Power Electronics

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Lecture - 34

Dynamic Model of Induction Motor Part – II

Last class we derived the real and the imaginary component of the stator and rotor voltage equation with respect to a general rotating DQ reference phase and the general rotating with a speed of omega rho; if you see here, d rho by dt is equal to omega rho. So with respect to, rho is measured with respect to our stationary alpha beta axis. Now, based on this equation, let us format our dynamic dq equivalent circuit.

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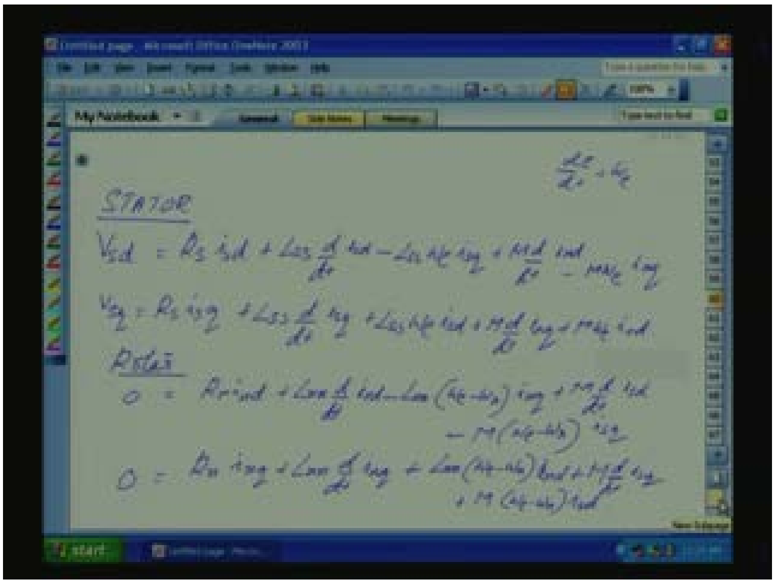

So, d rho by dt is omega rho; this is the general rotating reference frame speed. So, V is again, we will rewrite this one and formulate our dynamic equivalent circuit model.

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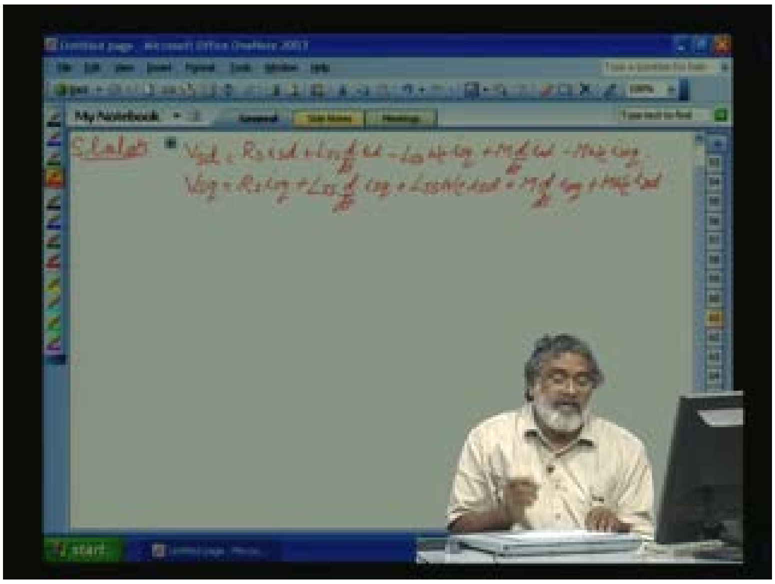

So, let us write down. Stator V_{sd} is equal to Rs isd plus Lss d by dt isd minus Lss omega rho into isq plus m into d by dt ird, ird means the rotor d axis current minus M omega rho irq; this the stator V_{sd}. Then stator V_{sq}, V_{sq} is equal to, see if you see here the stator, the d by dt isd, this will take care of the magnitude variation and this will take care of the speed variations.

But due to the speed variation; it will go to the, time will go to the q axis. So, V_{sq} is equal to Rs isq plus Lss d by dt isq plus Lss omega rho isd plus M into d by dt irq plus M omega rho ird; this is the stator. Now, let us take the rotor.

