

## Power Electronics

Prof. K. Gopakumar

Centre for Electronics Design and Technology

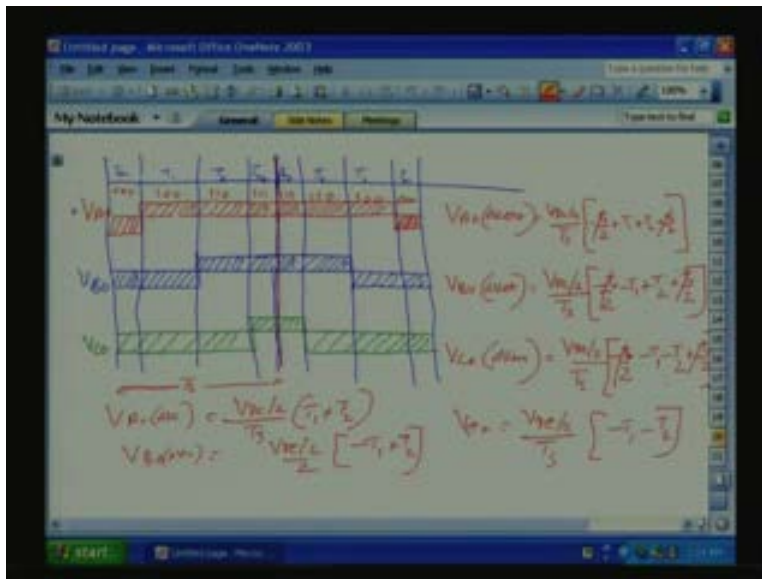
Indian Institute of Science (IISc.), Bangalore

Lecture - 28

### Space Vector PWM Part-3

So, last class we were trying to find out the volt second average that is the average variation of the pole voltage waveform for our space vector PWM. That means for switching between inner sectors the vectors forming the sector that is two active vectors and zero vectors.

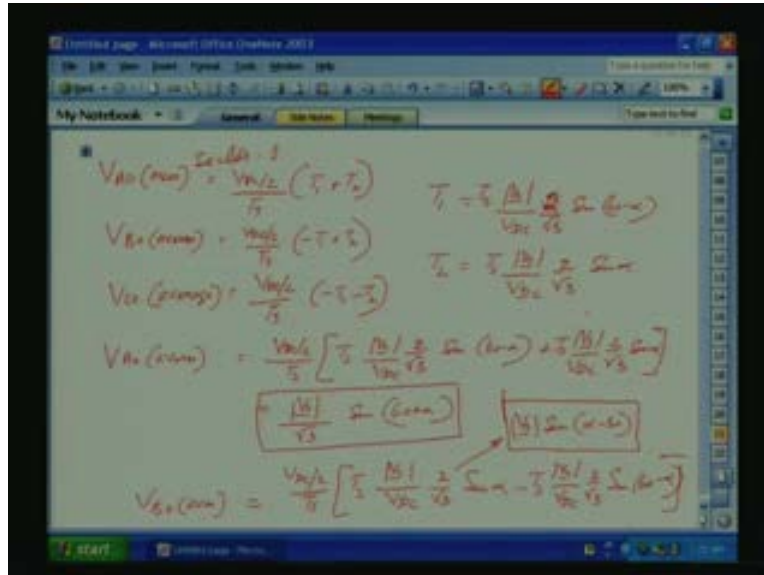
(Refer Slide Time: 1:11)



So, for sector one, so we found the  $T_0$  is divided into two parts that is  $T_{01}$  and  $T_{02}$  and traced at the start and end of the sampling period and the next sampling period, the sequence is reversed that means the first sampling period is  $T_{01} T_1 T_2 T_{02}$ , next sampling period is  $T_{02} T_1 T_2 T_{01}$  and  $T_1$  and  $T_2$  are the periods during which the active vectors forming the sectors or the sectors are switched and  $T_1$  is for the vector which is at the start of the sector. That means if the rotation is from the anticlockwise, it is start of the sector and  $T_2$  is the  $n$  vector.

So, for the general expression, we got for sector one  $V_{A0}$  average  $V_{B0}$  or  $V_{C0}$  average. If you see here, these equations are irrespective of the sector. So, if you know the  $T_1 T_2$  in a sector, we can find out the average value in that sector. So, let us see what is the average value in sector one. This is easily we can get by substituting the value of substituting values of  $T_1 T_2$ ;  $T_1 T_2$  we have already found out. So, let us go to the next page.

