

Power Electronics

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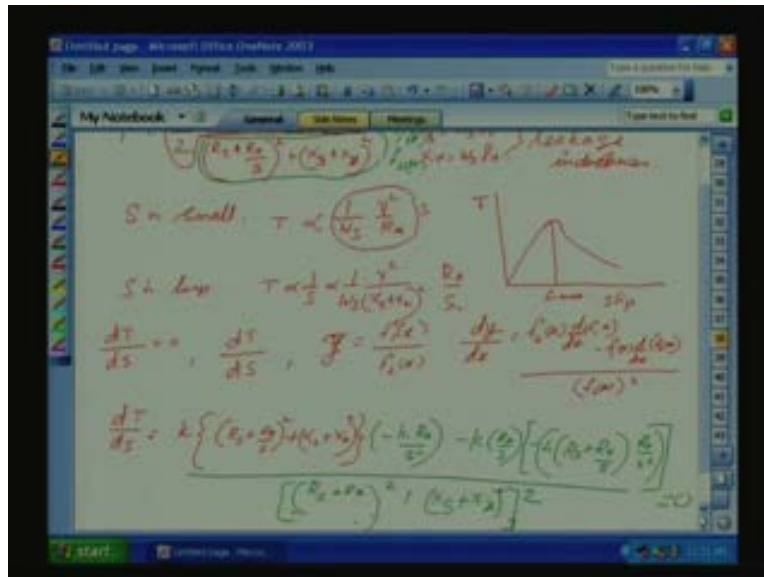
Indian Institute of Science (IISc.), Bangalore

Lecture - 31

Speed Control of Induction Motor Part-2

So, last class we derived the steady state equivalent circuit for the induction motor and from that we found out the torque speed characteristics. Now, today we have to find out the block diagram for or how we can do the speed control for the induction motor from the torque speed characteristics. So, last class we got T torque is equal to $\frac{P}{2}$, p is the number of poles, number of poles into 3 V square, V is the applied voltage into R_s plus R_r by S is the slip whole square plus X_1 plus X_2 square. X_1 is equal to $\omega S l_s$ sorry X_x and X_r and X_r is equal to $\omega S l_r$, l_r is the leakage inductance, these are the leakage inductances.

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Now, how we do the torque speed characteristics? First we will put, let us say S is small; S is small, we found T is proportional to 1 by $\omega S V$ square by R_r into S. So, that means these are all constant, T is proportional to S. So, the torque speed characteristic will be like this; here the torque with respect to slip. So, small value of slip, it will be like this; linear, proportional to slip. Then S is large, then we found torque is inversely

proportional to S and is proportional to 1 by omega S V square by X_x plus X_r whole square into R_r by S. So, this is we said, inversely proportional to S. So, it will happen something like this. So, in between it will go through a peak and come back.

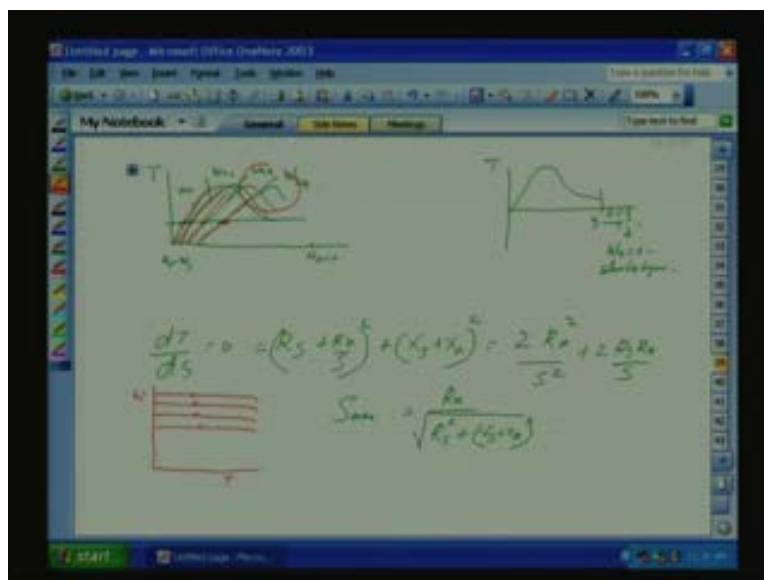
So, how to find out the X maximum? So, the standard book gives how to find out the X maximum but we will try to derive this one here. At this point it goes to a maxima and changes. So, we will say dT by dS is equal to 0 and we know T is this one. So, this implies, so what is dT by dS? dT by dS of the form, this torque dT by dS where we have to find out torque is of the form f₁(x) by f₂(x). So, function of x.

Let T is equal to this one, block **sorry** y is, so dy by dx is equal to f₂(x) into d by dx of f₁(x) minus f₁(x) into d by dx of f₂(x) whole divided by f₂(x) whole square. So, let us derive our x maxima here. So now, let us see dT by dS; dT by dS is equal to some k constant k into R_s plus R_r by S square plus X_x plus X_r whole square. This is our **f₁** f₂(x) that is a denominator. So, this will be our f₁(x) and the denominator, so denominator is the f₂(x).

