Power Electronics

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Lecture - 33

Dynamic Model of Induction Motor Part one

So, yesterday we talked about, we derived the space phasor based voltage equations for the stator and rotor with respect to alpha beta reference axis and alpha beta what we told is alpha is placed along the a space reference axis, so alpha beta we called the stationary references.

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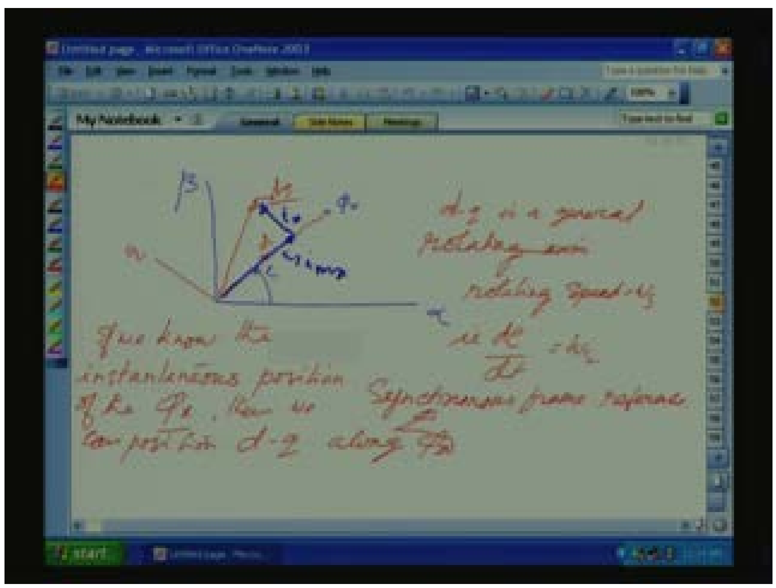

Now, finally what we want? We want the instantaneous position of a flux. So, we want a general rotating reference frame dq, dq such that d should be always placed along the si r axis. So, while doing so, the current i_s, stator space phase current we can split along this phi r and orthogonal to it and by controlling the orthogonal component, we can control the torque same like in a separate excited DC machine and by keeping the current alone si R constant under all dynamic conditions, then flux would be kept constant. So, we will get a performance similar to a separately exited DC machine. So, what we require here is instantaneous position of the si R is needed.

So now, let us derive the stationary reference model - alpha beta. Let us derive it with respect to a general rotating reference frame dq so that then we can appropriately place a speed of the dq so that we can get either alpha or if the speed of the reference frame is 0, then again we will get alpha beta; speed of the reference frame is equal to omega s, we will get synchronous reference frame and speed of the reference frame equal to omega r, we will get the rotor reference frame. So, what we are going to do now is a general dynamic equation with respect to a general rotating reference frame. So, let us go to the next page.

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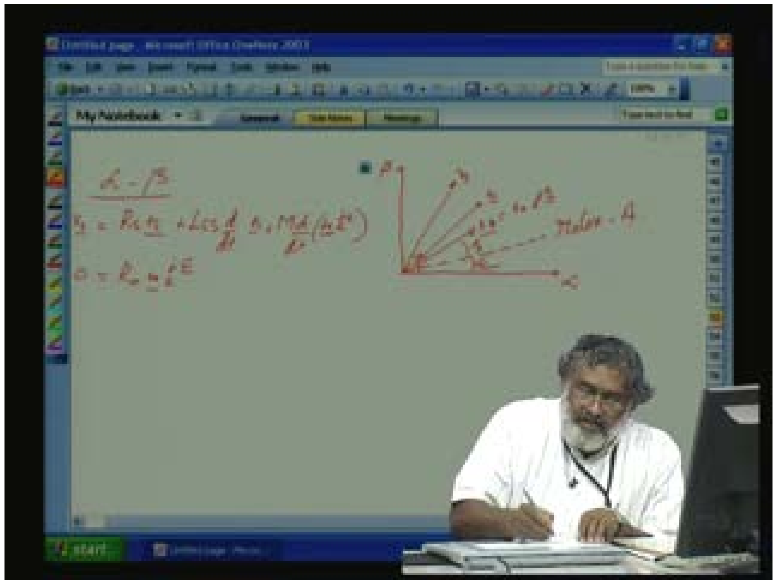

Let us again draw our axis; this is the alpha and this is beta. So, we already got voltage space vector space phasor based stator equation and the rotor equation, this is i_s. Then we said, this is the rotor A axis or rotor alpha axis, rotor A axis; I will make it bigger so that it will be clear, rotor A axis. So here, with respect to rotor axis, we have the current i_r, this is theta. So, equations we have derived.

