

Power Electronics

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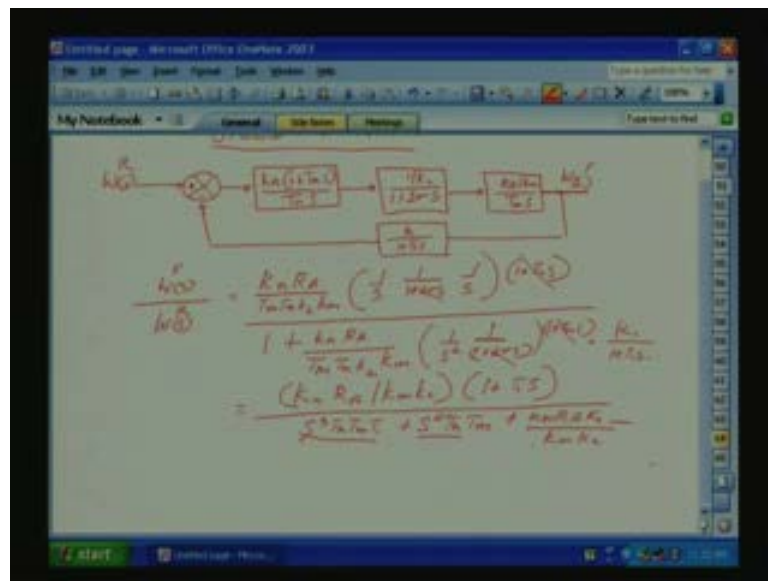
Indian Institute of Science, Bangalore

Lecture - 20

Dc Motor Speed Control Controller Design Part 2

See, last class we entered, we started at the end to design the, we had got the block diagram for the speed loop and we started to design the controller based on the pole zero cancellation and we found that pole zero cancellation will lead to instability or those we are losing the S time. Let us start with that one, speed control.

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Now, our reference is the omega S reference. Here is our PI controller that is K_m into 1 plus $T_n S$ divide by $T_n S$. Now, this will give the current reference and the current transfer function, approximate current transfer function which we can use in the speed loop is 1 plus 2 sigma S. The sigma we know, it is the sum of the smaller time constant that is the convertor lag and the current filter time constant. This we will give it to the machine, machine. If you see, this R_A by K_m by $T_m S$, this will be our speed feedback. This will come, feedback we have the gain as well as filter.

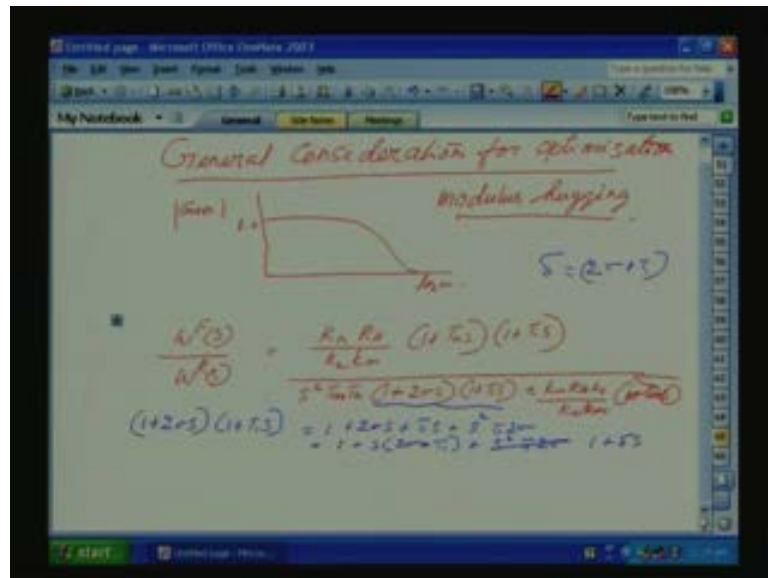
Now, when we are considering the speed, the pole zero cancellation that is T_n ; let us see let us write the transfer function for this one that is omega s of output feedback divided by reference is equal to $K_n R_A$ by $T_m T_n K_2 K_n$ into 1 by S 1 by 1 plus 2 sigma S to 1 by S into 1 plus $T_n S$, this is from here, this one, divided by this the, this the, this is the open loop gain that is the from this, from here to here, not the feedback loop, that is the $G(S)$

part plus 1 plus $K_n R_A$ by $T_m T_n K_2 K_m$ into 1 plus S square into 1 plus 2 sigma S into 1 plus $T_n S$ into our feedback filter that is K_1 by 1 plus $T_1 S$.