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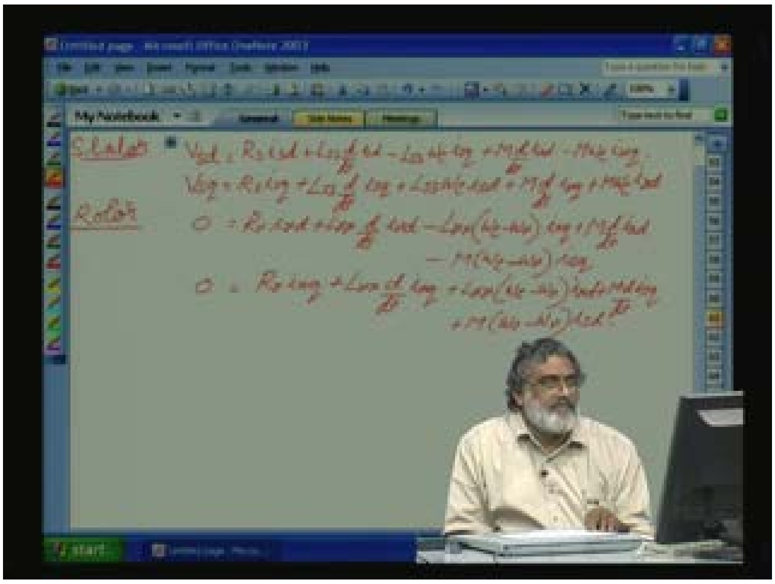

So here, dq axis is applied voltage is zero. So, that is equal to Rr ird plus Lrr d by dt ird minus Lrr omega rho minus omega r. See, the voltage time due to the rotation at the rotor is the relative speed. Rotor is already rotating with omega r and our rotating reference frame is omega rho. So, relative speed is omega rho by omega r. But if you see the stator, it is omega rho because stator is stationary. So the rotating voltage, the voltage due to the rotation, omega rho is depending on the speed of our reference frame. Omega rho minus omega r into irq plus M into d by dt isd minus M into omega rho minus omega r into isq; this is the rotor d axis.

Now, let us take the rotor q axis. This is equal to zero is equal to Rr irq plus Lrr d by dt irq plus Lrr into omega rho minus omega r into ird plus M into d by dt isq plus M into omega rho minus omega r into isd. Now, let us based on this dynamic equation d and q; see, what we did? First we defined the voltage and voltage space vector for the stator as well as the rotor with respect to the alpha beta reference frame axis. Then we transferred whole voltage turn to the general rotating reference frame axis dq, the speed is the position of the general rotating reference frame axis with respect to alpha beta axis is rho at any instant and D rho by the speed of rotation is omega rho. Then we separated into real and imaginary component; the d and q component.

Now, we got the dynamic equation control conditions. That means when the amplitude as well as the speed or the frequency varies. Let us take the first stator equation. So, there is a V_{sd} voltage we are applying.

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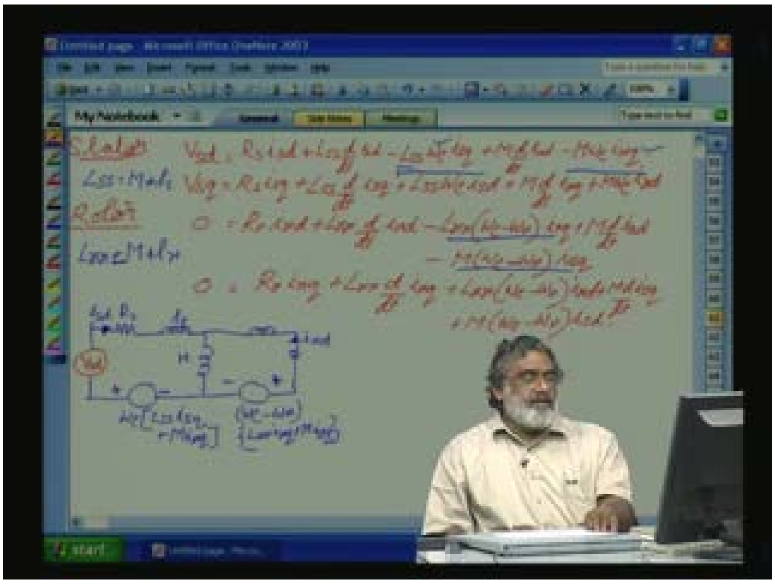

So, this is V_{sd}. Here, maybe we will use a different colour, so it will be clear, applied voltage. So here, there is a drop Rs isd; this is Rs, isd is this one, isd is flowing like this. Then Lss d by dt isd; so Lss is equal to, if you see Lss is equal to M plus leakage inductance Ls. So, let us draw the leakage inductance here. This is ls and there is Lss into isd. So, this isd is going to the mutual inductance also, then only you can say Lss. So, mutual inductance here, this is M, Lss isd.

