

Power Electronics

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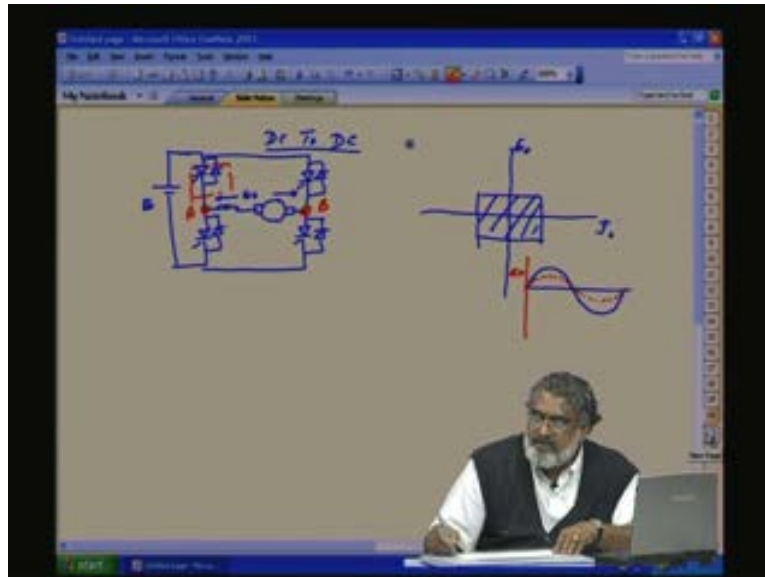
Indian Institute of Science, Bangalore

Lecture - 9

Sine Triangle PWM Control of Convertor

Last class, we talked about a four quadrant converter, AC to DC converter. So, the configuration, we start with thyristor. So, let us see, how the four quadrant converter looks like.

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You have the diode, so this thyristor with this diode together, it will act as a bidirectional switch. So, we want both voltage and current to be changed; in both directions, we require that is E_0 I_0 . So, for the other side also, we require. So, we will put one more link here. This is the configuration. So, we will be able to do the four quadrant operation. Both voltage and current, we can change it. See, four quadrant operations, both the voltage and current, we can change it independent of each other. At the same time, if you see here, the load is connected here; this is the wave form, input side and this is the output side, this we are talking about dc to dc converter, dc to dc.

So here, this is the output; output we can have both positive and negative, current also we can have positive and negative and previous class, we also studied about the PWM mode of

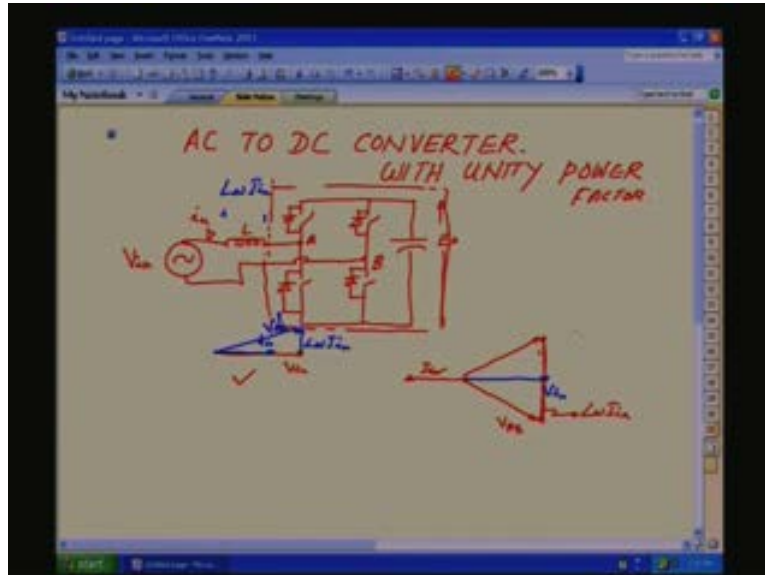
operation. So, PWM mode, we can have the E_0 , we can control. So, we can control in such a way, a sinusoid average, sinusoidal variation is possible, possible here, E_0 side; this is possible. Now, if that is possible; can we have a sinusoidal output voltage? We can have a sinusoidal output voltage like this. If the sinusoidal output voltage is possible, we can also have a sinusoidal output current. So, voltage and current depending on the load current **may be leading**, current may be lagging for an inductive load. So, by proper control, can we have a unity power factor also like this? It is possible, we will see that one. Then see, this previously, we talked about front end converter ac to dc converters. Now, here if you see here, if you see here, this E_0 with PWM control E_0 we are changing, in a PWM control, the fundamental is we are making us; the average value we are varying at a sinusoidal, so here.

