

Industrial Driver Power Electronics

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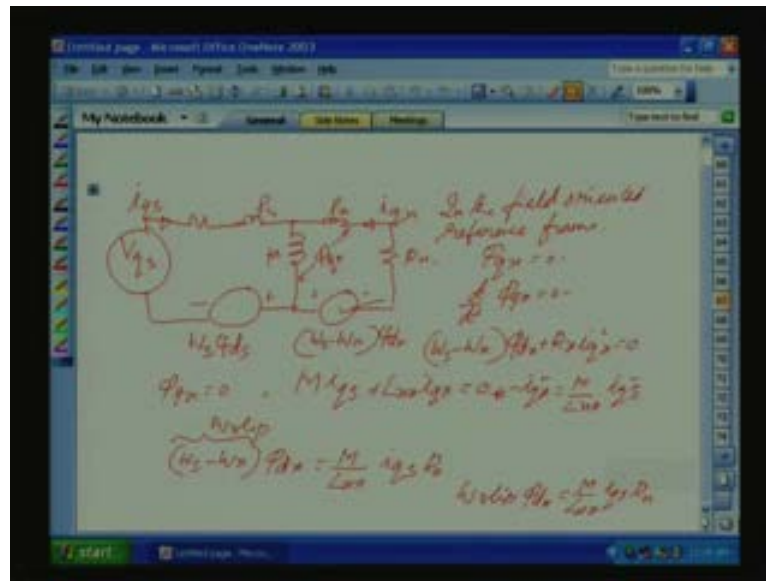
Indian Institute of Science, Bangalore

Lecture - 35

Vector Control of Induction Motor

Last class we found out the relation between the omega slip and the quarter age axis current that is i_{qs} in the field oriented reference frame that is when the dq axis is oriented along the flux space phasor.

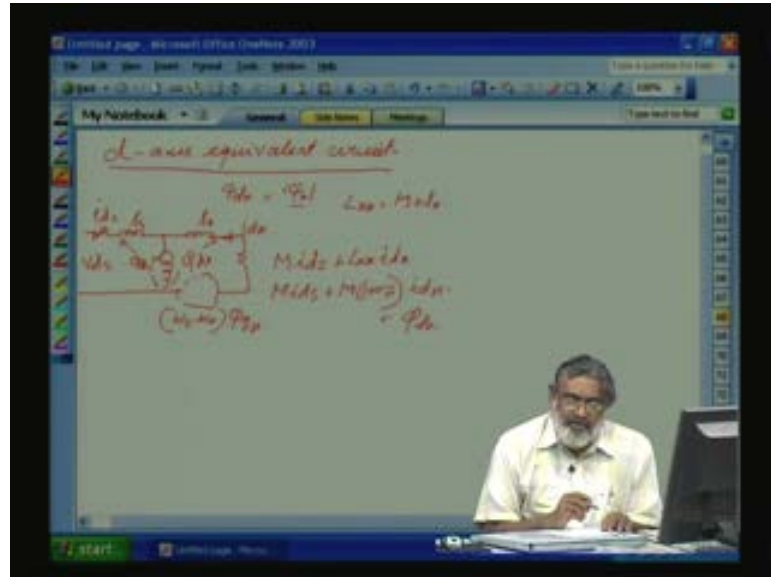
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So, when we found when the d is along the flux space phasor, the imaginary component ψ_{qr} is equal to 0, so $d \psi_{qr} / dt$ is equal to 0. So, from the rotor circuit, $d \psi_{qr} / dt = 0$, then the voltage coming across the R_r is the rotational voltage. So, that we equate it, then we found the i_{qr} in terms of i_{qs} that is from the flux equation, then we found omega slip is equal to proportional to i_{qs} . So, under field oriented control, omega slip is proportional to i_{qs} and we know that for a motor, a torque is proportional to slip.

Now, let us take the d-axis equivalent circuit under field oriented control. So, we will go to the next page that is the d-axis equivalent circuit, equivalent circuit.

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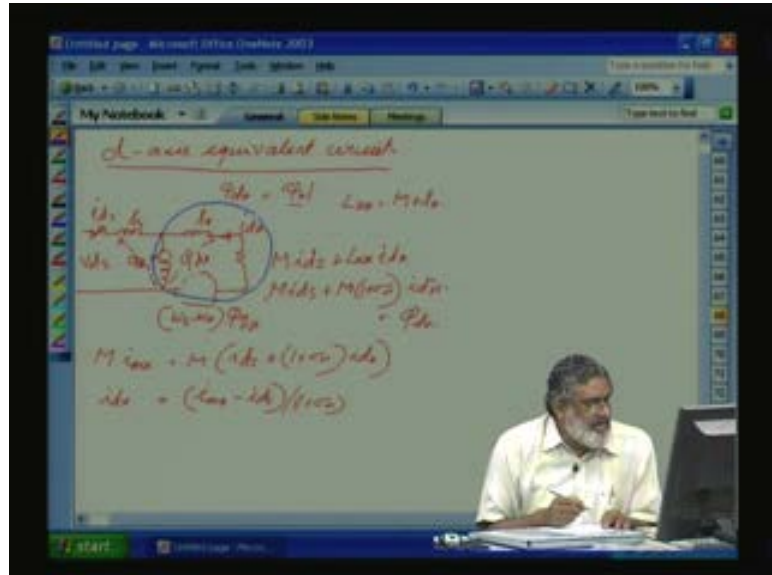


