

So, current controller, I will put as K_c into $1 + T_c s$ divided by $T_c s$. Now, comes the converter; so, we have to model the converter same like front end AC to DC converter. So, we will come to that one. So, converter; this is our converter. Converter will give the correct V_A , our armature voltage. So, we know the motor model now.

So, this minus the back emf that is E_b multiplied by $1 + R_A$ plus this armature time constant, this will give the motor current I_A . So, this I_A is the one we have to feed back here. So, this I_A is actual motor current. You want DC current and because of the converter switching, it has the ripple. So, this ripple we have to reduce it, eliminate it and also we have to bring it to the controller level. So, there is a gain and a filter, gain reduction, so K_2 $1 + T_2 s$. So, filter we are representing as a first order filter. This we will give it here.

Now, this I_A , we have introduced the electro mechanical time constant. So, R_A by K_m machine parameter, we will give the driving torque M_d , M_d minus here is our motor torque, this is M_d , here is our M_L , M_L into R_A by K_m square. This is our load torque. So, difference in the driving torque minus load torque into $1 + T_m s$ will give our speed ω_m . This is our ω_m mechanical speed.

So, this mechanical speed, this is our output; this mechanical speed, we have to bring down to the controller level and it is a torque, there is a filter will be there. So, this one more filter is required, first order filter, $1 + T_1 s$, this we will give it here. This is our complete closed loop diagram, so this is our ω_m . Now, how do you represent our converter?

See, let us take a three phase fully controlled converter. So, let us take for a three phase fully controlled converter for a typical firing angle α is equal to 60° if you see here, the ripple will be 6 times the fundamental frequency. So, ripple will be approximately something like this we will have, part of the line voltage, 6 times it will happen. So, 6 times means this duration is 60° . For 360° , this is 60° duration. So, 60° duration for a 50 hertz wave form, this will be equal to 3.3 milliseconds.

Now see, if we have initiated a firing here; the next firing happens here only. Now, a change, if you see, a change in the converter firing angle happens at every 60° , it is not instantaneous. That means a delay of 3.3 milliseconds. So, it can have a maximum delay of 3.3 milliseconds or minimum of 0. That depends on where the demand for change happens. So, let us take an average of 3.3 divided by 2; 1.1 milliseconds. That means if a command is given; after 1.1 milliseconds, a change is happening. This, we can approximate as a first order lag.

So, converter can represent a delay with a gain. Why gain is required? Here, we have the V_A that is the actual voltage. This is coming from the controller, so this is the V_A reference. This may be for the control actions, it may be plus or minus 10 volt and what is V_A ? So, let us take a 10 volt reference comes, we want if we use the cosine firing with linear control, output, maximum output voltage should happen.

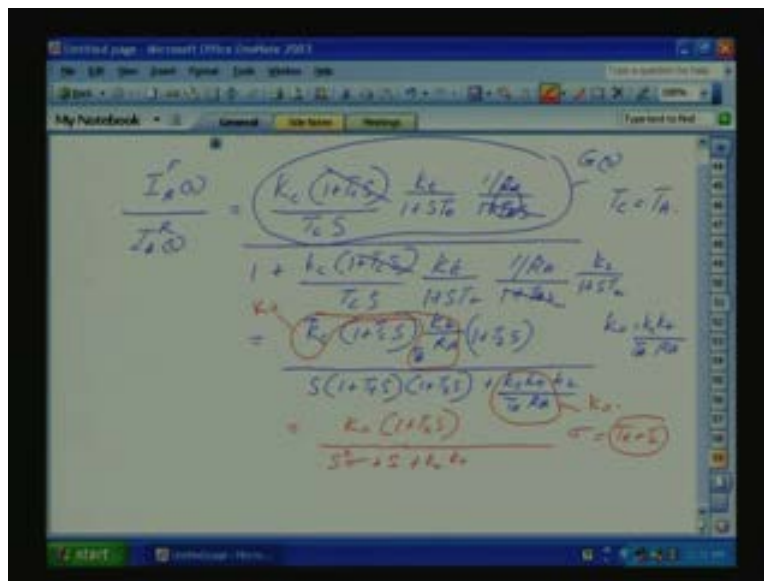
So, for a three phase converter, what is the output voltage? $\frac{2}{\sqrt{3}}$ into E_{rms} into $\cos \alpha$. So, maximum voltage happens when α is equal to 0. So, this will give the maximum voltage,

E rms. So, control 10 volts that means when V star is equal to 10 volts, actual V should be equal to 2 into 1.17 E rms.