Now, this is the current with unity power factor. Then the question is can you use this configuration for an ac to dc converter with a unity power factor? See, now the power flow can go from source to load as well as load to the source. Then the same thing we can use it for an ac to dc converter with unity power factor. Now for PWM control, why we require PWM? We want PWM so that the harmonics shifted to the high frequency side; we come to that one soon. So, this configuration, we can use it for an ac to dc converter with unity power factor.

So, instead of thyristor, we will be talking about here for high frequency PWM, we require; so we can use instead of thyristor, we can use IGBT's or MOSFET because thyristor require a force computation that we have not shown here. This thyristor require a force, so extra circuit is required. That we are not talking about that one.

Now, another thing is these two limbs; this limb and this one - A and B, this is one leg - A leg, this is B leg. These are independent of each other. See, that means the switching of A is not dependent on B. So, it can be independent. So, B side, we have introduced to have a negative voltage and current here. That means we want to generate, we want to run the system in all four quadrant operation. Now, let us talk about the PWM operation now, ac to dc converter. So, how an ac to dc converter with unity power factor; what we want? We come to that one now. Let us go to the next slide now.

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AC to DC converter with unity power factor, converter with unity power factor, power factor, the circuit: now ac to dc, so let us see this is our ac side. Then, as before, we have high inductance is there, to drop the ripple voltage, to drop the ripple voltage and also to reduce the ripple current. Then, so here, I will use each limb as switch IGBT or MOSFET, you can use it. So, I will show it as a symbol switch with freewheeling diode, freewheeling diode also required. This is connected here, this side will go here, A, B and you have the dc side, the capacitor here. This is our E_0 , this is input, our single phase ac V_{in} , this is inductance L .

Now, we are using PWM here such that across AB, there is a fundamental voltage, PWM, there is a fundamental voltage and the other frequencies, PWM voltage. We will come to that one. But for the analysis, **due to** with PWM, PWM switching for leg A and leg B, there will be resultant fundamental AB voltage which is we will be controlling. So, for unity power factor; how we have to control such that we get unity power factor? So, let us say, this is our V_{in} , V_{in} and let us say this is our I_{in} . I_{in} probably, we will mark with a different colour. So I_{in} , so I_{in} has to be in phase with V_{in} , unity power factor. So, this is I_{in} .

So, this I_{in} flowing through L , it will have a drop across the inductance and what is a drop? That drop will be $L \omega$ into I_{in} , the peak value of, we are trying to draw the phasor diagram, the peak of value of this current. Now, this V_{AB} , fundamental AB will be, so this voltage will be perpendicular to, this will be perpendicular to, this will be equal to this perpendicular to V_{in} , the drop across a inductance that is $L \omega$ into I_{in} . So, the V_{AB} fundamental will be here, this is the V_{AB} , V_{AB} fundamental. This is when the power flow from this side to that is source side to the dc output side.

Now, when the power is reversed; how will it happen, the phasor diagrams? Again, we will draw it here. This is V_{in} , this our V_{in} again, power reversal with unity power factor means I_{in} has to be in this direction. So, I_{in} we can draw with a different colour. Now, this is your I_{in} , I_{in} phasor and

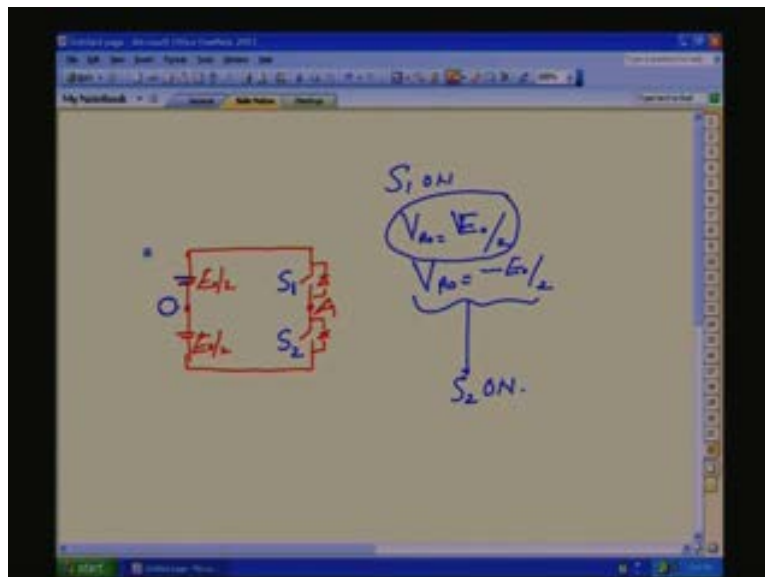
the V_{in} will be in this direction, $j\omega L I_{in}$ here, it will be in this direction. So, this is V_{AB} , this vector will be $L\omega I_{in}$, ω is the mains frequency.

