Power Electronics

Prof. K. Gopakumar

Centre for Electronics Design and Technology

Indian Institute of Science, Bangalore

Lecture - 25

Inverter – Current Hysteresis Controlled PWM

Last class we started with selective harmonic elimination; so this is another form of pulse width modulation wherein we are introducing the notches to the pole voltages such that it has the half wave symmetry as well as the quarter wave symmetry same like in the case of a sine triangle PWM. And, the notches are introduced with the purpose so that low frequency harmonics; let us take fifth, seventh may be eleventh, thirteenth harmonics can be eliminated and at the same time, the fundamental amplitude can be controlled depending on the frequency.

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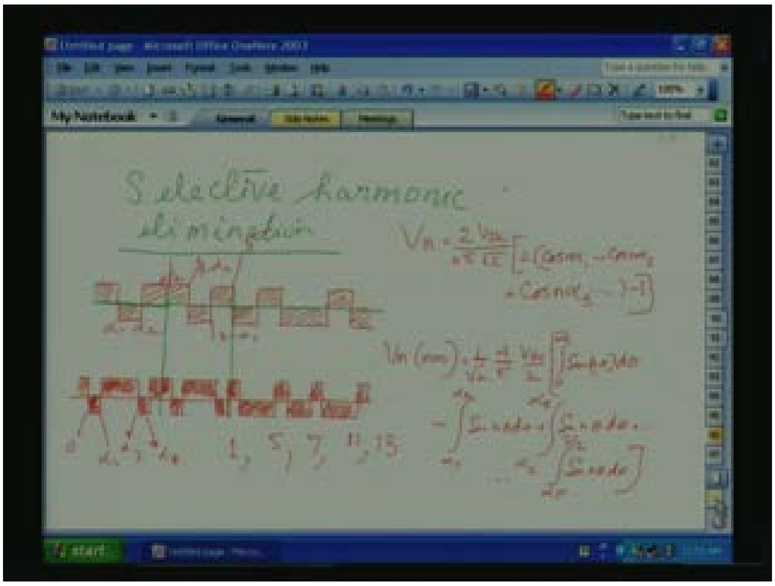

So, if you see the figure here, two wave forms are given; the first wave form here, we are introducing the notches at alpha $_1$ and alpha $_2$. Now, the second wave form we are introducing more notches. So, if you see, from pi by 2 period, we have the quarter wave symmetry; at the same time, the half wave symmetry we have, we can see here, this negative half wave is equal to the positive half. Now, we have to decide what should be the width of alpha $_1$ alpha $_2$ or angle at which alpha $_1$ alpha $_2$ should be introduced such that the pole voltage switches between the positive plus V_{DC} by 2 and the minus V_{DC} by 2.

So, let us do the Fourier series for the general, for a general PWM with general notch. So, the harmonic analysis if you use Fourier series, it will be that is V_n, V_n is equal to it can be written; 2 into V_{DC} our dc link voltage divided by n pi. Want to see the RMS value, so you have to divide by root 2 into 2 cos n alpha $_{1}$ minus cos n alpha $_{2}$ plus cos n alpha $_{3}$, this way it goes, finally it will be minus 1. This, how do you get it? This we are getting from the Fourier series expression.

So, Fourier series expression, we can write it like this; V_n, I am just repeating, the RMS value is equal to 1 by root 2 4 by pi 4 by pi V_{DC} by 2. So, this is the integration, 0 to alpha $_{1}$ sine m theta d theta, then minus alpha $_{1}$ to alpha $_{2}$ sine m theta d theta plus alpha $_{2}$ to alpha $_{3}$. See, here, it is a general thing; so depending on the notch, here in that case, 0 to alpha $_{1}$, here you have the alpha $_{1}$ is this is in the negative. So, you have to, negative you have to introduce here but here, alpha $_{2}$alpha $_{1}$ to alpha $_{2}$ is positive.

So, depending on the notch, you have to introduce the positive negative sign but this is the general expression; sine m theta d theta. So, this way it goes, finally you will get alpha p to pi by 2 that is why this 1 comes here, this 1; alpha by 2 sine m theta d theta. So, this is the general expression. So, for your notch width, accordingly you have to write the thing. 0 to alpha $_{1}$ is positive, alpha $_{1}$ to alpha $_{2}$ is negative, then alpha $_{2}$ to alpha $_{3}$, here it is alpha $_{2}$ to alpha $_{3}$, so this way it goes.

