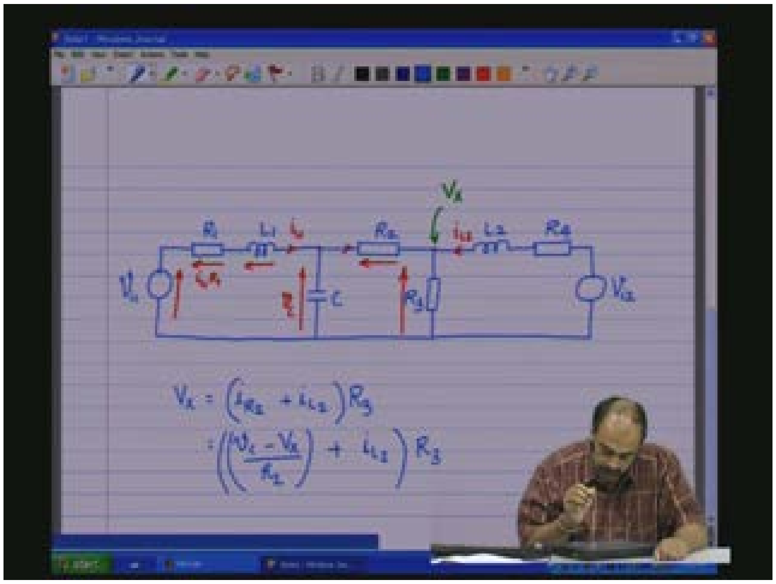

Now if you see V x is the only variable here in this equation and then V x can be expressed as a function of V c and i L 2 which are again state variables. So basically V x can be expressed as a function of the state variables. So let us see what happens. Now again I want to select this portion (Refer Slide Time: 19:04); let us copy, go to the next page and paste.

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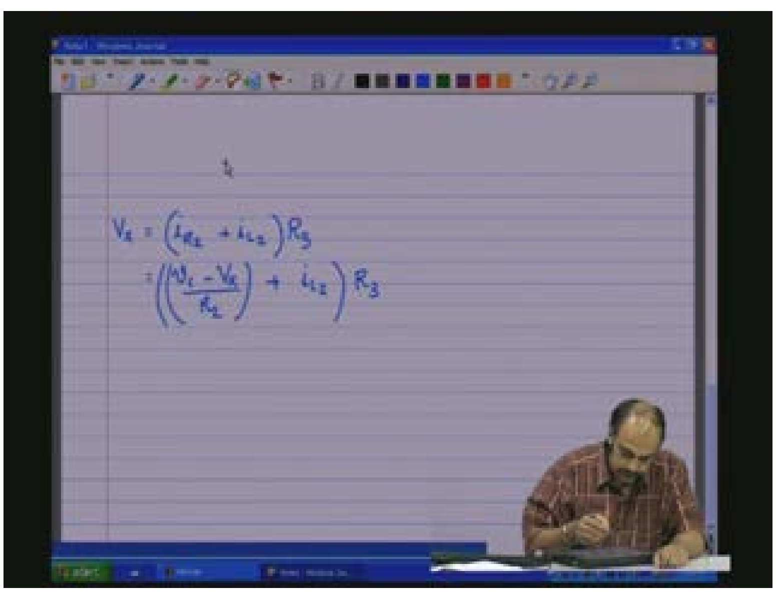

Now let us solve this for V x which will turn out to be V x plus V x R 3 by R 2 equals V c R 3 by R 2 plus i L 2 R 3 and V x is equal to [V c R 3 by R 2 plus i L 2 R 3] this whole thing multiplied by R 2 by R 2 plus R 3 from this equation (Refer Slide Time: 20:37) so from here this can be obtained.

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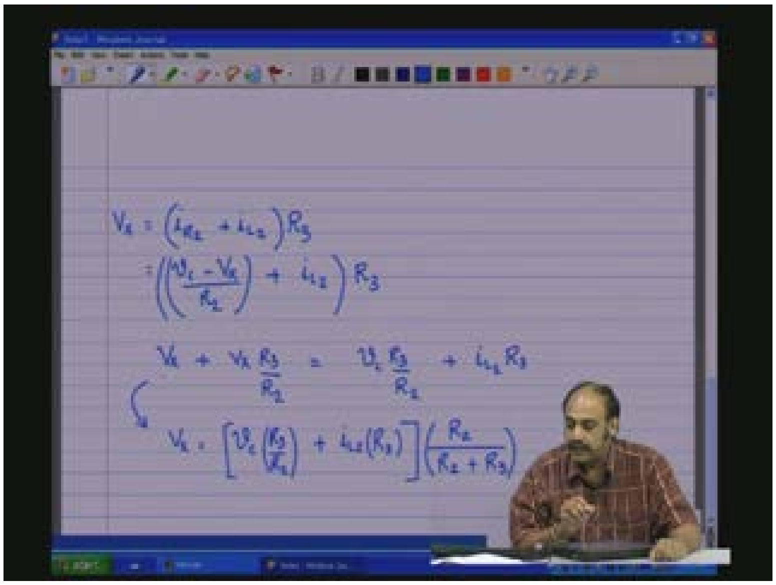

So we see here that V x is nothing but a function of V c and i L 2 both being state variables and some parameters. We could further simplify this. Let me use the selection tool, take this to the next page (Refer Slide Time: 21:09) copy, go to the next page, paste.

