

Industrial Drive Power Electronics

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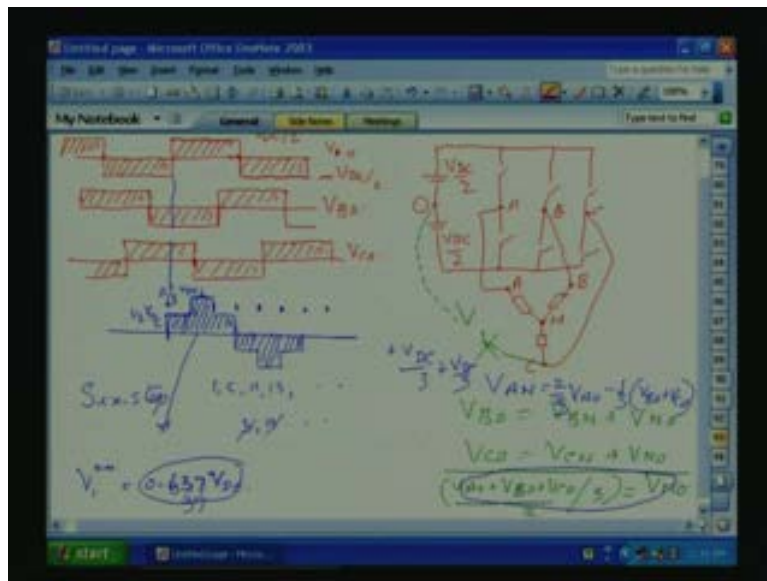
Indian Institute of Science, Bangalore

Lecture - 24

Inverter – Sine Triangle PWM

Last class we were talking about the three phase; how to generate three phase waveform based on individual pole voltages with 120, 120 degree phase shifted and for three phase, how to suppress the triple n order by removing the neutral. So, we will again start from there.

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So, let us draw the waveforms once again. This is the let us say, for A phase, the pole voltage waveform will be like this; this is the A phase waveform, square waveform of A phase waveform. Again we will draw it for continuity here; this is our V_{A0} pole voltage. Let us say, our ωt starts from here. This is V_{A0} , V_{DC} by 2 and minus V_{DC} by 2. And, V_{B0} pole voltage will be 120 degree phase shifted. So, V_{B0} will be 120 degree phase shifted, this will be, so with respect to 120 degree means it will happen here. So, this is our V_{B0} , 120 degree phase shifted, V_{B0} .

Now, V_{C0} pole voltage, again 120 degree phase shifted with respect to B phase. So, it will be from here. 120 degree phase shifted means let us draw the axis first, so 120 degree means it would be starting from here, this is our V_{C0} . Now, for the inverter we said, let us draw the inverter here, we assume bi directional switches A B C; A B C, so our load is like this. It comes here, it comes here. C means it comes here, this is our neutral and this is our fictitious

O, V_{DC} by 2 and V_{DC} by 2. See, this O, this is we are not using two DC link, V_{DC} by 2 and V_{DC} by 2. For analysis, we are trying to measure the various pole voltages with respect to the fictitious centre tap.

Now, we told before, this is derived from our basic configuration, our single phase half bridge converter. So, this connection, neutral connection we have removed and all the triple n will be removed. Now, we note that we have also derived our V_{AN} V_{BN} V_{CN} equations, V_{A0} sorry if V_{A0} is equal to V_{AN} plus V_{NO} and V_{B0} is equal to V_{BN} plus V_{NO} , NO is the voltage from here to here. So V_{A0} , if you measure this voltage is equal to V_{AN} plus V_{NO} ; similarly V_{C0} is equal to V_{CN} plus V_{NO} and by summing, we found that V_{A0} plus V_{B0} plus V_{C0} by 3 or by 3 is equal to V_{NO} . So, V_{A0} V_{B0} V_{C0} divide by 3 is equal to V_{NO} . Substituting this one, we know V_{AN} is equal to we have derived, V_{AN} is equal to I can write it here, V_{AN} is equal to we have derived; V_{AN} is equal to $2 \text{ by } 3 V_{A0} \text{ minus } 1 \text{ by } 3 V_{B0} \text{ plus } V_{C0}$.

