Power Electronics

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Lecture - 30

Speed Control of Induction Motor Part-1

So today, from today we will start the speed control of induction motor, DC motor. So, before that we will study the working principle of how the induction motor works and its steady state equivalent circuit and with the steady state equivalent circuit, how we control the machine and for high dynamic performance applications, what is the dynamic equivalent circuit model and from there, how we drive the machine for high dynamic performance application; we will study.

So, let us start the induction method; how the working principle of induction motor, sinusoidal steady state and induction model, we will start sinusoidal steady state induction.

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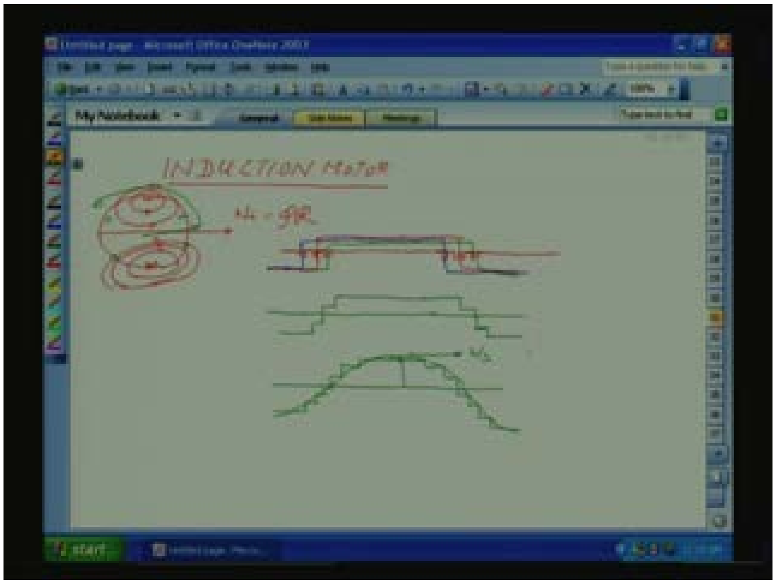

So, let us take the winding; how the winding is distributed in slots to get a sinusoidal distribution? So far, we said the distribution is sinusoidal. But with the distributed winding, the air gap flux distribution or MMF is a stepped wave from as close as to a sine wave. Let us take the phase, let us say this is one slot for A phase; A phase let us this way, so current is coming here and going like this, this is for A phase. Let us take one time or all or let us say all the times are embedded in one slot. So here, if you see here, the flux, the flux will be in this direction. So, you have the flux distribution like this, here also it will be like this; so the current movement is such that all the flux direction here will be like this. So, let say, this is the starter with uniform air gap, the reluctance is uniform throughout. So the MMF, so let us split this one and spread it; so if you see here, for the A phase, around 180 degree, you will have this one like this. So, if you go like this, the MMF will be like this throughout the air gap.

We know that MMF is equal to n into i is equal to phi into R. But this is a stepped wave; it is a rectangular waveform, the starter sinusoidal distribution. So, in the slots, we have different conductors in adjacent slots for phase A, they are short pitched also to get a nearly stepped waveform. So, let us take to distributed winding like this; 3 slots, so we have one here, one here, let us take this way, one here and one here.

So, MMF for this one, we will use different color. For this one will be, so same current, this conductors in this slots belong to A phase and they are series connected and same current is flowing through. So, the MMF for the conductors in the other slot will be like this. Then for this one, is are of equal amplitude; I have slightly changed the amplitude to make it visible. So, if the resultant MMF if you see here, you will get a stepped waveform here for the A phase with pre conductors like this; here it will be like this, then here it will be like this, then you will go like this, go like this so sorry here it will be like this, it will go like this. Now, so this type of stepped waveform for A phase will be.

When all the 3 phase are exited, then you have B phase around 120 degree phase shifter from here. 120 means it will be somewhere here it will be there, for the B phase. C phase again 120 degree means somewhere here it will be there. So, if you see here, all the windings together and if you see the MMF distribution in the air gap, you will have a nearly a sinusoidal a stepped waveform as close as to a sinusoidal will be here. So, this winding has to take care of this to get the nearly sinusoidal distribution of the MMF here.

So, this we can approximate a sine wave. So, as power electronics engineers, control engineers, we will not worry about the stepped distribution. We will assume this is sinusoidal distribution. So this flux, sinusoidal distribution; so we will be assuming the flux distributor, the sinusoidal distribution of the flux across the air gap and if the machine is exited with sinusoidal currents, this peak value slowly rotate with respect to a speed omega s, omega s is our singular speed or excitation frequency. So, this peak keeps on moving. So, that is equal to say that or we have a flux, space vector flux rotate smoothly, this way with omega s, the peak value rotates that means this will move like this.