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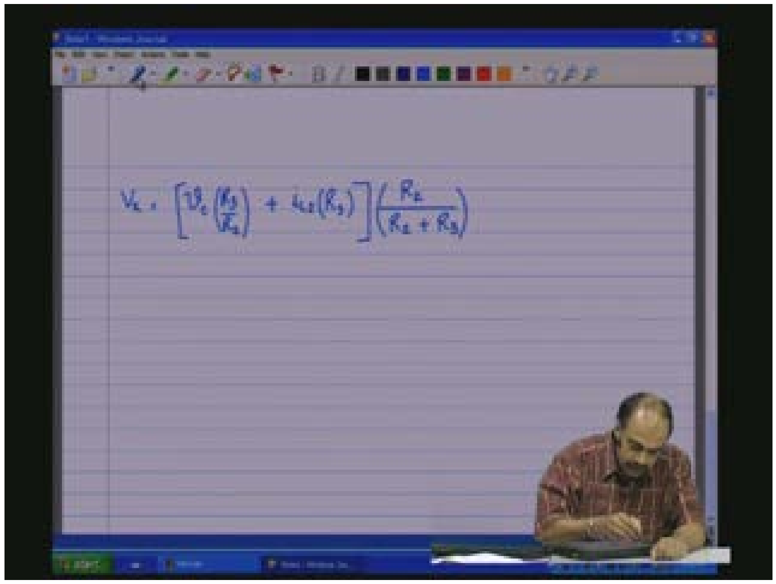

So V x equals V c R 3 by R 2 plus R 3 plus i L 2 R 3 R 2 by R 2 plus R 3. This is the equation for V x which is the intermediate variable that is expressed in terms of state variables.

Now coming back to the circuit here we need to express the third equation which is given here as….. let us paste it here (Refer Slide Time: 22:53). Now the third equation is the equation for L 2 L 2 di L 2 by dt is equal to the voltage across L 2. The voltage across L 2 is obtained by…….. now the current is flowing this direction let us say that there is an effort here like that, there is an effort of the source which is like that, the current direction is assumed this way and therefore we have the voltage across R 2 in that form in that direction and the voltage across L 2. And the voltage across R 4 of course we know that is i L 2 into R 4 and the voltage across this is nothing but V x. Therefore, we have V i 2 minus i L 2 R 4 minus V x.

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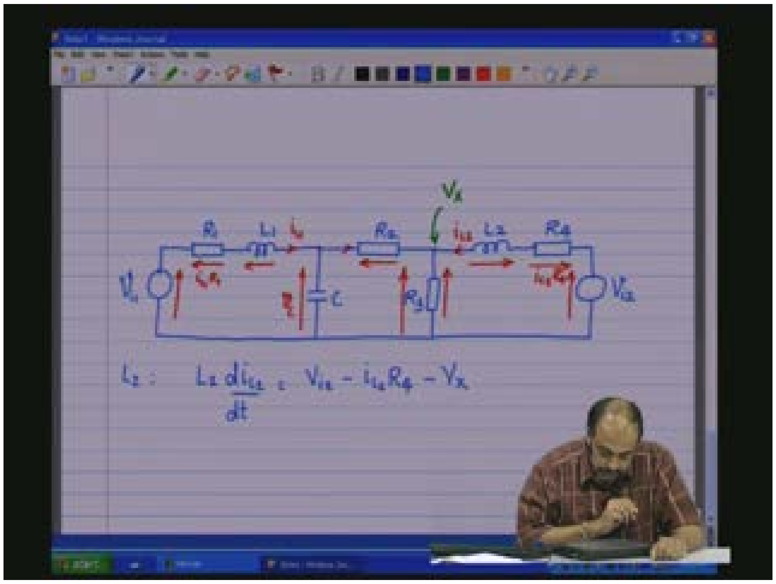

This is the equation the third equation. The first two equations we saw is here (Refer Slide Time: 24:19). The first equation is here: L 1 d i L 1 by dt which is equal to the input and the state variable and two state variables i L 1 and V c coming into the picture there. The second equation is V x sorry sorry it is not this the second equation is the Cd V c by dt equation which is equal to the current through the capacitance which is the i L 1 minus V c minus V x by R 2, here there is the new intermediate variable coming into the picture. However, we have shown that the intermediate variable can be expressed as a function of the state variables which ultimately is given in this fashion V x as a function of V c a state variable and i L 2 a state variable.

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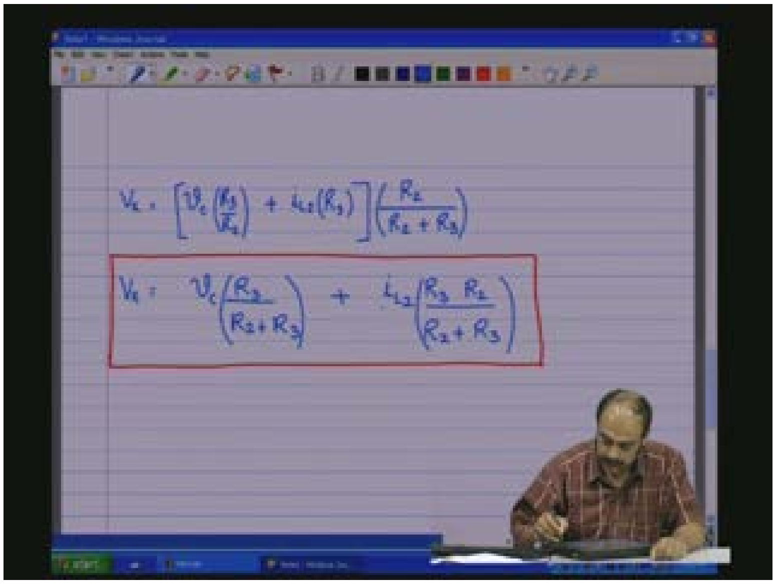

And the third equation is this which is revolving around the dynamic element L 2 which is L 2 di by dt equal to V i 2 minus i L 2 R 4 minus V x and V x again is the intermediate variable which is same as that expressed as which means all three variables all three equations are expressed as functions of input variables and state variables only and the intermediate variable being expressible in the form of state variables. This extra effort needs to be done in some of the circuits where some of the nodes are not directly a state variable. In such cases the intermediate variables can be used for such nodes or probably branch currents as the case may be and it will definitely be expressible in the form of expressible in the form of an equation an algebraic equation which is a function of the state variables, some or all are state variables.