We will again write the equation alpha beta, with a reference to alpha beta, we will repeat the equation. Vs is R_s i_s plus Lss d by dt i_s plus M into d by dt of i_r originally defined with respect to the rotor axis that is i_r into e raised to j zeta. We want to bring it the stator axis, so this i_r space vector, we have to multiply it by e raised j epsilon. So, angle is increased now; this is our epsilon. So, stator equation defined with respect to alpha beta axis.

Now, the rotor equation also we want to define with respect to alpha beta axis. So, that is R_r i_r e raised j epsilon; multiplied by e raised j epsilon, we will be defining with respect to alpha axis that means from the rotor axis, we are coming to the alpha axis plus Lrr d by dt i_r e raised j epsilon plus M into d by dt i_s e raised minus j epsilon. This is the stator current; this stator current i_s underscore, stator space vector defined with respect to the stator by multiplying e raised, it has gone to the rotor axis. So, you will get the current, the stator current, the component of the stator current which is contributing to the induced voltage in the rotor multiplied by M, M into d by dt. So, this is defined with respect to rotor axis.

Now, everything we have to define with stator axis, so it will be e raised j epsilon. This e raised j epsilon is to bring to the original rotor voltage equations defined with respect to rotor axis should be brought to the stator axis. Now, what we want? We want to bring everything to a general rotating reference frame.

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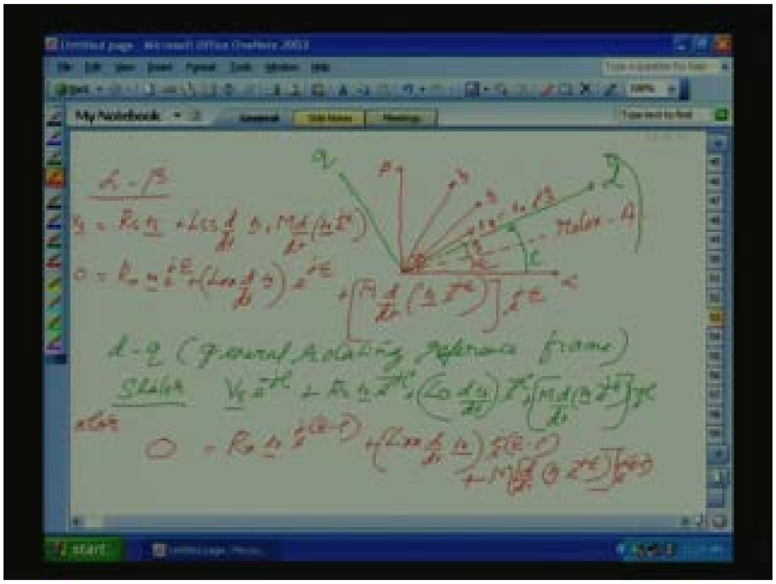

So, let us see the general rotating reference frame, we are not worried about the si r now. This is d and perpendicular to that one, this is q, this is the dq axis and we know instantaneous position of the dq with respect to the stator axis the angle position is rho. So now, dq axis, general rotating reference frame; now let us start from the stator equation to the general rotating; Vs originally defined with the stator axis we have to move to the d. Now the angle, this is the general rotation, anticlockwise rotation is the general rotation of all the axis.

So, when we go here, angle is that reduces to e raise to minus j rho plus R_s i_s underscore e raised minus j rho plus Lss. Now, this d i_s by dt, this is the complete voltage term; this voltage term, we want to bring it to the dq axis. So, this will be outside the d by dt time. Similarly, for the mutual inductance time also, M into d by dt i_r e raised minus j epsilon whole thing multiplied by e raised minus j; full stator equation we are transferred to the dq axis.