So if you see, this is for power. Here, V_{in} into I_{in} is negative, $V_{in} I_{in}$ will be power will be flowing in the other side that is dc side to ac side here. In this case, it is going from ac to dc. So, if you take with unity power factor as the power for a fixed I_{in} , the movement of V_{AB} will be along this direction for forward as well as reverse power flow. Now, as a I_{in} varies; I_{in} varies V_{in} will also vary, I_{in} magnitude varies. So, I_{in} magnitude depends on the load connected across E_0 . So, we have to control V_{AB} for any load. Any load that means what is meant by load here? You have a dc here, output dc current, so for that dc current E_0 into I_0 current, output dc current, there is a power. That power has to be match taken from the input current, input source that is V_{in} into I_{in} . So, I_{in} will be connected, I_{in} will be related to the output load dc current.

So, depending on the load, I_{in} will vary. So, for a particular I_{in} , for forward or reverse power flow, V_{AB} has to be moved along this direction. So, we have to control the PWM such that your V_{AB} moves along this direction for unity power factor. So, how do you do that one? That is more important.

So, as I told before, we are using PWM operation. So, let us talk about the PWM. In PWM, how we can control the V_{AB} , fundamental V_{AB} , amplitude? And also, I told previously, these two legs are independent of each other, we can control independently. So, how do you do that one? So, let us take only this part now that is a converter alone and study the PW. So, let us go to the next slide now.

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We will talk about the converter part and how we do the PWM. So, **the input** the output voltage E_0 , I will divide into 2 values of E_0 by 2, E_0 by 2 and this also E_0 by 2, E_0 by 2. So, this way, we will do it. Now, we have the switch here, we have the freewheeling diode here, this is our A.

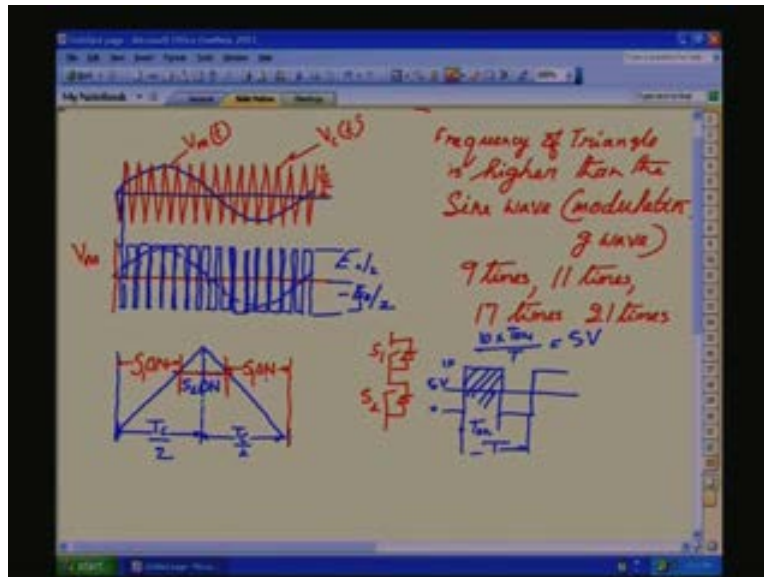
Now, other limb, we are not we are not talking about those. What we can use for A, the same thing can be applied to B leg also. Now see, even though the output is not divided as E_0 by 2, E_0 by 2; I have divided, for analysis I have divided into E_0 by 2 but the total output dc is still E_0 .

Now, let us take a condition when S_1 is on; S_1 is on, what is the voltage at A? The voltage at A means we have to measure the voltage with respect to a fixed point. So, what we will do? We will measure with respect to the fictitious center of E_0 . Here we represent it as O; this point, we will mark as O. That means only thing we have to measure the voltage with respect to a common reference point. The reference point is O that is that is a fictitious; it is not there, for analysis, we are doing it at the centre part of output E_0 voltage E_0 .