So, if I remove this one, we can write it neatly once again, I can write here; V_{AN} is substituting this value, V_{NO} in our V_{A0} equation we found out, we can find out V_{AN} is equal to $2 \text{ by } 3 V_{A0} \text{ minus } 1 \text{ by } 3 V_{B0} \text{ plus } V_{C0}$. Now, using this equation, let us find out our V_{AN} here, we will try to find out the V_{AN} here. So, let us draw the line here for X axis. Our omega t starts from here that is what we said here, from here. So, if you see the 60 degree interval period that is this one, this one; there is a change in pole voltage, every 60 degree interval period. So, the V_{AN} , that shows V_{AN} would be stable for stable for 60 degree interval period and every 60 degree interval, it will be changing.

Let us say, first 60 degree interval; V_{AN} is equal to $2 \text{ by } 3$ that is this one, $2 \text{ by } 3 V_{A0} \text{ minus } 1 \text{ by } 3 V_{B0} \text{ plus } V_{C0}$. So, $2 \text{ by } 3 V_{A0}$; so here V_{AN} , see V_{B0} V_{C0} are opposite that means it will cancel. Now, this is V_{DC} by 2, this is V_{DC} by 2; so V_{DC} by 2, so V_{DC} by 2 into $2 \text{ by } 3$, $2 \text{ by } 3$ is equal V_{DC} by 3. So here, it will be V_{DC} by 3, let us draw the waveform here; this is V_{DC} by 3, let us see V_{DC} by 3. This amplitude, I would say, this is V_{DC} by 3 from here to here.

Now, next 60 degree interval; V_{A0} and V_{B0} are negative, so minus V_{B0} plus V_{C0} that negative that is equal to minus V_{DC} by 2, minus V_{DC} by 2 that is equal to V_{DC} by 2, V_{DC} sorry V_{DC} by, V_{DC} , minus V_{DC} and minus 1 by 3, it will come plus it will come to plus V_{DC} by 3 sorry plus V_{DC} by 3. This is the effect of these 2 portions - V_{B0} minus V_{DC} by 3. Then our $2 \text{ by } 3 V_{A0}$, $2 \text{ by } 3 V_{A0}$ is again V_{DC} by 3 like here. So, V_{DC} by 3 plus V_{DC} by 3, it will be $2 V_{DC}$ by 3. So, it will go 2 times from here.

Now you see, it repeats, it repeats like this; this is the positive half cycle, then this is the negative half cycle. If you do the Fourier series of this one, this will not contain triple n order. That means this will contain fundamental, fifth, eleventh, thirteenth pole harmonics only, third, ninth all these harmonics will be absent. All these triple n orders will be across this V_N V_{NO} because triple n order if it is there in the phase voltage; all the triple n order will be in same phase and there is no return path, so the current, so that current will not have a return path, so current is not there. So, in this pole in these voltages that triple n order would not be there, that triple n order will be dropped across this point neutral and O, this is the A phase waveform.

Now, B phase waveform will be 120 degree phase shifted. That we can do; once you get the thing, other thing we can easily get. This is one third, $1 \text{ by } 3 V_{DC}$; this will be $2 \text{ by } 3 V_{DC}$, this side. So, if you do the Fourier series of this one, the peak fundamental V_1 maximum will be

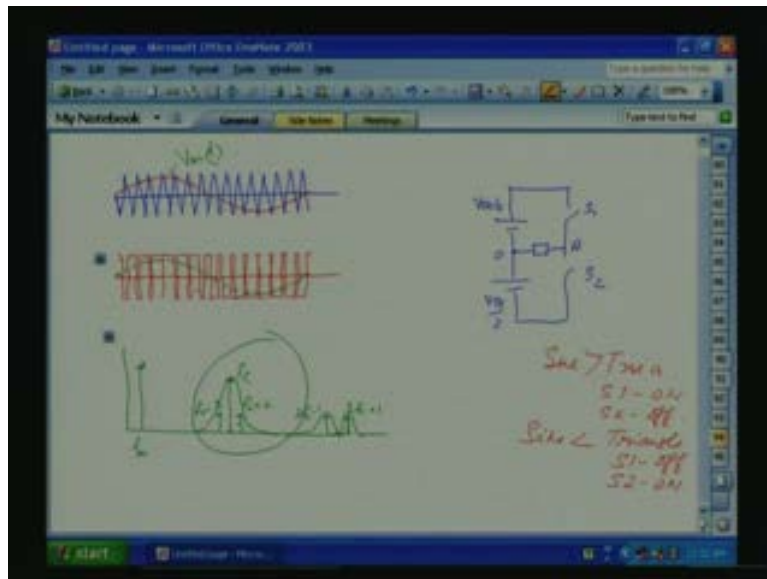
$0.637 V_{DC}$, this you can do the Fourier series and you can find out. So, for this waveform, the maximum, V_1 maximum will be $0.637 V_{DC}$.