Now, let us see the rotor; zero, squirrel cage motor, so then R_r i_r e raised j epsilon j into epsilon minus rho, this we can do it, we are multiplying by e raised minus … plus Lrr d by dt of i_r e raised j epsilon minus rho plus M into d by dt of i_s e raised j minus epsilon into e raised j epsilon minus rho. So, the rotor equations also we have transferred to the dq axis. So, our dynamic equation is complete. Now, we have to take the real and imaginary component. How do you take the real and imaginary component here? See, let us go to the next page.

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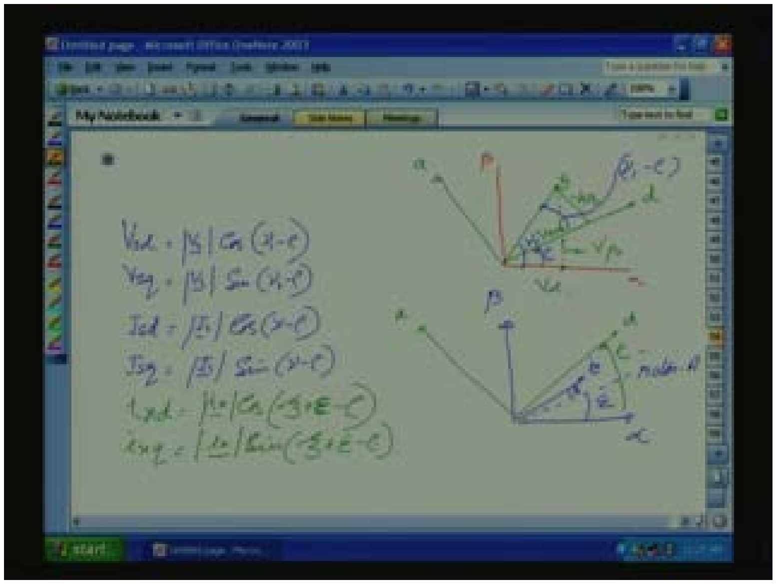

Let us draw again for our clarity, our alpha beta axis and this is our dq, general rotating axis. Suppose Vs is here, the Vs component along d and perpendicular to q, this is called Vsq and this is Vsd. Now, the same Vs if you project it to the alpha beta axis, the real or imaginary component, magnitude will vary. This is V beta because as it is parallel to beta axis, this is V alpha. So, the real and imaginary components are the projection of the voltage space phasor, the projection to the corresponding reference frame axis. Then dq is the real and the q axis gives the imaginary component. So if you see here, this is rho; this is rho, this is gamma one.

So, what is Vsd? If you know the alpha beta component, how do you find out the Vsd and Vsq? Vsd is equal to Vs mod cos of gamma one minus rho. What is gamma one minus rho? That is this angle, this angle is the gamma one minus rho angle and Vsq is equal to Vs mod sign gamma one minus rho; this is true with i_s also, i_s the angle is gamma. So, Isd is equal to I_s underscore mod cos gamma minus rho, Isq is equal to I_s underscore sin gamma minus rho.

What will be for the rotor? So rotor, again for clarity, I will again draw it here; this is alpha beta, this is our rotor axis, rotor A phase axis or rotor alpha axis, rotor A. This is making with respect to this one, this is epsilon and here is your I_r and this is zeta and our dq will be, dq somewhere here, d you only know the dq angle, this is q, dq angle is rho.

So here, ird is equal to i_r mod magnitude into cos of originally defined with rotor reference space axis that is shift to alpha axis epsilon, then we have to bring to the dq, then irq; this is also equal to i_r mod sin of… Now let us, our original equation, let us bring to the dq axis.