When switch, top switch is on that is I will make this one as S_1 , this one as S_2 ; when the top switch is on, the voltage A with respect to zero, this we will say pole voltage V_{A0} is equal to V_{A0} is equal to E_0 by 2. We will assume the switching are instantaneous when S_1 is off and S_2 is on. When S_2 is on, V_{A0} will be minus E_0 by 2, V_{A0} is equal to minus E_0 by 2. In this case, we are turning S_1 on; in the second case, here S_2 on. Now PWM operation, we have to switch on S_1 and S_2 such that the average variation at point A with respect to O should vary sinusoidal. So, how do you that one?

Here, we will use a sine triangle PWM technique. So now, let us go to the sine triangle PWM. In sine triangle PWM; what we will do? The sine wave, we are comparing with a fixed triangular waveform, waveform of high frequency and the sine wave is the fundamental wave what we want, the reference wave. Let us take a typical example, how the sine triangle PWM, we are comparing.

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This is the V_{A0} we want. So, we generate the V_{A0} . This V_{A0} will be compared with a fixed triangular, triangle waveform of fixed amplitude and fixed frequency. So, let us take triangle waveform. May be triangle waveform, we will draw with a different colour so that clarity will be

more. This is our triangle waveform, this we can generate with in an analog circuit or a microcontroller or in VSP. The generation of this sine triangle waveform is not that difficult but only thing is the triangle frequency is much higher than the sine frequency. Why we do that one? So that the harmonic frequency; you will know, how the harmonics are generated? The high amplitude harmonics will be shifted to the high frequency side which is the frequency of the triangle wave form.

So, high frequency amplitude, high amplitude harmonics are shifted to the high frequency side that is at the triangle side, then the impedance for this harmonics $L \omega$ will increase and the current ripple drawn from the mains will decrease. So, the current will be as close as to a sinusoidal wave. It will happen that way. So, this is the thing. So, we will call this waveform, the modulating wave. That is our reference sine wave and the triangle wave we call as carrier wave that is $V_C(t)$ $V_C(t)$, the amplitude of that one, V is equal to V_C .

Now, as I told, whenever the sine is greater than the triangle waveform, whenever the triangle is greater than the **sorry** whenever the sine is greater than the triangle; we will be switching the top switch one. Top switch is one means the V_{A0} will be V_{dc} by 2. So now, let us use a different colour so that it will be like this; this point that is from this point to this one, V_{dc} by 2. Then when the triangle is greater than the sine wave, it will be minus V_{dc} by 2. So, as we go along the sine triangle comparison for full one cycle of sine wave; the output waveform, PWM pulse width modulated, PWM pulse with modulated V_{A0} will be like this.

See, when it comes to the center, the V_{dc} , the plus V_{dc} by 2 is more and the minus V_{dc} by 2 is, the width of minus V_{dc} by 2 is smaller. So, you can see, the average value, if you see; this average value will be varying sinusoidal. So, we will study that one; is it following the sinusoidal way? And we also want, the output should be proportional to our $V_m(t)$, modulating wave that is our reference wave. So, how we can control the output V_{A0} fundamental with respect to our modulating wave? We will study now. Let us complete this one. So, negative side, it will happen this way.

So, this way, we will get the PWM waveform. So, if you see here, the positive portion from here to here, this will be **sorry** E_0 by 2 and this side will be minus E_0 by 2 that is I will just remove this one **sorry**. So, I will rewrite this one for clarity. So, this will be plus E_0 , E_0 by 2 and below it is minus E_0 by 2. So, you will get this type of PWM wave form and the average waveform, the average wave form will be proportional to our $V_m(t)$. This we will check now but this average variation will be proportional to our $V_m(t)$.

Now, let us check whether this average variation is proportional to $V_m(t)$. If it is proportional to $V_m(t)$; the moment $V_m(t)$ amplitude we reduce, the average variation amplitude also will reduce. The $V_m(t)$ if you can shift it with respect to, slightly shift it with respect to sine triangle waveform, again the average variation at the V_{A0} will also shift. Let us see whether it is true. So, what I stated now, by using this PWM, sine triangle PWM by switching the top switch and bottom switch with respect to the sine compared with triangle; we get a PWM wave form which I shown here, pulse width waveform like this. The average variation will vary sinusoidal and proportional to $V_m(t)$. Let us see, whether it is proportional to $V_m(t)$.