So, the pole voltages are switched in a rectangular pulse with the 180 degree on and 180 degree off with the magnitude is equal to V_{DC} by 2 and minus V_{DC} by 2. If you could do pole voltages, the phase voltage will have a six step waveform like this; this is six step waveform. So that means one cycle, to make a one cycle, it goes to one step, second step, third, fourth fifth and sixth; so you can say six step waveform. So, this is also called a six step waveform. This is the phase voltage waveform V_{AN} , six step and it will have maximum fundamental of $0.637 V_{DC}$, this you can $0.637 V_{DC}$, this you can do the Fourier series and we can find out.

Now, with three phase also like our single phase; frequency we can control by controlling the one duration half duration of our pole voltages, only thing the ABC has to be 120 degree phase shifted with, B has to be 120 degree phase shifted with respect to A, C has to be 120 degree phase shifted with B. So, frequency can be controlled but here also the problem is the voltage cannot be controlled.

Now, for three phase, let us say, if it is a motor drive applications or UPS applications, three phase; this voltage control we can use our sine triangle PWM. So, we have studied about the sine triangle PWM, let us revise once again that one, again let draw our single phase half bridge basic converter.

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This is V_{DC} , S_1 , S_2 , this is our load A, this is zero, this is V_{DC} by 2, this is V_{DC} by 2. We are using sine triangle PWM, so we will have a triangle waveform of fixed amplitude. Let us draw it; this is our triangle waveform and when we use a triangle waveform, the frequency of triangle waveform is much higher than the triangle waveform. Also, we will use the triangle frequencies and odd multiple of sine frequency.

So, the PWM, suppose you will have odd symmetry, so we will come to that one now. Now, let us draw our triangle waveform, sine waveform that is called the modulating waveform. The triangle waveform is called the carrier waveform, so let our modulating waveform will

be like this. Now, as I told before, the switch sequence, sine greater than the triangle; top switch is on, top switch is S_1 on and that time when S_1 is on, S_2 is off.

Next, when sine less than triangle that is our carrier wave S_1 off and S_2 on is the switching sequence. So, when S_1 is on we know it is pole voltage is V_{DC} by 2, this is minus V_{DC} by 2. So if you see, you will have, here if you see, sine is greater than the triangle; so, it is this point, so V_{DC} by 2. Then during this portion, here, sine is less than the triangle; so, you will get minus V_{DC} , this. So, if you see here, our pattern will be like this and the midpoint of pattern in a triangle period varies proportional to the amplitude of the sine wave.

That means the area of the portion increases sinusoidally. This we have also analysed in the context of front end ac to dc converter that the output fundamental will be proportional to the sine wave and using the odd symmetry here and with the large carrier frequency; this also, not only proportional in amplitude, the phase also in all respect, it will be exactly proportional to the sine wave fundamental.

So now, this is the negative portion, negative portion, negative side; that is the portion the pulse width waveform pattern after 180 degree exactly opposite to that of the pattern before 180 degree. So the fundamental, if you see here, fundamental we can use a different colour over clarity; so that fundamental will be like this. So, fundamental amplitude will be proportional or the fundamental will be proportional to our $V_m(t)$ waveform that is $V_m(t)$ modulating waveform.

So, by varying the amplitude, these fundamental can be controlled and also here, for three phase operation; what we do? We will add three more legs, same triangle we can use but the sine will be, we want the fundamental, so sine will be 120 120 degree phase shifted and exactly for three phase waveform. So, before coming to three phase waveform, let us make this harmonics of these waveforms, harmonics we have studied.

We also talked about the harmonics symbol in the context of content of the ac to dc converter; harmonics will have, they we have the fundamental that is fundamental frequency. This is F that is at f_m , then you have at f_c , then side bands f_c plus 2 and f_c minus 2, then you have at $2 f_c$ side bands that is $2 f_c$ plus 1 and $2 f_c$ minus 1. This way harmonic goes; again it will have the same repeated $3 f_c$ and $4 f_c$ you will have side bars, it will go. So, the harmonic amplitudes if you see here, it will decrease like this, this way it goes; the harmonics.