Now, if we use the pole zero cancellation that means if we are cancelling this with this one, larger time constant, then our final transfer function will be of the form, will be $K_n R_A$ by $K_m K_2$ into 1 plus $T_1 S$ divided by after the cancellation if you properly write down the transfer function, it will be S cube $T_n T_m T_1$ plus S square $T_n T_m$ plus constant $K_n R_A K_1$ divided by $K_1 K_2$. So, if you see here, the S term is missing. S square is there, S cube is there, S term constant is there. So S , so S time is missing in the characteristic equation that is the denominator. Hence, the speed control system is unstable. So, the loop should be optimised using a different approach.

But if you see, if you do not do the cancellation part and if you write down, the S time will be there, all the time will be there. That means system is stable. Now, the question is optimising. Optimise means we have studied in the context of front end AC to DC convertor, so we will again repeat that one here for clarity.

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See, general consideration for optimisation, for optimisation. That is that means, we are optimising in terms of the bandwidth. See, the dynamic performance of a control system is good with a controlled variable very rapidly reaches the reference input. Ideally, for any frequency of input variation that means if in the closed loop in the closed loop control system, the input we give any, ideally for any frequency of input variation. That means we give a signal with sinusoidally varying, frequency varying let the amplitude be constant.

Then output should track the input variable instantaneously. That is what we want. What we want when we vary the input, here variation we are trying to do with various frequency of signal with constant amplitude; then input should track, the output should track the input. That means for different frequencies of operation if the output tracks the input that means the ratio between the output and input will be one. So, but as the

frequency increases because of the system lag, slowly this system's response that feedback signal will not be following the output.

So, that way if that frequency and frequency component and as the input frequency varies, slowly the output gain will slowly decrease. So, there we talk about the bandwidth. So, for a practical system, in terms of frequency range, the modulus of the output, output gain should be very close to one over a wide frequency range that is bandwidth that means the ratio between the output and the input. So, if you see here, we want something optimised. We choose our controller such that the gain, modulus of the gain should be one for as fast as possible $\log \omega$ here.

Now, so optimisation aims at bringing the modulus of the frequency characteristic as close as to one or a wide frequency range. This is also called modulus hugging. This we have studied when we when we were going through the controller design for the ac to dc converter. The same technique we can use here.

So, what we want now? Let us find out the transfer function; without pole zero cancellation, let us write down the transfer function. If you say, so now without pole zero cancellation, the ω feedback S divided by $\omega R s$ is equal to $K_n R_A$ by $K_2 K_m$ into $1 + T_n S$ because of our PI controller and again because of our feedback time constant because of the filter. Then finally, it will be $S^2 T_m T_n$ into $1 + 2 \sigma S$ into $1 + T_1 S$ plus $K_n R_A K_1$ by K_2 into K_m into $1 + T_n S$.

So here, if you see here, if you expand this portion, if you expand this portion alone, let us do that one that is $1 + 2 \sigma S$ into $1 + T_1 S$. Then this will be L will be $1 + 2 \sigma S$ plus $T_1 S$ plus $S^2 T_1 2 \sigma$. See, this also we can write as $1 + S$ into 2σ plus T_1 plus $S^2 T_1 2 \sigma$. See here, these are filter time constants T_1 and 2σ that is because from the current control loop, these are smaller time constants compared to our electromechanical time constant. So, because this loop is this is coming in a loop where the maximum response depends on the electromechanical time constant; so, this also, we can neglect this one. On practical purpose, we can do it and this approximately, we can represent as a first order lag, again like a first order case.