(Refer Slide Time: 2:55)



Let us again write for our clarity;  $V_{A0}$  average is equal to in sector one, this is in sector one,  $V_{A0}$  is equal to  $V_{DC}$  by 2 divided by  $T_s$ ,  $T_s$  is the sampling period into  $T_1$  plus  $T_2$ . Now,  $V_{B0}$  average that is average variation that is why we are dividing by  $T_s$ , this will be equal to  $V_{DC}$  by 2 divided by  $T_s$  into minus  $T_1$  plus  $T_2$ . Since the  $T_0$  periods are equally divided at the start and end and for the start if it is 000 is used, end it is 111 and equal duration the volt second, they will cancel. So,  $C T_0$  will not come into picture for the average variation.

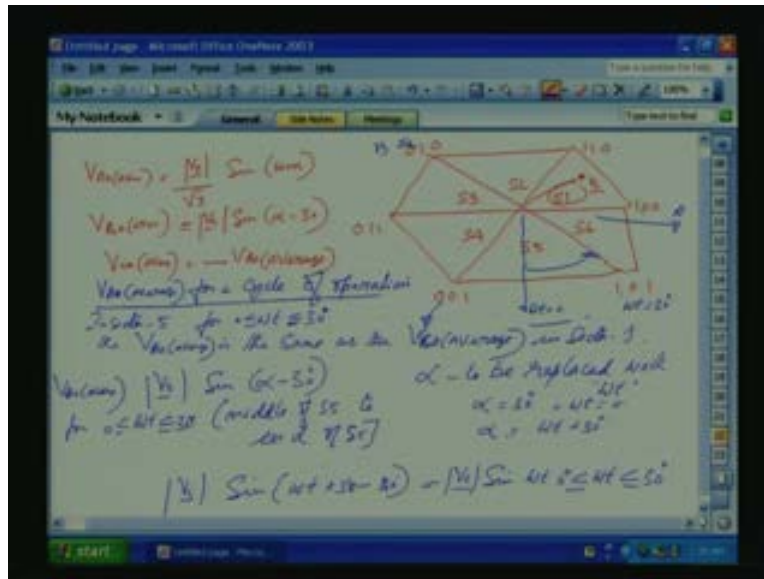
Now,  $V_0$  zero average equal to  $V_{DC}$  by 2 divided by  $T_s$  into minus, minus  $T_1$  minus  $T_2$  that means this also equal to minus of  $V_s$  average in sector one. Now, we have already found out the values of  $T_1 T_2$ . Now, three phase channel. So, for our reference, let us note it here,  $T_1$ .  $T_1$  is equal to  $T_s$  into  $V_s$ , mode of our reference space vector divided by  $V_{DC}$  that is our radii of the hexagonal structure,  $V_{DC}$  into root 2, 2 by root 3 that is  $\sin 60$  is there so that means  $\sin 60$  root 3 by 2; so this will be 2 by root 3 into  $\sin 60$  minus alpha. So, alpha is measured from the start of a sector. So, alpha can vary from 0 to 60 degree. So, we are finding out the average variation in a sector. The sector is an equilateral triangle formed by the active vectors and the zero vectors.  $T_2$  is equal to again  $T_s$  into  $V_s$  mode divided by  $V_{DC}$  into 2 by root 3 into  $\sin \alpha$ , this is our ...

Now, let us put these values  $T_1 T_2$  in equation 1 and 2. So, if  $A_0$  and  $V_{B0}$  average if we can find out,  $V_{C0}$  average we can find out; it is only equal to minus  $V_{A0}$  average. So, let us say  $V_{A0}$  average, this is equal to  $V_{DC}$  by 2 divided by  $T_s$  into  $T_1$  we have put,  $V_s$  mode divided by  $V_{DC}$  2 by root 3 into  $\sin 60$  minus alpha plus  $T_s$  into  $V_s$  mode divided by  $V_{DC}$  into 2 by root 3 into  $\sin \alpha$ . So, this we can, this is of the form  $\sin A$  plus  $\sin B$ , we can use the standard trigonometric relations and we can reduce this one to finally it will come to  $V_s$  by root 3,  $V_s$  mode divided by root 3 into  $\sin 60$  plus alpha.

Now, let us say what is  $V_{B0}$  average? So, this is the one we require. So, in a sector with alpha vary from 0 to 60 degree, the average variation of  $V_{A0}$  average varies like this that is  $V_s$  by root 3  $\sin 60$  plus alpha. Now, let us take  $V_{B0}$  average. Now, let us see the  $V_{B0}$  average is equal to again

$V_{DC}$  by 2 divided by  $T_s$  into  $T_s$  into  $V_s$  mode divided by  $V_{DC}$  into 2 by root 3 sin alpha minus  $T_s$   $V_s$  divided by we are substituting only the  $T_1 T_2$  values into 2 by root 3 sin 60 minus alpha; so, this is that. So, if you see here, this will reduce to  $V_{B0}$  average will reduce to  $V_{B0}$  average will be  $V_s$  sin alpha minus 30; this way it will happen this is  $V_{B0}$  average. So now, let us plot, let us find out this only 60 degree, so let us find other sectors how it works. So, let us go to the next page.