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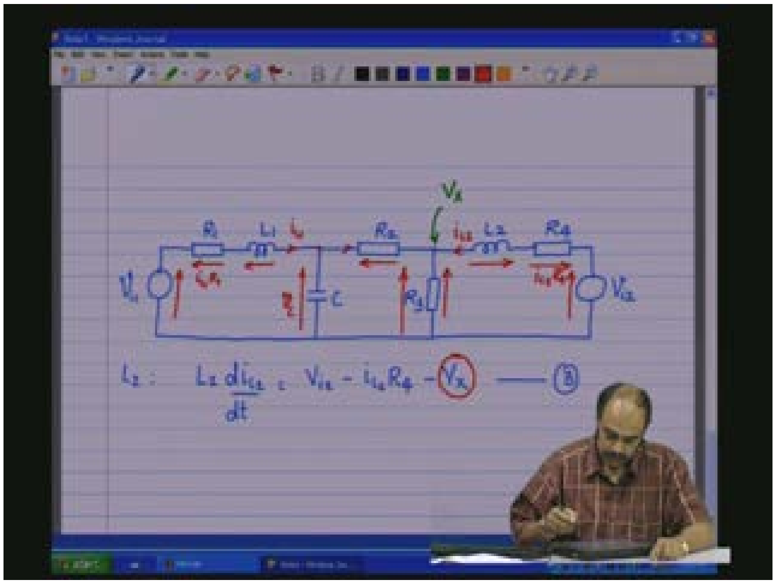

Now these equations 1, 2, 3 can now be put in the standard form of x dot which is equal to Ax plus Bu and y which is equal to Cx plus Du.

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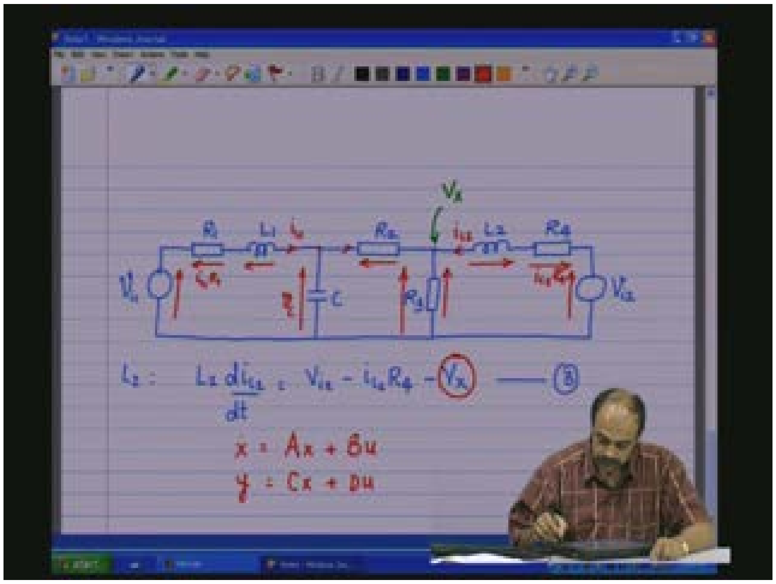

In this case the state vector, x is the state vector which is i L 1 V c i L 2, this is the state vector. There are three state variables, u is the input vector which consists of two V i 1 and V i 2. So you will find i L 1 dot i L 2 dot i L sorry 2 dot……….. I made a mistake here I apologise V c dot is equal to the A matrix which is basically the function of all the parameters of the circuit the R 1 R 2 all those things and you have i L 1 V c i L 2 plus the B matrix and V i 1 V i 2.

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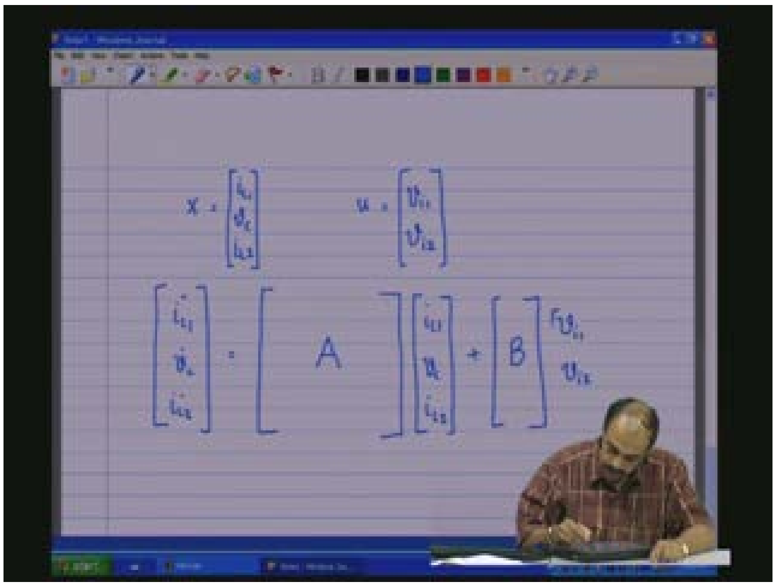

So here actually the A matrix is going to contain three by three elements, the B matrix is going to contain three rows and two columns hash one and the output y whatever the output y is supposed to be will contain the C matrix i L 1 V c i L 2 plus we have V i 1 V i 2 and as there are no direct feed forward terms that is zero and zero and here if it is…………. and here the V c if we want to consider it as an output variable then it would be this 0 1 0. So this would be your C matrix, this would be your D matrix, this is your B matrix and the A matrix all expressible in the standard state equation format.