One assumption also we made; we will make it, the frequency of the triangle wave form is very high compared to sine wave. Why that is required, that assumption? Let us take one triangle period, one triangle period. See, triangle which is going from negative maximum to 0 and then positive maximum and comes back, this one. So, if you see here, one triangle period, we call as the carrier period. So, when the triangle goes from minus negative to positive, this duration is equal to T_c by 2, half carrier period; similarly here also, T_c by 2, half carrier period.

Now, during one triangle period, any triangle, we are just expanded here; we are comparing one portion of a sine wave with respect to the triangle. Now, we will assume, during this one carrier period that is starting from here to here, T_c , we will assume the frequency of the triangle waveform, we will assume the frequency of the triangle waveform is so high during this period. The variation of the sinusoidal waveform, that part of the sinusoidal wave form during this period is constant. It is not varying much. So, for analysis, we can assume, we are comparing the part of the sine wave here.

See, assuming high frequency; see, this is our sine wave amplitude during one triangle period. So, what we said here? The frequency of I will write here, frequency of triangle or carrier triangle is higher than the than the sine waveform, sine wave. That is the modulating wave. See, modulating and carrier; these terminology, we borrowed from the communication paper, modulating, modulating, modulating **sorry** it has gone to other side modulating wave, relating wave.

So, how much time? Typically 9 times or 11 times, 17 times, 21 times; I am talking about the frequency of the triangle wave with respect to the modulating sine wave so that during a sampling period **sorry** during a carrier period that is one triangle period; the sinusoidal waveform, the part of the sinusoid which is compared with the triangle wave during that period is nearly constant. So now, as I told, whenever the sine is greater than the triangle, S_1 is on, the top switch is on that is this is our, with a diode, will mark it for clarity here; this is S_1 , this is S_2 . So, whenever the sine is greater than the triangle; top switch is on.

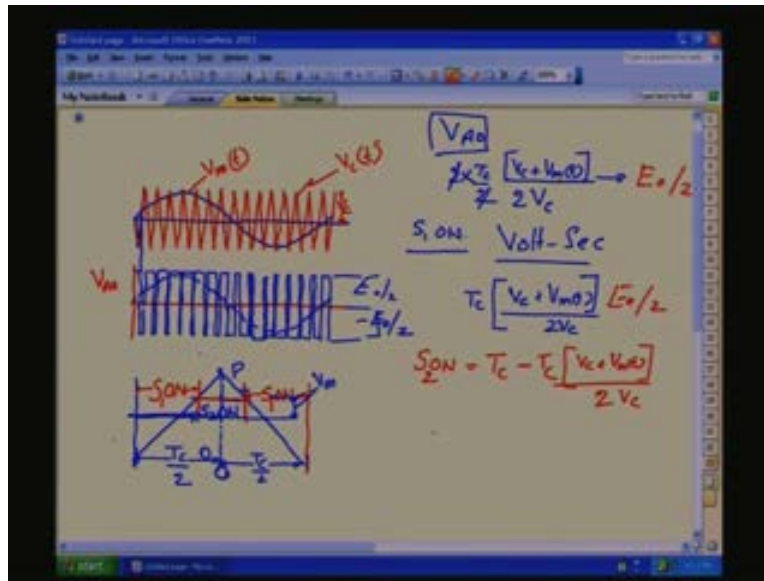
So, during this portion, S_1 is on; this is S_1 here, S_{10} from here to here. Also, when during the negative slope of the triangle from here to this period that is here also, S_1 is on and during this period, when the triangle is greater than the sine wave here, S_2 on, this period. Now, what we have to find out? Same like in SMPS; see if you see here, suppose you have a 10 volt, you want a 5 volt, how do you use it? Using a chopper; in a period, part of the time, we will be switching on the device so that the **output** input voltage of 10 volt will across a load and rest of the period, it will be 0.

So, if you see here, so we want the 10 volts, so we will we doing like this, we will be turning the device for half the period and half the period will be switch off. So, the output will be 0 to 10 during this period and rest of the period, it will be 0. So, the average value will be 5 volt. That is how you generate 5 volt here using volt second. That means volt second means 10 into T_{on} , this is T_{on} divide by total period will be equal to 5 volt if T_{on} is half of T. If T_{on} is 25% of T means the output voltage will be 25% of 10 volts that is 2.5.