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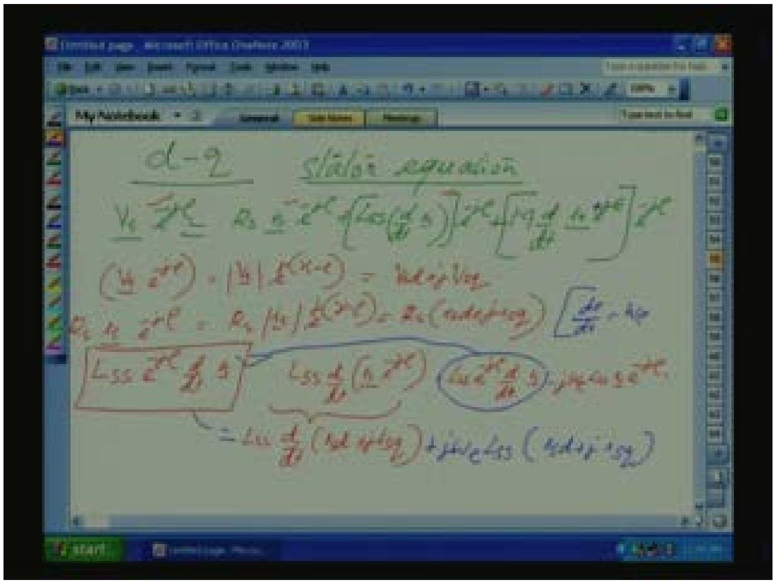

So, let us take our stator equation first, original stator equation dq, stator equation that is Vs underscore e raised to minus j rho plus Rs i_s e raised minus j rho plus Lss d by dt i_s e raised minus j rho plus M into d by dt i_r defined with respect to the rotor axis, transferred to the stator and then finally to the dq sorry this is equal to, not the plus here. Now, this we know it; Vs e raised minus j rho is equal to Vs mod into e raised j gamma one minus rho is equal to Vsd plus j Vsq; so this term is over.

Now, the second term that is Rs into i_s e raised to minus j rho that is Rs into i_s mod e raised j gamma minus rho that is equal to Rs into isd plus j isq. Now the previous one, now let us see, this term. This term if you see here, this e raised minus j rho is not inside, it is outside. See, if it had been inside, then the whole thing d by dt of Isd plus jsq we could not written but how to bring the Isd plus jsq? Let us take it, let us take the term that is Lss e raised to minus j rho d by dt of i_s, this term. This is the term we want, so what I will do?

I will put this e raised minus j rho also inside. So, that will be equal to that means we are going to find out Lss d by dt of i_s e raised minus j rho. This will be equal to Lss e raised minus j rho into d by dt of i_s. Now, the magnitude also we are going to differentiate, so we are taking all the dynamic conditions. Then what is d rho by dt? It is omega rho, so this will be equal to minus j omega rho into Lss into is e raised minus j rho.

So, if you see this equation, this equation we can write as Lss into d by dt of isd plus j isq and this is the term what we want, this is exactly this one. So, whatever this term, so this will be equal to we will bring this one this side so that means plus j omega rho Lss, this is again equal to isd plus j isq. So, the second voltage term is also solved, here, we have solved it.

Now, let us find out the rotor type. Same way, we will bring the minus j rho inside and then differentiate and subtract this term, the same technique we can use throughout. See, in the original equation Vs e raised minus j rho, this i_r e raised j epsilon because i_r originally defined with respect to the rotor axis; we have to bring to the stator axis. So, it is not minus j epsilon, this is plus and we are going to the dq axis that is why e raised minus j rho.

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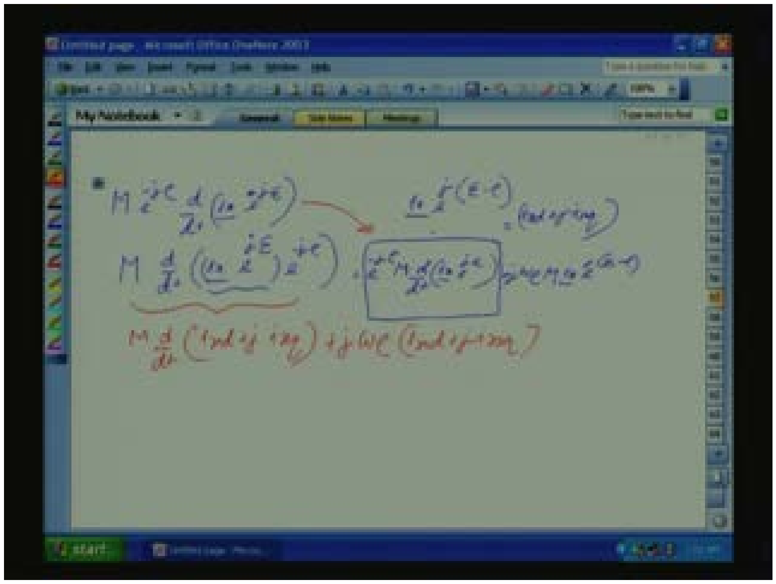