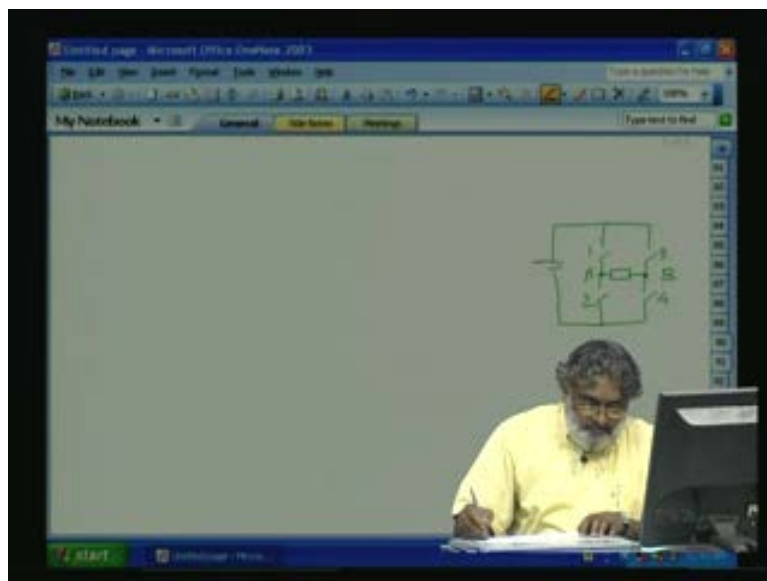
So, the purpose of doing the PWM is to control the output in proportion to our control voltage, control voltage, the $V_m(t)$ and also we want to shift the high order high amplitude harmonics to the high frequency side. So here, if the f_c is, take it 11 times, so the high amplitude and f_m is 50 hertz; the f_c will come or f_c will be at 550 hertz, 11 and 50, far away. So, if you see here, all the third, fifth, seventh, all the harmonics will be suppressed. So, what is the advantage?

In the load voltage, even though it is present in the load voltage, here, across here zero; the current drawn by this harmonics amplitude is much less and current will be nearly sinusoidal with a small ripple. Now, suppose if I am using for UPS application this one; so UPS you want a pure sine wave, so the purpose is to suppress the harmonic. Now, for UPS application, suppose if you want for some UPS application, you want the pure sine wave output and in some cases, we do not want to have high switching frequencies; so harmonic has to be suppressed.

So, sometimes we need to have some filter. So, filter size if the high amplitude harmonic switches shifted to the high frequency side as much as possible, the filter side especially the inductance part, that will be a LC part, inductance size and the size can be inductance value can be reduce and the copper size can be reduced, will be a saving. So, how to suppress this harmonic without resorting the high frequency switching?

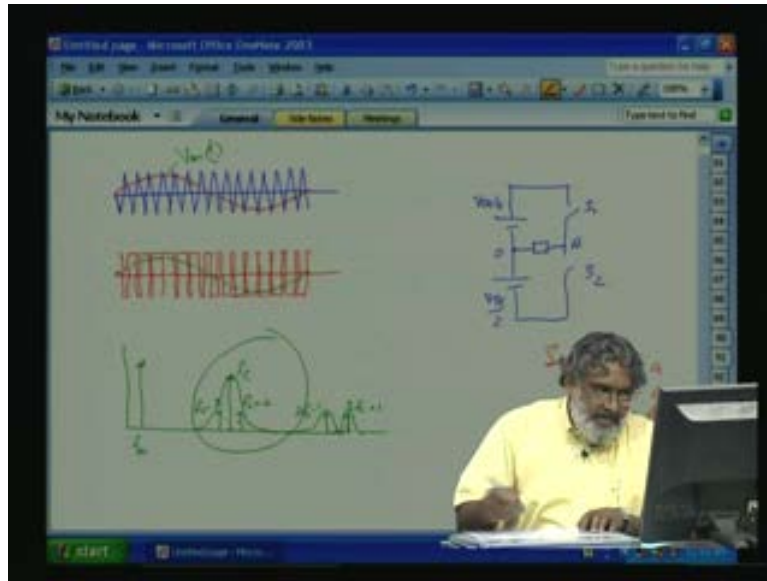
So, like our front end ac to dc converter; to remove this one, so with our sine triangle comparison with this one, we have got the fundamental amplitude variation controlled. Now, to limit this one, we require one more degree of freedom. So, let us see. In this case if you see, the output is going plus 1 and minus 1. Now, let us go to the next page, our converter is like this.

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We have, for UPS, let us say, this is the battery; so this is A, this is B, our load is connected here. Now, we know that this is 1 2 3 4, these two legs can be independently switched and we want independently switched so that the fundamental V_{A0} , zero means the fictitious centre tap here; V_{A0} and V_{B0} fundamental should at and the harmonics at the, harmonics at this should cancel.