Now, when it moves like this, what happens? So, flux is sinusoidally distributed, so if you see here, let us go the next page, we have a, now we will not show the stepped waveform, we will show it is sinusoidal distribution. So this is the flux, air gap flux is sinusoidal distributed.

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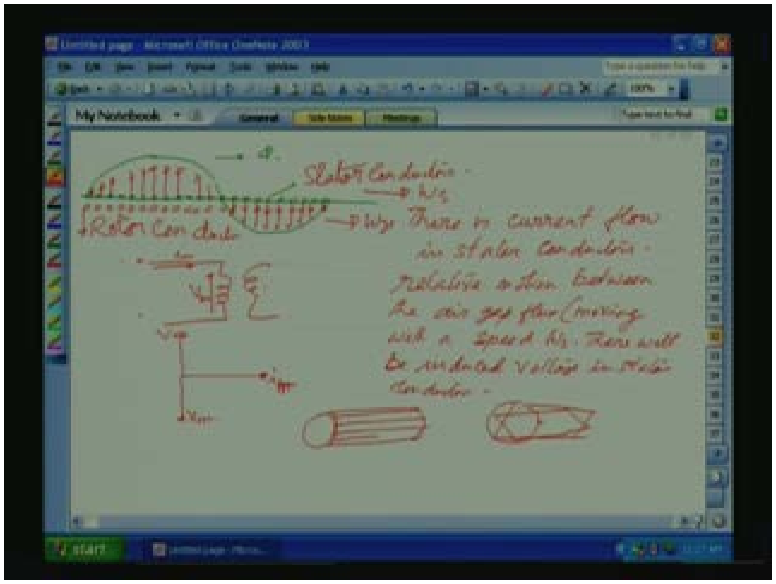

There is current flow in stator conductor - relative motion between the air gap flux (moving with a speed ω_s. There will be induced voltage in stator conductor -

Now, what happens? Let us take the starters, starter it has conductors, conductors in various loads, these are the conductors in various loads; starter, conductors. This starter conductor, there is a relative motion; the starter conductor is having current. Now, there is current, there is current flow in starter conductors and the flux is moving; there is a relative motion between a current carrying conductor and the air gap flux. So, what will happen?

Because of the relative motion between the air gap flux, air gap flux is moving with a speed omega s, omega s is our excitation frequency because of this relation relative motion, there will be induced voltage induced voltage in starter conductors. So, this conductor where the flux is maximum, you will have the maximum voltage. So, each conductor will experience a bar magnitude voltage and that voltage is proportional to the speed and also the flux, the flux at that point cutting the conductors. These are called bar magnitude voltages, these are called induced voltage.

So, let us take the machine is not loaded, induction machine. So, if you see here, we are applying a sinusoidal voltage here, across the machine and assuming starter drops are negligible. So, if you see here, then this induced voltage E_b sinusoidal distribution, this is V_m will be exactly opposite to V. That means we are sending a current, magnetizing current through the starter, no load. So, let us see this is i_m magnetizing current; due to magnetizing current, flux is generated and this flux, this i_m is generated due to the voltage V, these two are lagging by 90 degree, no load operation, no load, secondary.

Due to this flux, there is an induced voltage V_m; if you see here, according to notation, let the current is going like this i_m. So, one is leaving this one, one is entering this terminal, so notation this b_m induced voltage is equal to minus d phi by dt. So, this will be V_m. So, this V_m and with the applied voltage will be exactly opposite that is opposing each other; this is starter, under no load.

Now, the flux is rotating but we have rotor also we have conductors, the rotor slots we have rotor bars and short circuited at the ends. That means we have short circuiter bar that means to applied rotor voltage is 0. So, this short circuited rotor bars if you see here, this will act like a coil, these are short circuiter and then these are short circuited at the end. So, these are like coils, the various coils; see various coils will be there, all are short circuited, full periphery you can have slots like this if you see spread.

So, this rotor conductors; there is a flux moving air gap and there is relation motion between the flux and the rotor conductors also. Let us take the rotor conductors with a different color here. Let us see this is or assuming the rotor also having the same number of times as the starter; so we have rotor conductors here. But if you see, if the rotor is rotating with a speed, this is rotating with omega s, rotor is rotating with omega r, let us say omega r these are rotor conductors. Now, what will happen? Rotor will also get induced voltage, same bar magnitude voltage will the rotor also get. Now, rotor is short circuited, so each coil has an alternating voltage, this way, bar magnitude voltage. So, this coil will also have a current flowing through the coil and the coil means it has inductance and the resistance.