So, the gain will be this divide by 10, this is the gain. So, converter can be represented at a first order lag. How? Here, we will do convertor, we will put as, some gain; gain G, we will put as Kt, converter we will represent Kt by 1 plus S Tt, S Tt, this is for the convertor. So, this converter transfer function, we can represent as we can remove this one, we can represent the converter as Kt by 1 plus St that is here, Kt by 1 plus ST the t, T is the time constant, 1.17 millisecond, Kt is the gain that is V_A, maximum V_A divided by the control voltage, whatever you use, that is the gain.

Now, let us design our current loop first. So, that is current loop means this one. To design this current loop, let us write down the transfer function. Now, G (S) by 1 plus G (S); output is I_A, input is I_A reference and G (S) is equal to this. So here, when you design, there is a E_b is there. So, if for an applied voltage V_A, if E_b is there, the difference the voltage here is; as E_b increases, the voltage gets reduced. So, we will design for the worst case design, worst case design is current is large when E_b is equal to 0 that is starting, we are applying the voltage. So, we will assume E_b 0. Then our transfer function, the loop gain; the open loop gain is K_c into 1 plus S_t s plus this one Kt into 1 plus S_t into 1 by R_A by 1 plus T_A s and the H is equal to this one. So, if you take the transfer function, let us draw the input output relation. Let us go to the next page now.

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So, our I_A (s), this is our feedback, divided by I_A (s), this is our reference is equal to K_c into 1 plus T_c S divided by T_c S into Kt by 1 plus ST_t into 1 by R_A by 1 plus T_A S divided by, this our G(S), 1 plus G(S) H(S); I think, this will 1 plus again K_c into 1 plus T_c S divided by T_c S Kt by 1 plus ST_t into 1 by R_A divided by 1 plus T_a S, then H(S) that is K₂ into K₂ by 1 plus S T₂.

This filter time constant; the moment we introduce a filter, it gives a delay. So, this delay or the time constant should be much low, much much lower than the larger time constant in the loop.

So, this is we are introducing. This means this should not create any problem with the converter. Otherwise, unnecessary delay will happen here. So, this filter to remove the high frequency ripple due to the PWM action. So, this will not create any problem. Let us see what or later we will take a typical example and try to solve the thing. Then we will know it, typical values of this one.

Now here, see, we are using a PI controller here. In this PI controller, this $T_c S$, $1 + T_c S$, the time constant; because of the 1 plus, there is a pole introduced. Now, this pole, what we can do? This is in our control, $T_c S$ we can choose it such that we choose $T_c S$ such that one of the larger time constant here we can cancel this one, order system can be reduced, response will be still faster. So here, which is the larger time constant?

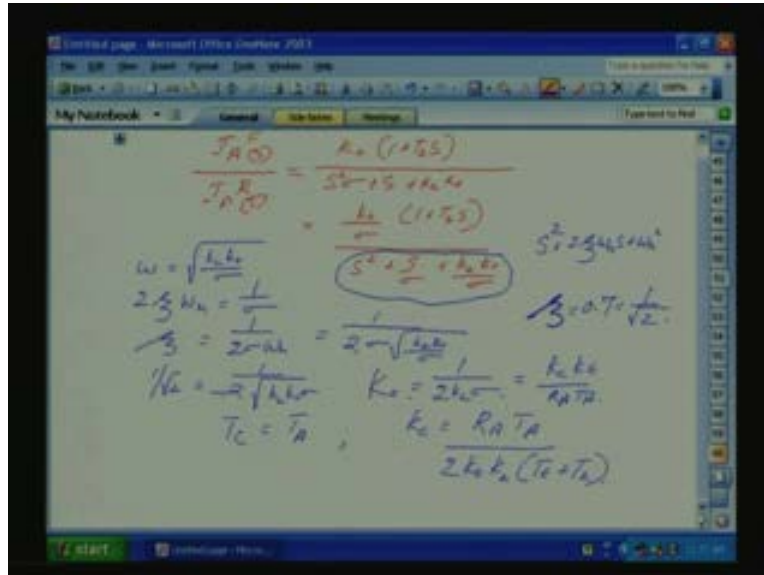
See, there is a converter delay is there, ST_t . But converter delay should be much much smaller than the delay used to the armature time constant. So, the larger time constant has to be this one. So, let us make T_c is equal to T_A . See, if the converter T time constant, the delay in converter is much higher than the T_A , the system will not work. Our controller has to be faster compared to any delay in the system. So, T_c is equal to T_A . So, this and this, we can cancel here. That means T_c is in our control if we keep it.