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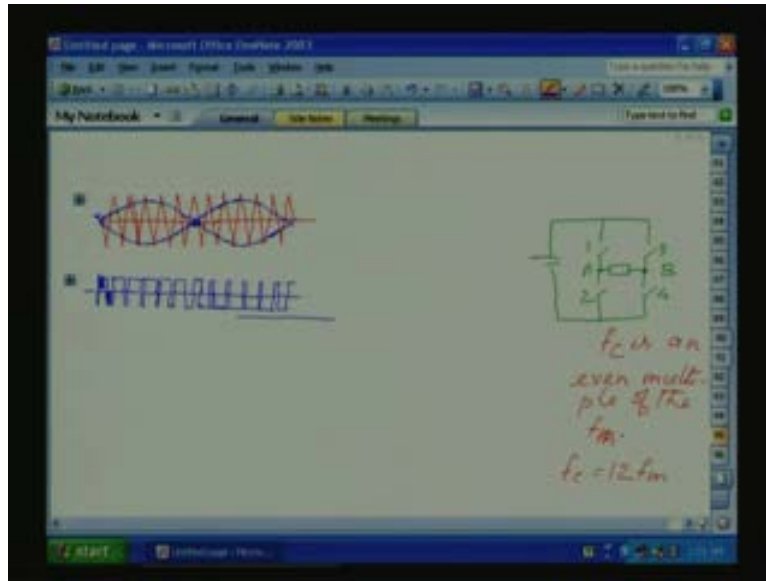


That means the fundamental should add and the harmonics, these harmonics should get suppressed and we are using the same PWM techniques, sin triangle PWM such that we have to use it such that the V_{AB} is equal to V_{A0} minus V_{C0} , V_{A0} is equal to V_{AB} is equal to V_{A0} minus V_{B0} if you have one more limb here. Now, the load is here, this is our B; this is not the way to connect when you have two limbs here.

Now, if you see here, the fundamental of V_{A0} and fundamental of V_0 should be, it should be in same phase **sorry** it should be in opposite phase because we are using V_{A0} minus V_{B0} and this we want to cancel, f_c , f_c to cancel. All these harmonics due to V_{A0} and V_{B0} should be in the same phase. So, when you subtract, same phase it will get cancelled because f_c waveform same phase means it is exactly they will cancel each other.

So, how to do it? So, let us go to our figure here, again let us draw our sine triangle. We will go to a different colour; this is our x axis, again this is our triangle waveform.

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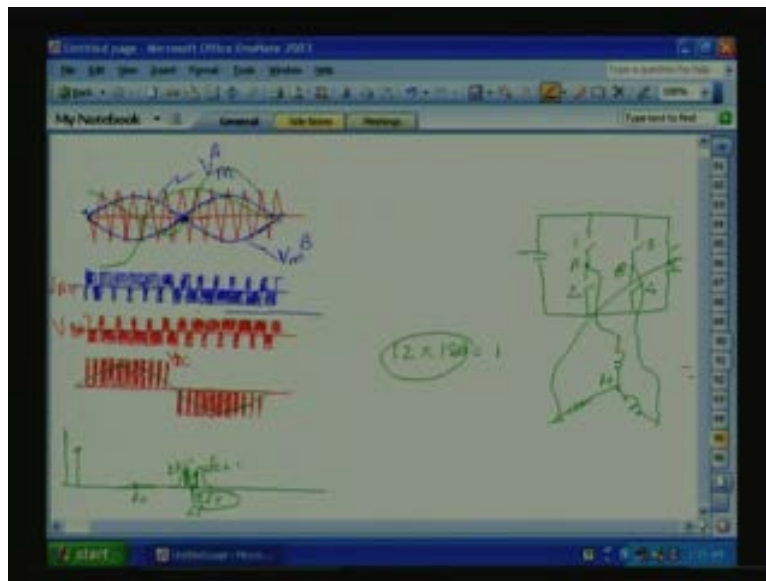
So, here also we will use a triangle waveform which is; the frequency of this carrier waveform is much higher than the sine wave. Now, here we will do one thing; the triangle frequency, the frequency of the triangle waveform that is f_c is an even multiple of our f_m , f_c is equal to f_c is an even multiple of the f_m that is sine wave. Previously, we were using 11 times, here we will use 12 times. Let us take that is equal to even multiple of not f_c , f_m that is that is equal to that is we can write here, that we can write, f_c is equal to let us say $12 f_m$, this is an even multiple.