That is $1 + \delta S$ where δ is equal to 2σ plus δ that is δ is equal to 2σ plus T . So, that means order of the denominator we can reduce it. So, still the characteristic equation is well within the frequency range of our system, system with which it responds. So, if you do that way, finally the transfer function we can write it like this. The whole purpose is to bring the whole system for modulus, I think whether we have to bring it either to the second order or third order because we have we know how to do the do the optimisation for two second order as well as third order. So, let us see what is the order of the system if you do it here.

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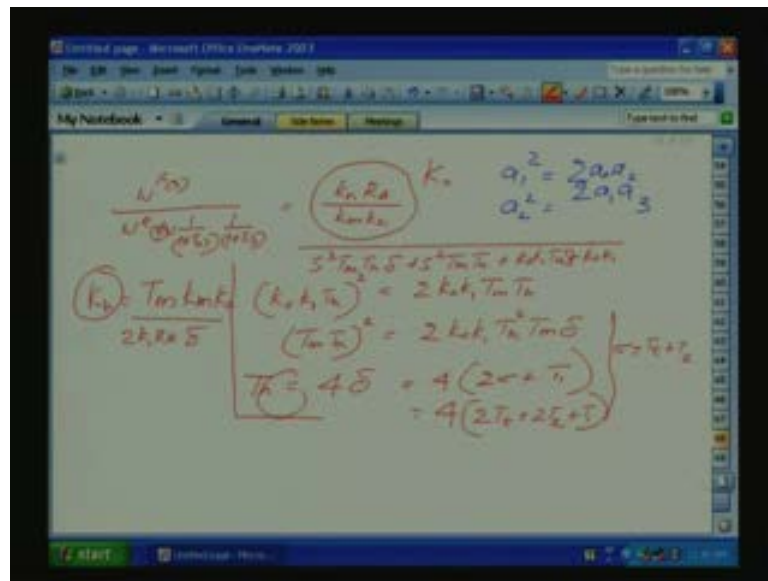
So, the final system will be $K_n R_A$ by $K_2 K_m$ into $1 + T_n S$ into $1 + T_1 S$ divide by S square $T_m T_n$ into $1 + 2 \sigma S$ into $1 + T_1 S$ plus $K_n R_A K_1$ by $K_2 K_m$ into $1 + T_n S$. Now, this we have approximated as $1 + \delta S$ where δ is equal to 2σ plus T_1 , this one. So, the final transfer function will be if you do the, if you properly simplify this one, the final will be $K_n R_A$ by $K_m K_2$ into $1 + T_n S$ into $1 + T_1 S$ divided by S cube $T_m T_n \delta$ plus S square $T_m T_n$ plus $K_0 K_1 T_n$ into S time is there now, plus $K_0 K_1$. So, the denominator if you see here, it is third order one.

Now, this $1 + T_n T_1 S$ and $1 +$ into $1 +$ this, this zero, this we can take care of like the current transfer one, transfer function by appropriately giving a delay to the reference input. So, the transfer function which has to be optimised for finally to be optimised for our closed loop control is this one. Because this zero, this we can eliminate the effect of this one by giving a filter, the input is filter using this time constant, using these two first order filters. So, we can take care of this one.

Now, let us take the denominator. Now, again let us go back to our optimisation. If you see here, the optimisation taken we know it, for the second order system, let us go back to the previous one; for a third order system, the $F j \omega$ if it is b_0 plus $j \omega b_1$ by a_0 plus $j \omega a_1$ plus $j \omega^2 a_2$ plus $j \omega^3 a_3$. So, one condition is at from the numerator. But here, we are not worried about in our system numerator because that we are taking care of by using a filter to the input, same like that our current controller. So, it is b_0 is equal to A_0 , b_1 is equal to A_1 .