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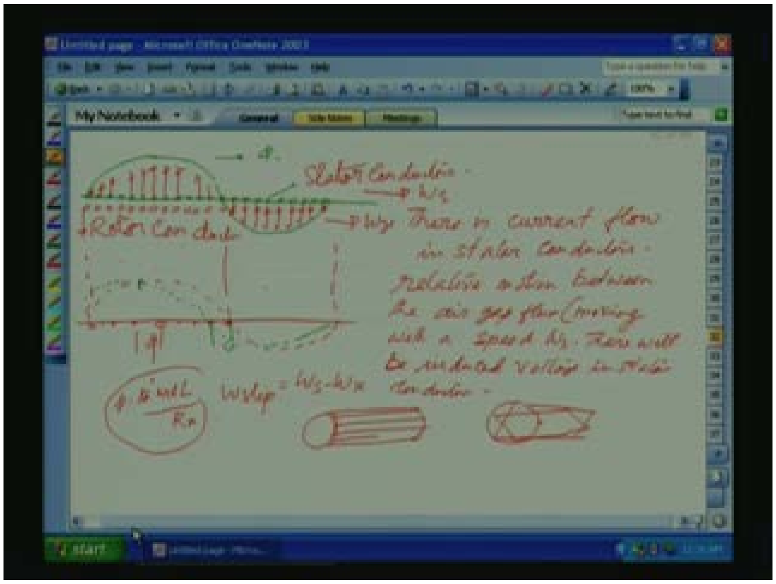

There is current flow in stator conductors. relative motion between the air gap flux (moving with a speed ω_s. There will be induced voltage in stator conductors.

So, that current will be lagging the voltage. So, what you meant by lagging? Let us draw that one. See, let us draw that one, let us draw the current; see this is the 180 degree, this is the 0 degree, this is the starter. So, if you see here, the rotor bar magnitude voltage will be like this; this bar magnitude voltage for the rotor also, this voltage will not be the same as the starter induced voltage. Why? Rotor is moved with the speed omega r. So, relative speed will be that is omega s minus omega r, this is called the omega slip.

So correspondingly, this voltage that is starter voltage and this voltage will not be equal but they are if you see here, they for the same conductors, if you see here, the rotor also will get this type of bar magnitude voltage and because of the rotor coil, you will get current in the starter rotor coils. That current will be lagging this bar magnitude voltage. What do you mean by that one?

Suppose, the conductor which is lying here at this movement is having the maximum voltage; lagging, lagging is equal to the rotor leakage inductance that is the phi will be tan inverse omega sl L divided by that rotor resistance r, that phi. So what is meant by lagging rotor current? Rotor also will have this bar magnitude current, sinusoidally distributed that is lagging the bar magnitude voltage by an angle by phi. What is meant by that one?

That means the conductors which is having the maximum voltage now will have the maximum current after phi. So, that means the bar magnitude current will distribution will be, for the conductors, it will be like this; this is the phi. That means now this conductor is having the maximum current; after sometime, this conductor will have the maximum current. So, current is lagging the coil. Now, what happens? If you see the equivalent circuit, let us draw the equivalent circuit here itself.

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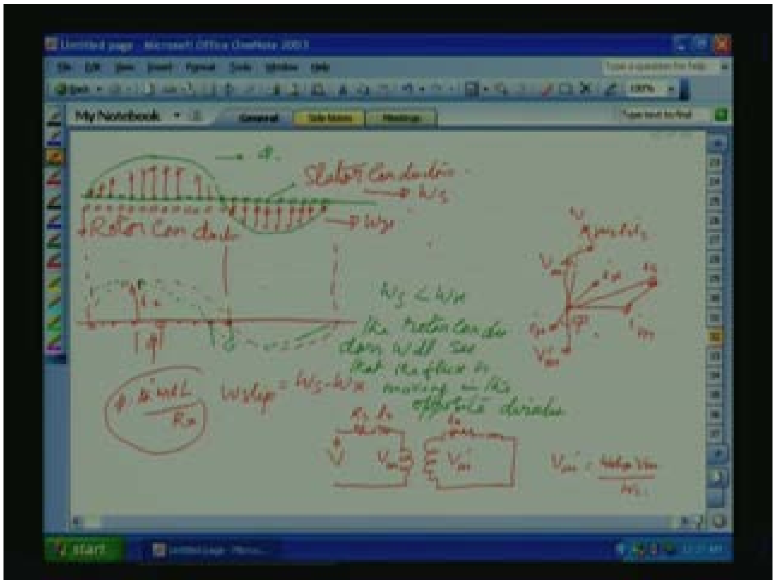

So, let us strike your phasor diagram now. If you see here, the phasor diagram is like this; see we have, originally we have the i_m which will generate the flux because of the flux we have the this i_m is generated by V applied voltage, starter voltage for, this is the single phase equivalent circuit; so i_m, these are 90 degree and this will i_m generate the induced voltage which is V_m, these are quarterly conversion, V_m V_r, V_m and V_r are opposite.