Now, let us draw our, also, for the A phase and B phase, the fundamental should be in the opposite phase. So, we will use 180 degree opposite sine waves or modulating for the two legs; so, that we can draw it like this. Let us take a different colour. So, we will use the sine wave like this; this is our modulating wave, let us this is for the modulating wave for pole A and pole B, we will exactly 180 degree phase shifted, we will use it; only thing, this is odd multiple. So, that zero crossing we know that it is happening somewhere here, somewhere here it is happening. So, this is the zero crossing happening here.

Now, if you draw the PWM that means f_c is $12 f_m$; so for the V_{A0} , the pole voltage waveform will be will be like this, we can draw it this way. This will be sine is greater than the triangle, top switch is on; so, it will go like this. So, the area or the width of the pulse increase in a sinusoidal wave; here it will revise. So, whenever sine is greater than the triangle, we are turning on the top switch; when the sine is less than the triangle, we are training on the bottom switch that is for the leg A.

So, the PWM pattern if you see here, these are the portions during which top switch is on, bottom switch is off. So, here the top switch is on, so you can see, the width slowly increase in a sinusoidal wave, here it is like this.

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So, if you see here, it slowly comes here, equal durations, at the cross over, equal durations, positive negative; then here, the negative side, it is slowly increasing. So, if you say, the fundamental of this one will be proportional to our V_M that is V_M for that is this is the one; V_M for A phase and this one is V_N for B phase. So, this is the way it changes. So, V_{A0} will be like this. Same way we can write the V_{B0} also. V_{B0} also, this is V_{DC} by 2, this is minus V_{DC} by 2. Now, for the other phase, we will do it like this, may be you can use a different colour.

So now, with respect to this sine wave; sine is greater than the triangle, top switch is on. So here, see this triangle is this way, so sine A, so sine amplitude is greater than, so if you see here, during this portion, sine is greater than the triangle; so the switch will be open and we will be turning on the top switch. So, the pattern will be like this. Here also, the area during a triangle period, it various proportional to our sine wave.

Now, the sine is, so this is the wave. Now, the area started increasing at the positive direction; so if you see these two patterns, the patterns are exactly the same with the 180 degree phase shift. So, this pattern and this pattern are exactly same with the 180 degree phase shift. So, this 180 degree phase shift happened because of the fundamental. So, fundamental is 180 degree phase shifted. So, the harmonic switch is 12 times that is even multiple; what will happen to that one? 180 into 12, it will be 360 degree that will be in the same phase. So, all the harmonics at f_c and it side bands will be in phase but the fundamental is opposite phase. So, if you subtract, if you see the subtraction, the final will be pulse amplitude of V_{DC} , positive direction. So, you will get something like this and subtracting V_{A0} ; this is V_{A0} , this is V_{B0} .

Now, let us draw the pattern V_{A0} minus V_{B0} , this height will be V_{DC} . So, let us draw the X axis. So, we are subtracting the instantaneous V_{A0} and V_{B0} waveforms. See here, during the positive cycle if you observe, the waveform is going from 0 to V_{DC} and the negative cycle, it would be going like this. So, this variation is also in a sinusoidal way and this fundamental of this one will be 2 times V_A , V 2 times the modulating for A because fundamental of V_{A0} V_{B0} are 180 degree phase shifted; so when you subtract, both will get added, so we will get an

increase fundamental here. At the same time, see we have used 12 times the modulating frequency for the carrier wave, so here the fundamental will be like this.

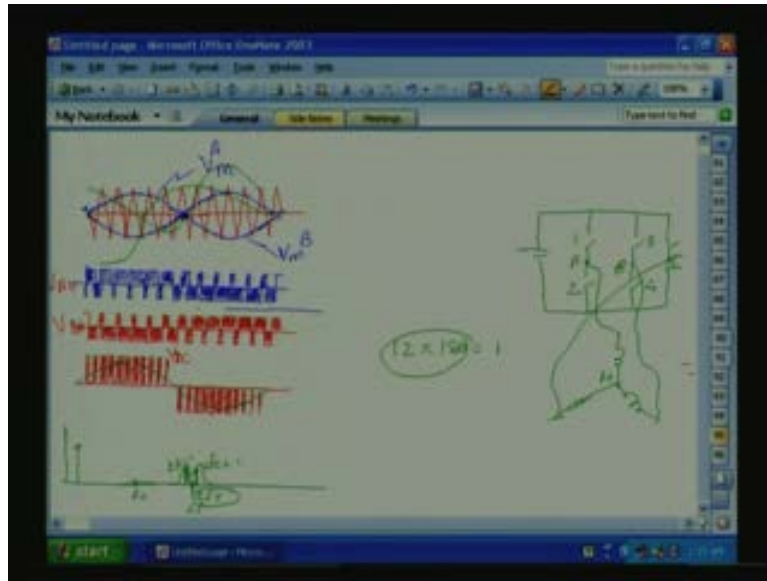
Now, so fundamental get added but carrier frequency at f_c , now the f_c if you see here f_c is equal to 12 times f_m , 12 times f_m and if you see the pattern here, these patterns V_{A0} V_{B0} are exactly same with the 180 degree phase shift. This 180 degree phase shift is with respect to the fundamental. So, fundamental is 180 degree phase shifted, so 12 times fundamental. How much it will be? 12 into 180 will be, this will be in same phase that is this is a multiple of 360; so this will be some multiple of 360. So, that means even harmonics, all even harmonics that is at f_c and its side bands will be in same phase and when you subtract, it will get cancel.