Then the final transfer function, we can write it. We can simplify this one, this will be K_c into $1 + T_c S$ into K_t by R_A into $1 + T_2 S$. So, this will be $1 + T_2 S$ divided by $T_A s$ into $1 + T_2 S$ into $1 + T_2 S$ plus $K_c K_t K_2$ by R_A ; these are all constant. Let us make this one $K_c K_t K_2$ by R is equal to K_0 . That means K_0 is equal to $K_c K_t K_2$ divided by R_A . So, this we can represent as K_0 . So, this $T_A S$, I can bring it here. $T_A S$ is here so that I can bring it here; numerator and denominator I can divide by $T_A S$. So, if you do that one, this will go. So, this will become S and T_A will come here, T_A will come here also.

So, $K_c K_t T_A$ by, so we will K_2 we will remove it. So, $K_c K_t T_A$ by R , this is K_0 ; so, if you see this one, if you put this one that means this is K_c , this is K_0 , also this is also K_0 . Then the final transfer function will be that is final transfer function is equal to K_0 into $1 + T_2 S$. If you see here, the moment we introduce a filter, it has given a zero. So, in the response, that can create overshoot. But, how to tackle that one; we will come to that one now.

This we can represent as $S^2 \sigma + S + K_2 K_0$ where σ is equal to T_t plus T_2 . Here, what we did? T_t plus T_2 , thus this $T_2 T_t$ is the converter lag and T_2 is our filter lag. These are smaller time constant, so the products we can neglect here and T_t plus the sum of this T_t plus T_2 , we can approximate as the σ here. That is what we did here. From this one; how to design our controller? So, this form, let us bring it to the standard form and let to sort out the, we will find out how to find out K_c and T_c . So, let us go to the next page now.

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See, again we will write it our $I_A (s)$ feedback divided by $I_A (s)$ reference is equal to K_0 into 1 plus $T_2 S$ divided by S square sigma plus $S K_2 K_0$. See, this again we can write it as K_0 by sigma into 1 plus $T_2 S$ divided by S square plus S by sigma plus $K_2 K_0$ by sigma. So, if you see, the denominator this one, this one is of the standard form. The response depends on the denominator is of the form S square plus 2 zeta omega N s, omega N s is a natural frequency, omega N square.

So, from this, equating these two; what is omega? Omega is equal to root of $K_2 K_0$ by sigma. And, what is 2 zeta? Omega N is equal to from this one, it is 1 by sigma. So, from this one, the damping factors zeta is equal to 1 by 2 sigma omega N . This is equal to 1 by 2 sigma into $K_2 K_0$ by sigma. Now, from the standard control system to get a proper response, this is a second order system. You know it, zeta is equal to 0.7 is a good, it will give a reasonably good response. This is equal to 1 by root 2 . So, 1 by root 2 is equal to 1 by 2 into root of $K_2 K_0$ sigma.

From this one, we can find out K_0 . Why K_0 we have to bring here? Because, K_0 contain the factor K_c ; so, K_0 is equal to 1 by $2 K_2$ sigma. So, this is equal to we are substituting, so this will be equal to $K_c K_t R_A T_A$. See, T_A , we have already put is equal to T_c . So, instead of T_A , we can replace it by T_c . So, from this one, we can find out K_c substituting for sigma. So, the final K_c , already found out T_c is equal to T_A . So, T_c we have already found out, T_c we have found out; T_c is equal to T_A . Now, what is, from these two equations, what is K_c ? K_c will be equal to $R_A T_A$ divided by $2 K_t K_2$ into T_t plus T_2 . So, the transfer function if you see here, this will give a second order system with good response.

Now, let us go to our transfer function and modify putting this value and let us modify it. So, let us go back to our old transfer function.