In the d-axis equivalent circuit, the ψ_{dr} is our ψ_r , flux phasor magnitude. Now, let us see the let us draw the d-axis equivalent circuit and from there we will derive the relations. This is mutual inductance, this is V_{ds} , this is ψ_{ds} that is ψ_{ds} which couples this mutual and the leakage, leakage is equal to L_r ; this is mutual, ψ_{ds} , this the rotational voltage term that we have derived previously, this is equal to $\omega_s \psi_{qr} - \omega_r \psi_{qr}$, this is i_{dr} . Now, in this equation, ψ_{dr} is equal to ψ_r .

What is ψ_{dr} ? ψ_{dr} is equal to that is this one ψ_{dr} which couples the leakage, rotor leakage and the mutual inductance that is equal to this is i_{ds} . So, ψ_{dr} is equal to $M i_{ds}$ plus $L_{rr} i_{dr}$. See L_{rr} is, the term L_{rr} is mutual plus leakage L_r . So, leakage we can always in induction motor equivalent circuit, leakage is represented as a fraction of the mutual inductance. So, this we can write as $M i_{ds}$ plus $M \sigma$ into i_{dr} , σ is the fraction 5% of the mutual inductance so that σ is the fraction is the constant into i_{dr} , this is our ψ_{dr} .

So, if you see here, this flux is the combination of inductance plus current inductance plus current and we want this flux to keep the flux constant under field oriented control. So, let us say constant means now i_s we have to split along the d_r and perpendicular to that one and as I told, when the parallel component, the component and the along i_{dr} when it is kept constant, the flux ψ_{dr} or ψ_r is constant because ψ_r is along d_r axis. So, let us say the current which is responsible for the rotor flux, we can write as $M i_{mr}$.

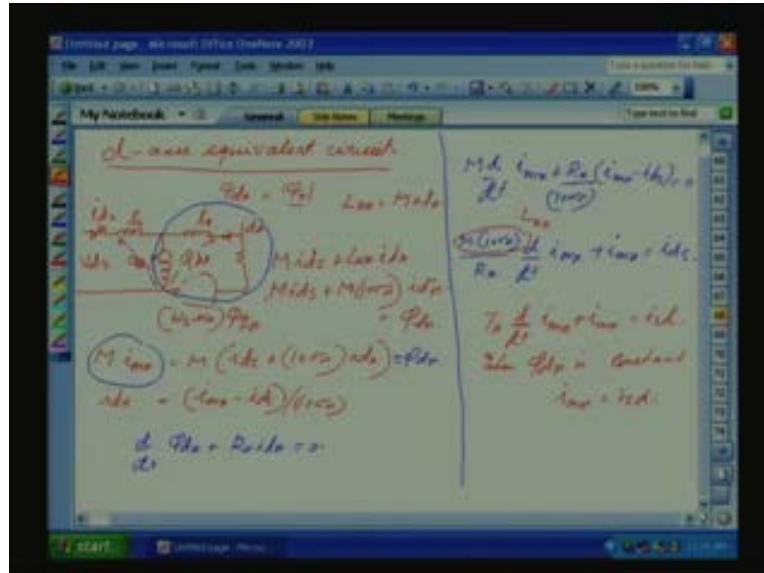
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This is from this equation; this is equal to M into i_{d1} plus L_{σ} into i_{dq} . So, from this equation, the i_{dq} which is very difficult to measure because we want all the quantities in terms of the stator quantities; i_{dq} is equal to i_{d1} , the flux producing component minus the stator i_{d1} by $L_{\sigma} + M$.

Now, let us go back to our rotor side, d axis rotor side. If you see here, ψ_{qr} is equal to 0; under field orientation, ψ_{qr} is equal to 0. Now, $\frac{d}{dt} \psi_{qr}$ we are keeping constant, so $\frac{d}{dt} \psi_{qr}$ is equal to 0; so what is our current or what is the voltage coming across i_{dq} ? i_{dq} is equal to $\frac{d}{dt} \psi_{dq}$ because ψ_{qr} is equal to 0. So now, if you write down the voltage loop equation here, $\frac{d}{dt} \psi_{dq} + R_r i_{dq}$ is equal to 0; this is the voltage loop equation.