Now, if you see here, these are voltage terms due to the rotation that means this one and this one that is this one and this one. This we can write down; if you see here, Lss plus omega rho isq plus irq, so this is due to the q axis. So, the rotational voltage due to the current in one axis will appear at the other axis, other orthogonal axis. So, before coming to that one, let us take the rotor term also. Rotor term Rr ird; so we have Rr here, Rr ird here also plus Lrr d by dt of ird. Lrr is equal to M plus lr, leakage inductance. So, we have the leakage inductance here. So, this also comes here. So, through the mutual inductance term here, both isd and ird is flowing. So, ird is here.

Now, what is this Lss omega sigma isq plus irq? This is we can say, the flux si sq. So, we will come to that one now so that we will follow. So, there is a rotational voltage term appearing at the stator due to the isq and irq. That means current flowing through the q axis equivalent circuit, these two and this is minus, that shows; see, we have taken current entering as positive, so this is minus means current comes, isd it comes like this. So, we can take this time as negative positive. So, current entering is taken as positive, so the term is like this. So, this is a function of omega rho, omega rho multiplied by Lss isq plus M into irq M into irq. What is this one? It will come to that one.

Similarly the rotor circuit, the D axis, Lrr that is this term, isq; here also it is negative, so with respect to ird if is a negative means we will put it like this and go. So, this is negative, this is positive because ird is going like this, ird is entering the negative side; so that conversion we follow. So, this will be equal to, this also rotational voltage term that is also equal to omega rho minus omega r into Lrr irq plus M into isq; this is the d axis.

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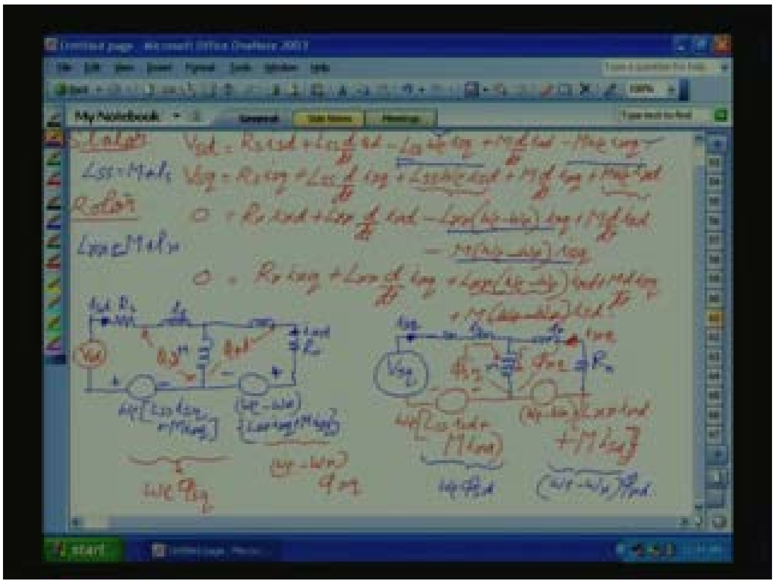

Similarly, we can write the q axis. Let us start with the V_{sq}, applied voltage V_{sq} and this is the isq. So, we have the Rs term here, Rs isq, again the leakage Ls and the mutual M here. Similarly, for the rotor side also you have leakage inductance lr and the resistance term Rr, this is Rr, rotor resistance. Here also the voltage term, so the voltage term polarity if you see here; for the q axis, this is the one and the next one is due to the rotational. So irq, M into irq; irq is coming in this way, isd isq is coming there, isq it goes through ls plus M that is why Lss.

Now, that voltage time, we will put it like this here, due to rotation. Stator side it is omega rho into Lss isd, Lss isd plus M ird plus M into ird; this is the term and polarity this is positive that means current is entering the positive terminal notation like this. Here also, rotor side also irq, there is omega sigma minus omega r, omega rho minus omega r that is this two terms Lrr ird plus M into isd, this is the term. So, if you see here, Lss isq, Lss isq is this one, ls plus M and isq is flowing here, then M into irq, irq is here. So, there is a current irq flowing here and isq flowing here.