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$$\frac{F(s)}{I_A(s)} = \frac{K_c(1+T_s)}{s^2 + s + K_0}$$

$$= \frac{1}{2K_2} (1+T_s) \frac{K_c}{s^2 + s + K_0 + K_c(1+T_s)}$$

$$= \frac{1}{2K_2} (1+T_s) \frac{K_c}{(1 + 2\sigma s + 2\sigma^2 s^2)}$$

$$K_0 = \frac{K_c K_t}{R_A T_A}$$

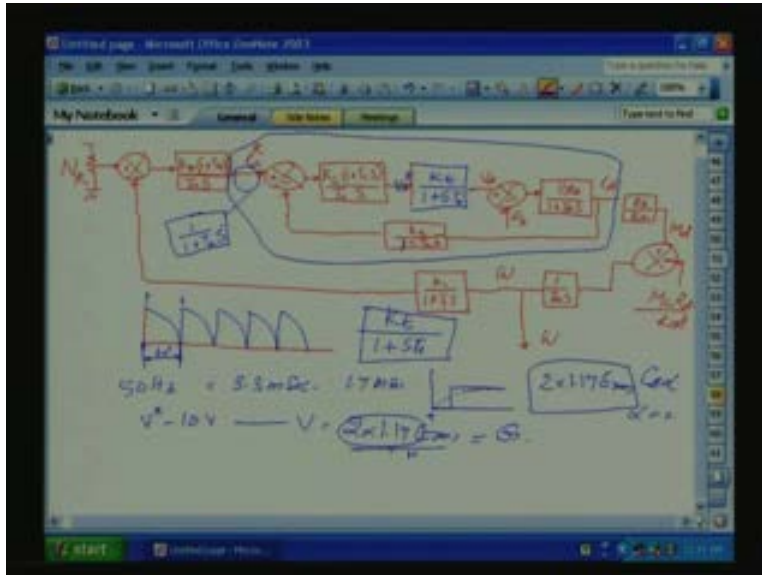
$$= \frac{K_c T_t}{2K_t (T_t + T_t)} \frac{K_c}{R_A T_A}$$

$$= \frac{1}{2K_2}$$

If you see here, I_A s reference by I_A s **sorry** I_A s feedback, output is there; output by input, this is equal to K_0 into $1 + T_2 S$ divided by S square sigma plus S plus $K_2 K_0$. We want to put the value of K_c and T_c and see how the response works here. So here, K_c , you know K_0 is equal to $K_c K_t$ by $R_A T_A$. This is equal to putting the value of K_c now, $R_A T_A$ by $2 K_t K_2$ into T_t plus T_t into K_t by $R_A T_A$. Here K_t goes, $R_A T_A$ goes; so finally K_0 will be 1 by $2 K_2$ sigma after our PI controller selection.

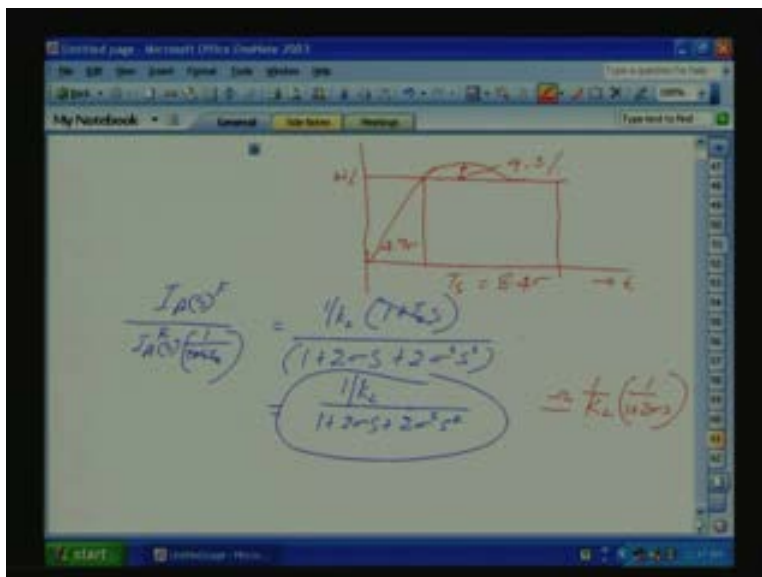
So this, if you put it here, this will be of the form 1 by $2 K_2$ sigma into $1 + T_2 S$ divided by S square sigma plus S plus $K_2 K_0$. This one further simplification, some further simplification, it will be equal to 1 by K_2 into $1 + T_2 S$ divided by $1 + 2$ sigma S plus 2 sigma square S square. Now, there is a pole here. This can create over shoot during the starting or any change it happens. So, it can get create large over shoot. So, to avoid this one; this happens because of the delay introduced in our feedback. Now, this can be cancelled, the same delay if you give to our reference also. That means our reference is we are going to pass through the same filter, the filter with a time constant of one by $T_2 S$. Then what happens? Let us see.