So, if you see here, the denominator, there is one more term is there that is into R_r by S; R_r by S is there, so this is this one. This should be, we will write, this is also equal to, let me complete that one, this is also equal to R_r by S into 1 by omega S is the correct expression. So k, all these values including omega S, we can put as k, constant. Then this R_r by S you will take it as this is our f₁(x).

So, f₁(x) if you see here, that will be minus of k by R_r by S square. Then minus k into R_r by S, this will be equal to minus 2 into R_s plus R_r by S into R_r by S square; so, this is the one, divided by R_s plus R_r by S whole square plus X_x plus X_r whole square; this is our; then whole square. So, this we are equating to 0, so this is the final equation, it is equal to 0.

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So, when you do this one, the final equation will be, let us say final equation will be dT by dS is equal to 0 is equal to R_s is equal to R_s plus R_r by S whole square plus X_x plus X_r whole square is equal to $2 R_r$ square by S square plus $2 R_s R_r$ by S . So, solving this one, we can find out the S that is X maxima that at this point that the peak value is equal to R_r by root of R_s square plus X_x plus X_r whole square; you will get that.

So, if you draw our torque speed characteristic, now it will be like this; this is T , this is S , it will be linear and inversely proportional. So, at this point, S is equal to 1. S is equal to 1 means this implies rotor speed is 0 or ω_r is equal to 0 that is the starting, starting torque. Now, for variable speed operation, how it is done?

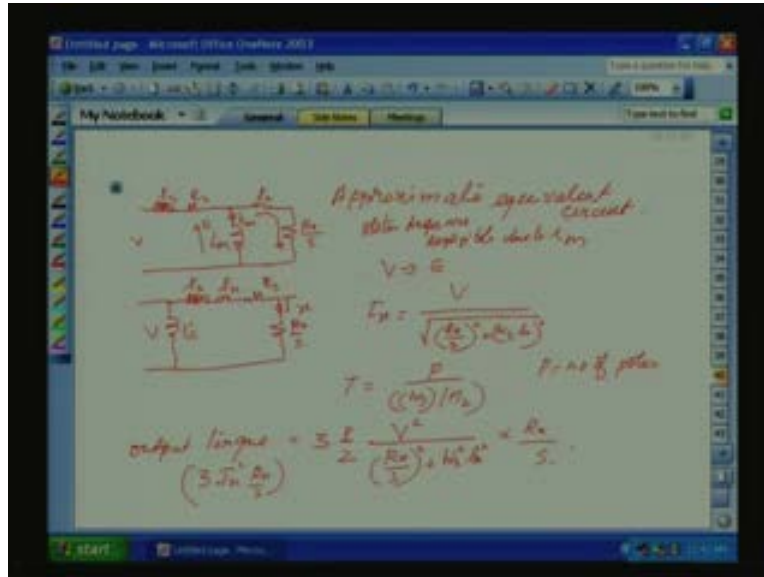
See, let us draw the torque speed characteristics in terms of ω ST . Here S is equal to 0, S is equal to 0 means ω_r is equal to ω_s , here ω_r is equal to 0. So, the characteristic goes like this. Now, for different ω_r , so if our ω_s varies; ω_s S is equal to ω_s , ω_r is equal to ω_c is here, so we can shift this curve like this; so, this is for ω_s , this is for ω_s , this is for ω_s , this is for ω_s . So, for the same torque, we can control the speed. See, compared to our separately excited machine; what we assumed?

Let us draw the torque speed characteristic like this; so this is our ω , this is our torque. So, if you see here, the curve will be with various ω , it will be going like this, this is where the peak value happens and with different speed, we can go to different speed of operation theorem. That means we will restricting our operation only in this region, this is the stable region; this portions we will not be using. Why stable region?

Suppose, let us see here, you are staying here; at this point, if you see here, the speed decreases. Speed decreases means torque is more, so it will pull you back here; if it the speed decrease here, speed increases here, **so torque, here the speed**, at this region or the speed increases, the torque at this, torque less than the low torque, so it will again come back to original speed. So, this is the stable region, so will be restricted to this region.

Now, with restricting to this region; how we can have a torque speed control for the induction motor? We will study now.