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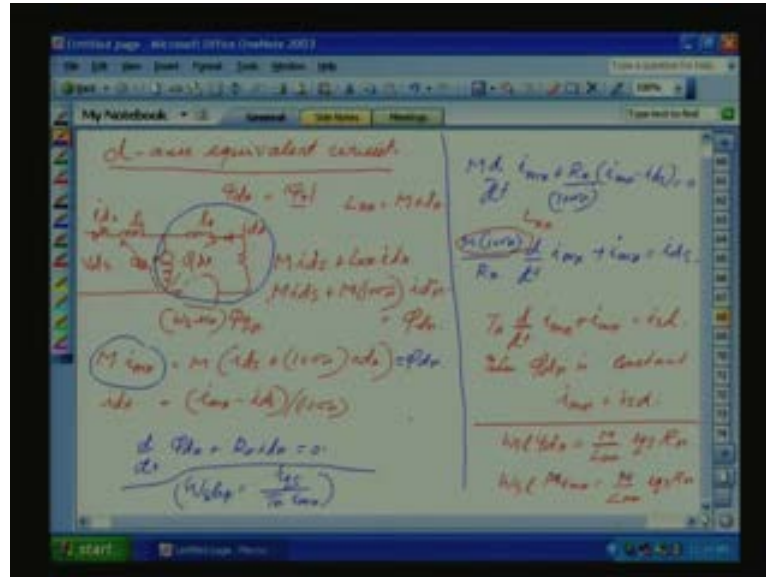


This is equal to, now ψ_{dr} we put as ψ_{dr} is equal to this is $\psi_{dr} M i_m$; so d by dt , we can write it here, M into d by dt i_m that is our ψ_{dr} that is we have got from this equation that is this equation. So, this is equal to ψ_{dr} . So, d by dt ψ_{dr} that is this one is equal to M into d by dt i_m plus $R_r i_{dr}$ that means plus R_r by $1 + \sigma_r$ into i_m minus i_{ds} is equal to 0. This we can multiply throughout by $1 + \sigma_r$ into R_r . So, this will be equal to M into $1 + \sigma_r$ divided by R_r into d by dt i_m is equal to plus i_m is equal to i_{ds} , this the equation we get.

What is M into $1 + \sigma_r$? M into $1 + \sigma_r$ is equal L_{rr} , this is L_{rr} that is mutual plus leakage. So, L_{rr} by R_r is the rotor time constant, rotor time constant that we can take as T_r into d by dt i_m plus i_m is equal to i_{ds} . So, if you see here, the i_m , i_m is equal to i_{ds} when the flux is constant that is when $\frac{d\phi_r}{dt} = 0$. When the flux is kept constant, i_m is equal to i_{ds} . When ϕ_{dr} is constant, d by dt i_m is equal to 0; i_m is equal to i_{ds} . Then why this first order equation happens?

This is why during the field weakening region, when we will slowly reduce the flux. So, when you slowly reduce the flux, the i_{ds} and the flux, the variation is proportional in the first order. Otherwise not in the flux weakening region that is in the constant torque region, i_m is equal to i_{ds} that is below the base speed i_m is equal to i_{ds} . So, this condition we will get. Now, we got the two equations. See, we have to control, we are injecting the current i_s into the machine and i_s we have split into two orthogonal components; i_{ds} into i_{qs} and i_{ds} is placed along the ϕ_r , rotor flux axis. If that is correct, the perpendicular component is proportional to slip and that will be proportional to **torque** and the parallel component, component along the ψ_r is the one which is responsible for the flux.

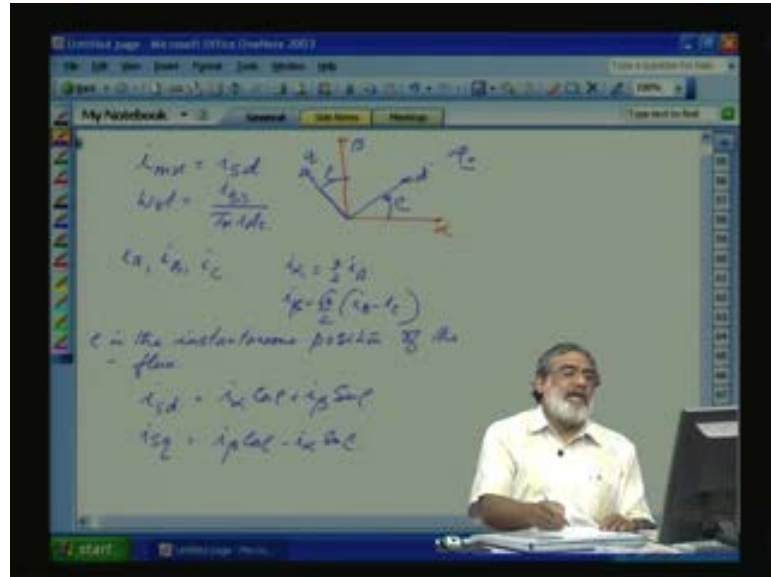
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So, both equations under the field oriented control, the right equations, the current equation and field oriented control are let us write down the previous equation; omega slip into psi dr is equal to we got in the last class $M L_{rr} i_{qs}$ into R_r . Now, if you see here, psi dr is equal to omega slip into $M i_{mr}$ is equal to M by $L_{rr} i_{qs}$ into R_r . Again, $M i_{mr}$, M we can cancel here in this equation, so M , M goes. So, what is omega slip? So, the final omega slip equation is equal to omega slip is equal to i_{qs} divide by see R_r by L_{rr} that is one by T_r so that is T_r , rotor time constant into i_{mr} ; i_{mr} is equal to i_{ds} when the flux is kept constant, so omega slip is proportional to i_{qs} . So, we got the correct conditions for field oriented control.