Now, let us take in the stator equation, the mutual inductance time, the voltage due to the mutual inductance that is M into e raised minus j rho d by dt of i_r e raised minus j epsilon. But we know that i_r, this is plus because we are trying to bring it to the rotor side, i_r e raised j epsilon minus rho is equal to ird plus j irq. See here, this i_r originally we defined with respect to rotor axis; then multiplying by e raised j epsilon, it will come to the rotor stator axis, then again multiplying by e raised minus j rho, it will go to the dq axis. So, the real component will be ird plus j irq.

But if you see here, e raised minus j rho is outside. So, we will bring this way here; M into d by dt of i_r e raised j epsilon into e raised minus j rho together here. So this when we differentiate, it will be, first let us take this will be equal to, first will differentiate with this term; so e raised minus j rho M into d by dt of d by dt of i_r, d by dt of ir e raised j epsilon. Then we will differentiate with respect to this one that is minus j omega rho M into ire e raised j epsilon minus rho.

So here, what we want is this term that is if this term; but we know this term is equal to M into d by dt of ird plus j irq. Now, this term will be, this we will bring it here, so this will be equal to plus j omega rho into ird plus j irq. So, this also we split into ird component along ird along d and q axis. The same thing we can do for the rotor axis also. Let us go to the rotor equation.

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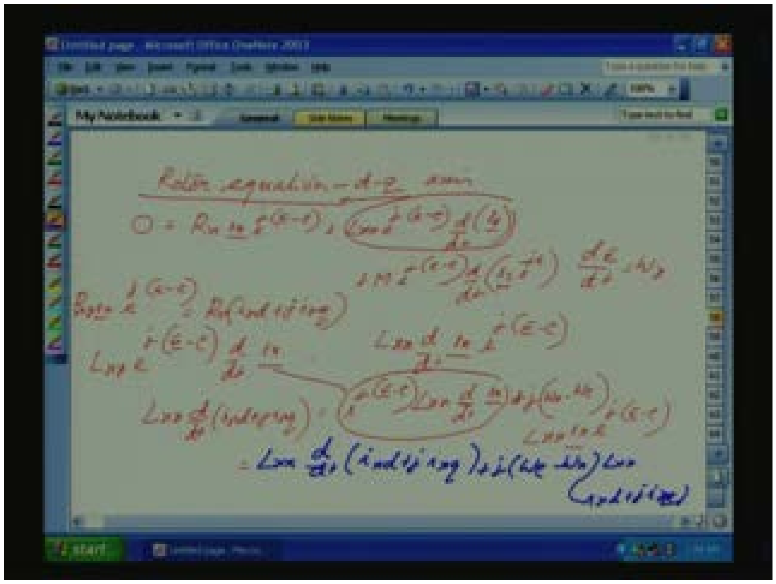

Now, the rotor equation; the original rotor equation is applied voltage is equal to zero is equal to Rr i_r e raised j epsilon minus rho plus Lrr e raised to j epsilon minus rho. This epsilon minus rho we will transfer all the rotor voltage and current equation defined with respect to the rotor axis, it will be transfer to the dq axis, d by dt of ir e raised minus j epsilon. See, the rotor equation is originally defined with rotor axis by multiply by e raised minus j epsilon, we are trying to bring it to the stator axis.