(Refer Slide Time: 09:33)



We know or let us draw our hexagonal voltage space vector structure; this will be like this. So, sector one, they are all equilateral triangle, my figure is not that clear but it is hexagon with equal radii, here. So, this is our sector one. Then sector two, sector three, sector four, sector five, sector six and switching sequence is 100 110 010 011 001 then 101. Now, so  $V_{A0}$  average in sector one,  $V_{A0}$  average is equal to the sampled  $V_s$  that is this is our  $V_s$  is somewhere here, so the sampled  $V_s$  magnitude into by root 3 sin 60 plus alpha and  $V_{B0}$  average is equal to  $V_s$  sin alpha minus 30 degree and  $V_{C0}$  average is equal to minus  $V_{A0}$  average.

See, we want the let us see the average variation for pole A. If you know the pole A average variation; pole B, pole C will be 120 120 degree phase shifted. So, in this phase, see our ABC phase will be so far for the conversion, if AB if this is A, B is in this direction, C is in this direction, C A B that means A is along this axis in this direction. Now A phase, so let us take the average variation along pole A,  $V_{A0}$  average for a cycle that means we are trying to plot  $V_{A0}$  average for a cycle of operation.

So, A phase is here; so here we are using space vector, average variation of the voltage space vector. So, when it is here, A phase will have maximum. So, let us take here A phase will be minimum. So, omega t is equal to 0 is here. So, the voltage space vector rotates from here; when it comes here, A phase will have maximum; here it will when it comes here, it will be again zero; here it will come, it will be negative and opposite. So, omega t C  $T_0$  starts from here. So omega t, first let us see omega  $T_0$  is equal to 0 to 30 that means in sector 5, what is the average pole

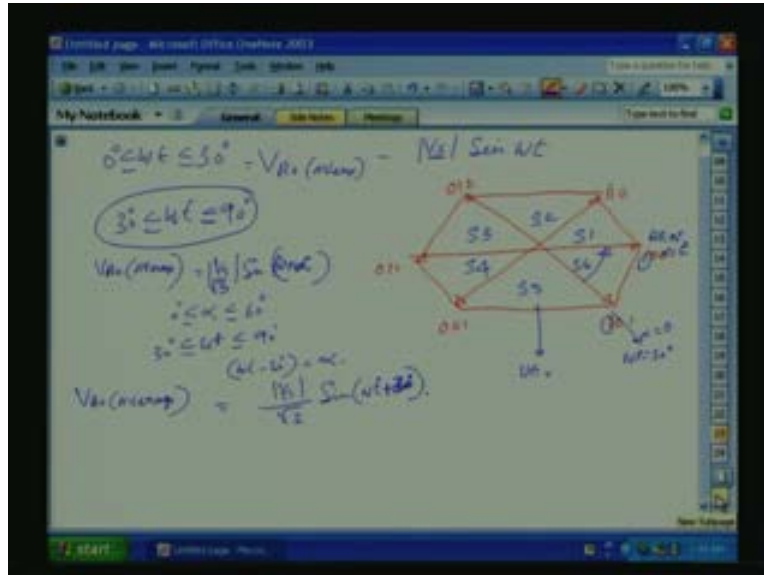
voltage variation for phase A from here to here or that means  $\omega t - C$   $\omega t - C$   $\omega t$  is equal to 0 to  $\omega t$  is equal to 30 degree. How to find out?

If you see here, the average pole voltage variation is that switching is from 0 to 1, 0 to 1 and we know for the inner sector, the inner sector if  $\alpha$  is taken from the start of the sector for here it is start of sector is here, for sector one it is here; the  $T_1$   $T_2$  are the same in any sector, only the switching vectors are different. Now, let us see find out the average variation  $V_{A0}$  average variation. It is varying, the switch vector 0 to 1 here. But we have derived the equations for sector one, so sector one it is 0 to 1 is B phase. So, in sector 5 for  $\omega t$  less than is equal to 0 to less than equal to 30 degree; see now we are talking about the  $\omega t$  general variable,  $V_A$  is or we will say the  $V_{A0}$  average  $V_{A0}$  average is the same as the  $V_{B0}$  average in sector one,  $V_{B0}$  average in sector one. So, what is the  $V_{B0}$  average? The  $V_{B0}$  average is equal to  $V_S \sin \alpha$  minus 30 degree.