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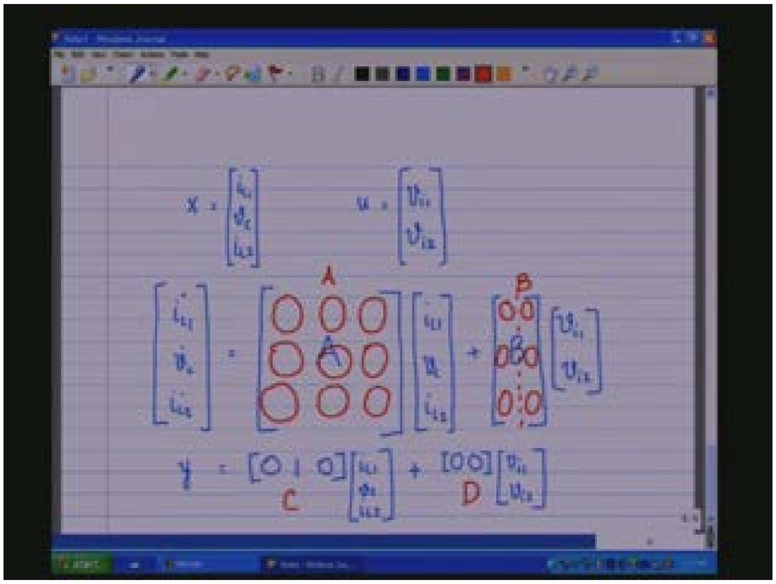

So now we have obtained the mathematical representation of the circuits whatever may be complexity of the circuits. Now we will be in a position to obtain the mathematical representation that is in the state equation form either by using intermediate variables or even without depending upon how the circuit topology is.

Now having once obtained the mathematical representation what do you want to do with them? Now you have the state equations with you which gives you the dynamic behaviour of the circuit, now what do you want to do with this mathematical equations? We need to analyse them, we need to analyse the equations so that we get a better insight into the circuit. So now we have the model. The model is in the form of a mathematical equation which is in our case the state equation. Now with this model we would like to now analyse the circuit analyse the circuit.

Now the analysis of the circuit; before was being done manually that you do a hand calculation probably, plot the graphs and then try to get the details or the features of the circuit. But now there are lot of computer software available which will do the bull work of doing the computations and then plotting the graphs so we need to supply of course the mathematical model that you would have obtained like we discussed just now and in the process of analysis there are few points that I would like to mention that the analysis is generally done in three domains or viewpoints. One is the time domain, the other is the frequency domain and the third is the pole-zero domain. It should be noted here but the circuit the signals all these are existing in the time domain only. However, the designer or the analyser or the person who is analysing can view the system or the signals by jumping to other domains and that is the frequency domain and the pole-zero domain to obtain some useful information about the circuit or the system or the signal which may not be so easily detectable in the time domain.

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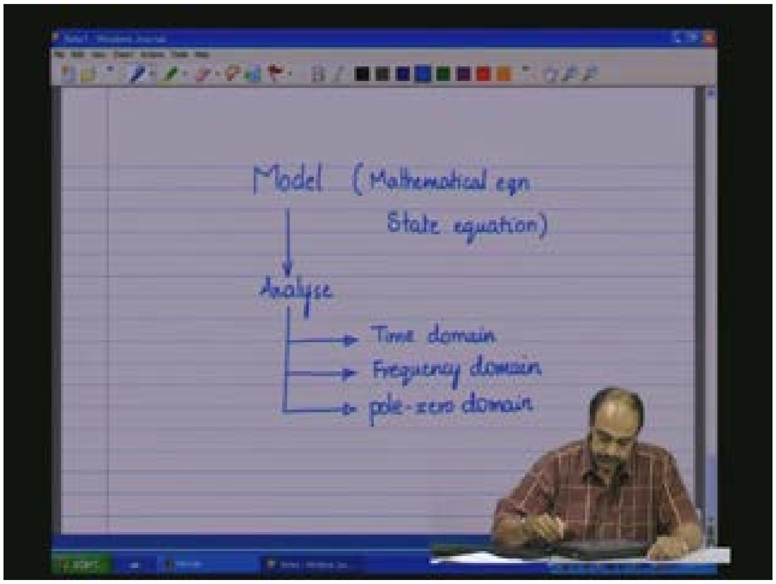

Therefore, in the time domain in the time domain we are plotting everything always with respect to the time as the x axis. The y axis would be amplitude; could be a current wave shape or could be a voltage wave shape or any other wave shape which is evolving with respect to the time; it could be the response of the system also like a step response. Like if you give an input as a step how will the output evolve? The output probably evolves like that and so on.

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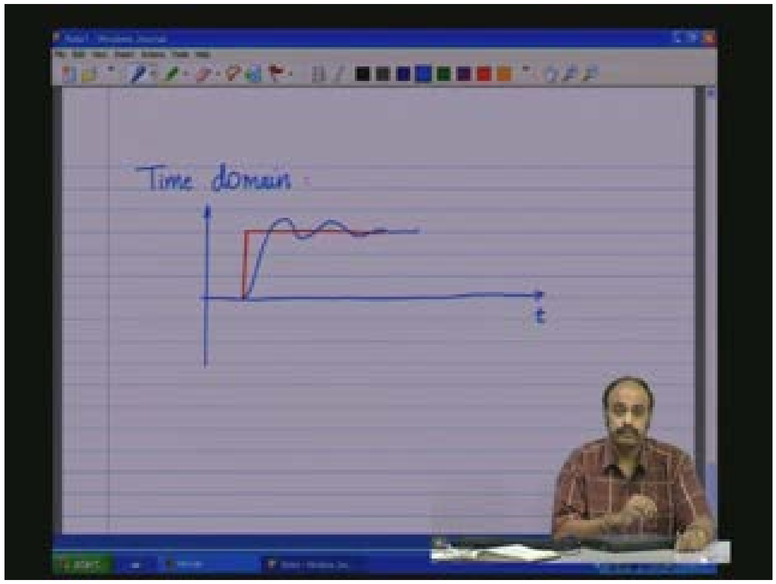

So these are the issues that we can see, the characteristics of the waveform in the time domain like the rise time the rise time or what is the peak overshoots or what is the steady-state accuracy accuracy or the settling time; how long the response of the system takes to settle down after the step has been initiated and things like this. Now these are some of the parameters but can be directly read out from the time domain wave shape.