Now, let us see under field orient control; how the machine, how the equivalent circuit, how the machine works? Machine model, if the field oriented control is correct, then the machine model is like this. See again, let us write our condition is i_{mr} is equal to i_{sd} , omega slip is equal to i_{qs} divide by $T_r i_{ds}$.

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What we did? We put, if you see our reference frame, so our reference frame if you see here; this is our alpha, this is our beta that we are transferring to a general rotating reference frame that is our d and perpendicular to that one is the q. So, the reference frame axis is always we placed along the phi r axis, phi r flux space phasor. So, this is rho, instantaneous angle, this is also rho.

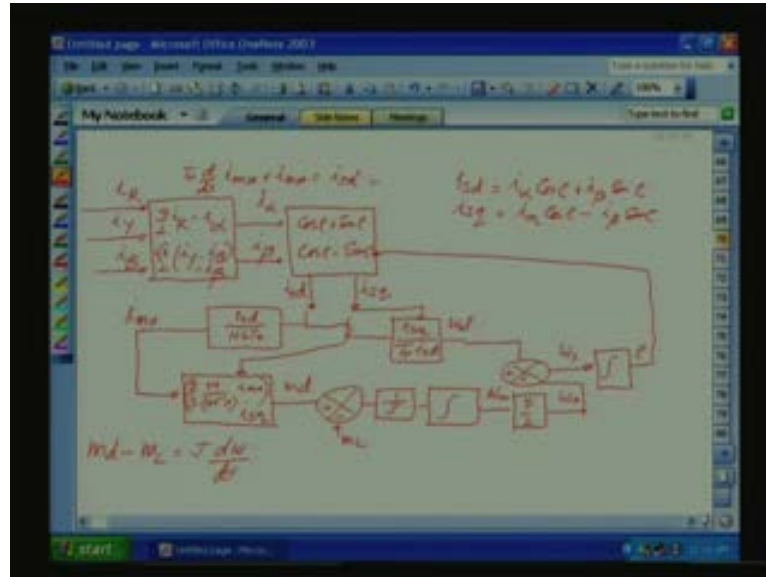
Now, if we know the abc component, we can find out the alpha beta component. See, we will be measuring i_a i_b i_c ; from i_a i_b i_c , what is the alpha beta component? We know that i alpha is equal to $\frac{2}{3} i_a$, i beta is equal to $\frac{2}{3} (i_b - i_c)$. So, alpha beta component, how do you get d and q provider rho is known, rho means the instantaneous position of rotor flux is known?

Now, under field oriented control, rho is the instantaneous position of the rotor flux of the rotor flux. So now, for field orientation, we have to find out the i_d and i_q . So now, our i_{sd} is equal to $i_{\alpha} \cos \rho$ that is $i_{\alpha} \cos \rho$ into plus $i_{\beta} \sin \rho$. From this equation, from this figure, $i_{\beta} \sin \rho$, this is beta axis, $i_{\beta} \sin \rho$ that is this perpendicular $i_{\beta} \sin \rho$ is this axis, this is our i_{α} from here.

Now, i_{sq} is equal to $i_{\beta} \cos \rho$ and minus $i_{\alpha} \sin \rho$ that is minus $i_{\alpha} \sin \rho$ that is from here, we can find out. See, $i_{\alpha} \cos \rho$ is compounded along this one; now this is i_{β} , $i_{\beta} \sin \rho$, this is also rho. So, $i_{\beta} \sin \rho$ is also equal to because this is parallel to, perpendicular to this one is over, parallel to this one. So, we can get this one.

Now, from this one, how do you find out the equivalent circuit? Let us try to draw the machine induction motor model under field oriented control. We will redraw the induction machine model when the correct field orientation is achieved. So, we are sending the three phase currents to the motor that is the i_R , then i_Y and then i_B .

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This, inside the machine if the field orientation is correct, the correct model under field orientation is this is $3 \text{ by } 2 i_R$ is equal to i_α and $\sqrt{3} \text{ by } 2 i_Y$ minus i_B is equal to i_β . So, we got i_α and i_β . Now, if the ρ information is correct, if you know the rotor flux position, instantaneous position of the rotor flux, ρ is known; then using this transformation, $\cos \rho$ plus $\sin \rho$ and $\cos \rho$ minus $\sin \rho$ we will get i_{sd} and i_{sq} , i_{sd} and i_{sq} .