So, when you expand everything, you will get, two times you will get it because alpha $_{1}$ is coming here, alpha $_{2}$ is coming here, alpha $_{3}$ is coming here and next side also it will come here, so two times that will come. From this one, let us write down the equation for harmonics one, harmonics fifth, harmonic seventh that means the harmonics order of fundamental, fifth, seventh, eleventh and thirteenth; how the equation will be? So, let us go to the next page. Now, see let us start with the fundamental.

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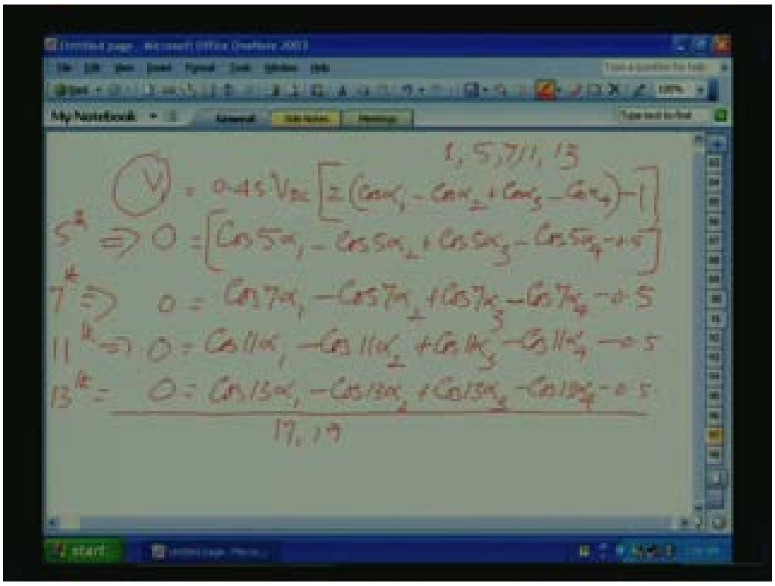

So, fundamental will be V_1 is equal to 0.45, we can expand the previous waveform into 2 cos alpha $_{1}$ minus alpha $_{2}$ plus cos alpha $_{3}$ minus cos alpha $_{4}$ minus 1; this is the fundamental equation. So, if you see, we want to control eleventh, we want to control if you see here, fundamental, fifth, eleventh, fifth, seventh, eleventh and thirteenth. So, if you see here, the angles required; we require how many equations? Five equations are required to control. So, correspondingly we require starting from 0, then alpha $_1$, alpha $_2$, alpha $_3$, alpha $_4$, 5 angles starting from 0 is required.

Now, let us take the fifth harmonic. Fifth harmonics will be, we want the fifth harmonics to be 0, so I will put it 0. So, the equation will be cos 5 alpha $_1$ minus cos 5 alpha $_2$ plus cos 5 alpha $_3$ minus cos 5 alpha $_4$ minus 0.5; this is the equation, this for the fifth harmonic. Now, let us take for the seventh harmonics, for n we have to replace instead of 2 we have to replace. So, this will be in the original previous equation if you do it, this will be cos 7 alpha $_1$ minus cos 7 alpha $_2$ plus cos 7 alpha $_3$ minus cos 7 alpha $_4$ minus 0.5; this is eleventh.

Let us take the thirteenth harmonic sorry eleventh harmonic. That also we want 0; so we want all the low order harmonics from 5 to 13 to be 0. Then this will be cos 11 alpha $_1$ minus cos 11 alpha $_2$ plus cos 11 alpha $_3$ minus cos 11 alpha $_4$ minus 0.5. Same way, we can also write the thirteenth; that also we want to be 0. So, it will be cos 13 alpha $_1$ minus cos 13 alpha $_2$ plus cos 13 alpha $_3$ minus cos 13 alpha $_4$ minus 0.5.

So, if you see here, we have five equations and we have 1 2 3 4 unknowns to be find out. From this five equations, you have to find out what is alpha $_1$ alpha $_2$ alpha $_3$ alpha $_4$ such that given is our required value and this harmonic are 0. So, if the V_1 is fixed, let us take for an output frequency is fixed, then V_1, the required RMS we can it and we can solve this one.

Now, take a case where a general variable speed operation, let us take we are using the harmonic elimination for a three phase motor control application where the frequency varies from 0 to 50 hertz. Let us take a frequency resolution; we are in a step of increasing the speed, 1 hertz. So, 50 times we have to calculate the alpha $_1$ alpha $_2$ alpha $_3$ alpha $_4$ and V_1 because for difference speed of operation as the motor increases, the fundamental amplitude varies in proportion to the speed. So, 0 to 50 if you are increasing the speed in one step in one hertz that means 50 steps required, 50 times we have to solve this one because the V_1 is varying and we have to compute alpha $_1$ alpha $_2$ alpha $_3$, then we have to store this part in a memory and output it.