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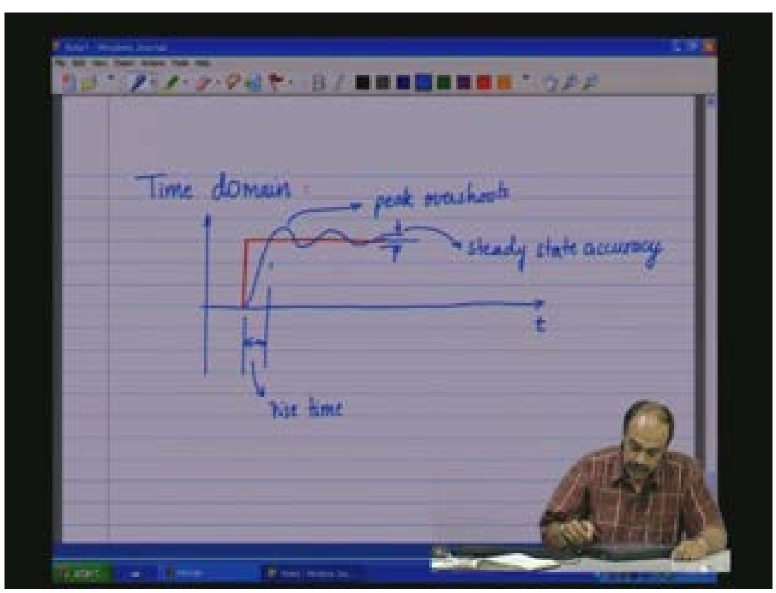

In the frequency domain the information that is not directly evident in the time domain is the harmonic contents, the frequency spectrum and how the gain of the system varies with frequency. These are some of the characters which cannot be obtained directly from the time domain response. So, in the frequency domain one would like to plot the waveforms with respect to the frequency omega. Omega is basically the radiant frequency.

Here there are two major plots that need to be viewed: one is the magnitude plot the magnitude plot and the other is the phase plot with respect to the frequency omega; meaning how does the absolute value of the gain vary with frequency? Is it a low pass filtering effect and at what frequency will the 3dB point 3dB point occur, at what frequency the zero crossover occurs. So these are important characters characteristics of the magnitude plot: what will be the gain or the attenuation of a particular circuit at various frequencies.

So, corresponding to these frequencies what will be the phase plot and how does it look like at various frequencies? Is it going to be in phase or is it going to be having no phase shift with respect to the input waveform? is the response having 90 degrees phase shift or 180 degrees phase shift with respect to the input waveform etc. So these are issues that you can obtain, these are the information that you can obtain from the frequency domain plots.

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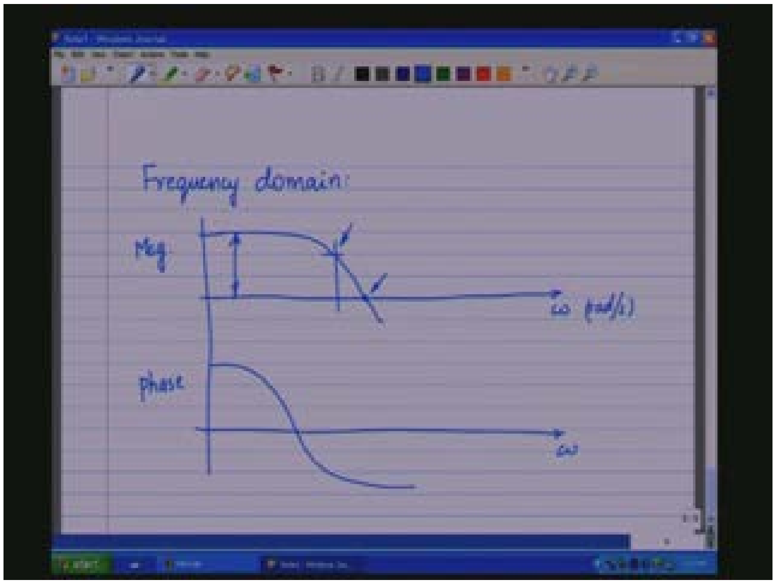

Then the third domain is the Pole-Zero domain. The Pole-Zero domain it is a Cartesian coordinate system where the x axis is named sigma and the y axis is omega and any point in this coordinate system S is sigma plus j omega and this is called the S-plane. So the S-plane is basically a representation of the physical system which will give you an idea of the poles, location of the poles, zeros of the system. The poles of the system are nothing but, in the time domain the exponential behaviour or e to the power of………. if there is a value a; a pole of value a only then this will result in a time response of e to the power of minus at. So we know that if there is a disturbance in the system now that disturbance will decay with this time constant a time constant which is 1 by a. So, each of the pole point here represents an exponential decay mechanism such that any disturbance or an excitation will decay at a particular rate.

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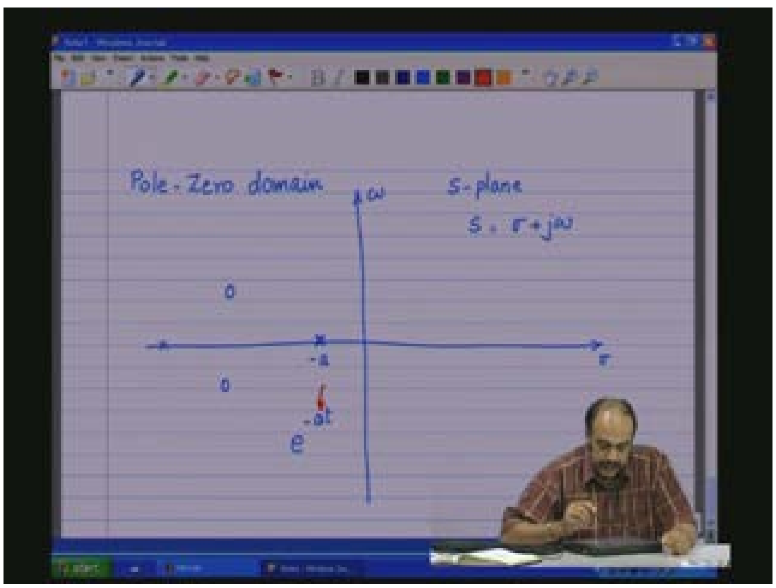