So, if you see here, what is this one? This is called that is isq plus irq that is called si sq flux si sq, phi sq. Similarly, this one is equal to phi si rq; this is the flux coupling, this mutual and this is lr. Similar way, we can say this one is phi rd flux, this is phi sd, the flux which is coupling this ls and due to this ls and the current flowing through it, this is phi sd.

So, if you see here, what is this equation? Omega rho, this equation is equal to omega rho into phi sq, this one. And, what is this one? Omega rho minus omega r into Lrr irq, Lrr irq means plus M isq; Lrr irq is this one total flux plus M into isq. So, this is si rq; this is equal to into phi rq. So, due to the flux and the rotation, in the q axis, there is a voltage time in the d axis stator and the rotor.

Similarly here, what is this term? This term will be if you see here, this is omega rho into Lss isd that is this one plus M into ird that is this is phi sd, omega rho phi sd and similarly we can find out this term will be omega rho minus omega r into phi rd. So, our dynamic equivalent circuit is ready. This can also we can represent in this, in a matrix form. See, using this equation, let us say, let us go to the next page now.

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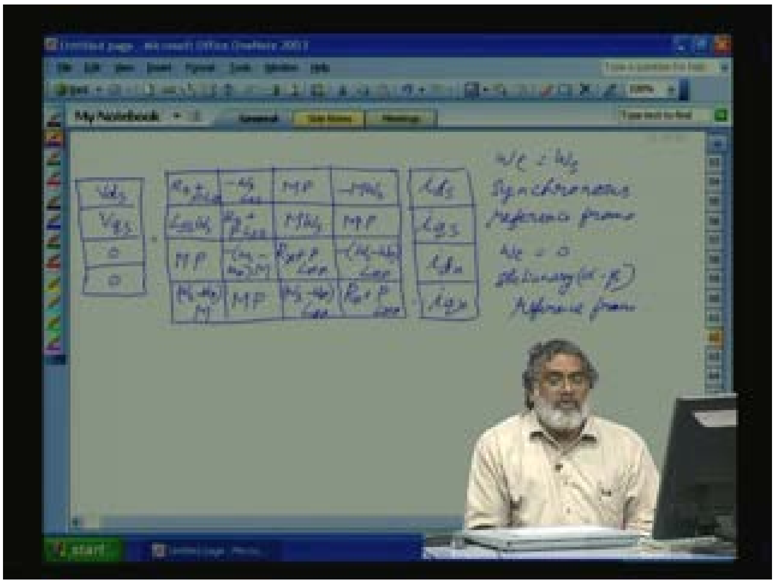

See we have, let us take omega rho is equal to omega s if you put it our dynamic equivalent circuit will be with respect to synchronous reference frame. Omega rho is equal to zero, then it will be stationary reference frame, stationary that means omega rho is equal to zero means dq will be aligned with alpha beta that is stationary alpha beta. the reference frame.

So, let us write down in matrix form, the equation. This will be we have four stator voltages that is Vds Vqs and the rotor zero is equal to you have four currents also, so this is a 4 by 4 matrix; 1 2 3 4, here also 2 3 4, this is our ids, iqs, idr, iqr. So Vds, the first term; Rs plus d by dt of Lss, d by dt we will say P Lss, P is an operator, P is equal to d by dt. Then this one will be minus omega s Lss, term will be here. Here will be M into P, P is d by dt; then minus M omega s that is a stator d voltages.

Then stator q voltages are Lss omega s, then Rs plus P Lss, P means d by dt, d by dt of iqs. Then M into omega s, see that is we are taking everything into synchronous reference frame that is the subscript s, synchronous reference omega s. Then M into P, P is the d by dt. Again here, M into P, rotor the relative speed is omega s minus omega r minus of omega s minus omega subscript r into M. Then rotor resistance and the self-inductance storm Rr plus P into Lrr, then minus omega s minus omega r into Lrr. So, rotor d axis voltage term is over.

Now, rotor q axis; this is omega s minus omega r into M, then M into P, omega s minus omega r Lrr. Then Rr plus P Lrr, P is the d by dt is an operator. See, we have the, we got the machine dynamic voltages within matrix form. So here, we can either use equivalent circuit or this one. If you see here, what is the torque produced? The torque produced is power divide by the frequency of the operation.