So, if you see the FFT spectrum for this one, we can draw the FFT spectrum for this one; you have the fundamental, when all at f_c will not be there, it will be highly suppressed, f_c will not be there and you have the next harmonic happens at only side bands of $2 f_c$, here. This is f_c plus $2 f_c$ plus 1 and $2 f_c$ minus 1. So, it is shifted. So, the higher amplitude harmonics here, it is shifted to the $2 f_c$ side, here. So, these types of waveforms are unique, these are called unipolar waveforms. Unipolar, that means one side it is going to positive, other it is negative. So, the shape is because compared to this one, the f_c is cancelled. So, what do that one; we can use this one for UPS applications, what is advantage?

Then the filter required is cut off frequency; you have to design with $2 f_c$, so filter size requirement can be reduced and so this way appropriately phase shifting sine triangle carriers, sine triangle waveforms and appropriately choosing the triangle carrier frequency; many ways we can control the harmonics. Another way of controlling the harmonics we have studying in the context ac to dc converter. So, the whole purpose here was to control the fundamental and suppress the low frequency harmonics.

Then, is there any other way we can do it? Can we do it? And so far, PWM technique we have talked about single phase case. For three phase case, same triangle but the modulating wave should be 120, 120 degree phase shifted, then we can appropriately switch the pole voltage because for a modulating wave, what we are or see, for a leg, when the modulating wave or the sine wave is greater than the triangle wave, we are switching on the top one; when the sine is less than the triangle wave, we are switching on the bottom one. So, the sine wave if it is 120, 120 degree phase shifted, the same technique can be used for our three phase case. That means for the three phase case, we will be adding one more limb here.

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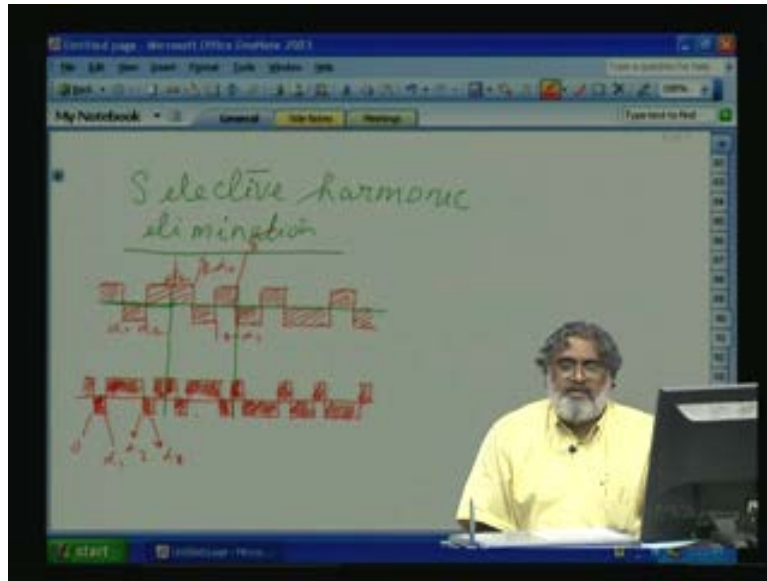
Now, our load connection is not like this. So here, this is our B, three phase this is our C; our load connection is like this. Schematic representation for three phase it is like this; B will be like this and C will be like this, this is our N will now be connecting here. So, the pole voltage ABC, so all the switching depends or we are concentrating at the pole voltages, harmonics are suppressed, triple n harmonics are suppressed because the neutral is not there. So, pole voltages will be switched with respect to sine triangle PWM.