Then finally, if you take the mod $F j \omega$ in our front end ac to dc controller, we got this equation; $a_0^2 \omega^2 A_1^2$ divided by $a_0^2 + \omega^2 a_1^2 - 2 a_0 a_2 \omega^4 + a_2^2 \omega^6 - 2 a_1 a_3 \omega^8 + a_3^2 \omega^{10}$. So, we will be using to make the modulus as close as possible to one. For wide frequency range, we can eliminate this one, we can eliminate this one also using the condition a_1^2 is equal to $2 a_0 a_2$, a_2^2 is equal to $2 a_1 a_3$. Now, let us use this one, for our compensation. So, let us write down this condition and our transfer function and then try to do the speed loop.

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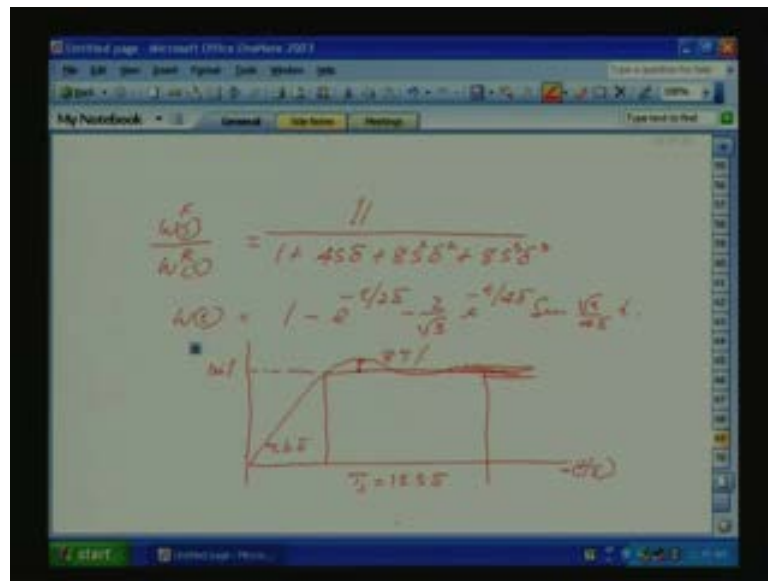
This will be equal to omega F (s) divide by omega R (s), omega R (s) we are passing through a filter. So, this will be 1 plus T_n S into 1 by 1 plus T₁ S. Then the final transfer function will be K_n R_A by K_m K₂. Let us call this one as a constant K₀ divided by S cube T_m T_n delta plus S square T_m T_n plus so this K₀, K₀ K₁ T_n plus K₀ K₁. Now, the condition for optimisation we got. What is the, what are the conditions? We will write it here.

That is one condition is a₁ square is equal to 2 a₀ a₂. Now, a₂ square is equal to a₂ a₁ a₃; so this shows, from this one, a₁ square which is a₁? This is, S time is missing here. There is an S time here **sorry** there is an S time here, there is an S time here that we forward right, S time is here. So, a₁ is the constant of the S time. That is K₀ K₁ T_n. So, if you see there, K₀ square is equal to 2 K₀ K₁ T_m T_n and a₂ square S square time that is T_m T_n that is T_m T_n square is equal to 2 K₀ K₁ T_n square T_m into delta.

So now, from this one, we can find out our T_n. That T_n we can substitute it here. Then from that K₀, we can find out K_n. So, if we do it simplifying, T_n will be we will get T_n is equal to 4 delta. Delta is equal to 4 into 2 sigma plus T₁ and we know sigma is equal to sigma is equal to convert or time cons the current filter. So, this is will be equal to 4 into 2 T₁ plus 2 T₂ plus T₁, this is our T_n. Then substituting this one, finally our K will be K_n that is our PI controller, K_n, P gain, K_n will be T_m K_m K₂ divide by 2 K₁ R_A into delta. This will be the K_n. So, we got K_n; optimisation, we can get T_n.

Now, let us substitute this one back to the equation and simplify it. The system will be the final, our omega, let us go to the next page.