Now this gives lot of insight about the system's dynamic behaviour and the disturbance rejection behaviour which will not be obtained from the other two domains. And this domain will also give you a good aid to position your poles and zeros such that any disturbance and any noise rejection and any decaying properties can be planned and designed and thereby the values of the components, parameters can be chosen.

Of course it should be noted that in a physical system all the poles and zeros will exist only on the left half of the S-plane because only in the left half of the S-plane any pole or a zero would have e to the power of minus at which decays but if it is on the right half of the S-plane let us say point b then it would have a time domain response e to the power of bt which means it is growing with time; any disturbance will not decay but grow with time as b is positive and it is an unstable system and therefore it is not a system which can exist in nature. So all physical systems will have poles and zeros located on the left half of the S-plane and how close or far away from the omega axis or the y axis will determine the features and characteristics in the damping ratios, decaying nature all those issues which can be used for synthesis of the circuit.

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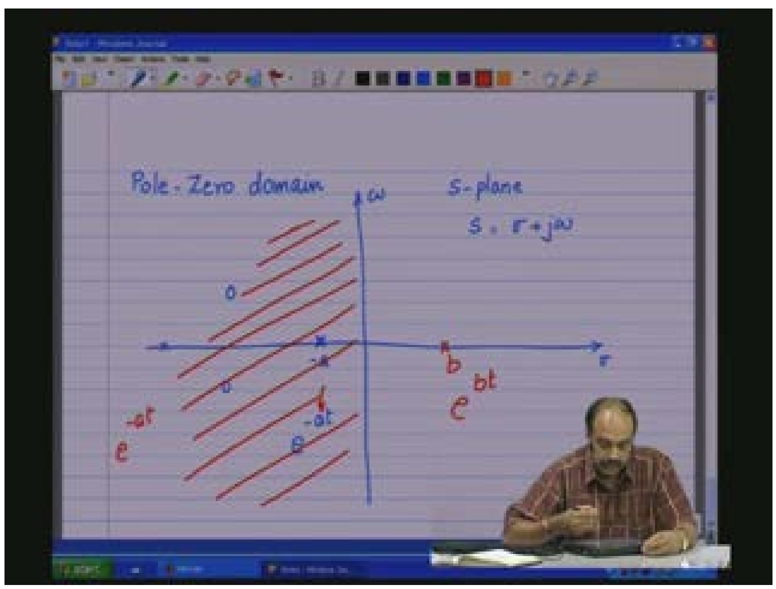

Therefore, we have three major viewpoints. Any system, any circuit will exist only in the time domain. However, apart from being viewed in the time domain, meaning the response of the circuit apart from viewing it in the time domain the response can also be viewed in the frequency domain and also there is one more domain which is very important in characterizing the system is the pole-zero domain which describes the system in a slightly different way in the S-plane; which gives you a behaviour of the excitation capability, the decay of disturbances, the decay of the noises, the decay of a sinusoid, the amplitudes of a particular disturbance which will affect the output all these issues can be studied from the pole-zero configuration or the pole-zero pattern on the S-plane. And one could also design the values of Rs, Ls and Cs such that the pattern of the pole-zero can be modified to suit a particular design requirement or design specification.

These are the three domains that one should be able to move about and understand for any mathematical representation or any model and one should seamlessly be able to jump from time domain to frequency domain, frequency domain to pole-zero domain, pole-zero to time and so on and so forth without much problems, then the understanding of the whole system or circuit will be more complete.

Now what we shall do is to analyse the system and we said that we are going to analyse the system in these three domains that is the time domain, the frequency domain and the pole-zero domain. And to analyse, as I was saying there are many software packages available and out of which for this discussion and all the future discussions we will mainly be using the MATLAB/ SIMULINK environment.

The MATLAB/SIMULINK environment is what we will be using for doing most of the bull work of the analysis. We of course will provide the model that is the mathematical representation of the equations. There are the equations that we have obtained from the circuit after having done the process of the state equation extraction.

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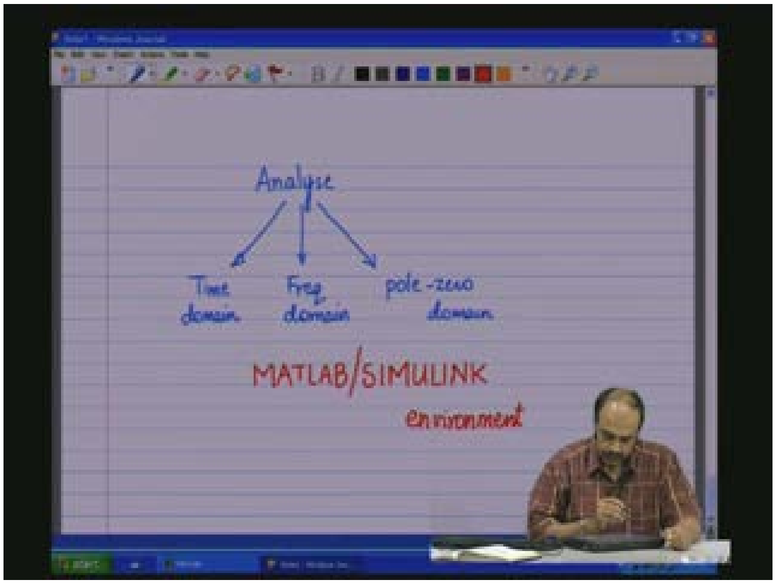

So let us take a simple example circuit. We had previously in the last session worked on a very simple RC circuit. If you remember recall that this RC circuit was analysed sorry was modelled and the state equations for this were also obtained. This of course has a direction as shown, you have the voltage across C and it there is a state variable and there is one equation here (Refer Slide Time: 50:14) which is given by C dV c by dt which is equal to V i minus V c by R. So C dV c by dt is the current through C which is nothing but Vi minus V c by R. Or putting it in our standard form V c dot will be equal to minus 1 by RC into V c plus 1 by RC into V i and let us say V 0 is our output which is given in this form plus 0 into V i. So we have our A matrix, the B matrix, the C matrix and the D matrix.