When the sine is greater than the triangle top, for the A phase, sine is less than the triangle, bottom switch. For B phase, same triangle waveform like this but the sine will be 120 degree phase shifted somewhere here; that we will be using for B. C phase again, it will be another 120 degree like this, something here; so, that we will be using for the C phase. So, my waves, sine waves are not correct but you can draw it. So, this way, we will be controlling the ABC sequence.

So, as I told, the whole purpose was to control the harmonics and also to control the control the low order harmonic and also to control the fundamental. Then what we have achieved? We have introduced the notches instead of the rectangular pulse; to equal all rectangular pulse, we introduced notches, in between notches. That means positive side, part of the time it will give negative and the negative side, it is part of ring goes positive. So, notches are introduced so that the harmonics are suppressed. So, can we introduced notches in such a way, some of the lower harmonic can be suppressed and the fundamental can be controlled? This is called selective harmonic elimination.

See, lot of work, literature, in literature you can see lot of, earlier, lot of work on selective harmonic elimination, mainly from academic research institutions. But implementation of this work, especially for very base speed applications, more for us, specific applications with fixed fundamental frequencies operation, it may be possible. But for variable frequencies of application, the selective harmonic elimination that means deliberately introducing notches requires lots of offline computation and so it is seldom used in dry environment. But still it is there in literature; we will study about that one.

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Selective harmonic elimination, harmonic elimination: here, we are deliberately introducing the notches in such a way that the waveform, the positive; we are trying to get us fundamental sinusoidal variations, so the notches are introduced such a way, so the waveform positive half cycle that is first 180 degree period and the second 180 degree period have correct symmetry of a sine wave. So, how it is introduced? Let us see some typical pattern for a selective harmonic elimination.

See, we can now, let us take care of X axis like this; see we introduced notches, so this is our π by 2 period, this is our π . So, let us draw our waveform. Now, here notches we will introduce like this, exactly π by 2, that symmetry. If you fold it, this path exactly come at that is π by 2 side, it will go there, this is π . So, we introduced notches like this. So, this is the positive half period, top switch is on, this is bottom switch, this is bottom switch is on, here this is the positive half cycle. This exactly will be exactly represented the second half cycle in the opposite way. So, in the opposite way, it will be like this.

So, we have introduced at the π by 2 sides, notches at α_2 α_1 and here it will be π minus α_2 and here it will be that is this point will be π minus α_1 ; this is one. We have introduced in a half period that is in a half period, two notches; one notch here, negative and one notch here and the negative side also, one notch here, one notch here, two notches with that points at α_1 and α_2 .

See, we can introduce more number of notches. So, introducing more number of notches, we can control more number of harmonics. So, let us introduce some more notches and see how the typical pattern looks like. So, we will go here, this is our axis. So, we are talking about with a single pole based on our basic configuration, single phase half bridge converter, this can be extended to the other legs also. So, the pole voltage A; positive, negative, this way, like this it goes and it comes here. So, this is plus V_{DC} by 2; during this period, minus V_{DC} by 2 and this is the quarter wave.

So symmetry, quarter wave symmetry; see, exactly fold like this, quarter waves will merge, same like our sine wave and the half way symmetry existing for the sine wave should be

there. That means this positive path should exactly go to negative side. So, we have introduced how many notches in a quarter periods? See previously, in a quarter period, one notch. Here, in a quarter periods, we have introduced two notches, two notches; this is zero, this is α_1 , this is α_2 , this is α_3 and rest is symmetry from this axis.

So, introducing more notches, you can control more harmonics and at the same, we have to control the fundamental, fundamental as required by the load. So, we will find out, from this one, we will write down the harmonics, Fourier series of this one, we will try to find out what is the fundamental and what are the α_1 α_2 α_3 required for a particular waveform to suppress some of the low order harmonic. So, that means if you have to control a fundamental, you require one equation; if you want to control fifth and seventh, we require one or two more equation. So, we require three conditions are required to suppress fundamental and fifth and sixth, seventh.

So, if more harmonics to be control, you require more notches, more equations is there. So, this way, for more harmonics, more equations need to be sold and this is only for with respect to one fundamental. So, to variable speed operation with different fundamentals, we require different conditions for the notches. So, it requires lot of offline computations, so it is seldom preferred in drives and applications. But, how to find out the suppressed harmonics and how to control the fundamental for this empty harmonics elimination with the general equations; we will study in the next class.

Thank you.