But already we have multiplied e raised j epsilon minus rho, so this will i_r mod e raised j zeta sorry this is originally defined with respect to the rotor voltage equation because due to the rate of change of current of the rotor current which is defined with respect to the rotor axis that is i_r. Then plus M into e raised j epsilon minus rho into d by dt of i_s originally defined with respect to the starter axis, now will be defined with respect to rotor axis. So, this is with respect to rotor axis, this is also with respect to rotor axis.

So, multiplying by e raised j epsilon minus rho, we will go to the alpha beta axis and then again from minus rho, it will go to the dq axis. So, this rotor equation is defined with respect to the dq axis. We know this one, first equation; Rr i_r e raised j epsilon minus rho is equal to Rr ird plus j irq, this is Rr.

Now, let us take this term, second term that is Lrr e raised j epsilon minus rho into d by dt i_r. See, this i_r is originally defined with respect to the rotor axis. Now, both the stator and the rotor axis we define with respect to the stator alpha beta axis that is why we have multiplied outside by e raised j epsilon. Now, the whole equation, we are transferring to the general rotating reference frame that is why we are multiplying by e raised j minus rho.

Now again, this we will write as, this inside will bring it and then differentiate that is Lrr d by dt of i_r e raised j epsilon minus rho. This we can write as this is equal to, first we differentiate with the i_r space vector. So, e raised j epsilon minus rho into Lrr into d by dt i_r, then plus j epsilon minus rho that will be equal to j into omega r, rotor speed d sigma by dt is equal to d epsilon by dt is equal to omega; so d epsilon by dt that is omega r minus omega rho into Lrr into i_r e raised j epsilon minus rho. So, if you see here, this equation when you bring it inside, this we can write as Lrr into i_r Lrr sorry d by dt into ird plus j irq.

So, this term will be, this the one we want that is this term; by bringing this one this side, this will be equal to finally that term will equal to Lrr d by dt of ird plus j irq. When this bring here, this will be we can write plus j into the omega rho minus omega r into Lrr into ird plus j irq. So now, let us take the mutual inductance term in the rotor equation that is this term. Let us take go to the next page.

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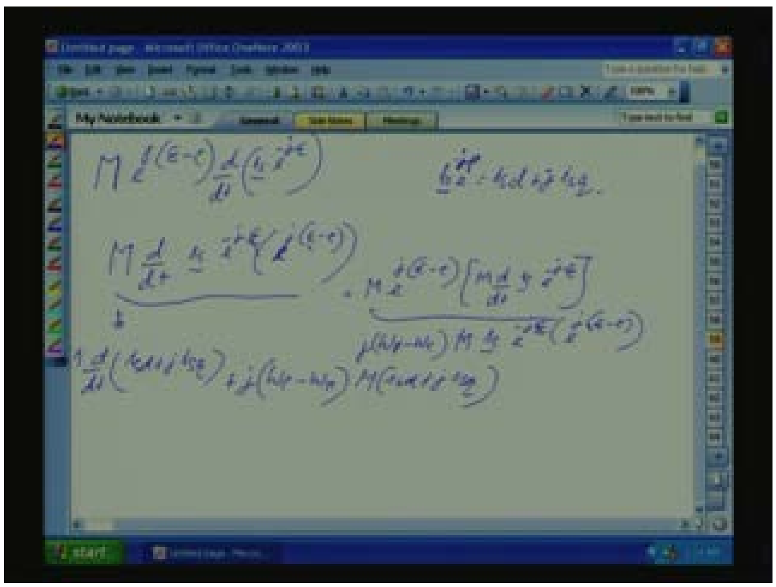

M into e raised j epsilon minus rho into d by dt i_s originally defined with respect to the stator axis, now the component of this one along the rotor axis will be equal to e raised minus j epsilon. So, this is the voltage term M into d by dt i_s e raised j e raised minus j epsilon, this is the voltage term induced in the rotor. Now, this one we want to transfer to the stator axis so that is why we have multiplied by e raised j epsilon and then again multiplying by e raised minus j rho, we are bringing to the dq axis. So, here also we know i_s e raised j e raised minus j rho is equal to isd plus j isq.