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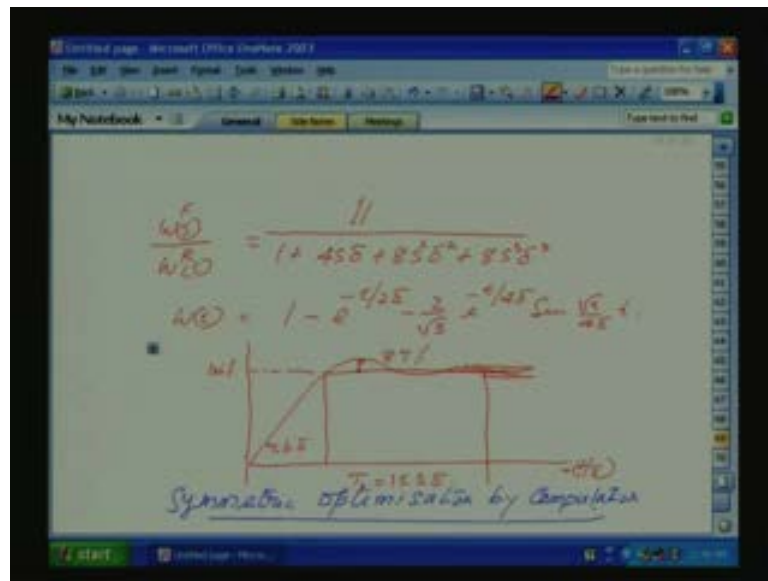


Final omega (s) feedback divide by omega (s)^R will be of the form 1 by that means this omega R is after filtering through 1 by our pervious numerator, numerator means 1 by T_n S into 1 plus, 2 filters of 1 by T_n S into 1 plus T₁ S or we can have another filter 1 by 1 plus T₁ plus T_n also is possible. Now, this will be finally of the form 1 by 1 plus 4 s delta plus 8 s square delta square plus 8 s cube delta cube. So, this one, what do the transient, this is the Laplace domain; what is the transient response of this one with a for a unit step?

We can find out the output for a unit step omega t will be of the form 1, we can find out e raised to minus t by 2 delta minus 2 by root 3 e raised to minus t by 4 delta into sine root 3 by 4 delta into t. So, if you plot the response, see let us take here t by delta as the x axis. So, this is our 100%. So, this will be approximately if this is the 100%, this will come within 2% of our final value, 2% of the final value that is the settling time, approximately 13.3 delta settling time and this will be approximately 7.6 delta and we get an overshoot of approximately 8.7%. So, this a good way of we have optimised the system. This is called the modulus hugging.

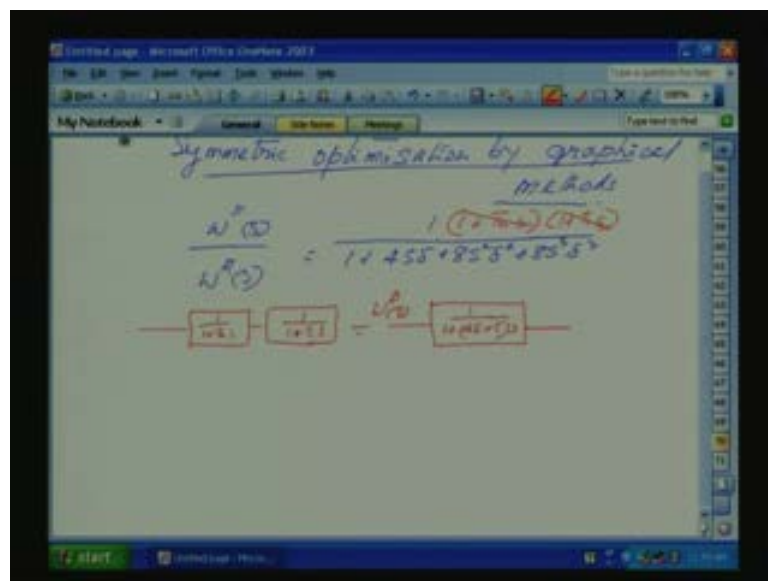
Now, there is another way, also it is called symmetric optimisation by computation. This is called this type of optimisation literature is called symmetric optimisation by computation.

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We can write it here, symmetric optimisation by computation. So, this can also be done also using graphical methods. That also we will study so that we can use these techniques appropriately whenever it is needed. Let us go to the next one.