So, if you see here, in this equation, the power, the torque is power divide by the speed. So, power means voltage into current. So, the torque produced and that voltage which is responsible for the torque is due to the rotational voltage because of the rotation, we get the torque. So, all the rotational voltage is multiplied by current that is the power which is responsible for the torque production. So, if you see here, which are the ones here in the equation? One term is here, one term is here, then here, here, then here, from the rotor side this one, this; these are the terms.

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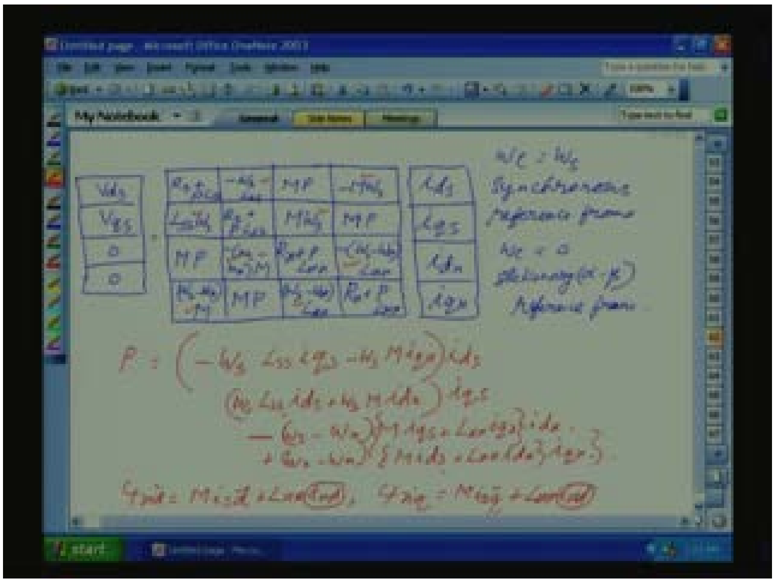

So, let us take out the power; P is equal to we can write minus omega s into Lss. So, this into iqs, this the voltage at the d axis, then there is another voltage time minus M omega s into iqr that is minus omega s M into iqr. So, this is the voltage term occurring at d and the current at the d is ids. So, we have to multiply this one with ids to get the power at the d axis. Then the q axis stator, again, this will be equal to omega s Lss ids that is this voltage term. There is another voltage time here, plus omega s M and the current is idr, this is the voltage term, so multiply it by iqs; this is the power at the stator.

Now, let us take the rotor, rotor dr that is minus of omega s minus omega r into M into iqs. Again, there is minus by omega, so that we can club together plus this term, Lrr into iqr; the whole thing is multiplied by - now this current is in the d axis - by idr. Then again, the q axis plus omega s minus omega r into M into ids that is this current M into ids, then omega s minus omega r plus Lrr into idr, multiply iqr; this is the total power. But we know that see this iqr and idr, rotor current is very difficult to measure. So, we have to replace iqr idr in terms of rotor flux but rotor flux information we want. But from the previous equation we know that psi rd is equal to M into isd plus Lrr ird. So, from this one; what is ird?

We can find out what is ird. We can find out ird here in terms of isd and the rotor flux. Similar way, si rq is equal to M into isq plus Lrr ird. So, from here, ird we can select based on this one, si rd and si r because we want the rotor flux information.

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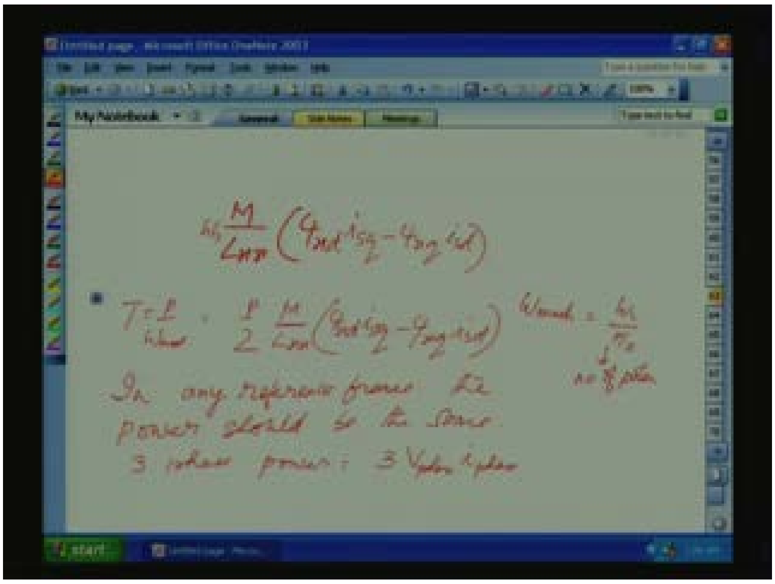