M into d by dt of i_s e raised to minus j epsilon into e raised to j epsilon minus rho; this will be equal to, first we will differentiate with the first term that is equal to M into e raised to j epsilon minus rho, M into d by dt is e raised minus j epsilon. Then, then we will differentiate with respect to this one. That will be equal to omega r minus omega rho j into M into i_s e raised minus j epsilon into e raised to j epsilon minus rho. This term, we know it is equal to M into d by dt of isd plus j isq because this two cancels, here also this two cancels. So, this term is by bringing this one here, so this will be equal to j omega rho minus omega r into M into isd plus j isq.

So all the terms, all the voltage terms, we have brought into the real and imaginary components; we have to subtract out the real and imaginatic component. Then we will get the real and dynamic equations, in dq reference term the real and imaginatic component for the stator and rotor. So, let us do that one.

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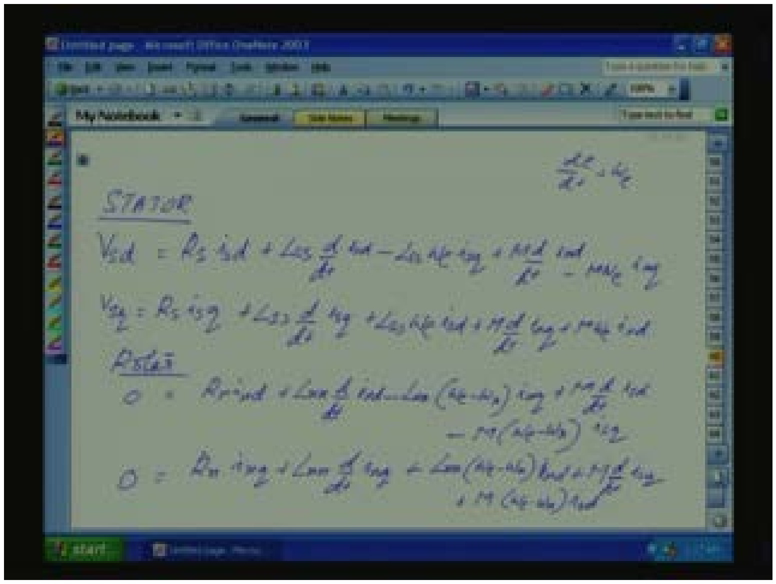

So, the final equation when we transfer to the real and imaginatic component; the stator will be, final equation - Vsd is equal to Rs isd plus Lss d by dt isd then minus Lss omega rho isq. So, real component, the j component we will put into that is Vsq component, we are separating now, d component and q component; plus M into d by dt of ird minus M into omega rho into irq, omega rho is the subscript here sorry this is the subscript, d rho by dt, d rho by dt is the omega rho, omega rho.

Now, Vsq Rs isq plus Lss d by dt of isq plus Lss omega rho into isd plus M into d by dt of irq plus M omega rho ird; stator dynamic equation is ready. Then rotor, Vrd is equal to zero, we are not applying any voltage. So, this will be equal to Rr ird plus Lrr d by dt of ird minus Lrr into omega rho minus omega r into irq plus M into d by dt of isd then minus M into omega rho minus omega r into isq. Now the q axis, again zero is equal to Rr irq plus Lrr d by dt irq plus Lrr into omega rho minus omega r, omega r is the rotor speed into ird plus M into d by dt of isq plus M into omega rho minus into omega r into isq. So, we got the dynamic equation in dq reference frame.

Now, dq reference frame is at the dq reference frame is rotating with a general speed omega rho. If it is not rotating, that omega rho is equal to zero. Then the equations are defined with respect to the original alpha beta axis. So, making omega r is equal to zero, then everything is in alpha beta axis; the d component become Vsd component become Vs alpha and Vsq component become Vs beta. Now, omega rho is equal to omega s, then we will get the synchronous reference frame that means our dq reference frame is rotating with respect to the synchronous rotating reference frame. The speed of rotation is omega s.

Now, what we want? We want to align the dq which is rotating with synchronous speed to our phi r, axis phi r space vector axis and phi r is also rotating with similar speed; this we will study in the next class.