So, in some cases, suppose we have to eliminate the next harmonic source that is the seventeenth and nineteenth, then number of equations will be more. So, this will increase the computation, it requires lot of offline computations storing but what we want in a control? Quickly online correction that is the best way; so, that is not possible here. So, even though selective harmonic elimination is available in literature, it is seldom used in motor drives application just because of extensive offline computation. But in multilevel, in the context of multilevel inverters, still this harmonic eliminations are coming with PWM technique but we will not coming to that one now.

So, only what we want to remember here is to suppress the low order harmonics, we have to introduce the notches with the proper symmetry, then write down the Fourier series and what are the harmonics to be eliminated along with the required fundamental, we have to write this equation and solve it. Here, as I told before, for three phase application, the sine triangle PWM is the best, the best approach wherein we have the fixed triangle frequency and whatever the amplitude fundamental that is the V_1 what we required and with that frequency, we generate the corresponding sinusoidal waveform with 120, 120 degree phase shift and do the appropriate switching.

So, whenever we want to vary V_1, just vary the modulating wave, corresponding modulating wave amplitude. So, automatically the fundamental is generated and at the same time, harmonics are shifted to the high frequency side which is proportional to our f_c. So again, for three phase case; let us go to the next page, how the harmonic elimination for the three phase case.

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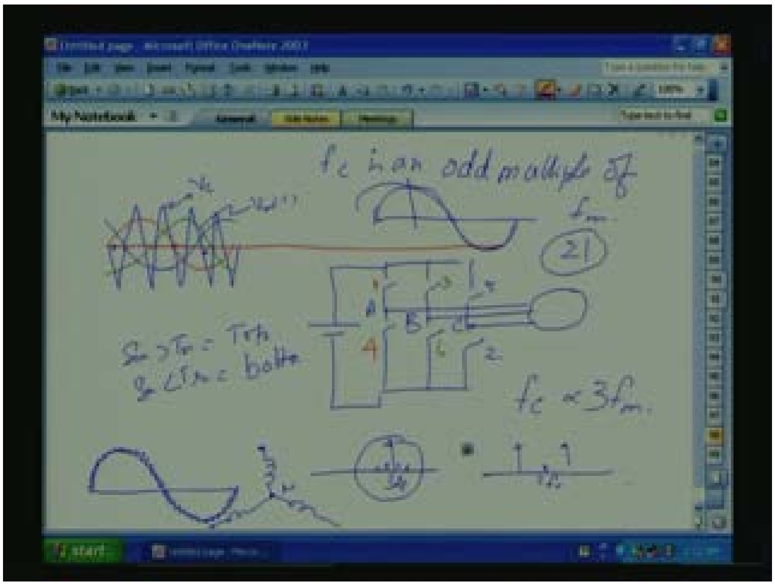

Let us draw the triangle wave. So, let us take the corresponding modulating wave; this is the modulating wave for A phase, the B phase will be 120 degree phase shifted. So, let us go with the different colour, this is the B phase; then C phase will be, this amplitude of A B C phase, even though it is 120 degree, amplitude are the same. That depends on the variable the speed operation or what is fundamental we require. Now, this is the C phase.

Now, this we would be comparing with the fixed triangle frequency. So, triangle frequency will be high frequency triangle waveform. See, here for motor drive applications, this is called the V_c, carrier this is called the carrier wave and these are the V_m (t). So, three phases; so for the inverter, we have the 3 legs like this and V_{DC} and 3 loads like this. Let us say, if it is for the motor, it is more. So, for the A B C phases, depending on when the triangle is triangle is greater than the sine wave, the top switch is on; when the triangle is less than the triangle, we will be using this one. So, the same sequence if you say, it will go, 1 3 5.

Now here, this is the bottom one; so if you see our previous sequencing, it is numbering is like this. So, when the corresponding when the A phase sine is greater than the triangle, sine is greater than the triangle, top is on; sine less than the triangle, the bottom switch is on. So this, independently we carry for the three phases A B C.

Now, how to choose the triangle waveform? For motor drive applications, we want the PWM waveform have the same symmetry like the sine wave, we should have the quarter wave symmetry and when you fold it, would exactly match and the negative half should be equal to the positive. So, we will always use fc is at odd multiple of f_c is an odd multiple of odd multiple of f_m or modulating waveform. So, that will and let us take 21 times that is the high frequency you can say, 21 times and also will use it is 3 times, f_c is proportional to 3 times f_m 3 times. What is the advantage?