So, by varying the T_{on} period, we can vary the output voltage here. The same way here, we are trying to turn on the device here. S_2 is turned on during this period, S_1 is turned on this period

and S_1 is turned on period. So, same like our previous SMPS case, we should we should find out what is the volt second or the average value from the volt second, average value during one sampling period. What is the purpose? We want to find out this average value; is it proportional to the $V_m(t)$, part of that sine wave during that period? So, that is what we want to find out. Let us take S_1 on period, the volt second during S_1 on period.

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Now, let us see find out the volt second during one triangle period and find out the average value during a triangle period. See, if you see here, when S_1 is on; so this slopes if you take, from here to here, it is the slope. It will go minus V_c to V_c . So, total this height is that is I would say O, here, O to T sorry P, here that is this height. So, from this point to the vertical height, O to P that is this point is $2 V_c$, $2 V_c$.

So, when the vertical height goes to $2 V_c$; what is the X axis? X axis is equal to T_c by 2. So, for $2 V_c$, it is T_c by 2. Then, what is for this height? This height is V_c plus V_m . V_m , we are measuring from the zero that zero sides, from the zero axis side, the zero voltage level. So, this much distance if you see here; this is V_m , this is V_m , this is equal to V_m during that period and from here to the midpoint, here, midpoint of the vertical height is equal to V_c . So, for $2 V_c$, the period is equal to T_c T_c by 2. Then what is the period? That is to find out the period from here to here, S_1 from this period we are finding out, this period, from here to here. What is the period? It is equal to so T_c by 2; for $2 V_c$, it is T_c by 2.

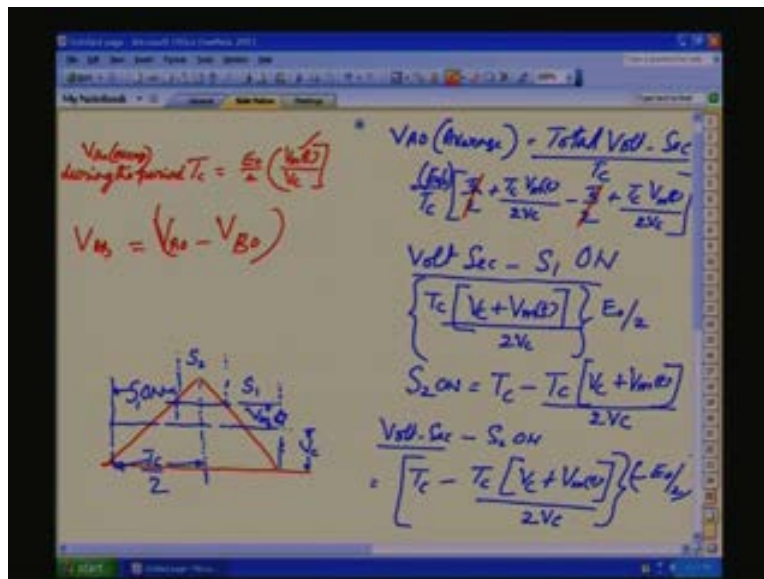
Then, what it is for V_c plus $V_m(t)$, $V_m(t)$ at that instant? That is the amplitude of the sine wave. During this portion, V_{A0} , during this period, V_{A0} sorry V_A during this period, we are talking about the V_{A0} , during that period, it is equal to V_{DC} by 2. But if you see here, during this period also S_1 is on from here to here and this period and this period because of the symmetry of the wave form and assuming the amplitude of the sine wave is constant, this period from here to here

and this period from here to here, both on periods are equal. So, the total on period is equal to 2 times T_c by 2.

Now, during the S_2 on period, the output voltage will be, so what is the total area volt second? So, this period multiplied by V_{DC} 2 by is the total volt second during on period. So, S_1 on, the volt second that is voltage into time is equal to when they multiplied to, this 2 and this 2 will get cancel. So, T_c into T_c into V_c plus $V_m(t)$ divide by 2 V_c into sorry not it is, our notation is not V_{DC} , it is E_0 sorry E_0 by 2. So, this is E_0 by 2. So, here also it is we have to write E_0 by 2. Here, it will be into this is the volt second, E_0 by 2.