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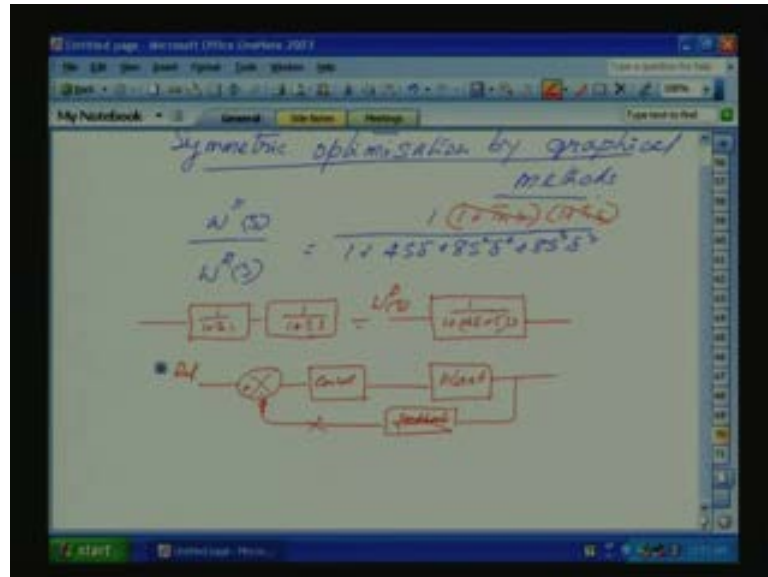


Symmetric optimisation by graphical methods: so the pervious transfer function that is omega feedback (s) divide by omega^R (s) we got it like this that is 1 plus 4 S delta T 1 plus 4 S delta T that is 8 S square delta square plus 8 S cube delta cube and previously if you know, we we had this denominator here. The denominator was what was the denominator? The denominator was we will write it differently, that was 1 plus T_n S into 1 plus T₁ S and I told effect of this one, we can remove by reference we are, the reference we are passing through this filter. That is if you have the reference is here, we will pass through two filters of the form; 1 by 1 plus T_n S, 1 by 1 plus T₁ S and T_n we got 4 delta.

So, this also can be approximated with one prime constant; summation we can take it, 1 by 1 plus T_n is equal to 4 delta, 4 delta plus T_1 into S so that a filter of the time constant of 4 delta plus 1 omega R if edit and give it here. Then effect of this one; this will be cancelled and final transfer function will be like this and we have how the response of for this one, found out using the computation method. Now, we will talk about symmetric optimisation by graphical method.

See, if you see the close loop controller, let us go back to the close loop control system.

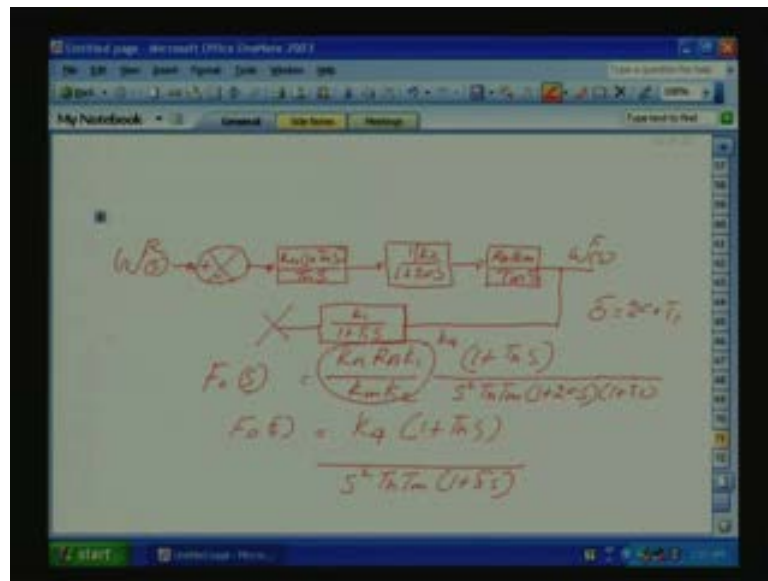
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See, you have the reference coming here, reference; there is our reference, then we have the controller, then the controller or plant transfer function, then our feedback is coming here. So, what we want? Let us find out the open loop gain. Open loop gain means we are removing this one. For ideal control, this feedback what is coming here, it should always track the reference. Also, since it is going to control and plant which has frequency depend elements and reference also varying with frequency; so feedback signal, you should not give any phase shift here.