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So, in our pervious transfer function, if we go there; so here, at this point, this 1 by T_2 happens because of our filter time constant we are smoothing the current. So here, if I put a filter with time constant 1 by 1 plus T_2 S, then we can cancel the effect of this one. Let us see, how we can do it. See, let us write down or let us go to the next page now.

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See here, I_A s feedback divided by I_A s reference; now I_A s reference is passing through a filter. So, we have to, this also it would be 1 by 1 plus S_2 s, this is our new reference, we are putting a filter. This will be equal to our 1 by K_2 into 1 plus S_2 s divided by 1 plus 2 sigma S plus 2 sigma

square S square. So, this will go here, this and this will get cancelled. So finally, our transfer function will be 1 by K_2 divided by 1 plus 2 sigma S plus 2 sigma square S square.

So, what is this mathematical manipulation mean? See, the feedback, the moment you put a filter, there is a delay in acquiring the signal coming to the control input due to the delay, delay is due to the filter and it is equal to 1 by 1 S_2 s. So, it has created a 0 here. Now, the references also if we delay with the same, so these delays are introduced by the filter which will not come into the diagram if you have the current loop. That current loop dynamics is much much lower than this filter time constant or the current loop time constant is much much higher than this filter time constant. So, slightly delaying the reference will not affect that one. So, we have put the filter, same filter we have put at the reference also. What happened? So, the transfer function finally has some of the form like this.

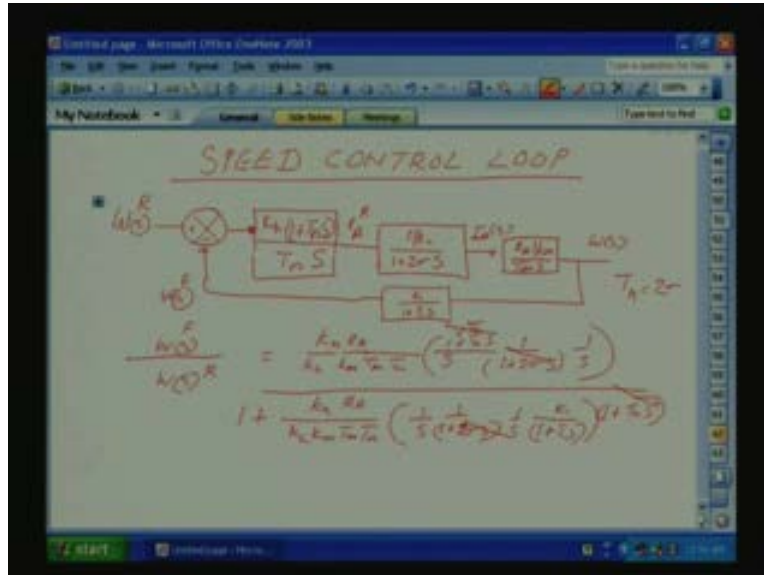
Now, for a unit step input, we can find out the inverse Laplace transform of this one and we can find out, how the response will be. If you plot it, it will be like this, the response; unit step input will be of the form like this. So, we want any change in I_A reference, it should go, this is our 100%, it has to come here. So, what happens? This will response will go like this, come here and come like this. So, this is slowly **sorry** this is slightly large I have put. This only around this much, this peak is around only 4%, 3% 4.3%. That is this is around 4.3%, slight over suit will be there and this if you take T this one, this is around 4.7 sigma and final settling time, approximately this will be settling time will be 8.4 sigma. This is a good response with proper compensation.

Now, current loop we have designed. So, what is the transfer function of the current loop finally? 1 by K_2 by 1 plus 2 S sigma S plus 2 S sigma square S square. This S square means the frequencies of operation. Compute this one and will come at the higher higher end and this current loop is inside a speed loop. So, speed loop, the frequencies of variation that bandwidth is much much below this one. So, effect of this one can be neglected and this current control loop that is this, it can be when we are going to inscribe this one into the voltage control loop, we can approximate this one by 1 plus K_2 into 1 by 1 plus 2 sigma S because this current loop is coming inside a speed loop where the time constant of the speed loop is much much higher than compared to the current loop.