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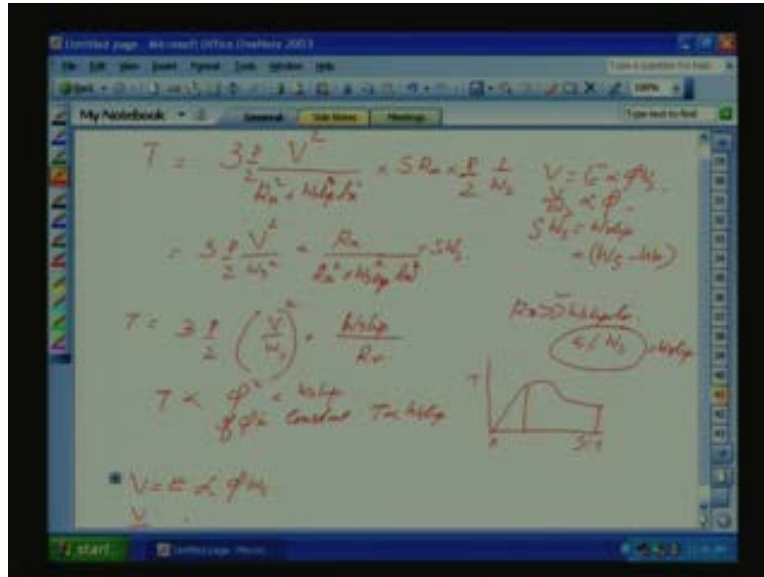
See, this is our equivalent circuit. This is our inductance L_s , R_s , this is our mutual inductance L_m , this is our L_r , then R_r by S , this is our applied voltage V . We will assume first approximation; this is the magnetizing current i_m , so typically for a magnetizing current will be much smaller compared to the load current. So, we will assume the drop due to i_m at the starter is negligible. So, we will assume the E starter drops are negligible due to i_m , then V is approximately equal to E ; so our approximate equivalent circuit is like this.

But due to the rotor current; rotor current can also flow through the starter, so we will not neglect that one. So, our approximate equivalent circuit is like this; then R_r by S , R_s , L_s , L_r . Now, if you see here, output power, again we will see up from the approximate equivalent circuit, this will be here, approximate equivalent circuit. Again, let us see, what is the I_r ? I_r is equal to V_1 by V_1 by root of R_r by S square plus $\omega_s L_r$ square.

So, how we have done? We said V , V applied voltage, so this V , applied voltage V that is this one, V same as E , this is our E ; so instead of V , we can put E . So, V is approximate to E ; so the I_r current here, I_r is equal to V divided by root of R_r square this one and torque is equal to P by ω_s into ω_s divided by P by 2, p is the number of poles. So, from this one, for the three phase system, output torque; this is for the three phase system it will be 3 into 3 by 2 into V square divided by R_r by S whole square plus $\omega_s L_r$ square into L_r square into R_r by S that is equal to 3 into I_r square into R_r by S ; from this one we have derived this one.

Now, let us see, how this one we can approximate for a speed control.

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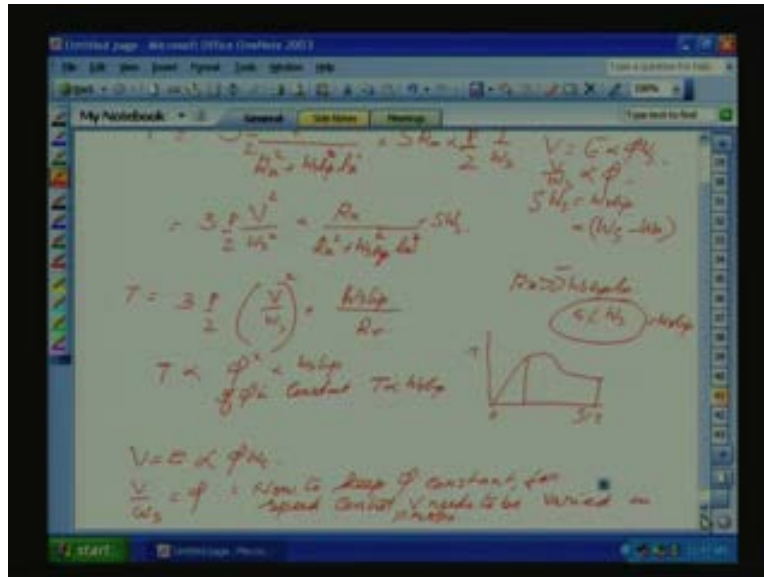
So now, torque is equal to, this can again furthermore, 3 into V square by R_r square plus now I am bring the omega slip square here and l_r^2 square into, previous case I have multiplied by S square, here also we are going to multiply it by S square; so it will come $S R_r$ into p by 2 into 1 by omega S because we know it S into omega S is equal to omega slip is equal to omega S minus omega R. So, this also again we can modify; 3 into V square, I put omega S square here that means one more omega S we have added at the denominator, so have to put that omega S at the numerator, omega S square into R_r by R_r square into omega slip square into l_r^2 square into S omega S.

Now, let us talk the torque; we can assume R_r is much greater than omega slip into l_r . See, omega slip is the difference in speed between the starter and the rotor. So, it typically it will be 5% of omega S, only will be equal to omega slip. So R_r , we can assume R_r is much greater; so we first approximation is equal to 3 into, see here P by 2 is also there, that part, here also P by 2, 3 into P by 2 into V by omega S whole square into, see R_r we assumed much greater than omega slip. So, this can be approximately it will be S omega, finally it will come to omega slip divided by R_r .