Even if with, it is gain, same gain comes here and if it gives a phase shift of minus 180 degree, negative feedback become positive feedback. So, the loop gain once it comes for unity gain, this phase difference should be should not be close minus 180, it should be far away from 180 degree. So, if you can do that also that is also another optimisation. So, let us take open loop gain. So, let us take our transfer function. Go back to our transfer function, older transfer function and find out the open loop gain.

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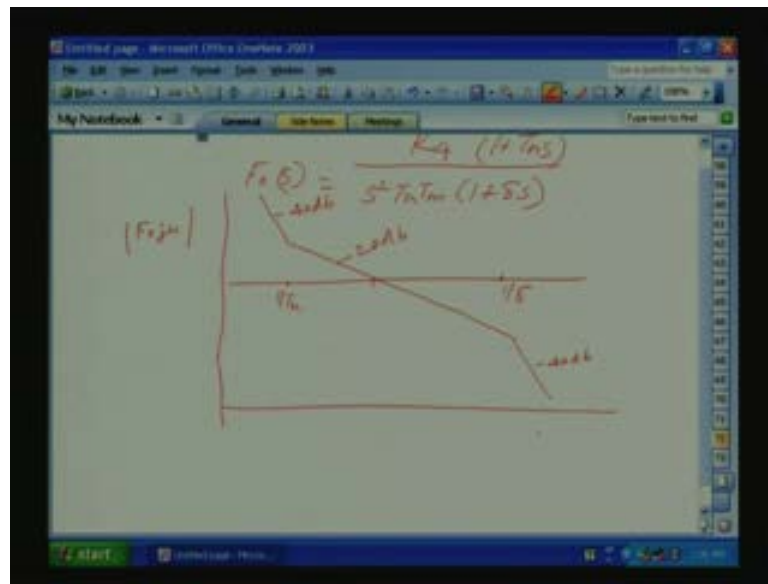
Yes, let us write down the block diagram once again that is K_n into $1 + T_n S$ divided by $T_n S$, then our current controller loop that is 1 by K_2 by $1 + 2\sigma S$. So here, 1 by 1 by K_2 current reference and finally R_A by K_m divided by $T_m S$, here is our omega S feedback, then our filter. Filter is equal to K_1 by $1 + T_1 S$.

Now, to find out the loop gain, we are breaking here. So, let us find out when the signal comes here, what is the gain and what is the phase shift. So, phase shift when it when it becomes phase shift for the unity gain, see when it comes here, let us find out the loop gain and the phase shift here. So, when you do that one, let us draw the Bode plot first. Let us find out the loop gain first; then from the loop gain, we will draw the block diagram that is open loop gain.

So, open loop gain will be that is $F_0 S$ is equal to $K_n R_A K_1$ by $K_m K_2$ into $1 + T_n S$ divide by S square $T_n T_m$ into $1 + 2\sigma S$ into $1 + T_1 S$. So here also, we can use the approximation; δ is equal to $2\sigma + T_1$ and this one $K_n R_A K_m$ by K_2 this one, let us call as K_4 . Then the final transfer function will be that is open loop gain $F_0 S$, S will be equal to K_4 into $1 + T_n S$ divide by S square $T_n T_m$ into $1 + \delta S$. This we are trying to find out another way of optimising the loop by choosing K_n and T_n using the symmetric optimisation using graphical method. So, this is the open loop gain.

Now, from the control system point of view, the system will be stable, this open loop gain. At the unity gain cross over frequency, it goes by minus 20 db per decade and at the gain cross over frequency if we can have a sufficient phase margin, the system is stable, we can stabilise the system. So, let us write down the, let us draw the Bode plot of this one.