Now this V_m, this V_m is the voltages which introduce the rotor also. So, this rotor will have a current which is lagging the V_m by phi; this is phi, angle phi, this is the current, peak value of the current, peak value of the current is here. Let say, this is I_r, peak value, so we will make the I_r here. Now, what happens? There is a current distribution in the quarter circuits, the same way the current distribution i_m in the starter. So, starter has produced a sinusoidal distribution of the flux. Same way, rotor can also produce a sinusoidal distribution of the flux. Then what happens?

The original air gap flux will get disturbed. So, what the starter will do? Flux cannot change instantaneously for steady state operation. So, what happens? Starter will also to counter the effect, it will produce equivalent opposite current i_r. Now because of the rotor current, the starter current will be i_m parallel to that addition we will do; this is the i_s, new i_s will be generated

Initially, we assumed starter does not have or starter drops negligible but starter has resistance on inductance. So, if you see here, the equivalent circuit, we are applying V here; it has resistance, leakage inductance and the magnetizing inductance. Now, this is the rotor current. Rotor will also have induced voltage, so this induced voltage depending on the speed that is the slip, rotor also will have an induced voltage here. If this is equal to V_m, this is equal to V_m dash that is proportional to slip and you have resistance and inductance here. So, this drop will be equal to the omega slip; this is the current i_r, this is V_m dash. If V_m, if omega s is equal or with the speed omega s, we will get omega m. V_m dash will V_m dash which approximately we can find out, V_m dash will be equal to omega slip into V_m divided by omega s; this way you will get it.

Now, if the starter also has a leakage inductance, leakage inductance; the starter voltage, applied voltage has to be for this distribution, not be V. So, then our applied voltage V, this is V_m, we have starter drop along is that is R_s, R_s leakage inductance l_s, then perpendicular to that one, j omega s l_s into i_s; that drop will be there. So, depends on the i_s, this V may be here, this is may be our V. So, this is the steady state equivalent circuit.

Now, let say, this is for motoring operation; now, let us say, for the same slip, motor is rotating in the opposite direction. For the same slip, motor is rotating in the opposite direction or we can say let us I am slowly reducing the speed, omega s, somehow if I can reduce the speed let us say, such that the relative speed omega s omega r is greater than omega s and omega s minus omega r, absolute value of the slip is the same.

So, what happens? If you see here, the rotor conductors, we will see the flux is rotating in the opposite direction because omega r is greater than the omega s. So, if you stand on the rotor conductors when the omega slip is omega s is less than omega r; the rotor conductors we will see that the flux is rotating, flux is moving in the opposite direction, flux is moving in the opposite direction, opposite direction. What is meant by this one? Let us go to the next page.

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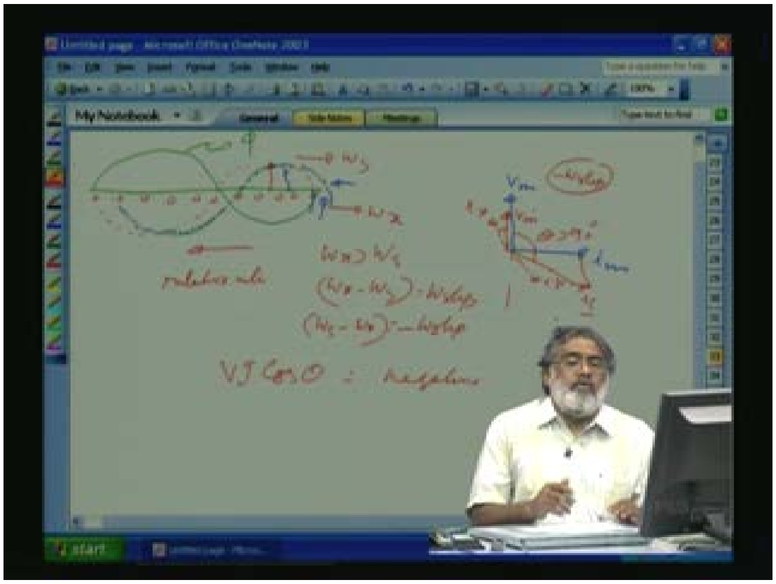

See, we will again let us draw our bar magnitude voltages here. See, this is the flux, this is the flux; from the starter, it will be moving in this direction. Now the rotor conductors, rotor conductors, they are moving with the speed omega r; this is omega s. Now, we will say omega r is greater than omega s and omega r minus omega s is equal to omega slip is equal to same as our omega slip, same as before. Now, the relative motion is in the opposite direction. So, what will happen? Rotor conductors, we will see bar magnitude voltages in the opposite direction; when the slip is negative that means slip omega s minus omega r is equal to minus omega slip, slip is negative.