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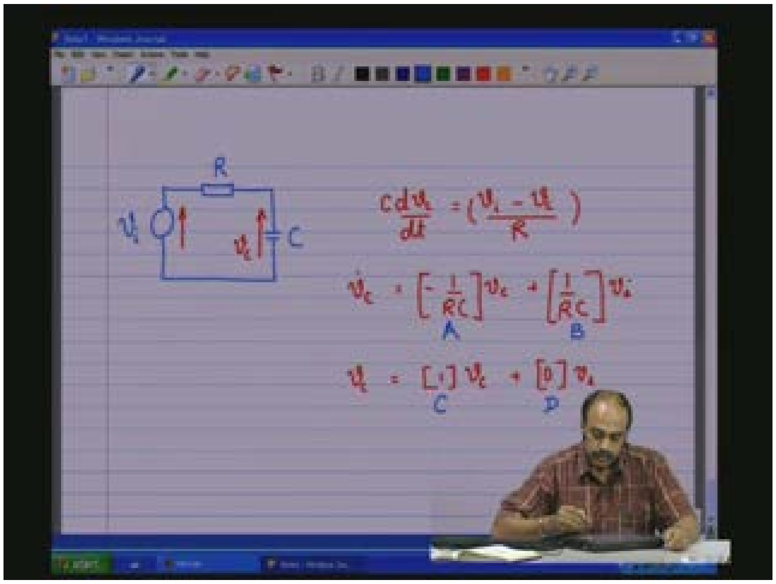

So what is necessary to be inputted to the software environment is the A matrix, B matrix, C matrix, D matrix and the values of the parameters R and the C. So if you go in to this MATLAB computational environment the command window looks like this (Refer Slide Time: 51:57). This is the MATLAB command window. This is where all the equations are typed in and the results are viewed. This is an interpreter based language so you could probably type in some variable equals to into something; it will compute like an interpreter. You could also use these variables for something else like for example; another variable b which is a to the power of 3 something like that.

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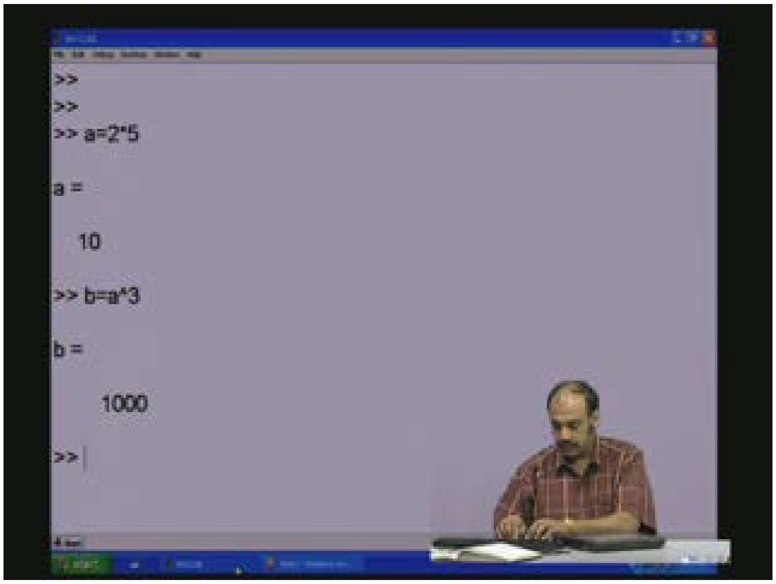

This is how the computational environment works. Now this is very suitable for our purpose of analysing the circuits which are of course being represented as state equations and the mathematical equations can directly be ported in here like for example let us say R is equal to 10 ohms sorry R equals 10 ohms, C equals let us say 10 Microfarads therefore I am writing it as 10 e minus 6 and then the A matrix can be written as within the brackets square brackets minus 1 by R into C this would be the A matrix and the B matrix is nothing but negative of the A matrix which is like that and of course the C and the D matrix are very trivial which is like that and D matrix which is 0.

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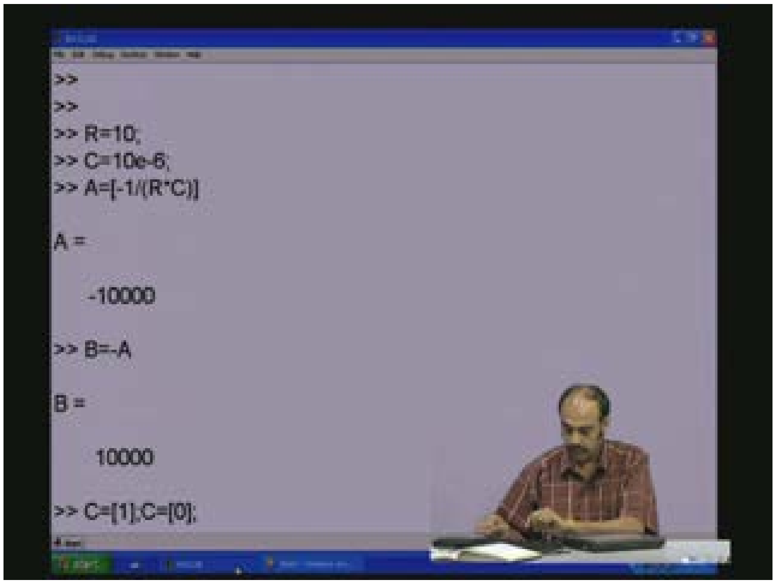

So in this way we can enter and obtain the three matrices which is the A B C D sorry D matrix.