Then what will happen when S_2 is on? S_2 is on, the time period is this is during S_1 is on, this period. So, T_c minus this period we will give you that is the period where S_1 is on that is this period, we will give the S_2 on period. So, S_2 on is equal to total T_c minus T_c . So here, let see clarity, I will remove this diagram. So, we have more clarity here. So, S_2 on period, so T_c minus and T_c minus, there is a total carrier period triangle period minus the on period of S_1 on period that is what we got; T_c into V_c plus $V_m(t)$ that is the $V_m(t)$ is the sine amplitude during the triangle period during the comparison under consideration now; $V_m(t)$ divided by 2 V_c and this is the S_2 on period. Now, the volt second multiplied by minus E_0 by 2.

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So, for clarity, let us go the next page. I have redrawn the sine triangle comparison during one sampling period. Now, we know the volt second, volt second when S_1 on, volt second when S_1 on is we got it that is T_c into V_c plus $V_m(t)$ divided by 2 V_c the whole multiplied by E_0 by 2 volt second S_{10} . Now, what is the volt second during S_2 on? The period, the time period during each S_2 on multiplied by minus V_{dc} by 2. We know that when S_2 in on, when the triangle is greater than the sine wave that is here, during this portion; so thus S_2 will be on and output voltage V_0 will be minus V_{dc} by 2.

So, what to the S_2 on period? S_2 on period is very easy to find out. We know the S_1 on period; T_c minus that one. So, S_2 on is equal to T_c minus T_c into V_c plus $V_m(t)$ divided by $2 V_c$. This is the S_2 on period multiplied by minus E_0 by 2 will give the volt second. So, the volt second during S_2 on, when S_2 on is equal to T_c minus T_c into V_c plus $V_m(t)$. $V_m(t)$ is a function of time at which a triangle period, we are sampling. That is why we have put as a function of time divide by $2 V_c$ whole multiplied by minus E_0 by 2.

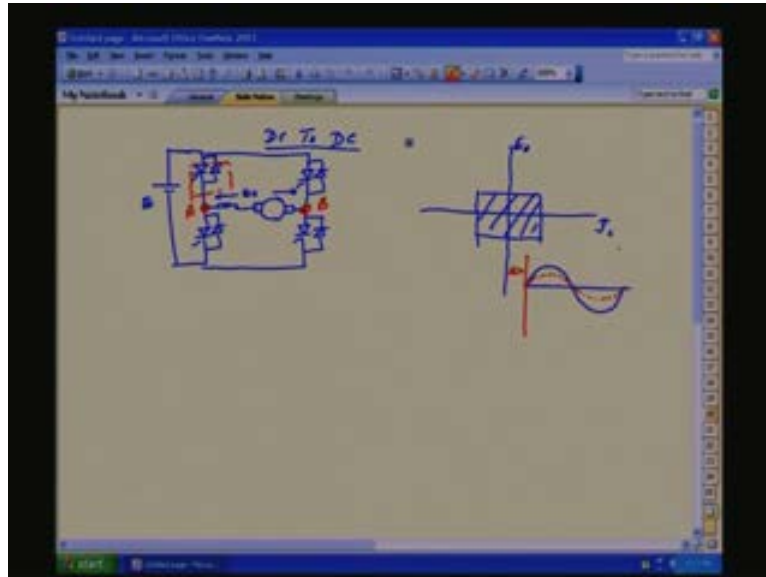
Now, the average value is the summation of these 2 volt second divide by T_c . So, V_{A0} average for a period T_0 , I can write it here so that it will be clear; V_{A0} average, total volt second divide by T_c is equal to total volt second divide by T_c for triangle period that is the average value. So, how it will be? If you sum these 2 and multiply if you see, it will be equal to 1 by T_c that is the average value into T_c by 2 plus T_c into $V_m(t)$ divide by $2 V_c$ then minus T_c by 2 plus T_c into $V_m(t)$ divide by $2 V_c$; we are taken care of the positive and negative multiplied by E_0 by 2, here I will write the E_0 by 2.

So, this will be, so if you see here, T_c by 2 T_c by 2 get cancelled. So, if I you say, this one and this one get cancelled. Then the whole V_{A0} average, I will write it here; V_{A0} , average value of V_{A0} during a sampling period **sorry** during a sampling period means during the triangle period, during the period T_c during the, during a period, period T_c is equal to V_{A0} average will be equal to from this two equations, it will be E_0 by 2 into the value of sine wave at that instant divide by V_c , V_c is the carrier period.