May be, current loop if you say around millisecond, there may be it can be around second the speed loop. So, this we can approximate, 1 by K_2 by 1 plus 2 sigma S or if you want to really stimulate, you can as well put this one also stimulate in the full system and see the response and if you put this one, there will not be, as speed loop, there will not be much change. But we have taken the exact modelling of the current control, current control blocks to take care of the $K_c T_c$ design. Once that is designed, we can approximate this one with this one.

Now, let us take what is our speed loop with this one? So, let us take now go to our speed loop now.

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Speed control loop; let us take our speed reference, ω_s reference that is our input. This we are comparing with our speed feedback, we can use a tacho encoder, whatever it is and this speed error, we are giving to a controller. So, this is K_n into $1 + T_n S$ divided by, I will just increase this one, $T_n S$; we are using a PI controller here, let us see how to design.

So, as I told before, any change in the speed, any mismatch in this reference speed and feedback speed is due to a difference, a mismatch in the driving torque and the load torque. So, load torque means the load torque or when the acceleration or deceleration that the required or the torque required for acceleration or deceleration that is J into $d\omega$ by dt plus load torque. So, that is a complete torque we are talking about depending on the load condition.

So, any change in this speed reference that shows the driving torque and the load torque, there is a mismatch in the drive driving torque and load torque. That means we have to give the correct current value. So, this controller output as I told before, this will give our current reference. So, this is our I_A reference, this we will give it to the converter, converter we have model and the converter and the current controller, we have model to give the correct converter voltage and we got tune the $K_c T_c$ and we go and we got the correct response for a change in I_A reference with respect to I_A feedback. So, the final transfer function, what we are using? We are using a first order lag also here; high frequency times, we are neglecting, σS . This is the $I_A s$.

Now again, for the worst case design; this will this is our actual $I_A s$ multiplied by R_A by K_n divided by $T_m s$, this is our ω_s . This again, we are a gain change will be there to bring it to the controller level K_1 divided by $1 + T_1$. If you see here, this is this way. Now, let us see what is the transfer function, input output? ω_s feedback divided by ω_s reference, $G(S)$ by $1 + G(S)H(S)$. So, this will be K_n by K_2 , R_A by K_n , then T_m also T_n into $1 + S$. Here, one plus $T_n s$ is also there, $1 + S$ into $1 + 2\sigma S$. This $1 + S$ is due to this one; there is one more S , here $1 + S$. This is $1 + S$, this is $G(S)$ divided by $G(S)H(S)$ that is again $K_n R_A K_2$ $K_n T_m T_n$ into $1 + S$, $1 + 2\sigma S$ and again $1 + S$, then our filter.

Here, if you see here, 1 plus K_2 is also there, K_1 , K_1 by 1 plus $T_1 S$ multiple multiplied by this 1 plus numerator. If you see here, all these S multiplication, it will result in a fourth order system. So, that will be very difficult to compensate. So, let us take some engineering approximation. Here also, this T_n , like before the pole zero cancellation, let us decide this T_n is equal to 2 sigma S . That is a larger time constant.

So, that means let us take a case, T_n is equal to two sigma. Then this one and this, we can cancel. Here also, this one and this one, we can cancel. Then finally, what will be the transfer function? Let us go to the next page and write it.

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$$\frac{\omega^F}{\omega^R} = \frac{k_n R_A}{k_2 k_m T_m T_n} \left(\frac{1}{s^2} \right) \quad T_n = 2\sigma$$

$$\frac{1 + \frac{k_n R_A}{k_2 k_m T_m T_n} \left(\frac{1}{s^2} \right)}{\left(\frac{k_n R_A}{k_2 k_m T_m T_n} \right) \left(\frac{1}{s^2} \right) + k_2 k_m T_m T_n}$$

$$= \frac{k_n R_A (1 + T_1 S)}{k_2 k_m T_m T_n (1 + T_1 S)^2}$$

$S^2 (1 + T_1 S) = S^2 + T_1 S^3 + S$

Here, if you see here, this is equal to that is again omega (S)^F divided by omega (S)^R and we have put T_c is equal T_n is equal to our 2 sigma. So, this is of the form $K_n R_A$ by $K_2 K_m T_m T_n$, T_n is the electromechanical time constant into 1 by S square divided by 1 plus $K_n R_A$ by $K_2 K_m T_m T_n$, we can bring the K_1 also here for a filter gain. Then the whole thing will be 1 by S square into 1 by 1 plus $T_1 S$. If you take this numerator to this side, then final transfer function will be let us write down, this is $K_n R_A$ by $K_2 K_m T_m T_n$. So, we are multiplying this denominator with one and then we are using this factor to multiply the numerator and the denominator.