Now again, there is a rotor conductor that is inductance and resistance, so it will take a lagging current. Here, what is the lagging current means? Now, the relative motion between flux and the rotor is, it is now in this direction, relative motion. So, the lagging means if you say this conductor has the peak value now; according to the current, the current will be lagging now, the lagging direction is in this direction; this is the rotor current. So, this is the rotor current, bar magnitude current; so this is the phi now.

So, as the relative motion is in this direction if this voltage is having the maximum, the maximum current is in this direction. So, it will come here after some period. But if you stand on the starter, see starter has, it has provided the i_m now; again we will let us assume the starter drops are negligible. So, V is here. Now our V_m, dash V_m dash is in this direction, this is the V_m dash for omega slip negative, for minus omega slip.

Now lagging current, as far the starter is concerned, if you look from the starter, this will take a leading current, the rotor is taking a leading current in this direction; this is the i_r, now to counter this one, starter will produce equivalent opposite current in this direction, i_r.

So, if you see, i_r plus is, this is the i_s; so new i_s, as far the starter is concerned more than 90 degree, this theta is greater than 90 degree. So, slip is named, so import power is equal to V_i cos theta, theta more than 90 degree, so is equal to power will be, theta is more than 90 degree means this will become negative. So, power flow from the rotor side to the starter side. So, this happens when during the slip is negative, this is the way what is happening inside.

But as far the rotor is concerned, rotors will always taking lagging current only but look from the starter, now the lagging direction is changed because of the relative motion changed, the starter we will see, rotor is taking a leading current and approximately it will equivalent opposite. So, net effect current will be more than 90 degree. So, theta more than 90 degree, it will be minus cos theta. So, it will become minus V_i cos theta, it will become, so power becomes negative; this happens during regeneration.

Now, we have approximately come to the steady state equivalent circuit where the machine is excited with sinusoidal currents. Now, how we have the speed control?

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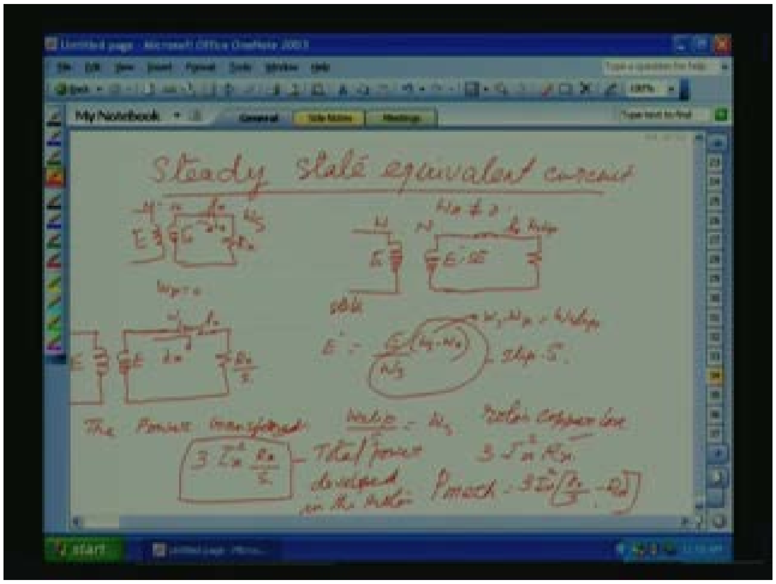

So, assuming starter will have a voltage E, starter and the rotor has same number of turns. Now, if relative motion between the starter and that is omega r is equal to 0 let us take, rotor we have, this is the rotor R_r l_r. Now, omega r is equal to 0, then the applied voltage V_m and V_m; so instead of V_m, I will put the excitation voltage now as E; so, that would make it simpler. Let us see, this is E, this will also have E, omega r is equal to 0. That means this drop, the i_r current; here also the frequency is omega s.

Now, omega r is not equal to 0 that is rotor is running, omega r is equal to non-zero but still the starter will have the voltage E; this is the starter. We will assume same number of turns for the starter as well as the rotor. So, rotor will have a voltage. What is the voltage? It is proportional to the slip. What is slip? Slip is equal to, so what is the voltage E dash?

E dash is equal to E by omega s into omega s minus omega r. So this turn, omega s minus omega r, that is a ratio; you will say, the slip s. So, E dash is equal to SE you will get.

So here, this leakage inductance l_r, here the frequency is equal to omega slip. Omega slip is equal to this is the omega s minus omega r is equal to omega slip; this is the ratio s. Then you have the resistance r; omega r is equal to not zero. Now see, starter also starter has E, rotor has SE. So, we can make starter and the rotor same induced voltage by multiplying or dividing throughout by s. If you see here, dividing throughout by s; this is also E, this is also E, then omega slip divided by s, s is equal to omega s. So, the starter frequency also becomes omega s l_r but now the new resistance becomes R_r by s.