See, i_{sd} will be equal to $i_\alpha \cos \rho + i_\beta \sin \rho$ and i_{sq} is equal to $i_\alpha \sin \rho - i_\beta \cos \rho$; from the phasor diagram, we can derive this one. So, we will get i_{sd} and i_{sq} . So this i_{sd} , we know that $i_{sd} = \tau \frac{d}{dt} i_{mr} + i_{mr}$ is equal to i_{sd} . So, from this equation, i_{mr} when i_{sd} pass through a first order filter, you will get i_{mr} . So here, this one i_{mr} is equal to $i_{sd} \text{ by } 1 + s \tau$. From this equation, you will get i_{mr} .

The moment i_{mr} is there, the torque equation we have derived, torque is equal to $\frac{2}{3} M \text{ by } 1 + \sigma r$ into i_{mr} into i_{sq} , i_{mr} into i_{sq} ; i_{sq} we got here. This torque equation, this is m_d , driving torque minus load torque, this the load torque. We know how to relate the speed and the driving torque and the load torque, this equation we can use; $m_d - m_L$ is equal to J , moment of inertia d into $d \omega$ by dt . So, this we can multiply by $1 \text{ by } J$, gain function. Then we can integrate it, here you will get ω mechanical speed multiplied by $P \text{ by } 2$, P is the number of poles, you will get the ω_r .

Now, from i_{sq} i_{sd} , we know if the slip is equal to, field orientation is correct, slip is equal to $i_{sq} \text{ minus } \tau_r i_{sd}$, so i_{sq} is available here, so ω_{slip} is available, $\omega_{slip} \text{ plus } \omega_r$ will give ω_s that is the speed of rotation. When you integrate, you will get ρ that is the rotor flux position. This ρ , you will feed it here so that from the look up table we can generate $\cos \rho$ $\sin \rho$. So, if the field orientation is correct, this is the machine model under field orientation. So, let us write the block diagram for field oriented control.

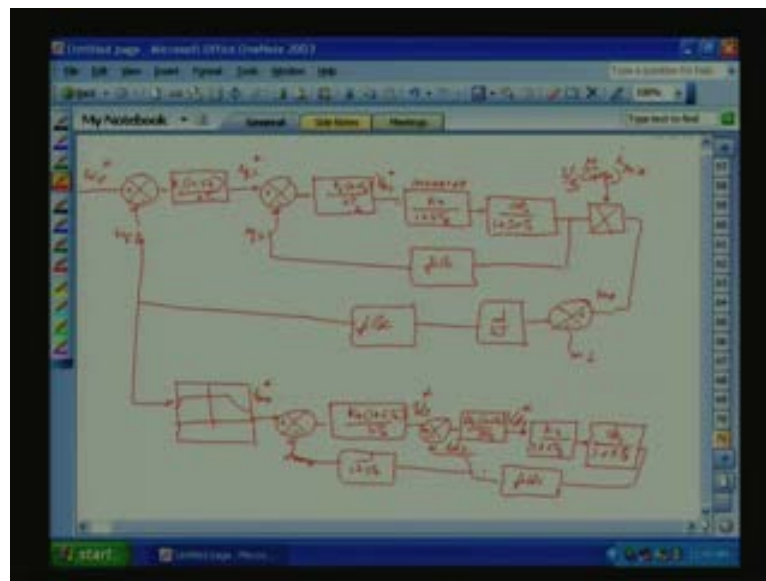
come to that one. Now see, assuming rho is available see this three phase current again we are converting ABC to alpha beta we will convert it then alpha beta to provided d is known rho is known we will find out alpha beta to dq so dq we got dq this is isd actual current isd this is isq when why when we pass through $1 + sT_r$ we will get the correct imr that you feed it here now isq you got it

We know the equation isq divide by imr torque you will get the slip this slip is a taco for the machine from there you can get the ω_r here you will get ω_s you integrate you will get rho that rho you feed it here now this omega you can give as a feedback here also so our vector control is correct current his initially current hysteresis control we can use it because it will give fast response so high dynamic performance app application we can use this one this is the block diagram for a field oriented current using current hysteresis PWM control.

We can use also space vector PWM also then we can from the current PI control we have to get the required voltage control so this way we can get the field oriented controls block schematics for the field oriented control is like this so this we can simulate using MATLAB and Simulink before an actual implementation is being done. So, this is current hysteresis controller. Now, if you want to use voltage control, how it can be done? See, we got field oriented control; now how do you decide your controller for the separately excited DC machine? We have found out some set of rules to find out the PI controller.

So, let us draw the closed loop schematics; from there, same techniques what we use separately excited DC motor, we can use it here also. So, let us see the close loop controller block diagram.

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See, we have the ω reference and this is the ω feedback. So this, we will give to a speed controller, PI controller so that we will make it as K_1 into $1 + sT_1$ by sT_1 ; this is the

PI controller. This output will give our i_{qs} star. This i_{qs} star for a voltage control, we have check i_{qs} and i_{qs} feedback, it is coming here. This we will give it to current controller that is K_2 into 1 plus sT_2 by sT_2 ; this controller output if you use current hysteresis controller, this output i_{qs} we can directly convert it to, using i_{qs} and i_{ds} , we can directly control into i_{as} reference, i_{bs} reference and i_{cs} reference.