Final power equation will be M by Lrr into psi rd isq minus irq isd; this way final equation will come. But if you see here, this has to be and multiplied by omega s, omega s will come. Now, for the torque, we have to divide by the mechanical speed. Omega mechanical is equal to omega s divide by P by 2. So, T is equal to p by omega mechanical, p is the number of poles; so p by 2 will be equal to p by 2 M by Lrr into si rd isq minus si rq isd. It will get in this form, torque; so when we divide by the omega mechanical, p is the number of poles number of poles.

Now, let us see, we have transferred everything from the three phase to two phase. So, any transformation, the power balance should be met. That means in any reference frame, the power parlance should be equal, the power should be equal. In any reference frame, the power should be the same that is the power balance should be met; the power should be the same. Let us say whether alpha beta transformation and find it to dq; whether the power balance is met.

Now, for a three phase power based on single phase equivalent quantity is equal to 3 into, for RBC system, 3 into V phase into i phase. Suppose that the same voltage waveform, we are sending it to let us say to a resistive load to make it simple. So, this transformation, the voltage and current transformation, it is not independent of the load. So, let us take a resistive load. So, three phase into i phase; three times for three phase power.

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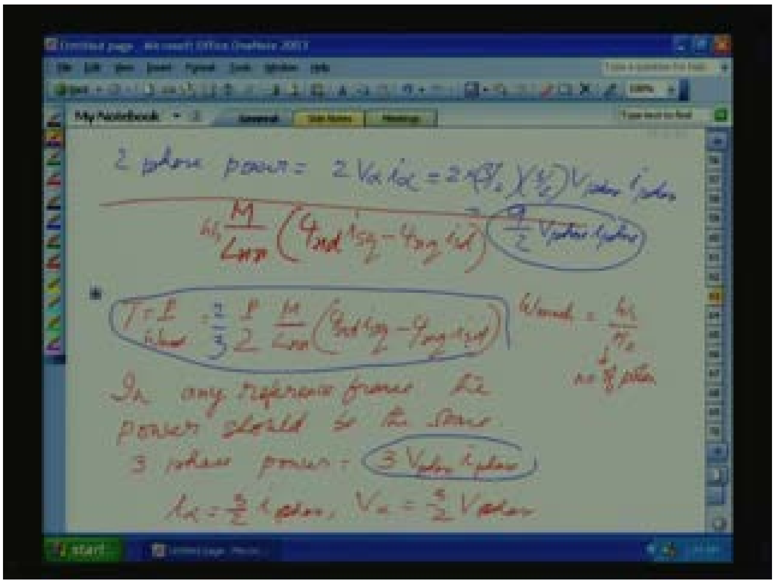

Now, in our transformation we have found out that i alpha is equal to 3 by 2 i_A, A phase or V alpha is equal to 3 by 2 V_A, V phase, here also we will say i phase. Now, alpha beta means two phase power. So, two phase power will be, the two phase power is equal to 2 into V alpha into i alpha, same resistive load we are trying to find out the power in the alpha beta reference frame. This will be equal to 2 into 3 by 2 into 3 by 2; 3 by 2 into 3 by 2 into V phase into i phase equal to 9 by 2 into V phase into i phase.

So, the power balance; for the three phase, it is this one and for the two phase, it is this one. So to meet, see here the transformation we have taken, we have not taken into consideration the power balance, you simply assume the number of turns are same for the alpha beta and transfer the voltage and current. Now, to make the power balance in our or to make it 3 V phase into i phase in the two phase quantity, we have to multiply by 2 by 3. So correspondingly, torque also we have to multiplied by 2 by 3. So here, in our transformation, this is the final - 2 by 3, p by 2 is the number of M by Lrr, this is the torque developed in the system in terms of rotor flux and the stator currents. See, we got the transformation equation dynamic condition under dq reference frame.