For motor drive applications if you see here, as I told in the previous class, we are connecting like this; the neutral is not connected anywhere. So, in the PWM waveform, V_{AN} V_{BN} if the triple n voltage is there, triple n current will not flow because the neutral is removed. So, if you use 3 times the fundamental at odd multiple and 3 times the fundamental, this carrier frequency at f_c, f_c is 3 fc and its side bands will be completely, the currents in the phase will be completely suppressed. This is one way of suppressing the effect of low order harmonics. Because the neutral is removed; the harmonics, the currents, the current harmonics at 3 times f_c that is the third harmonic, f_c and its side bands will not be there. So, the next higher harmonics will be at the side bands of 2 f_c and its side bands. Now this way, we can use it, sine triangle PWM.

Now, the purpose of the PWM is to have a nearly, so even though we are controlling the voltage, what we require is nearly sinusoidal current, we require nearly sinusoidal current. So, with the switching, the current will be the ripple current will be it is following like this. So, the current is nearly sinusoidal and the harmonics are shifted to high frequencies. For sine triangle PWM, we require a triangle waveform and a sine wave; we have to generate, it can be generate and for very high frequencies of operation, f_c is very high.

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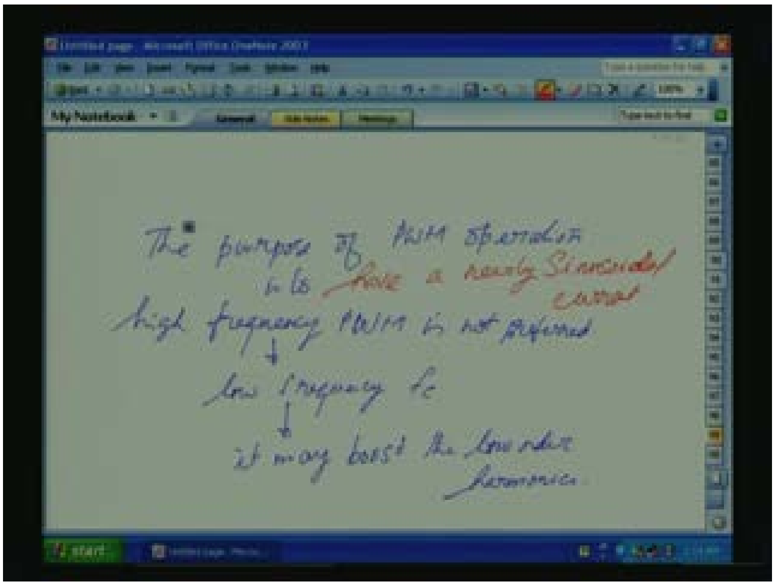

The synchronisation that is the zero crossing of the triangle and sine need not be, do not have to worry but for motor drive applications; let us see for motor drive applications, high frequency PWM self …((20:51)) preferred. High frequency PWM is not preferred. So, this will lead to low frequency f_c carrier f_c. But low frequency fc means the side bands, it may boost the low order harmonics. So, these are the problems. Also, the sine triangle PWM, synchronization is required that is exact zero crossing should be matching, that we will not be talking it now. But the whole purpose of PWM, the whole purpose of PWM is to have, the purpose of PWM operation is to have a nearly sinusoidal current, sinusoidal current. Why?

So, sinusoidal current will get the constant torque, ripple torque will not be there, harmonic losses will be there, efficiency is more. Then another PWM scheme is very popular, it is called current hysteresis type control. This is used in high dynamics performance applications and it is very easy to implement. So, we will talk about current hysteresis controller now, current hysteresis controlled PWM.

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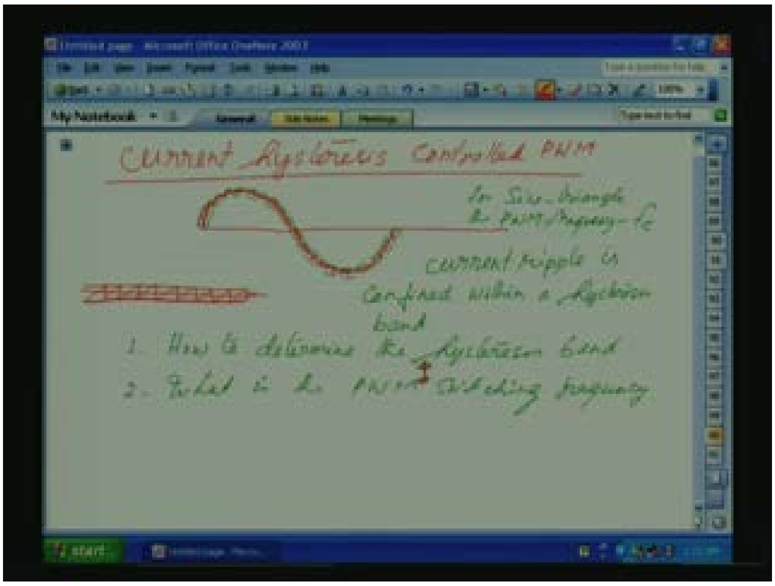