Now  $\alpha$ , we have to replace with  $\omega t$ ,  $\alpha$  to be replaced with our general variable  $\omega t$  with  $\omega t$ . So, what is the relation between  $\omega t$  and  $\alpha$ ? So, in sector five if you take when  $\alpha$  is equal to 30; see,  $\alpha$  we are measuring from the start of the sector, so from here to here it is 30 degree. When  $\alpha$  is equal to 30,  $\omega t$  is equal to 0. So, what is the relation between  $\alpha$  and  $\omega t$ ? So, this indicates that  $\alpha$  is equal to  $\omega t$  plus 30 degree. So, when  $\omega t$  is equal to 0,  $\alpha$  is equal to 30; then  $\omega t$  is equal to minus 30  $\alpha$  is equal to 0. So, that means in this equation,  $V_{A0}$  average in sector 5 from  $\omega t$  is equal to 0 to 30,  $V_{B0}$  average for  $\omega t$  less than equal to 0 greater than is equal to 0 less than equal to 30 degree that means from here to here, end of the sector; middle of the sector 5 to end of the sector 5 that means middle of  $S_5$  to end of  $S_5$ , sector 5.

What is the equation? We have to simply replace  $\alpha$  with  $\alpha$  in terms of  $\omega t$  that is equal to  $V_S$  is equal to  $\sin$  of  $\omega t$  plus 30 minus 30 degree. So, that is equal to  $V_S \sin \omega t$ . This shows this is only valid for  $\omega t$  less than equal to 30 degree less than equal to 0 degree. Now, the next, what we have to find out? What is the  $V_{A0}$  average from  $\omega t$  is equal to 30 to 90 degree? So now, if you know that relation 0 to 90 degree we have found out, so if the sinusoidal variation if the average variation is sinusoidal variation, then automatically we can draw the other waveform from the symmetry. So, let us find out what is the average variation for  $V_{A0}$  from  $\omega t$  is equal to 30 to 90. Again, let us go to the next page.

(Refer Slide Time: 18:41)



These are hexagon with radii equal to  $V_{DC}$ ; this is the radii, this is 100, 110, 010, 011, 001, 101. Now, we have found out  $V_{A0}$  variation from omega t that is omega t is equal to 0 to omega t is equal to 30 degree, we know it,  $V_{A0}$  average. Now, we have to find out omega t less than equal to 90 degree greater than or equal to 30 degree that means this interval. So, this is if this is sector 1, sector 2, sector 3, sector 4, sector 5 and sector 6 and omega t less than or equal to 30, less than equal to 0 degree, the  $V_{A0}$  average,  $V_{A0}$  average we got it from the previous slide, it is equal to  $V_s$  amplitude of the sampled reference phase vector into sin omega t. So, this you do not confuse with the continuous sin omega t for full cycle; this sin m, this equation is valid only for our omega t that is what do you mean by omega t? Omega t is the speed with which our reference space vector rotates, 0 to 30.

Now, we have to find out for omega t 30 to 90 degree that is in  $S_6$ .  $S_6$  if you see, the variation of the variation of the switches is from one to one. This is same as in sector one also. So, that means here the variation with respect to alpha, the alpha measure from the start of the sector, the  $V_{A0}$  average is same as in sector 1. Only thing we have to change the origin.  $V_{A0}$  average is same as the sector 1 that means  $V_s \sin$  alpha minus 30 degree. So, if you see here, for alpha varies from alpha varies from in sector 6 is the same as alpha varies from sector 1 because the vector change is one to one here and here also one to one.