So, this implies for low values of slip, low values of slip means our torque speed characteristics is like this and this is our slip S is equal to 0, S is equal to 1 here. So, low values of slip in this region; this is valid, this equation. So, this implies that torque is proportional to, what is V by omega S? V by omega S, and we assumed V is equal to E applied voltage, V is equal to E, E is from the induced voltage; that will be proportional to flux into our omega S. So, this implies that V by omega S is proportional to phi. So, V by omega S square is proportional to phi, mantis flux, air gap flux maximum value; phi square into omega slip, so torque.

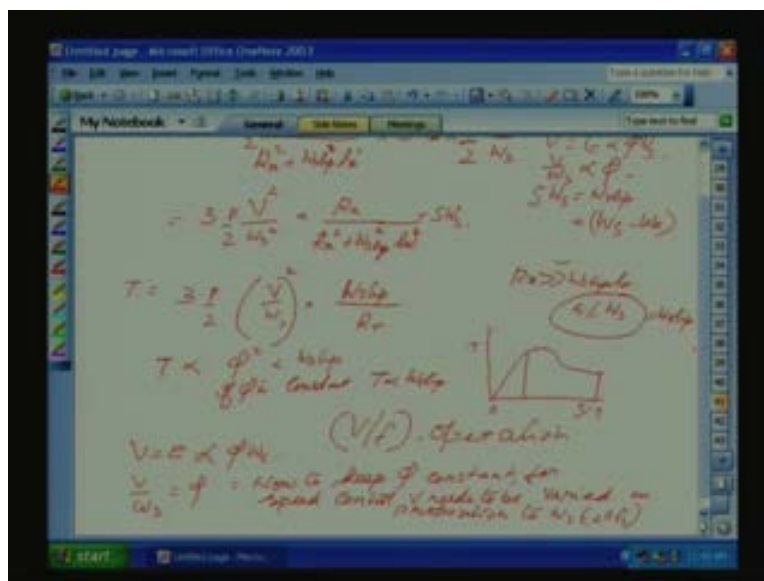
If phi is constant, if phi is constant, torque is proportional to omega slip. See, to make the phi constant; see as I told from this equation, V is equal to E proportional to phi into omega S. So now, that implies that V by omega S is proportional to phi.

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Now, to keep phi constant for speed control, V needs to be varied in proportion to omega S, in proportion to omega S.

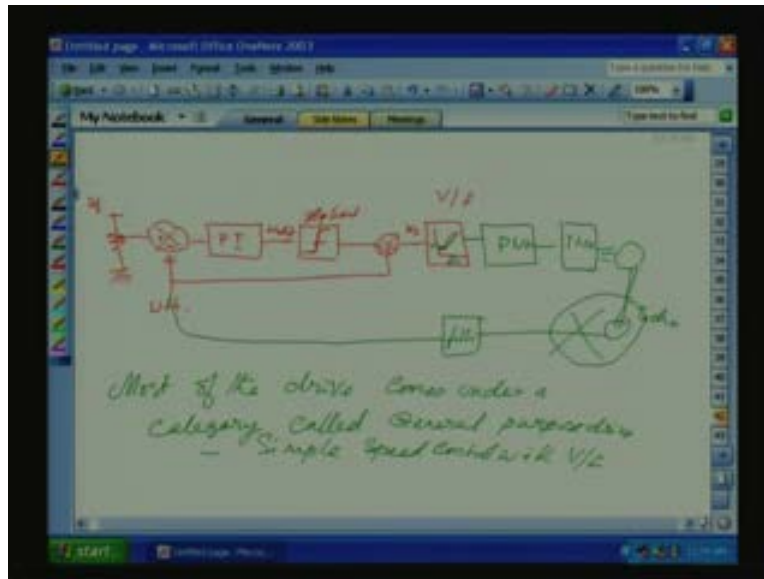
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What is omega S? Omega S is also omega S is equal to 2 pi f; if it is 50 hertz, 2 pi into 50. So, for the speed variation, as omega S varied; the applied frequency varies, V also

should be varied. This is called V by f operation. So, V by f operation is called to keep the flux constant. Now, how we can do it? We have to find out for low values of slip, from the steady state equivalent circuit; so what all things required for a speed control now? We want to keep V by f constant.

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Now, how do you control it? How do we can control the speed feedback? So, this we can do it like this; we have the speed reference, this is our speed reference, this we will be **sorry** this is our speed reference, this we will be giving to plus minus. So, we will use a PI controller here because speed is compared with this one. Now, this speed feedback that is omega feedback if there is a difference between the reference and speed feedback, that shows there is a change in the driving torque and the low torque. That is why the mismatch in speed happens and also we know that torque that means we have to adjust the torque.