So, when we do this one, power balance should be there; power before and power after should be the same, R_r by s. Now, the resistance has become R_r by s; so what is the power transferred? Let us see, the power transferred, the power transferred from starter to a rotor to the air gap, transferred. Let us take for one phase it is this is i_r is equal to i_r square into R_r by s is the power developed in the rotor. For the three phase it is 3 that is the total power developed. This is the total power in the rotor.

See, we have divided by s to make the induced voltage same as the rotor that is for convenient for analysis. But now that reflected a new resistance R_r by s. But the rotors copper loss is the same. What is the rotor copper loss? A rotor copper loss, loss is equal to 3 into i_r square into R_r, this is independent of the frequency. So, total power generated is this one, copper loss is this one; so total power minus a copper loss is the power which is responsible for the mechanical power that is a rotation. So, P mechanical is equal to 3 i_r square into R_r by s minus R_r. So, what is this one? Let us bring that to a simple form for analysis.

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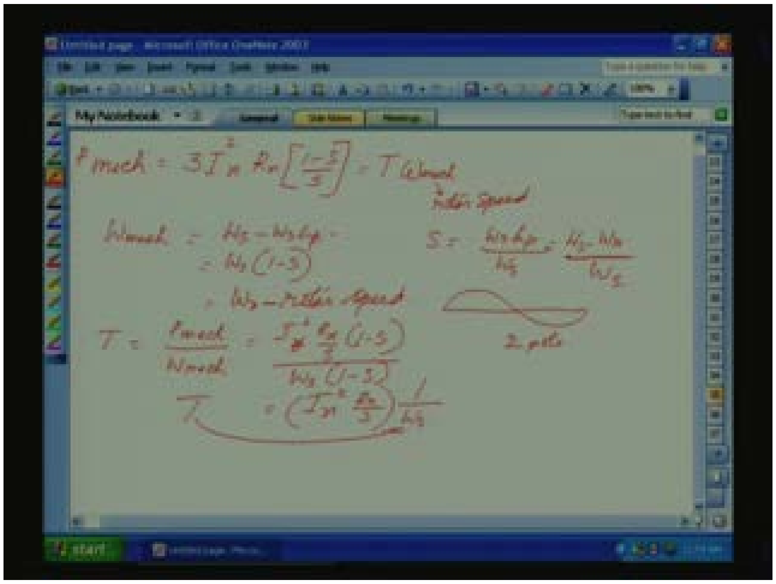

$$P_{mech} = 3 I_r^2 R_r \left[\frac{1-s}{s}\right] = T\,\omega_{mech}$$

rotor speed

$$\omega_{mech} = \omega_s - \omega_{slip}$$
$$= \omega_s (1-s)$$
$$= \omega_s - \text{rotor speed}$$

$$s = \frac{\omega_{slip}}{\omega_s} = \frac{\omega_s - \omega_r}{\omega_s}$$

2 poles

$$T = \frac{P_{mech}}{\omega_{mech}} = \frac{3 I_r^2 \frac{R_r}{s}(1-s)}{\omega_s (1-s)}$$

$$T = \left(I_r^2 \frac{R_r}{s}\right)\frac{1}{\omega_s}$$

So, P mechanical is equal to 3 into i_r square into R_r into 1 minus s by s, the previous equation if you simplify, you will get like this. See previously also, we know this is equal to torque into omega rotor, omega mechanical speed. So, what is omega mechanical? That is omega rotor that is omega mechanical is the rotor of speed. So, omega mechanical is equal to omega synchronous minus omega slip that is equal to, in s, omega s into 1 minus s it will be, slip where s is equal to omega slip divided by omega s is equal to omega s minus omega r divided by omega s; this is the excitation frequency, this is the rotor speed, this r is our mechanical speed.

So, omega mechanical is equal to omega s minus this one is equal to our omega r, this is equal to rotor speed. In all our analysis, we assumed that rotor conductors have the same number of turns as the starter and the distribution is also the same as starter and we have assumed that the distribution for the full cycle is like this; this shows two pole, two pole, so omega r, the omega r is equal to omega rotor speed. So, what will happen if number of poles is more? We will come to that one later.

So, T is equal to torque, develop T is equal to why we want torque? We want the torque speed characteristics of the induction machine, steady state for speed control that is some purpose. So, T is equal to P mechanical by omega mechanical that is equal to I_2 square I_r square R_r by s into 1 minus s divided by omega synchronous into 1 minus s. This will be equal to I_r square into R_r by s into 1 by omega s is equal to T, torque developed; this is the torque developed. So, we have related between the torque and the omega synchronous. Now, let us see, how we do the speed control for the machine?