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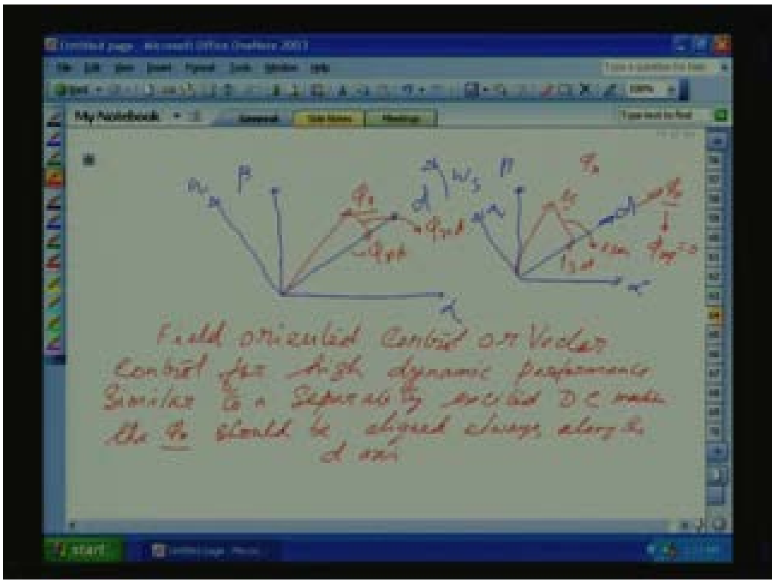

Now, what we want? This is the alpha beta and this is our d and q, dq is rotating, synchronous reference frame we made it. Now, we want to, suppose our si r rotor flux is here, somewhere here; this is our phi r flux space phasor that is phi r is equal to phi rd plus phi rq plus j phi rq. So, which is phi rd? phi r, you project the this one to this one, this is phi rd and this will be equal to phi rq; this our orthogonal component.

Now, for field oriented control, field oriented control or vector control for high dynamic applications, performance similar to a separately excited DC machine, DC machine; what we want? The phi r flux space phasor should be aligned always along the d axis. So, what is meant by that one? See, if you see here, see we will just rewrite this one here, that means in our new reference frame alpha beta; this is our d, perpendicular to that one we have the q and our si r, this is a synchronous, psi r should be always along this one.

So, what is meant by this one? phi r along this one means in the new reference frame and if it is i_s here, this is our i_s, this is our new isd and this is the isq; in this phi r, the phi rq is equal to zero. That means the new reference frame phi r magnitude is equal to phi rd. So, we have to beat this condition that means we should know the instantaneous position of the psi r and we have to align the psi r along our rotating, synchronous rotating dq axis.

Now, in this case, let us see what is our equivalent circuit? So, let us go to a new reference frame and find out what happens in the new reference frame. So, we will draw this one here, our alpha beta; alpha, this is beta.

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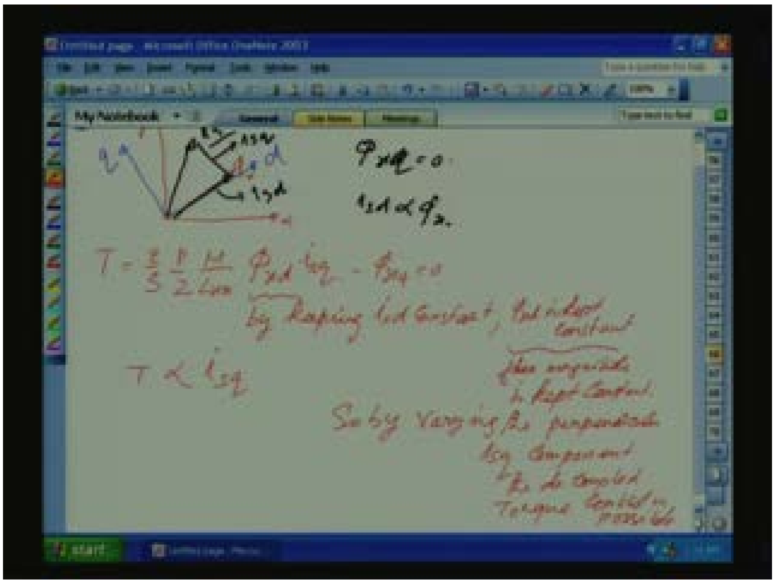

Now, our d axis, this is our d axis, this is our q axis and si r also along this one, phi r. Now, we have to resolve the current space vector, the current space vector is here. This is our i_s will be splitting along d as well as q, this is isd and this is isq. So, isd component will be proportional to the responsible for the flux phi r. Now, in the new one also, here in the new reference frame; phi rd phi rq is equal to zero. So, let us go to our torque equation, original torque equation; torque equation when psi rq is equal to 0 is 2 by 3 p by 2 M by Lrr. Final equation will be phi rd into isq because phi rq is equal to 0.