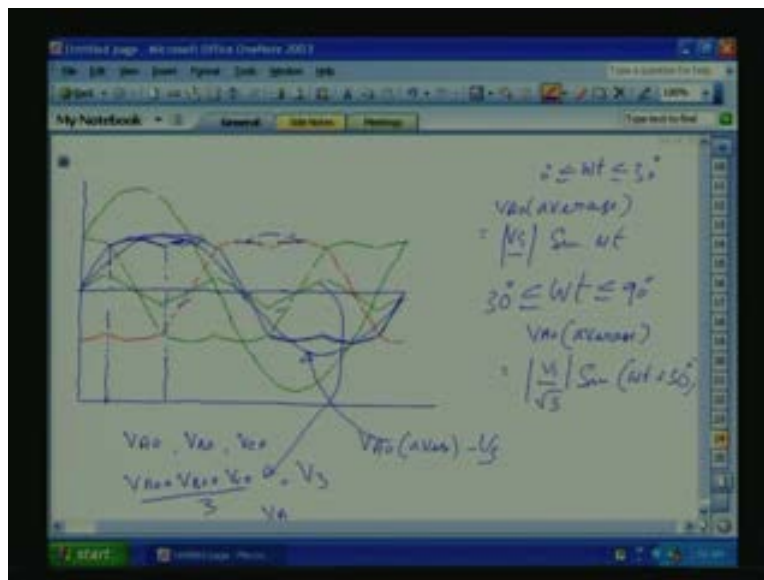
So, alpha varies from less than or equal to 0 degree, less than equal to 60 degree but omega t varies from less than equal to 90 degree less than equal to 30 degree. This shows omega t minus 30 degree is equal to alpha  $_0$ . That means when omega t is equal to 30, alpha is equal to 0 here that is here. Omega t is equal to zero means alpha is equal to 0 and the omega t is equal to 90, alpha equal to 60. Here, omega t is equal to 90 degree; alpha is equal to 60 degree.

So here,  $V_s$  by or here one mistake is there,  $V_s$  by root 3 is also there that I forgot to write; this is  $V_s$  by root 3 from the previous equation. So now, the  $V_{A0}$  average is equal to  $V_s$  mode divided by root 3 into sin alpha is equal to omega t minus 30. So, this will be equal to sin omega t minus

60 degree. So,  $V_{A0}$  average in sector 1 is  $V_S$  by root 3 sin 60 plus alpha, sin 60 plus alpha. Now, alpha is varying from 0 to 60 degree that is from here to here and omega t is varying from 30 to 90 degree; so what is the relation between omega t and alpha? Omega t 60 plus alpha that will be equal to omega t minus 30 is equal to alpha that means when omega t is equal to 30, alpha is equal to 0; when omega t is equal to 90, alpha will be equal to 60 degree that is from here to here. So, when you substitute this one, omega t minus 30 here, alpha; equation will be sin omega t plus 30 degree.

So,  $V_{A0}$  average for omega t varying 30 to 90 degree is equal to sin omega t plus 30. So, what will be the type of waveform this will generate? Let us try to plot the waveform, write down the waveform.

(Refer Slide Time: 24:59)



The required wave from for omega t is equal to omega t less than 30 degree greater than 0 degree, this is equal to  $V_{A0}$  average is equal to  $V_S$  the sampled  $V_S$  amplitude into sin omega t. Now, omega t less than equal to 90 degree greater than equal to 30 degree,  $V_{A0}$  average equal to  $V_S$  by root 3 into sin omega t plus 30 degree. So, these are the two equations, 30 degree. So, let us plot this waveform. So, let us draw the angle; this is our y axis and this is our X axis.

So, alpha is equal to 0 to 30 degree that means here, this is  $V_S$  sampled value of  $V_S$  plus sin omega t that means part of a sin omega t with amplitude is equal to sampled value of  $V_S$  and from 0 to 30 degree. So, this will be of this form, this one. So, this is our 30 degree. After this, from here to 90 degree, this is part of  $V_S$  by root 3, now the amplitude is reduced;  $V_S$  by root 3 sin omega t **sin omega t** plus 30 and omega t varies from 30 to 90 degree. If you plot it, it will be hope something like this. If you fix  $V_s$ , then  $V_S$  by root 3 is this amplitude. Now, if 90 degree; so this again repeats, so if you see, if you repeat this one for symmetry, it will go like this, 0.

So, the variation of our, see we expected a sinusoidal variation and we got something like this; this is for the A phase. B and C, it will be 120 degree phase shifted like this will be there. If you

draw it, we can get it. Let us say the B phase; B phase will be like this, 120 degree phase shifted,  $V_{B0}$  average variation, then  $V_{C0}$  average will be like this. We were trying to expect or we are expecting a sinusoidal variation and we got a waveform like this. So, what all this contains? Let us see.