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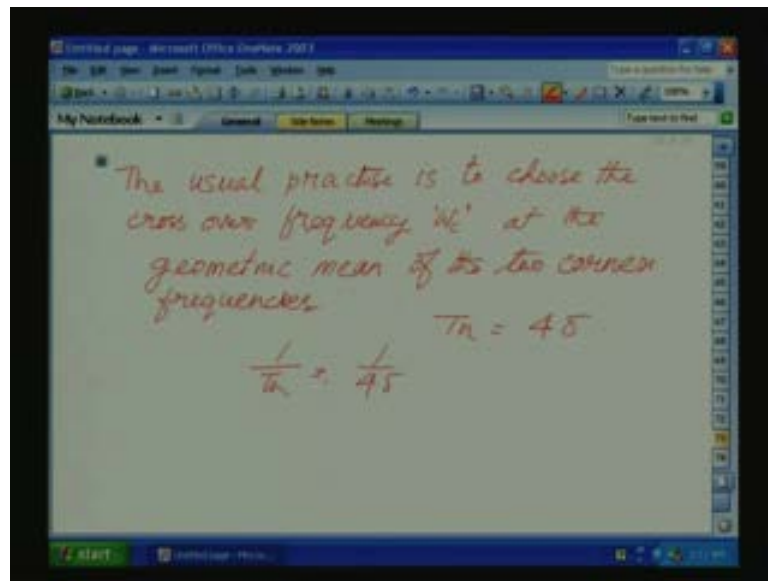
Again, the loop gain, this will be equal to K_4 divide by S square $T_n T_m$ into $1 + \delta S$ and numerator we have $1 + T_n S$. Now, if you see here; there is a 0 at T_n , there is a pole at δS . Now, we want 20 db per decade. So, if you $F_o j \omega$, if you place T_n here, 1 by T_n and 1 by δ , with respect to 1 by δ ; see upto here, it is minus 40 db per decade because of the S square, then the moment the zero comes, it will shift to till it reaches here 20 db per decade. Then again, go minus 40 db because of this effect.

So here, the effect of pole will come. So here, minus 40 db per decade, this is minus 20 db, this also minus 40 db per decade. So, we have to choose T_n such that now if you draw the phase margin, see minus 40 db per decade, so upto, upto this point, slowly it is minus π . Then because of the affect of this one, it will slowly go. Again, the effect of the zero comes and finally again it will go to minus db.

So, we choose the T_n such that at this point, we will have the maximum phase margin. This is called the phase margin. So, we should have the maximum phase margin, phase margin at this gain crossover. Now, we have to design our T_n such that we have, it should cross the gain crossover frequency, this axis at minus 20 db per decade and you should get sufficient margin. So, this is in log scale.

So, let us take, so usually we will design such that it will be the, usual practice is to choose the crossover frequency ω_c that is ω_c here at the geometric mean of the two carrier frequencies. That means let us write down that one. Let us go to the next page.

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The usual practice is to choose the crossover frequency ω_c , our ω_c at the geometric mean of two corner frequency. This is ω_c is we are playing to place at the geometric of the two corner frequencies, two corner frequencies. Now, from our pervious example, the T_n we can approximately choose as, that 1 by T_n , T_n is equal to 4 delta; we can use that one here itself also, T_n is equal to 4 delta. So, if you choose 1 by T_n is equal to 1 by 4 delta, so we already choosing this, approximately. Usually, this 1 by T_n should be away from the 4 del 5 to 6, that margin.

So, if you take this one, 1 by T_n is equal to approximately 1 by 4 delta, then let us find out using this one, let us substitute this one in the loop gain and let us find out what is the phase margin we are getting and based on this one at the gain cross over where the gain will be 1, let us find out what is K_n and with that K_n let us find out what is the phase margin we are getting.

So, if you are not getting the proper ah phase margin, again we will try to adjust this T_n with respect to delta, 1 by T_n with respect to 1 by delta. So then, we will see, so this is a trial and error. So since we have already got a T_n value from the previous thing, we will use that one and we will see whether we are getting the correct one. This, we will study in the next class.