So, if you see, the average value will vary when $V_m(t)$ vary because these two rest of the periods are constant. So, during assuming high frequency sine triangle PWM such that during a triangle period, the sine amplitude is not varying, we are assuming constant; so that is why we said high frequency PWM such that during sine triangle PWM, the average value of the output V_{A0} PWM wave form will be proportional to $V_m(t)$. So, to control the $V_m(t)$ or control the average value V_{A0} average, you wave the $V_m(t)$ amplitude. So, it will be proportionally varying. Now, this way, we will be controlling.

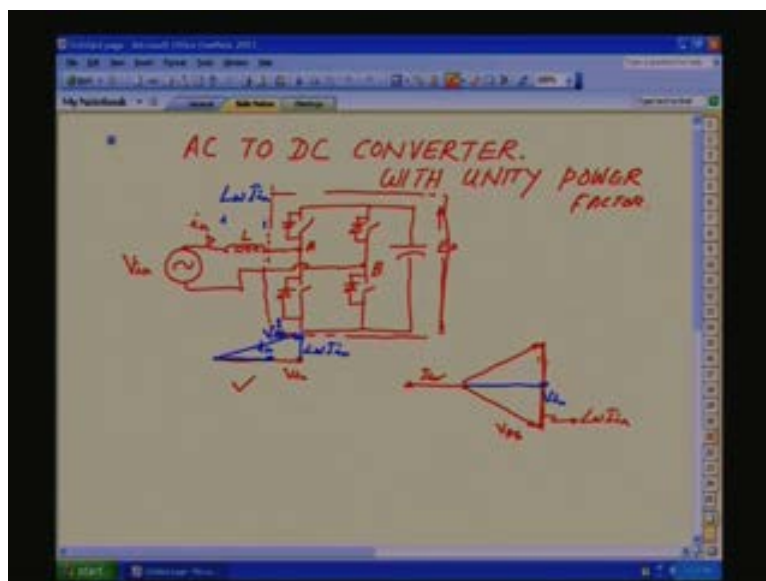
Then, the same way, it is for leg A. In the same way, we can control the leg B also; so that is V_{B0} . So finally, what to the AB? V_{AB} will be equal to, V_{AB} will be equal to V_{A0} minus V_{B0} .

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If you refer the previous figure that is here; so the voltage V_{AB} , we are independently controlling V_{A0} . A_0 means the zero with respect to the fictitious center point of this power supply. So, V_{A0} minus V_{B0} will give V_{AB} . So, we have to generate the reference waveform V_{AB} such that we get unity power factor, unity power factor as I shown here. So, V_{AB} has to be moved in this direction **sorry** the tip of the V_{AB} should be moved in this direction depending on I_A such that unity power factor, we will get.

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Now, see average value, we got. Now, we went for the high frequency PWM, sine triangle PWM; how about the harmonics? See, if you analyze this one through Fourier series, its highly it will get into lot of complication. But in a simple way, we can analyze how the harmonics, how the harmonics are present during a sine triangle PWM. See, this I am taking from a famous book by Professor Ned Mohan. So here, we can analyze that one.

Now see, sine triangle PWM as I told is the fundamental waveform, the average waveform is proportional to $V_m(t)$, our sine wave or whatever the waveform you are trying to compare with a triangle wave form. Now, where is the next higher order harmonics? Because the output waveform is a pulse width modulated waveform, pulse width modulated wave form, that is what we have shown before and we have talked only about the fundamental.

So, apart from the fundamental, if you have the harmonics, this harmonics will be, the high harmonics amplitude will be shifted to the triangular frequency. So, the advantage is if you use a high frequency triangle waveform, if possible if your power converter permits; the next high amplitude harmonics will be at the triangular frequency. So, the current drawn due to this harmonic voltage will be much less because of the $L\omega$, ω is more, harmonic that is the amplitude is, that the frequency of that one depends on the triangular frequency.

So, $L\omega$ will be higher, the current drawn due to their harmonic, harmonic will be much less. So, the ripple current is much lower and output you will get nearly sinusoidal current. So, in the next class, we will study briefly, simple thumb rule for a sine triangle; how the harmonics are placed? Now, then for our front end ac to dc converter, how do you use the PWM such that some of the harmonics can be suppressed. So, as the harmonics are suppressed, the current drawn from the mains or the ac side will be more sinusoidal with unity power factor that is what we want. That we will talk in the next class.