So, in current hysteresis controlled PWM, nearly sinusoidal current that means to generate sinusoidal current or the output current waveform; it is used in motor drive application. So, we use PWM so that the current ripple is confined to a hysteresis back so that the average variation of the current is nearly sinusoidal, this is the thing. So, we switch such that we will introduce envelope along this reference current such that current is that means we are introducing an envelope. So, we will mark with a different thing; we are introducing an envelope here, this is called the hysteresis band such that the current is, the current ripple is not allowed to go above or below this band. So, current is always current. The current ripple in hysteresis controller, the current ripple is confined within a hysteresis band.

Now, what are the things we require? How to design, one? How to determine the hysteresis band? This is very important so that why? See, now we do not know what is the switching frequency. Previously for sine triangle PWM, the switching frequency was f_c for sine triangle the PWM frequency is f_c, here we do not know what is the switching frequency. This is very important, so depend on the devices and switching losses, the switching frequency should be controlled that means the switching frequency depends on the devices. Then, what is the switching frequency, PWM switching frequency? That, we do not know. So, this is dependent on the hysteresis band.

So, we have to design our hysteresis band. These are related, depend on the hysteresis band so that switching frequency is controlled. Now, this also depends on the type of load, load with inductive or load with back emf load. So, let us design. How to design the hysteresis band for a particular switching frequency and what are the merits and demerits of current hysteresis controlled; this we will study now.

So, let us take our basic inverter schematic that is our half bridge configuration and let us define or see hysteresis boundary, now we are going for a general boundary; this is a sinusoidal reference current and a boundary along the sinusoidal reference. We can also have a dc current and we can design a boundary here so that the replace within this one. So, that is also possible. So, let us take general case for a half bridge converter.

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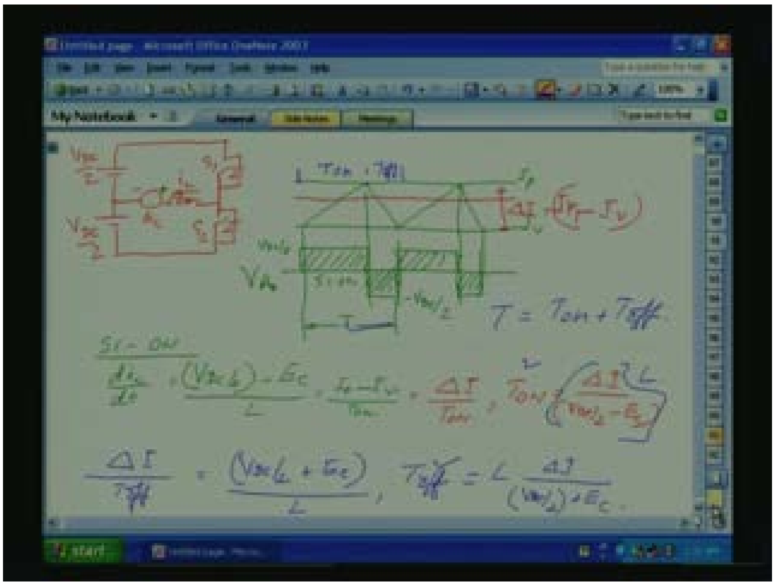

Let us draw our basic single wave, the basic inverter configuration. This is V_{DC} by 2, this is also V_{DC} by 2, let us take a load with back emf that is E_c and the let us take a dc model. This is S_1, this is S_2. It has the freewheeling diode also there. So, it is bidirectional switch. Now, let us say we want a current that is I_L your dc; so current has to be controlled depending on the dc, depending on the load. So, this is the current V_1 and we will introduce the hysteresis band so that the current is controlled, the ripple is controlled within this band.

So, let us say we will enlarge for analysis, let us say this is the band. So, this is the I_P, this is the I_V, upper band and lower band. Now, let us say the switching sequence; we know when the S_1 is on, current will flow through I_L. So, I_L will keep on increasing. The moment it touches the I_P, we are turning off S_1 and turning on S_2. Turning on S_2 means what will happen? This V_{DC}, lower V_{DC} by 2 and the back emf will oppose this current. So, the current will slowly decrease. So, the current will slowly decrease, load current.