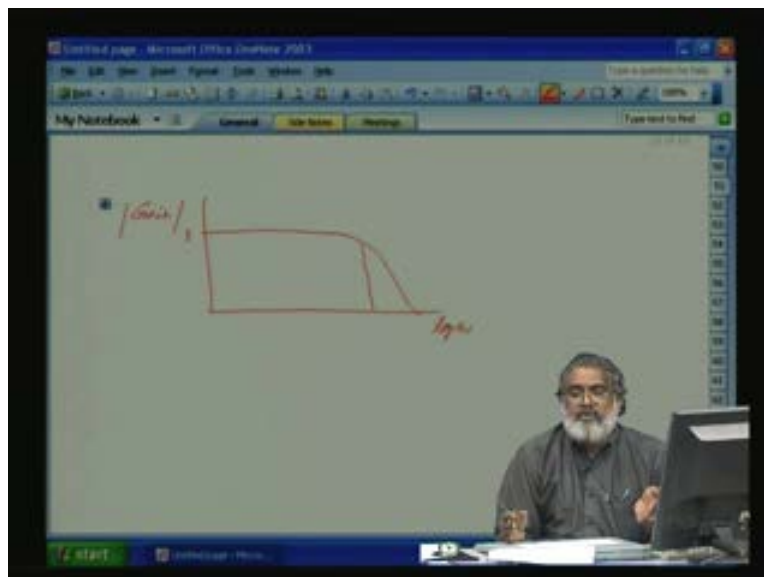
So, when you use this one, here 1 plus $T_1 S$ will come. This will produce L ; see same like our current controller, the delay, here the delay as introduced a lead here. Now, $K_2 K_m T_m K_s$, this will go and in the denominator, you will get a K_2 , see all these things will go here. Then the denominator will be $K_2 K_m T_m T_n$ into S square into 1 plus $T_1 S$ plus $K_n R_A K_1$. See, this also we can bring to, 1 plus, bring this one and 1 plus form, we can give. But if see here, if you expand this S square into 1 plus $T_1 S$; see let us expand S square into 1 plus $T_1 S$, if you see, it will come S square plus $T_1 S$ cube and the S square is there, S cube will be there, constant will be there because of the, that constant we will use; this S time will be missing.

So, S time that will give the, contain the damping factor ζ . So, this will not give a stable response for any change in the input speed. So, we cannot use a pole zero cancellation, we cannot use it. That means this, we cannot use it here. So, we have to retain the, we have to retain the $T_c S$ and we have to design the controller in or still we have to use some other approach, not the pole zero consideration. See, last time when we studied a front end AC to DC convertor, so without pole zero, how we can use the cancellation for input output relation? We said we used a optimisation technique where we want to adjust the T_c and K_c such that; so now with T_c and if the PI controller, if you write down the transfer function, the stress time will be there. So, system gives a, system is stable, it gives a good response. Now, we have to optimise.

So, optimise here, what we want? We want the input output to follow for any change in the input. Any change in the input means for any frequency of variation of the input, output should follow it as closely as possible. That means the output, the gain of the output, modulus of the output by the input should be 1 for large frequency of variation. We cannot have any frequency of variation, so there is a limit that is called the band width.

So, for wide frequency of variation, we should get that modular should be unity. So, if we can achieve that one, that is a, that way we can use it. Here, we said the general, we will write that one now, what we want here.

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See, one that we have written in the context of our front end AC to DC convertor, what we want is the gain if you take the gain bases frequency. So, we will put $\log \omega$ here and the gain that is ratio of the modulus of the input output gain. So, for any frequency of the input, the output follow very, output follow the input as closely as possible, immediately follows the gain will be 1. So, that frequency, you want this for wide frequency of variation. So, that means bandwidth, after sometime it will fall down.

So, you want to make it for wide frequencies of variation, for that type of compensation. That is we will use, this called the modulus optimisation. This is will use it here without going through the pole zero cancellation. This we will study in the next class. The same way we will we will look whether it is system is second order or third order system because we have derived the conditions for second order and third order system. Then we will see the system and we will try to bring it to some second order or third order with appropriate approximation, then we apply the rule and see the band width and then we will find out the transfer function based on that one, then find the response. This, we will study in the next class.