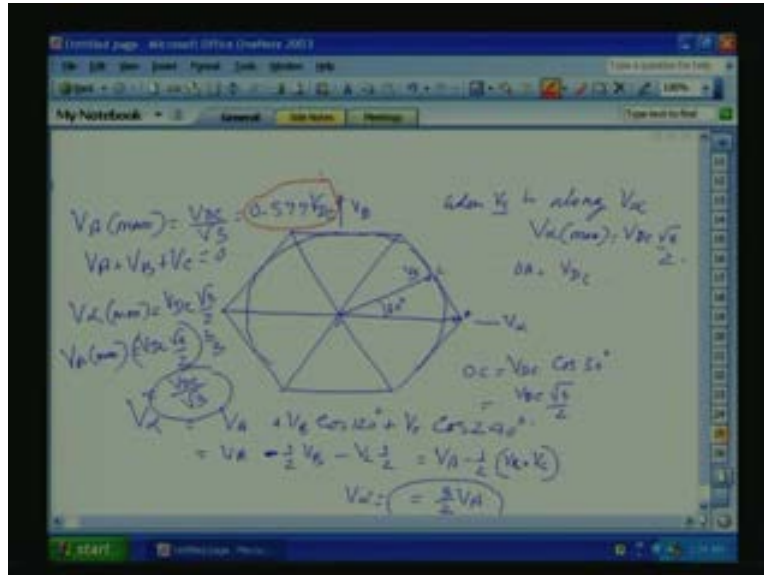
So, if you plot the line to line voltage, let us take the line to line voltage for if you use, see we know  $V_A$   $V_B$   $V_C$ ; if you can plot the line to line voltage, typical line to line voltage will be something like this it will happen. See, line to line voltage is sinusoidal; line to line voltage is the subtraction of two pole voltages. So, something common to this both the waveforms; that waveforms get subtracted and we got the line to line to voltage. Now, what is that one?

Let us sum all the three phases; if you sum all the three phases, what will you get? If it has contains a triple n harmonics, you will get a waveform like this. So, that means the average variation contains a triple n order. So, triple end order even if it is there, it will not produce an average voltage variation, average voltage variation only due to the fundamental. The fundamental component of this wave form will be like this; let us the typical fundamental component if you see that we can get from the line to line to voltage. The typical fundamental component for the A phase will be this one. So, that is sinusoidal.

So, how to find out? See, we got the  $V_{A0}$   $V_{B0}$  pole voltages;  $V_{A0}$  average variation,  $V_{A0}$   $V_{B0}$   $V_{C0}$ . Now  $V_{A0}$   $V_{B0}$ , add  $V_{A0}$   $V_{B0}$ ; if it contains only positive and negative sequence, then  $V_{A0}$   $V_{B0}$   $V_{C0}$  will be 0. But here it is not 0, we got a triple n order here, all the multiplication of the triple n, third ninth all that harmonics are there. So, you sum it and divide by 3 that is you get this waveform that is this waveform. This waveform let us call  $V_3$ ; this you subtract from  $V_{A0}$  average that means  $V_{A0}$   $V_{A0}$  average minus  $V_3$ , the triple end order you remove it, then that will be our the sinusoidal waveform, this one. So, what is the advantage, the sinusoidal waveform?

Let us say, even though triple n is there, the fundamental only will generate power. So, the moment we add the triple n, the peak of this one will be slightly suppressed. See, this is the level, this level that is this level. The sin was there, it would have here. So, that means and for sin triangle PWM if you use if you use the same way with the triangle, we get extra modulation that means some more we can increase. So, the space vector computed sin triangle, we get a boost in the voltage. So, how much that boost in the voltage? Let us find out.

(Refer Slide Time: 33:16)



This is our V alpha, this is our V beta and maximum voltage space vector, we can get the maximum inscribed circle. This amplitude let us say this OA, OA is equal to our DC link voltage  $V_{DC}$ . So, what is the radii of the maximum inscribed circle, this one? That is inscribed circle touch the hexagonal peripheral here. So, this will be equal to 30 degree, this will be equal to that means OC, OC is equal to  $V_{DC}$  into cos 30 degree. That means cos 30 equal to root 3 by 2,  $V_{DC}$  into root 3 by 2. So, as the voltage space vector rotates, that is our  $V_S$  rotates when the  $V_S$  is along V alpha, as it rotates; we will have the V alpha, the magnitude of the alpha component will be maximum. So, V alpha maximum is equal to  $V_{DC}$  into root 3 by 2.