Again, the moment it comes here, again we are turning on. So, it will go like this, so this way it will go like this. So, the raising slope and the falling slope need not be equal. So, falling slope will be here faster because of the E_c is opposing, E_c plus V_{DC} by 2, both are opposing. So, this is the current ripple. Let us see what is the switching sequence based on this one. So, when the positive slope, we are turning on S_1, S_1on, this is S_1 on. During the negative slope that is V_{DC} by 2, we are seeing the voltage V_{A0}, this is V_{A0}, when the S_2 is on; wherein this period, it is minus V_{DC} by 2. So, this way we are switching. This is also PWM operation, pulse width these control, we are depending on the current hysteresis band.

Now, let us see for a hysteresis band I_P and I_V; what is the turn on and turn off period? If we know the turn on and turn off period, we can find out the switching frequencies. The period is

T_{on} plus and T_{off} that is one period that is this is the T period. So, T is not, we can find out the switching frequency. Let us take when S_1 is on, S_1 on, the di by dt, d i_L by dt is equal to V_{DC} by 2 minus E_c divided by L. If you see here, this is opposite; plus is here, here minus, so V_{DC} by 2 minus m by L.

Now, from this one, this is also equal to what is d i_L? d i_L is equal to I_P minus I_V, I_P minus I_V divided by T_{on}, S_1 node T_{on}, what is I_P minus I_V? That is delta I. So, I_P minus I_V is equal to, delta I is equal to I_P minus I_V; this is equal to delta I by T_{on}. So, from this one, we can find out T_{on}. How do you find out T_{on} from this equation? T_{on} is equal to from this equation, T_{on} is equal to I_P minus I_V that is delta I divided by V_{DC} by 2 minus E_c. So, E_c is the back emf of the machine. So, the PWM period is assuming it is so high, during this period E_c is not varying, E_c is constant; so, that is we are opposing E_c. So, we can find out T_{on} here.

Now, let us take the T_{off} period; T_{on} we have found out, let us go to the T_{off} period. So, T_{on} we can again, T_{on} we can write here; this is delta I by V_{DC} by 2 minus E_c. So now, let us write the T_{off} period, what happens during the T_{off}. Let us take the T_{off} period, so T_{off} period, again the slope, slope is equal to delta I divided by T_{off}. So T, this is if you see here, this is T_{on} and this is T_{off}. So, delta I divided by T that is the slope, delta I by divided T_{off} is equal to now it is V_{DC} by 2 plus E_c.

So here, the slope is negative but we are not taking the negative into consideration because we only want the T_{off} period. So, we are taking the absolute value, V_{DC} by 2 plus E_C divided by L. So, L into di by dt f that is voltage across inductance is equal to V_{DC} by 2 E_c when this is turned on, this whole voltage has to drop across this one. Now, from this one, we can find out T_{off} is equal to what? T_{off} is equal to L into, here also see that delta I by T_{on} is equal to you have to multiply the L is also there that is missing here, this L has to be here, this L has to go here. So, this is also all multiplied by L, L has to be here. So, here also L into delta I divided by V_{DC} by 2 plus E_c. So, we got the T_{off} and T_{on}. Now, we know the period T, what is T_{off}? T is equal to period is equal to T_{on} plus T_{off}. So, let us go to the next page now.

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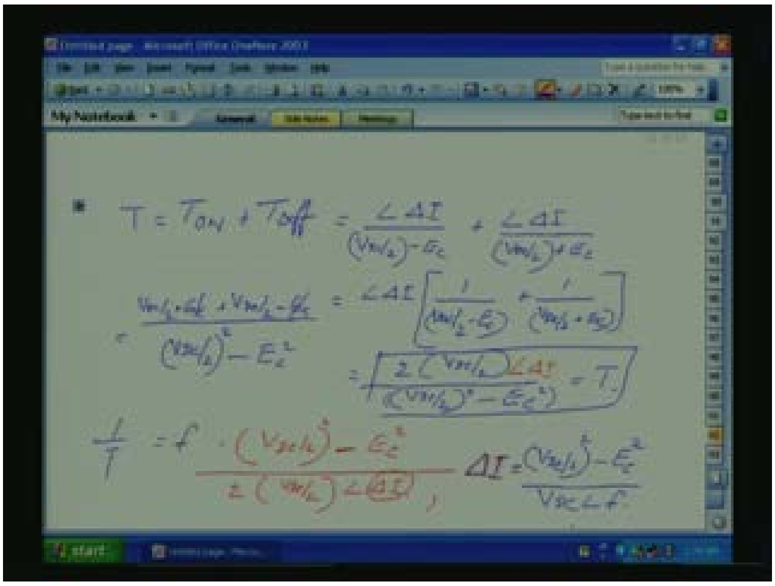