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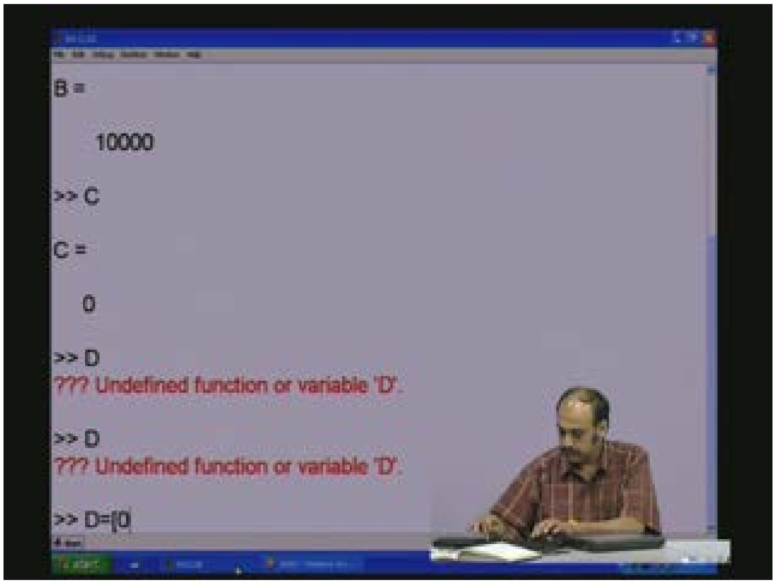

Now once the A B C D matrices have been entered they can now be used for all the manipulations mathematical manipulations that this software permits. which of course I will discuss that in some detail in the next class

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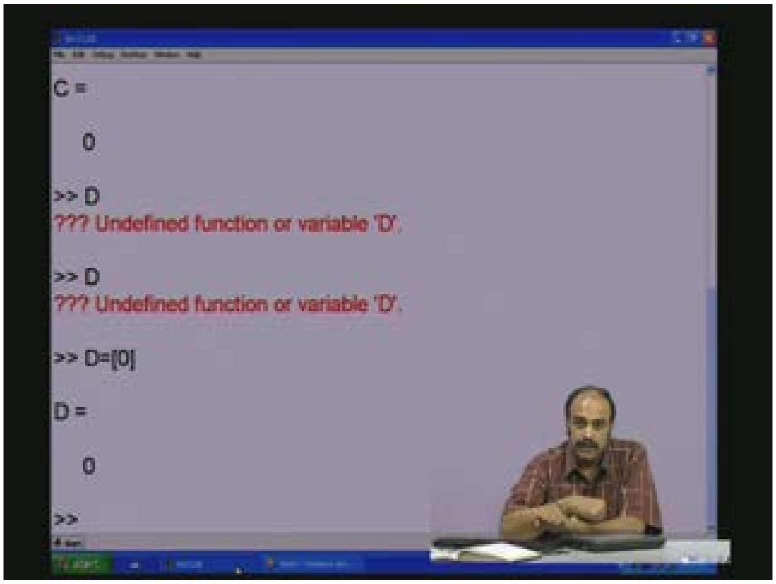

Coming back to the board here there are three domains as I told you: the time domain, the frequency domain, the pole-zero domain. After having inputted the A matrix, the B matrix, the C matrix and the D matrix into the MATLAB we will use three major tools. One is the step function to obtain the step response of the system, the second one will be bode or the frequency to obtain the frequency and the phase plot of this particular system that you have modelled and third the pzmap which will give you the plot of the pole-zero location of the system. So these are the three main functions that we will be utilizing in trying to analyse the system and trying to get the features of the circuit that you have modelled.

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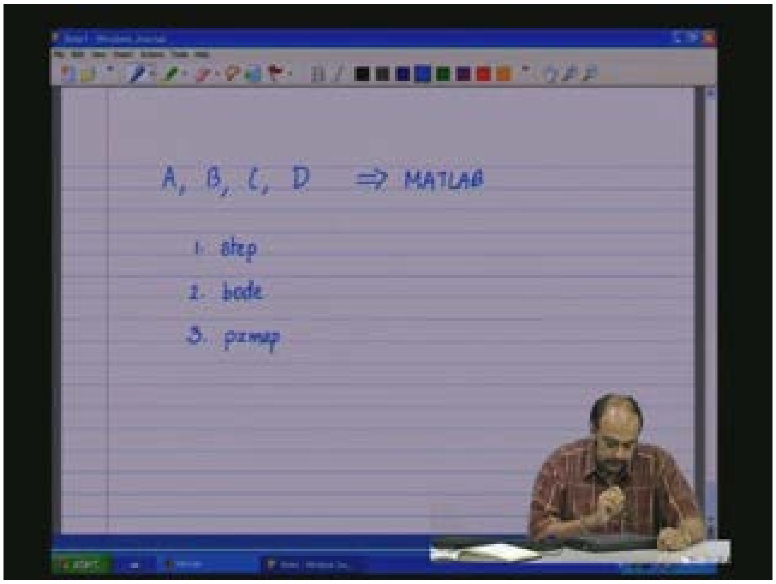

In the next class we will try to input the mathematical models of somehow the circuits that we have already obtained the state equations for; and in MATLAB try to analyse the system in the time domain using the step function, the frequency domain using the bode and the pole-zero mark to obtain the pole-zero domain properties. So the step could be for the time, bode for the frequency and pzmap for the pole-zero domain picture. So we close the session at this point. Thank you.