But if you want to, if you want to control a phase up to PWM, then we require the voltage reference. So here, we will have i_{qs} reference and i_{qs} feedback, then one more hysteresis controller, this output will give our V_{qs} reference. This **feedback** we will give to a PWM converter, so converter has to be or inverter, inverter we will again pass through; we will modulus a first order lack, so we will say inverter K_3 by 1 by sT_3 , this the inverter, this is our inverter.

So, same like our PWM based, on the PWM frequency, we can find out approximate one sT_3 . So this, it will go to the machine, machine again it will be same like separately excited DC motor 1 by R_s divide by 1 plus S leakage inductance, time constant due to the leakage inductance that is sT_s we will put it; this we will give the output i_{qs} . So i_{qs} , may be gain and filter if is required, we have to put it here and feed it here. So, this current control, we can same like a current loop and outer speed loop.

So, this i_{qs} multiplied by 2 by 3 M by 1 plus σr into i_{mr} is our torque, this torque m_d minus m_L 1 by SJ , SJ the moment of inertia, you will get the speed feedback; speed feedback again if you are with a filter and gain that first order, we can model the first order. So, we will get outer speed loop and inner current loop same like in a separately excited DC machine. And for the dq axis d axis, see you have the current reference, under field weakening it will go inversely proportional both forward and reverse. So, this will give depending on the speed, our i_{mr} , i_{mr} reference plus minus i_{mr} feedback. This we will give to one controller that will give our i_{ds} reference that is K_4 into 1 plus sT_4 divide by sT_4 . This will give our i_{ds} reference, i_{ds} reference minus i_{ds} feedback. If we are using a space circle PWM, then we require V_{ds} , so one more PI controller.

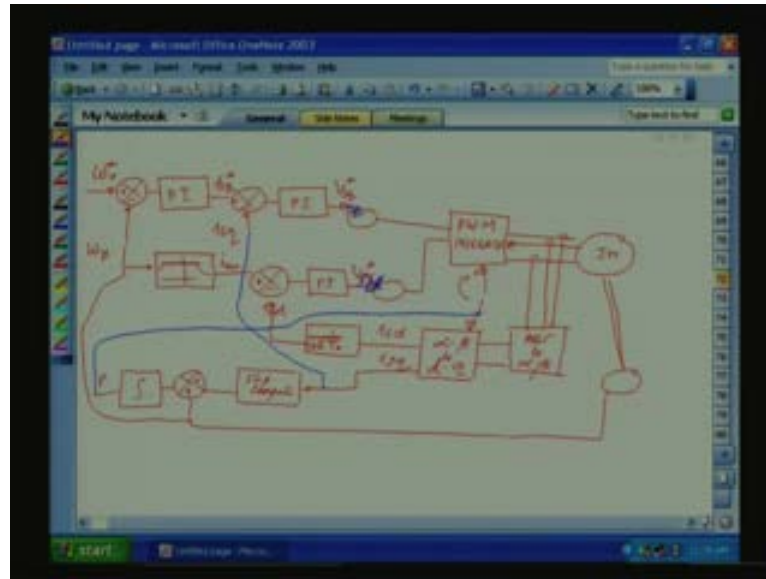
So, this is K_5 into 1 plus sT_5 , sT_5 , this will give V_{ds} reference; again the converter, V_{ds} reference, the same converter, there is the same K_3 by 1 plus sT_3 . It will pass through 1 by R_s . Here, this is not s σT_s because q axis because of the leakage time constant that and this is mainly the mutual inductance time comes. So, this is 1 plus ST_s , T_s is the stator time constant; this one stator, that mutual plus leakage because this is the flux producing component. So it will, mutual inductance time come into picture. So, this will come here and again if you are using a filter once again, you can feed it here; this is our current reference.

Now i_{ds} , 1 by 1 plus ST_r is our i_m . So, this loop also we can design if you know the input output relation, same like what we used for the separately excited DC machine or front end AC DC converter, you can use this one here. So, this way, we can have a vector control we can have it. The most important thing is the ρ , instantaneous position of ρ . If ρ is correct, field orientation is correct, then we can only, then we can use this closed loop control for block diagram for designing the control parameters. So, the most important thing is finding out the ρ .

See, this field oriented control, we have used the current hysteresis controller that means here we have the i_{sq} and i_{sd} reference we generate; then using ρ , we convert it into i alpha i beta then alpha beta to abc, then abc current and the abc feedback that is abc feedback we will be the simple hysteresis controller. But as I told, the problem with hysteresis controller previously, it is wide switching frequency variation and optimum PWM switching is not required.

So, instead of if you use a voltage control converter with a spacer PWM or sine triangle PWM, then we want the reference to the PWM is not current, the voltage. Then the field oriented control scheme will be the block diagram will be like this. We will go to that one now, next page we will go where the command voltage as the command variable for the inverter. So, field oriented control with a voltage as the command variable.