So T, the total period T is equal to T_{on} plus T_{off} is equal to L into delta I by V_{DC} by 2 minus E_c plus L into delta I by V_{DC} by 2 plus E_c. This can be this can also be written as L into delta I into 1 by V_{DC} by 2 minus E_c plus 1 by V_{DC} by 2 plus E_c. So, this so we can reduce, this is a plus b into a minus b. So, this also we can write as V_{DC} by 2 whole square minus E_c square. Here let us, so here it will be this is a plus b into a minus b; so here it will be V_{DC} by 2 plus E_c, here it will be V_{DC} by 2 minus E_c that is from this one. So, E_c and E_c will go and the final equation will be 2 into V_{DC} by 2 divided by V_{DC} by 2 whole square minus E_c square. This is equal to our T.

We got T; once T is known, we can find out the switching frequency that is our PWM frequency. So, what is the PWM frequency here? PWM frequency is equal to 1 by T equal to our f, here we have we have to put the LDI; LDI is missed here, so LDI is also there, L delta I that is also there. So, f is equal to what? Inverse of this, f is equal to inverse of this one that is V_{DC} by 2 whole square minus E_c square divided by 2 into V_{DC} by 2 into L into delta I. So, what is the purpose of deriving this equation?

See, we have brought the hysteresis band that is delta I into this one and the frequency PWM switching frequency related to the PWM switching frequency P. So, this equation shows for a particular delta I, for inverter, V_{DC} by 2, the DC link is fixed. Assume for a DC motor, if E_c is fixed, then the frequency is related to delta I. So, what is the relation? Again, this we can again rewrite in different form like this.

Now, delta I is equal to, we will write in different colour; delta I is equal to what? V_{DC} by 2 whole square minus E_c square divided by, see 2, 2 goes, divide by V_{DC} Lf. So, this is the relation; delta I and f. So, if you see, if you want high frequency PWM that means f is more; high frequency, we require delta I will be very small. So, for frequency to be reduced, we do not want high frequency PWM, then the delta I will increase. The moment delta I increase means the current ripple amplitude will increase, so there should be a compromise.

So, from this one, for a fixed DC and fixed business band and for a fixed switching frequency, delta I can be found out. With a fixed tolerance, tolerance band fi, the switching frequency f depends on the output voltage required. Now, let us take E_c, E_c is varying; let us take this is our for the three phase motor operation, this is that E_c is the single phase voltage that is the back emf of voltage from one phase. So, E_c will be varying sinusoidally, back emf. Then what happened to the frequency? So, this is for switching frequency.

Now, for variable frequency, motor drive application; if you can derive for one phase, we can derive for the other phase also.

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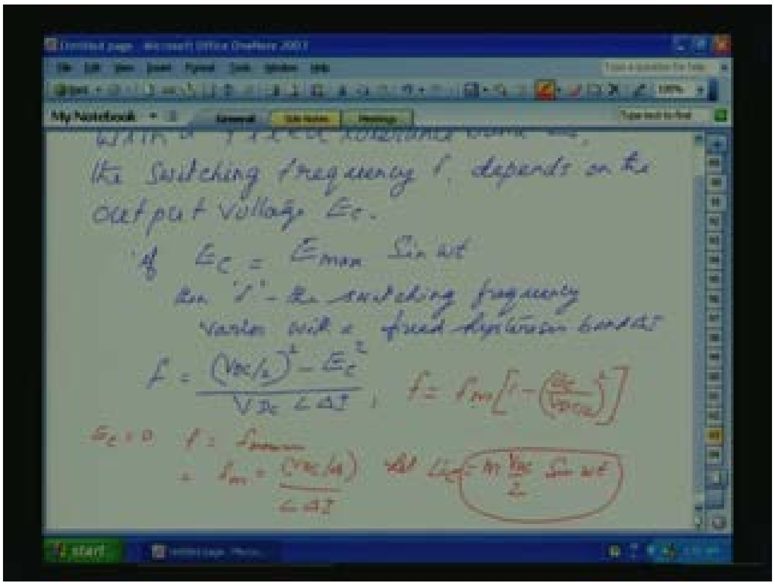

So, with a fixed tolerance band, tolerance band, with a fixed tolerance band, delta I; the switching frequency, the switching frequency f depends on the output voltage, load voltage, output voltage. Here, we have represented in our figure as E_c, output voltage E_c. Now, let us take E_c, E_c is varying sinusoidally. That means for a three phase operation where the single phase voltage is varying sinusoidally, A B C varies with the 120, 120 degree difference. So, let us take a sine, for a sinusoidally varying, if E_c is equal to some E maximum into sine omega t; see we from the DC model that is we made from the symbol analysis that E_c is fixed, we found out how the delta I and f is varying. Now, E_c we make it a variable that is sinusoidally varying signal. That way we are introducing the, slowly introduce the hysteresis band required for motor drive application. Then what happens?