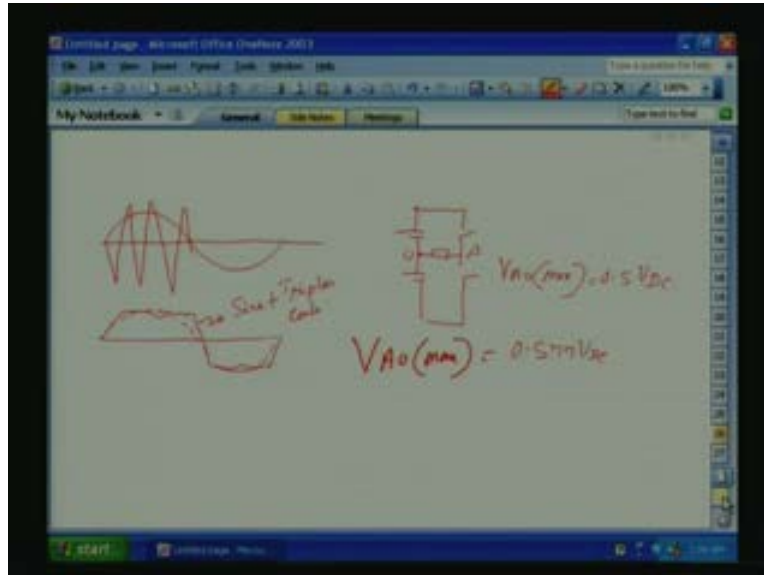
So, V alpha component we got. Now, we can find out what is the  $V_A$  maximum. That will give the maximum, maximum force will from our space vector PWM. So, let us go back to our alpha beta transformation. So, previously we found that V alpha is equal to  $V_A$  plus  $V_B$  cos 120 degree plus  $V_C$  cos 240 degree. So, what is cos 120? This is equal to  $V_A$ , cos 120 is minus 1 by 2,  $V_A$  minus 1 by 2  $V_B$  cos 240 also minus 1 by 2 that is minus  $V_C$  1 by 2. This is equal to  $V_A$  minus 1 by 2 into  $V_B$  plus  $V_C$ .

Now, we are talking about the sinusoidal components. For sinusoidal components,  $V_A$  plus  $V_B$  plus  $V_C$  equal to 0. So, this will be  $V_B$  plus  $V_C$  equal to minus  $V_A$ . So, minus of minus, this will become plus. So, this will be equal to 3 by 2  $V_A$ ; so, V alpha maximum. What is V alpha maximum? V alpha maximum is equal to  $V_{DC}$  into root 3 by 2. So, what is  $V_A$  maximum?  $V_{DC}$  into root 3 by 2 into from this equation V alpha is equal to, so if V alpha is not, VI is equal to 2 by 3 V alpha. So, V multiplied by 2 by 3 here. So,  $V_A$  maximum is equal to  $V_{DC}$  by  $V_{DC}$  by root 3; this is the maximum,  $V_{DC}$  by root that is  $V_A$  maximum using space vector PWM is equal to  $V_{DC}$  by root 3 is equal to 0.577  $V_{DC}$ .

So, you get, in space vector PWM if you read our sin triangle PWM, we get extra boost in this. How this extra boost has come?



(Refer Slide Time: 38:08)

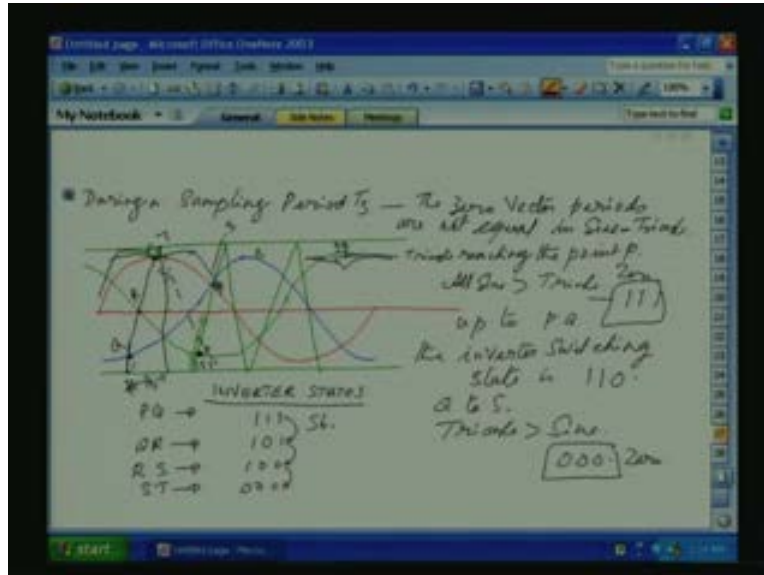


For sin triangle PWM, when we use sin triangle PWM, you have the sin wave and you have the triangle wave and if you take the basic inverter configuration, one pole is 0,  $V_{A0}$  maximum; the sinusoidal average variation maximum is equal to  $0.5 V_{DC}$ , sin triangle. Now, space vector instead of the triangle, what is the wave form we are getting? Fundamental is a triangle waveform but actual variation is a sin plus a triple end content that means something like this; this is the one, this is sin plus a triple n content that is multiple of 3, triple n content.