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So, here also, we have the reference, speed reference, this is our ω_r reference, then speed feedback is there that we will generate from encoder or a tacho, then this PI controller output. As I told, any change in the speed, command speed and the reference indicates a difference in that torque command and the load torque. So, we have to give the correct i_{sq} which is proportional to the torque. This i_{sq} we will compare; see now, the voltage we are injecting to the machine using PWM control voltage we are controlling. So, we know with that correct i_{sq} is coming. So i_{sq} , this we will take it reference, i_{sq} feedback is required. So, this output PI controller, we will give our V_{sq} star.

Similarly, we have the speed feedback or speed reference, we have the look up table for i_{mr} , this is during the field weakening region, i_{mr} will come here; during steady state of an operation, i_{mr} will be equal to i_{sd} . So, we have the i_{sd} feedback here, i_{sd} . This is the PI controller, this we will give you our V_{sq} star. So, intentionally put this one; there is some more term also we need to add here. So, this we will talk about later. Let us say V_{sq} is coming, this V_{sq} will go to the PWM modulator and inverter, PWM, inverter and control.

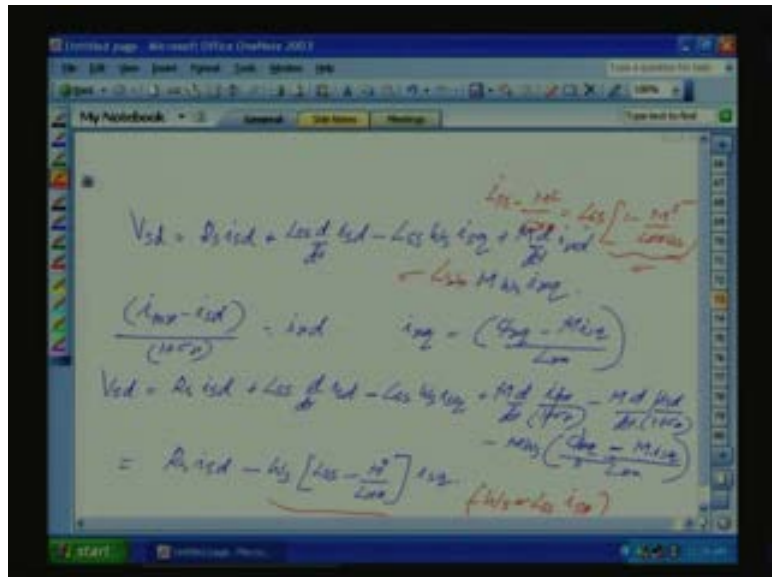
We require the rho signal also, so Vs_q Vsd we got it, here we require the rho also. This it will go to the motor induction motor; from the motor, we will send the current, all the three sensors are not required because i_a plus i_b plus i_c is equal to 0. So, two current sensing is required. But for clarity, we have put three here.

These are in abc, so we will convert abc to alpha beta. Again, alpha beta to dq, here also we require rho information, this rho is required here also. Here we get i_{sd} feedback and i_{sq} feedback. We know that i_{sd} is equal to i_{mr} in the steady state if any transient flux weakening is there, this i_{sd} by 1 plus ST_r; this is the correct i_{mr}. So, we will leave it here. So, feedback and i_{mr} will be the same.

Now, this i_{sd} you know, how to compute the slip information. Previous block diagram, we have found slip is equal to find out slip computer, slip and we have the t_{aco}. This two added, we will get omega s. That integrate, we will get rho; this is the rho we will be feeding it here so and again this feedback we will close it, omega r.

Now this Vs_q Vsd, this is coming from i_{sq} feedback, i_{sq} feedback here also that feedback term, we will close that one, it is coming from here. So, you will get how to compute Vs_q Vsd? See, what is this one? See, any change in the i_{sq} and i_{sq} feedback, immediately it should reflect on Vs_q. This shows that means Vs_q is dependent only on i_{sq} and here this is not Vs_q **sorry** this is Vsd and Vsd **sorry** this is Vsd, Vsd we are taking from the i_{mr} reference and i_{sd} feedback and the PI controller. That means any change in the i_{sd} reference and i_{sd} feedback should reflect to Vsd.