The inverter switching frequency here, if E_c is then f, the switching frequency varies for a fixed hysteresis band, with a fixed hysteresis band delta I. So, let us go back to the previous equation, f is equal to V_{DC} by 2 whole square divide by E_c square divide by V_{DC} into L delta I. So, let us take when E_c is equal to 0, E_c is equal to 0, then f is equal to f maximum that is the maximum switching frequency is equal to, how much it will come? Is equal to f_m is equal to V_{DC}, we can square, E_c is equal to 0, this will become V_{DC} by 4 divide by L delta I. This is the f_m. So, this we can replace it as f is equal to f_m into 1 minus E_c by V_{DC} by 2 whole square.

So, why this introduction is drawn? So, if E_c varies sinusoidally, there is an E_c varying in area sinusoidally, during the f_m will, the frequency will vary around f_m. So, E_c can go both positive and negative; so this switching frequency f will be modulated around a value and the maximum value is f_m. So, let us write down, let E_c is equal to we know for sine triangle PWM, the maximum value of the sinusoidally e maximum, it vary depending on the modulation index.

So, modulation index for sine triangle PWM we said for a half bridge converter, we can go from 0 to 1. So, let like E_c is equal to m V_{DC} by 2 into sine omega t. Now, substitute this one, we will get the, we will get the frequency variation of f with respect to the back emf of E_c. So, here what we found; out for a back emf load, for a sinusoidally varying back emf load that means for a sinusoidally varying back emf load, the switching frequency varies for a fixed delta I, delta I is the hysteresis band. So, for a constant switching frequency, we have to vary delta I.

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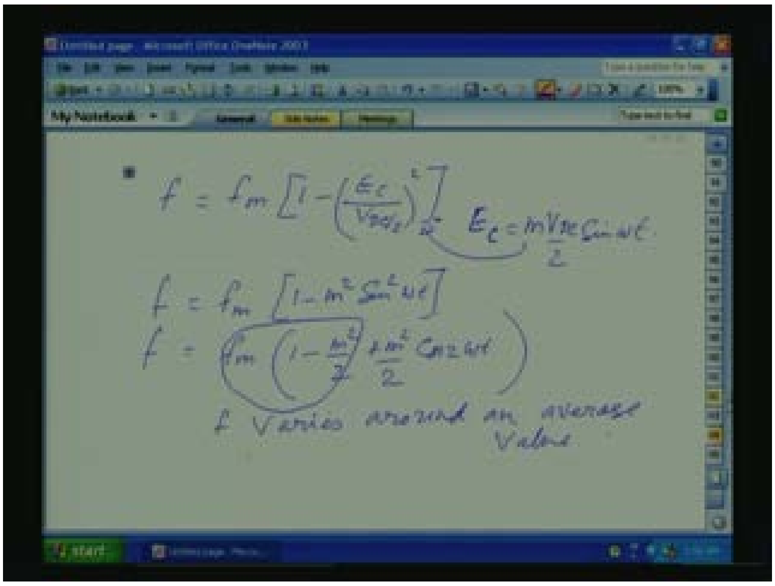

So, now let us find out how f varies for a back emf load E_c; E_c varies like with a modulation index of m into V_{DC} by 2 sine omega t. So, from this one, we can easily find out how the output switching frequency varies. Again, let us go back to our equation that is f is equal to let us go back to our equation f is equal to f_m into 1 minus E_c by V_{DC} by 2 whole square. Now, E_c is equal to m into V_{DC} by 2 sine omega t. Let us substitute this one, in this one. So here, f will be equal to f_m into 1 minus m square that is the modulation index sine square omega t; this is the frequency variation.

See sine square omega t, we can again represent as 1 minus cos 2 omega t by 2. So, this can be represented as f_m into 1 minus m square by 2 plus m square by 2. So now, when we substitute the value of E_c into this one, the final equation is like this. So, if you see here, the switching frequency f varies sinusoidally varies around an average value here. So, f varies around an average value.

So, let us find out what is this average value? How it depends? And, it depends on the modulation index. So, for variables speed operation, we have to design the hysteresis band either for the maximum switching frequency or the average switching frequency, you can decide. So, let us see how this switching frequency variation; is it varying two times the frequency with respect to the back emf flow, let us study in the next class.