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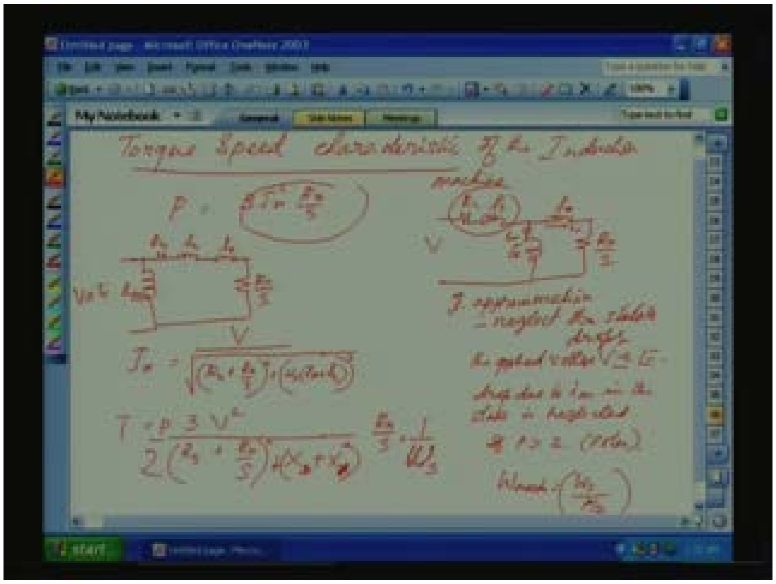

So, we want a torque speed characteristics of the induction machine with the sinusoidal excitation that is also called steady state excitation. So, the power transferred across the air gap, P is equal to 3 I_r square into R_r by s. So, let us assume the equivalent circuit is starter drop. See, now multiplying by s, starter and rotor, we have brought it to the same frequency. So, this is l_s R_s, this is l_r, E is the same for both; then you have the R_r by s. See, this is the applied voltage V.

Now, let us assume applied voltage V is equal to E that means the starter drops are negligible, first approximation. See, very high speeds of operation, 50 hertz, 40 hertz, 30 hertz of operation, we can assume starter drops of negligible; so, first approximation. Neglect the starter drops, so starter drops are negligible, so applied voltage is equal to E. So, our magnetizing m is here, let us say this is l_m, then you have seen starter drops are negligible as far as the V and E are concerned. That means what you mean by starter drops are negligible means? The applied voltage, V is equal to approximately our E. But due to current, there is starter drop is there. So, this starter drops, same current, so when we say these two are equal, the same current is, the current flowing through R_r, s also going through starter. So, starter inductance, resistance we can put it here, then l_r, R_r by s.

So, by putting this inductance here we are assuming V is approximately equal to E. But still there is starter drop here. So, if you see here, i_r is what? See, we have to find out i_r. If you take this one, this is i_m, this is i_r. So, to find out i_r, assuming V is equal to this way, so i_r will be equal to this is good approximation is equal to V divided by from this equation, V divided by root of R_s plus R_r by s whole square plus omega s into l_r plus l_s whole square; this is our i_r, this approximation we made V is equal to Em, so the same current passing the rotor, it is coming through here also.

So here, what we are doing is the i_m during loading, whenever the R_r is your rotor current is there that means there is a loading. So, the magnetizing current is magnetizing current is much less than the load current. So, the drop due to the magnetizing at current at the starter is neglected. So here, what is that? The drop due to i_m in the starter is neglected; that is an approximation we have done.

Now, this is i_r; from this one, what is torque? T is equal to 3 i_r square that is this equation that is equal to 3 V square by R_s starter plus R_r by s, this is slip, this notation subscribed only for the starter, this is the slip into square plus let us say, this I will make it as X_1 plus X_2 square. X_1 is the impedance that is X_1 is let us say starter l_s omega, let us say X_2 is l_s omega or we can say s X s plus X_r; that will be better, starter plus V, into R_r by s. So, this is the power, power divided by what we require? Omega s, this is from the previous equation, omega s. This is true if we have or number of poles is 2. Suppose the number of poles is more than two than if P is more than 2, number of poles, P means poles, number of poles, then omega mechanical, the mechanical speed will come down; so this one.

So equivalently, omega s divided by P by 2 is the omega mechanical. So, here it will be that means divided by p by 2, it will go to the numerator. So, with number of poles, this is the torque equation, steady state. When the machine is excited with sinusoidal currents that is using, that is called the steady state equivalent circuit. That means sinusoidal excitation means the frequency is varying, the inductive drop is due to the omega, l omega, i_r into drop. That is why called sinusoidal excitation; so, this is the equation. Now, let us see how to find out the torque speed characteristics here.