Now, we know the torque is proportional to slip. So, this output will give the slip. So, any change in the speed reference and speed control indicates, there is a change in torque that means there is a change in slip, so you have to give the correct slip. So, slip also we have to limit it. As I told, slip will be 5% of omega S. So, you should have a positive slip or negative slip, that limiter should be there. This is called slip limit. Then this slip limit we got; so now we add to due to an inverter with correct switching frequency that is f_s , so this and this should be added, omega feedback we can add this one, this will give our omega S.

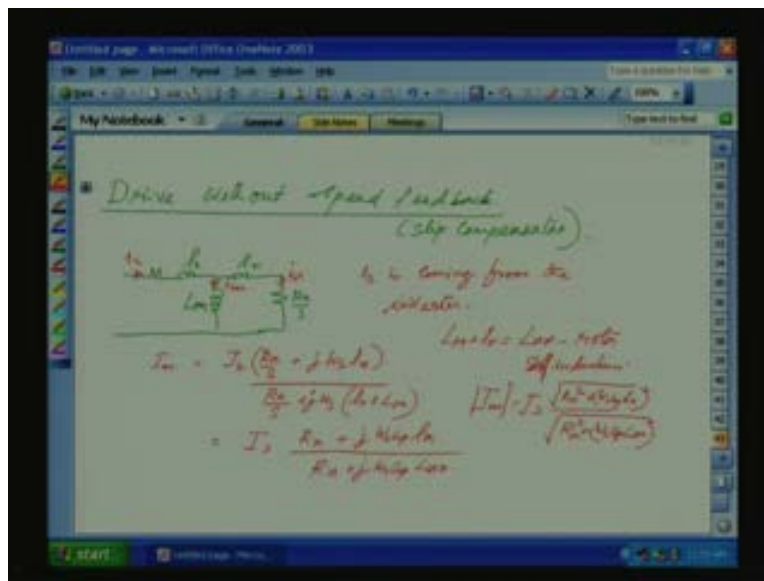
Now, V by f operation; so as the omega f varies, we have to generate the corresponding V, so this way; this is our linear control that is V by f operation but see this V by f operation, we assumed that starter drops are negligible. But low frequencies of operation when V is proportional to phi into omega S, omega S is very small, this voltage E will be very small and the applied voltage will become equal to the starter drop. So, low

frequency we will give an approximately around low speed upto around 5%, we will give a boost here to take care of the starter drop, see here. So, the characteristics goes like this; this is V, this is a look up table we can generate.

So from, once V E is known, we can generate, the f_s is known, we can generate a equivalent sine wave form and we can compared with sine triangle or from the V , we can find out the, we can go for a space vector PWM. So, that we have studied now, so this is the PWM generator. See, this F_s we can put into your look up table and we can just generate a sine wave and multiplied by the amplitude of V , so E compared with the triangle that is also possible, PWM waveform, this we give it to inverter. This is our inverter, inverter will go to the motor and the motor is connected to a tacho for speed measurement and tacho feedback will gain with the filter, we will give it here. This way we can control the speed. See, this is called V by f operation.

Now, for many of the general purpose application, see most of the drive, they are called, comes under a category called general purpose drive that means these are simple speed control is only required, simple speed control with V by f , V by f operation; V by f operation, keep the flux cost that is especially for fans, industrial fans, a general purpose drive is sufficient. So, to make a cost effective solution, many application, they do not want a tacho; so, that can save the thing.

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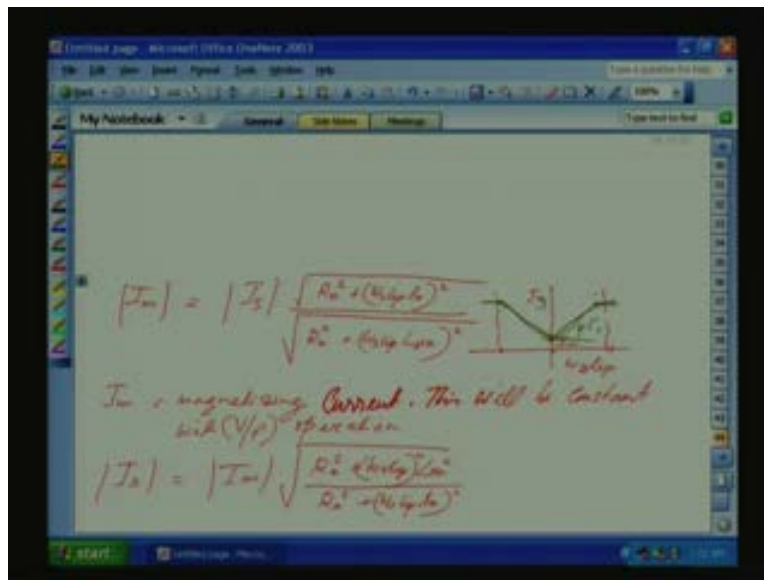


So, drive without speed feedback; how it can be done, drive without speed feedback? That means previously we have found out the slip from reference speed and the feedback speed, now we have to find out a different way of finding out slip. So, this is called slip compensation that means slip compensator; the tacho we have to remove and we have the indirect way of we have to find out the slip. How we can find out the slip? Let us say, let us go to the, again we will use the steady state equivalent circuit.