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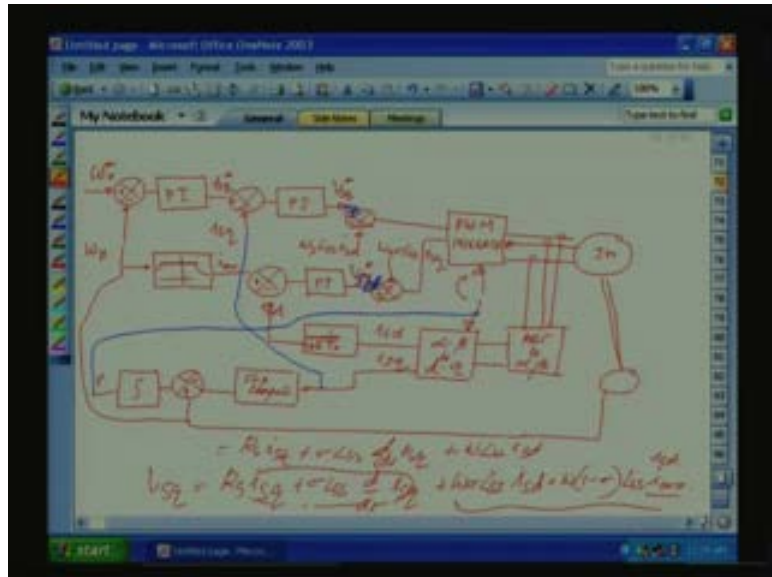
But if you see the actual Vsd Vs_q equation, let us go back to the actual Vsd Vs_q equation; here we can see Vsd is equal to R_s i_{sd} plus from the original, plus L_{ss} d by dt i_{sd} minus L_{ss} omega s i_{sq}. So, it shows Vsd, there is a i_{sq}; any change in the i_{sq} component, it will reflect on Vsd plus M into d by dt i_{rd} minus M into omega s i_{rd}. See, i_{rd} i_{rq} terms, the rotor

currents we can represent in terms of i_{sd} and i_{sq} . We know that $i_{mr} - i_{sd} / (1 + \sigma_r)$ is equal to i_{rd} . So, substituting this one here, also we know that i_{rq} is equal to $\psi_r / L_{rr} - M i_{sq} / L_{rr}$ and under field oriented control, ψ_r is equal to 0. So, let us substitute this one and write V_{sd} .

So, then V_{sd} is equal to $R_s i_{sd} + L_{ss} \frac{d}{dt} i_{sd} - \omega_s L_{ss} i_{sq}$ plus here we want to change i_{rd} , i_{rd} is equal to that is this term $M \frac{d}{dt} i_{mr} / (1 + \sigma_r) - M \frac{d}{dt} i_{sd} / (1 + \sigma_r)$, then minus $M \omega_s i_{rq}$ that is $\psi_r / L_{rr} - M i_{sq} / L_{rr}$. ψ_r is equal to 0 under field oriented control. Also, we can say during steady state, $\frac{d}{dt} i_{mr}$ is equal to 0; i_{mr} is equal to i_{sd} , so $\frac{d}{dt} i_{sd}$ is also 0 here, we can assume that one. So, this goes, then finally this will be equal to $R_s i_{sd} - \omega_s L_{ss} i_{sq} - M \omega_s i_{sq} / L_{rr}$, this M and M will come, M^2 into i_{sq} .

See, this L_{ss} / L_{rr} , this term, this term, $L_{ss} - M^2 / L_{rr}$, we can write as $L_{ss} / (1 - M^2 / L_{rr} L_{ss})$. So, this become a constant that we can say $\sigma_{L_{ss}}$, this will become $\sigma_{L_{ss}}$. So, this equation that is this equation, we can effectively write as $-\omega_s \sigma_{L_{ss}} i_{sq}$. So this, we will put as feed forward time in the controller here, i_{sd} term.

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So, in the V_{sq} term here, there is a feed forward term. See already, i_{sq} we know it; so, we can use this one $\omega_s \sigma_{L_{ss}} i_{sq}$ here, we can feed it here, $\omega_s \sigma_{L_{ss}}$ into i_{sq} . So, what is the advantage, this feed forward term? Then this PI controller has to change only for any variation in the i_{sd} only, any i_{sq} variation it will immediately affect through this one. Similarly, we can find out the feed forward term from the dq for the V_{sq} , we can find out from the equation, we can the same derivation and we can use it.

See here, we will see, if we go to the equation V_{sq} , V_{sq} is equal to $R_s i_{sq}$ plus σ . Finally, it will come $\sigma L_{ss} \frac{d}{dt} i_{sq}$ plus $\omega \sigma L_{ss} i_{sd}$ plus ω into 1 minus $\sigma L_{ss} r_{imr}$. If you from the matrix term that equation dq model if you get the V_{sd} equation, if you get finally, you will get this one. See here, ωi_{mr} is equal to i_{sd} ; so σi_{mr} we can replace it i_{sd} in the steady state operation, i_{mr} is equal to i_c . Finally, this equation will be equal to $R_s i_{sq}$ plus $\sigma L_{ss} \frac{d}{dt} i_{sq}$ plus $\omega \sigma L_{ss} i_{sd}$ plus ω it will come because i_{mr} we are taking into i_{sd} . So, this term together, it will come here.

So this time, we can use a feed forward term here, this is also plus here, this is also plus here. So, here we can use the feed forward term from the i_{sd} that is $\omega \sigma L_{ss} i_{sd}$ here. So, what is the advantage? So, this PI controller has to vary, V_{sq} has to vary only if variation in the i_{sq} . So, the variation for the PI limit only due to the i_{sq} here and in i_{sd} variation, we have feed forward here. So, it need not come through the PI control, so it will also increase the response of the system. This way for voltage controlled inverters with voltage as the variable for the PWM, we can use these types of closed loop schemes for the vector control application.