Then the  $V_{A0}$  maximum is equal to  $0.577 V_{DC}$  that means instead of using this sin wave, if you use this sin plus triple n; what we can or we have some extra modulation. If you see here sin triangle PWM, extra range is possible here because this is flattened here. So, that is why we are getting maximum modulation of  $0.577 V_{DC}$ .

So, let us see, what is or let us study what is the similarity between sin triangle and the space vector PWM. Let us go back to our sine triangle PWM.

(Refer Slide Time: 40:11)



I will extend this one for your clarity. This is A phase, this is our B phase; let us say this is our C phase. Now, let us say sin triangle PWM, let us say triangle PWM something like this it goes. See, we are using a high frequency triangle waveform; just for the analysis, we expanded a waveform like this. Now, our triangle waveform is going from here to here, it goes like this; this is our A B C till it reaches this point, till it reaches this point, all the phases are let us mark this point is some P, till it reaches P point, triangle reaching, triangle reaching the point P. Here, all the ABC phases are above, the magnitude of till that one, all the sin greater than the triangle.

So, what happens? All the top switches are on; all the top switches are on, we are marking as 111. Now, from P to Q, P to Q let us for my analysis P to Q, here both are equal; P to Q what happens? Till that one, A and B are above triangle but C is below. So, upto here, this is the sequence, the inverter sequence will be upto here, upto PQ, the inverter switching state is 110. Here, triangle upto the P, it will be 111.

Now, above that one, see here both are equal, so above that one, the Q to S, Q to S triangle is greater than the sin wave. So, all the switches will be it will be 000. So, Q to S if you see here, Q to S, these are zero period; this is zero period, this is also zero period. So, if you see, in sin triangle, the zero periods which are see vector if you say here, this vector switching let us take another switching sequence, let us take another sequence here. Let the triangle is going like this, so this I will mark that some triangle for hit us let us say P Q then R, then S.

So P Q region, P Q region, the inverter states is all 111 because sin is greater than that triangle. Then Q to R, Q to R, B is below triangle; so this will be 101. Then R to S, then let us see this is T, R to S, R to S C has also gone the below the triangle. So, that means 100. Then from S to T, it is 000. So, if you see here, same like space vector PWM, it is going from zero state to active vector, then the next active vector, then here. So, it is switching between 101 to 100 that means 101 to 100, it is switching in  $S_6$ ,  $S_6$  same like sin triangle space vector PWM but the zero periods are not equally distributed in a sampling period.

What is the sampling period? Now, the triangle which going from P to T that is this period, half of the triangle is our sampling period; so during the sampling period, during a sampling period  $T_s$ ,  $T_s$  in sin triangle period, the zero vectors are zero vector periods are not equal, zero vector periods are not equal in sin triangle, in sin triangle PWM. But in space vector, they are zero. But otherwise if you see when it goes from zero to active vectors, only one inverter leg is switched. So, all other factors are taken here, only the zero vector periods are not equally distributed. When the zero vectors are equally distributed, the average variation in the space vector becomes something like this. So, that shows this get flattened here.

So, we can extra boost **sorry** for the triangle waveform, it will be something like this, so here. So, we can get extra boost is possible but triangle only this much only possible. So, this extra boost is possible that will make the modulation index slightly increased in modulation in sin triangle PWM. So, we have talked about the space vector PWM. In space vector PWM, we require the instantaneous amplitude of  $V_s$  and the alpha that is the sector **in which it states in it** in which it is at the sampling period it is there and the alpha angle is required to find out the switching intervals that is  $T_1 T_2 T_0$ .

Once you do the switching intervals in various sectors, we will get average variation sinusoidal with triplen addition. So, we can get the extra boost in the fundamental voltage because trip triplen  $V_A V_B V_A$  plus  $V_B$  plus  $V_C$  is not equal to 0. So, that will not contribute to the space vector generation. So, we get an extra boost in the modulation index and for the space vector PWM, we will get  $T_0 0.577 V_{DC}$ . For the same with the sin triangle, it is  $0.5 V_{DC}$ .

Now, next class we will study how to generate the PWM based only on or there are various schemes available in literature, here we will proposes scheme based only on sampled reference phase amplitudes not based on  $V_s$ . We know that  $V_s$  is generated from  $V_A V_B V_C$ ; so instead of sampling  $V_s$ , we will sample at their instant the  $V_A V_B V_C$  amplitude and from that one  $V_A V_B$  will be rotating sinusoidally. So, from the  $V_A V_B V_C$ , we will find out the timings  $T_1 T_2$  a simple and fast algorithm for **in digit** implementation; we will be studying in the next class.

Thank you.