So here, this is our L_m leakage, $I_r R_r$ by S and here you have the i_m , here you have i_s and here you have i_r . So, i_s is we are supplying from the inverter, i_s is coming from the inverter. So, what is i_m ? Assuming sinusoidal currents are flowing the machine, we will take the fundamental component from the PWM. So i_m , the peak value of sinusoidal i_m is equal to i_s into R_r by S plus j into ωS into L_r that is i_s is coming, the current flowing through this one is equal to i_s into impedance of this one divided by the total impedance. So, this will be equal to simple network problem plus is equal $k \omega S$ into L_r plus L_m . So, we can put this L_m mutual inductance plus rotor leakage inductance is equal to L_r^r , this is rotor self-inductance.

So, this implies this is equal to i_m is equal to i_s into R_r , see I_m by S , so R_r plus $S \omega S$ is equal to slip, so this is to bring the slip inside, a slip is required, torque is preferred to slip that is the **synchronous** frequency divided by the rotor frequency into L_r divided by R_r plus $k \omega$ slip into L_r^r . So, for sinusoidal excitation, the i_m , the maximum value of i_m , the peak value of the i_m , $i_m \text{ mod}$ is equal to i_s into root of R_r square plus ω slip into L_r whole square divided by R_r square plus ω slip into L_r^r whole square. Now, for a machine, i_m gives the magnetizing current. So, let us go to the next page.

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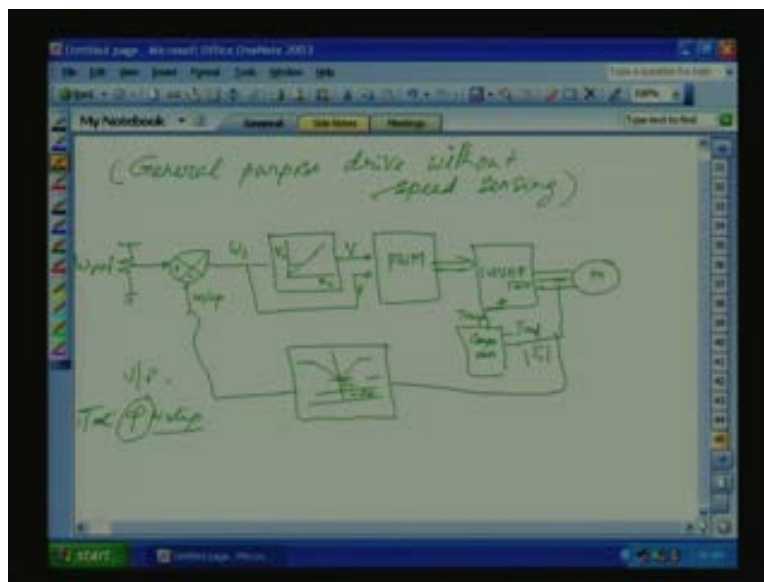
Let us write down, the mod i_m is equal to $i_s \text{ mod}$ into root of R_r square plus ω slip L_r whole square divided by root of R_r square plus ω slip L_r^r whole square. Now, i_m is the magnetizing current, so this will be constant that is the peak value of the sinusoidal value, the sinusoidal current; constant with V by f operation. So now i_m ; what is i_s now? So, we have to find out i_s for a particular slip; i_s is equal to i_m into root of R_r square plus ω slip square into, from this equation, from the this equation, L_r^r square divided by R_r square plus ω slip into L_r square.

So, approximately if you plot this one for a machine, we can compute for a machine, parameters are known. So, if you know with slip, ω slip we are restricting between

plus or minus 5% of ω_s ; so this will be approximately, curve will come, this is our i_m . So, as the slip varies, torque varies, i_s will vary along this point and we are restricting to point. So this one, for general purpose V by f drive application, we can also approximate like this, linearize it, linear curve that means i_s . When ω slip is maximum, i_s is maximum.

So, i_s is equal to, this value i_s gives $i_s i_m$ plus I_r value; so i_m is this value. When ω slip is zero, there is no load current; the i_m is equal to i_s . As the slip varies; i_r varies, from here, I_r varies, i_s also varies. So approximately, this curve, we can generate in a simple control system. How do you generate a closed slope control with this one? So, let us see.

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See, we do not or we want only general purpose drive with slip compensator or without speed sensing, without speed sensing at cost of active solution. So, you have a speed reference, this is our speed reference that is our speed. But for inverter, we require the similar speed. So, to the speed reference, we will add the slip; this is our ω slip, this is our ω_s . So, we have a look up table here. For any motor, what is V versus f?