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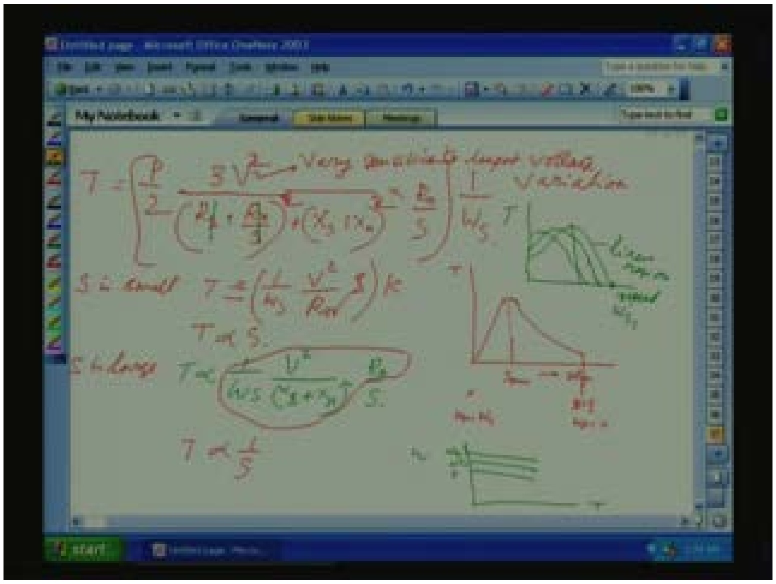

$$T = \left[\frac{P}{2}\,\frac{3V^2}{\left(R_s + \frac{R_r}{s}\right)^2 + (X_s + X_r)^2}\,\frac{R_r}{s}\right]\frac{1}{\omega_s}$$

Very sensitive to input voltage variation

s is small $T \approx \left(\frac{1}{\omega_s}\,\frac{V^2}{R_r}\,s\right)K$

$T \propto s.$

s is large $T \propto \frac{V^2}{\omega_s (X_s + X_r)^2}\,\frac{R_r}{s}.$

$T \propto \frac{1}{s}$

So here, if you see here, the torque T is equal to p by 2 3 V square divided by R_s plus R_r by s whole square plus X_s plus X_r whole square into whole square into R_r by s, whole thing divided by 1 by omega s. See, if you see here, this torque is this shows very sensitive to input voltage variation. That means this shows torque is very sensitive, V square comes here.

Now, let us say V is constant, let us take s is small; s is small means what we can do? This we can neglect, s is small means this will be very high and this we can remove, this also we can remove, then approximately T is equal to 1 by omega s V square by R_r into s that is in proportion and the proportionally constant into some constant K that is because of the P by 2, 3 all those things.

So here, what it shows? For is s is small, T is proportional to s. So, let us say torque speed characteristics for small slip; let us say this is torque, this is slip, for small slip torque is proportional to this one. Now s is large, s is large if you see here, then this is not the one we are going to remove; s is large, then we can remove this one. So, this is a plus here, rotor resistance is this also we can approximately we can remove this one. When s is large torque will be proportional to 1 by omega s into V square.

We can take a typical machine parameter and we can check this one. Approximately, you take a 50 hertz operation, omega s is equal to 50 hertz operation, V square divided by X_1 X_s plus X_r whole square into R_r by s. So, this shows torque is inversely proportional to s. That means T is inversely proportional to s for high speed. So, that means it will come; inversely means it will come. So, in between, it has to go through a peak and come for your continuous operation. So here, you will get the X maximum. So, this is slip.

Now, this is slip can be a maximum, s here, s is equal to 0 here means s is equal to 1. s is equal to 1 means what? Omega r is equal to 0 that means this is the starting speed, omega r is equal to 0, omega r is equal to omega s. So, torque speed characteristics will be like this. So, now let us write this one, torque speed characteristic.

See, this will be torque speed characteristic will be if you draw it, it will approximately that is what finally we want. We will write torque and speed here that is the omega s. When you do it, see this is the same curve, it will go like this. If you see here, this is the linear region, this is for one omega s. Suppose if our omega is changed, omega s and this let us see omega s_1; now omega s_2, then it be something like this or with various omega, it will go like this. So, that shows this parallel curves you will get.

Same like DC motor, DC is the stable operation. See, if you represent it like this that is torque omega, we are this is the linear region, this is the linear region and we will be operating in this linear region. So, torque speed characteristics will be approximating something like this, it will go like this. This part we are using it. So, how do you control the speed here, speed control for torque speeds characteristic like this? And, we are restricting only to the linear region; this we will study, this is for omega s_1, omega s_1, omega s_2, omega s_3, we will study in the next class.