So, in this equation by keeping isd constant, phi rd is kept constant. So, flux is flux magnitude is kept constant. So, this shows torque will be proportional to isq. So, by varying the orthogonal component, perpendicular isq component, the decoupled torque control is possible, torque control is possible. Now we say, si qr is equal to 0. Let us go back to our, now how the equivalent circuit will work? Let us go to our equivalent circuit. Let us go to the next page. So, our equivalent circuit is like this.

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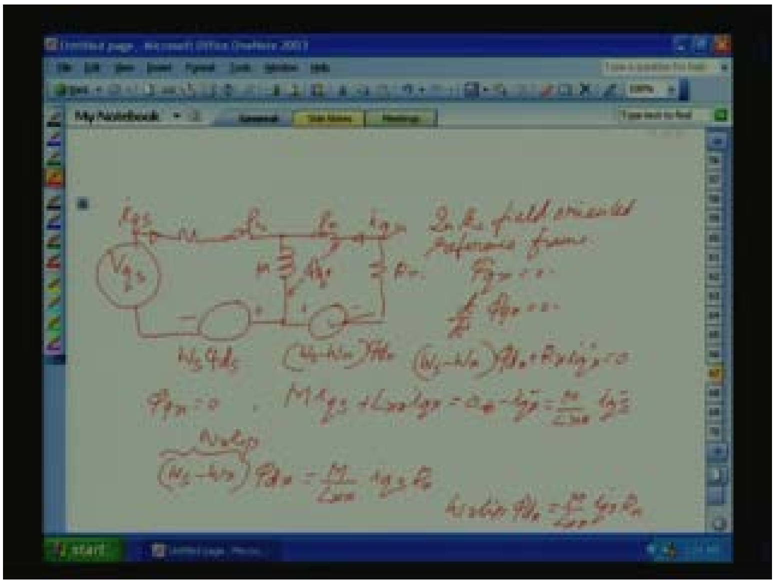

Let us take the q axis, Vqs, this is minus plus omega s si ds, then leakage inductance Rr, this is our iqr, this is our iqs, this is omega s minus omega r into si dr. Now, si qr is equal to zero. In the field oriented reference frame, si qr is equal to zero. Which si qr? si qr is the one due to these two fluxes; this is M lr ls. What it shows? That means d by dt of si qr also zero. So, the voltage coming across the resistance Rr is equal to the rotational voltage, this one. So, if you write down the voltage loop equation here, let omega s minus omega r into phi dr plus Rr iqr is equal to zero. That means rotor circuit equation if you take it, your d by dt psi r is equal to zero that means this rotational voltage will be dropped, should be dropped across the resistance Rr; so from this one, iqr.

Now, si qr is equal to zero that is what is si qr? M into iqs plus Lrr iqr is equal to zero. From here, from this one, minus iqr is equal to M by Lrr into iqs. So, we got iqr in terms of iqs. What is omega s minus omega r? Omega s minus omega r is the slip. So, omega s minus omega r into phi dr is equal to from this equation that is from this equation is substituting iqs that means M by Lrr iqs Rr. What is this one? This is slip, this one is the omega slip.

So finally, what we get? Omega slip into phi dr is equal to M by Lrr into iqs into Rr. So, si dr is constant, psi dr is constant; so, that means omega slip is proportional to iqs, the orthogonal component. Here, omega slip into phi dr is equal to M by Lrr into iqs into Rr. That shows phi dr keeping constant; by keeping the isd component, phi dr is keeping constant, then omega slip is also proportional to iqs.

Omega slip is proportional to iqs; previously we have shown under field oriented control or field oriented equivalent circuit, the torque is proportional to iqs. That means the torque is slip is proportional to the torque, slip is proportional to the torque by keeping phi dr constant. Then by varying the orthogonal component, we can vary the slip and in turn we can control the torque. So, we will continue this analysis field orient control in the next class.