ω_s , till low speed, we give a boost of 5%, then it will go linearly like this. This is our V_s . So, V_s will come here, ω_s is also comes here, PWM generation. Let us say sine triangle PWM with V and ω_s which is proportional to V by f. We can have a sine look up table, amplitude is controlled by V, frequency is controlled by f and it can be compared with a triangle waveform. So, sine triangle PWM, we have studied. So, this will go to the inverter, inverter to the motor.

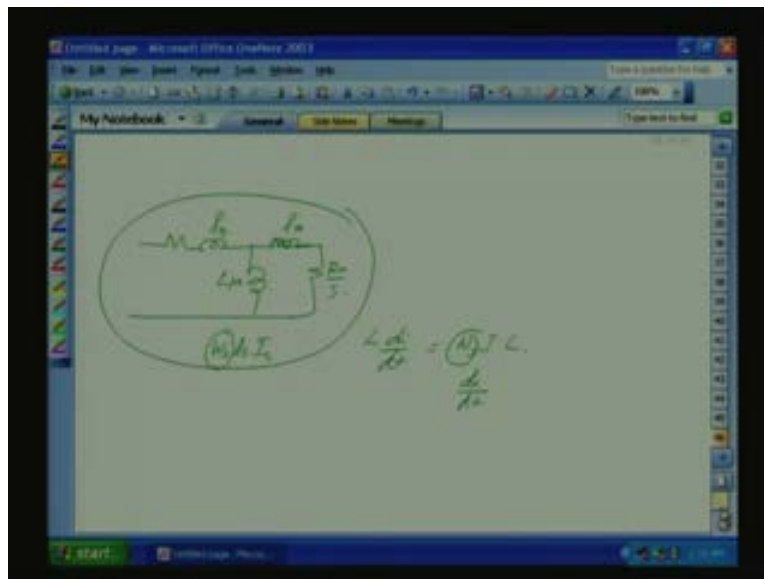
See, for any motor, protection is required. So, current sensing is a must so that any **our** current happens, immediately you have to switch off the system. So, from the current reference, this is our absolute value, i_s we are sensing. The same i_s we have compared with the I reference value, maybe this I reference is around two times or 1.5 times the

maximum load current you set it, a comparator will be there; when it goes more than that value, we trip the system, tripping, signal will be given here that is trip signal.

Now, this same i_s , we bring it here, we have look up table from our slip compensator, this is the look up table with slip, this is our i_m omega slip. So, from here for the machine for the particular i_s , we can find out the slip from here look up table and feed it here. So, this is a general purpose drive. This PWM generation, whole thing can be used for micro controller or in a DSP environment also very easy to implement. So this way, it goes.

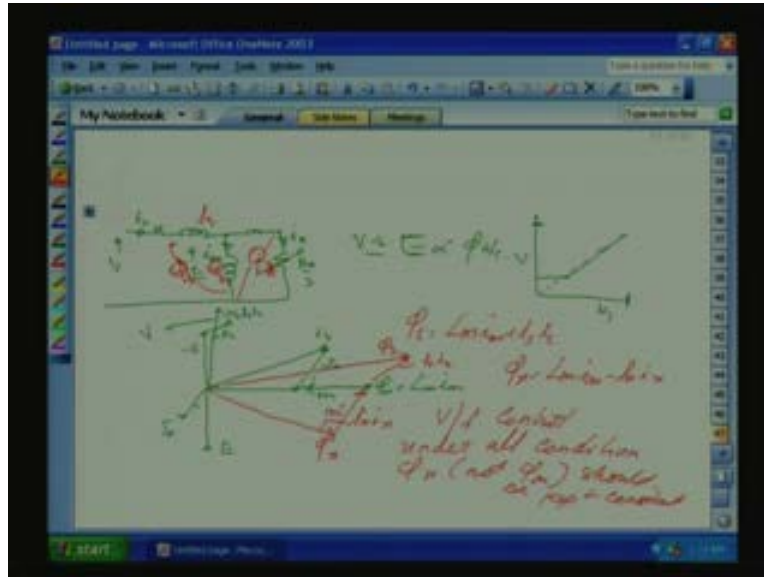
So, for throughout the modulation of region, throughout the region, we will keep the V by f constant, this is V by f control. Now, we said, the torque is proportional to linear region, it will be proportional to s into omega slip that is torque. So, sV kept constant, using V by f and slip torque is controlled with a slip so that means the correct V by f is given to the machine with slip information and the rotor speed.

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But if you see here, we have assumed sinusoidal steady state equivalent circuit that means our equivalent circuit is like this; so this is our inductance L_s , this is L_r , L_m . So, we found the drop across the L_s equal to $sL_s i_s$ that means for a sinusoidal excitation, the rate of change of current or the voltage is only due to the variation of s . But many drive application, there is a steady state, many drive application, the amplitude also can vary. So, $L \frac{di}{dt}$, we are taken as $sL i$. There can be change of not only with frequency, amplitude also can change. That means the $\frac{di}{dt}$ terms not only the frequency, the amplitude also can vary that is a dynamic condition. Then the steady state equivalent circuit is not the correct one for the dynamic conditions. So, what is meant by dynamic condition? Let us see.

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So, our steady state equivalent circuit is like this; so this is our i_m , this is our i_r and this is our i_s and first approximation we said E and V , we said V is equal to E , stator drops are negligible and E is proportional to ϕ into ωS . So, we approximated in a V by f control that is $\omega S V$, we approximated like this. For low speed, we give a boost like this; so our V by f curve, it goes like this. So, this is our high speed operation, this is the maximum rate at voltage for V .

So, that means if this assumption is correct, we are keeping the i_m , the voltage flux air gap flux is constant. But if you see here, the current which is producing the torque is the i_r . So, if you see the phasor diagram for this one, see, you have the i_m here, i_m , we have the E here, this is our minus E . Why minus E ?

See, if you see according to notation; the current is going from V , here it is entering E . So, current leaving we will take positive, current entering we will take negative that is it has gone here. So, minus actually when we compute the V , we have to take the minus one, I am take it. So E , so with to this E region, it will have a current i_r here. So, stator will to compensate, it will generate another i_r , minus i_r and you will get the i_s here, this is the i_s .

Now, **added** to this one, $i_s R_s$, $i_s R_s$, then ωS , $j \omega S$ that is 90 degree $\omega S L_s$ into i_s . So, this is our V , this is our V , actual V . When you assume this stator drops are negligible, this V will be along E ; this is the i_m . Now you see where is the air gap of flux? Air gap of flux will be proportional to i_m . So, let us say our air gap flux will be flux will be ϕ_m is equal to L_m into i_m , i_m the peak value; this is the peak value of the air gap flux. But if you see the flux which is coupling to the stator is the flux due to the leakage plus i_m that is this is the flux, let us say this is i_s , this is the flux coming here, this is our i_s , air gap flux.

Similar way, the flux which is coupling to the rotor conductors, the leakage flux, this leakage flux will not be, it is only coupling to the rotor conductors, it will not come to the stator through the air gap. So, this air gap flux is the one which coupled both the stator conductors and the rotor conductors. So, the total flux which is coupling to the rotor is the rotor flux, let us say ϕ_r is this one. I have put ϕ_m here, so ϕ_m , we will put ϕ also here; this is ϕ_r , this is ϕ_m , this is ϕ_s .

So, let us find out where is our according to this notation, where is our i_s or i_r ? But ϕ_s , the ϕ_s is equal to $L_m i_m$ plus L_s leakage inductance plus i_s . So, $L_s i_s$ will be parallel to i_s , so it would be along this direction. So, for a particular value of this is $L_s i_s$. So, our ϕ_s will be here. Where is our i_r ? i_r will be, current operation is in this direction, the i_r will be equal to $L_m i_m$ minus $L_r i_r$. So, minus i_r , minus i_r means i_r is in this direction, so minus i_r will be i_r into i_r into L_r will be in this direction, parallel to the i_r , this is $L_r i_r$ and our i_r will be, our ϕ_r will be in this direction. That means ϕ_r is equal to $L_m i_m$ minus of $L_r i_r$. So, that is this way it happens, i_r will be here.

So, if you see here, this ϕ_r and $L_r i_r$ will be 90 degree that means the current same like separately excited machine because of the winding separates, the flux is orthogonal to the stator current vector. But here in the induction machine, for all dynamic conditions, to control the rotor and keeping the flux constant; so that means this $L_r i_r$, L_r is a leakage inductance and is a value; this i_r current vector will always perpendicular to the i_s because if you say ϕ_r , ϕ is L_m this one and $d\phi_r$ by dt is the voltage is coming across R_r by S and R_r by S is the resistance, so voltage and current will be in the same phase and ϕ_r is sinusoidal excitation, ϕ_r we will be 90 degree phase shifted. So, that shows ϕ_r and i_r will be 90 degree phase shifted. So, that means for V by f control under all conditions all conditions, ϕ_r and not ϕ_m , should be kept constant should be kept constant.

So, now the question is for good dynamic performance that the flux should not vary, the flux which is responsible for the torque component that is i_r should be kept constant for dynamic conditions. Then only we can have a high dynamic performance from the induction machine similar to a separately excited DC machine. This we will study. Then, the steady state equivalent circuit is not valid because in the steady state equivalent circuit, the $d\phi$ by dt or di by dt times we assume that only the frequency is varying magnitude is not varying. But for all dynamic conditions, the magnitude variation also along with the frequency variation we have to take into consideration. Then, we require a dynamic equivalent circuit; from the equivalent circuit, what is the condition, the V by f that is constant flux operation under dynamic condition that means rotor flux keeping constant, how to achieve? We will study in the subsequent class.