Course Name: Design of Electric Motors

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Week: 09

Lecture: 50

Title: Equivalent Circuit Parameters of Induction Machine

Greetings to all in the last lecture we have discussed the resistance leakage inductances of stator as well as rotor with respect to the various leakage flux components and magnetizing inductance as well as magnetizing current. In this lecture we will discuss the equivalent circuit analysis of an induction machine. If we will consider the three phase balanced machine where the mechanical displacement of the windings as well as electrical current displacement of the windings is 120 degree. In that case the equivalent circuit for an induction machine with respect to per phase is like this we can see in this slide. This is the per phase equivalent circuit of an induction machine. Rs represents the stator side resistance, Xls represents the stator side leakage reactance component and jXm represents the magnetizing reactance and Rc represents the core loss component and respective currents Ic and Im with respect to the magnetizing and core loss component and Isis the stator side current and V phase RMS is the applied voltage and the current referred to the stator side we can see Ir dash the rotor current referred to the stator side is Ir dash and the induced voltage at the stator side as well as rotor side is E 1 and E 2 and rotor side resistance, rotor side leakage reactance component and mechanical load as well as loss components we can see in this equivalent circuit.

If the system or three phase machine is balanced the equivalent circuit is same for all three phases. If the three phase machine is not balanced either it is mechanical displacement or electrical ah current thing we have to consider the different equivalent circuits per phase. If we will refer the rotor side variables to the stator side the referred rotor resistance will be Rr dash that is the resistance which is referred from rotor side to stator side with some transformation ratio that transformation ratio we will discuss in the coming slides and Xlr dash is nothing, but reactance referred to the stator side and Rr dash into 1 by s minus 1 is nothing, but mechanical load and loss component and if we will neglect the core loss component the equivalent circuit will be in this fashion this is the equivalent circuit with respect to the IEEE standards. Why we have to study the equivalent circuit means to analyze the performance behavior of a machine with respect to the copper loss in stator as well as rotor maximum and full load torques and output power calculation with respect to the copper losses and power factor at full load as well as no load condition.

After knowing all these parameters like resistances inductances or reactances and resistance and reactances at the rotor side we can analyze all these performance parameters that we will see 1 by 1. First we will discuss the stator resistance per phase the general equation for a to calculate the resistance of an ah winding that is rho l by A we have discussed the equations related to the stator resistance per phase already this is the final equation. Our phase is equals to rho l length of the total coil divided by ac is nothing, but area of cross section of each conductor we can see right side here the single turn or 1 coil consist of N number of turns and if all coils placed inside the machine we can see the image at the bottom side. How to calculate the length of a coil means the length of coil is nothing, but 2 times the core length if we will place this coil in a machine core the length of the core is nothing, but le. So, 2 times the length of core we have to consider and then the end winding length 2 times the end winding length.

So, this end winding length we have to calculate depends upon the arc length like pi D I s by P. So, number of poles is nothing, but P and D I s is the inner diameter of stator this is the pole pitch pi D I s by P is nothing, but pole pitch into short pitch factor for a full pitch this short pitch factor will be 1 for full pitch winding for short pitch winding depends upon the short pitch angle let us say the coil is short pitched by 1 slot. So, slot angle equals to 30 degrees means for an example. So, substitute that 30 degrees here and find the short pitch factor and substitute in the length of the coil equation and the additional 20 to 30 percent of core length why we have to consider means we can see here some straight portion of the coil and this is the portion where the end winding is sitting the arc length and this is some portion of conductor straight portion of the conductor which is coming out of the core we can see here with respect to the machine some coil length is coming out of the core the useful conductor length sitting inside the core will be le the remaining straight portion of the coil length will be 20 to 30 percent of the machine we have to consider for a large mission we can consider 20 to 30 percent, but for small missions we have to consider slightly higher up to 30 percent we can consider. In order to meet the current rating let us say each coil is carrying 5 ampere to meet the 10 ampere rating we have to connect 2 wires in parallel.

So, this winding is carrying 5 ampere and this winding also carrying 5 ampere. So, total we can see 10 ampere. So, effective resistance if you have to calculate then 1 by parallel number of circuits into actual resistance of a phase that is 1 by c into rho l by a will give the effective resistance of a phase with respect to the parallel connected windings or parallel connected circuits.

Then how with respect to the material we have to choose the resistivity of the material for stator side windings most of the cases we will utilize the copper the copper resistivity value is 1.68 into 10 power minus 8 ohm meter and for rotor bars squirrel cage rotor machine we can see here the rotor squirrel cage rotor the resistivity for aluminum bars will be 2.65 into 10 power minus 8 ohm meter. So, other materials also we can see conductor semiconductors and insulators, but most of the electrical machines we will see the copper windings. Then the leakage inductance or leakage reactance with respect to the stator the leakage reactance is equals to 2 pi f s into Lls, here Lls is the stator side leakage inductance and f s is the stator frequency. The leakage inductance of the stator consist of 5 components first one is with respect to the slot leakage component and second one is with respect to the end winding third one is harmonic or belt leakage inductance fourth component is the zigzag leakage inductance component and fifth one is the skewing effect based inductance. So, by considering all this 5 leakage inductances we have to calculate the stator leakage reactance.

The equations associated with this 5 leakage components we have discussed already in the previous lectures. Same equations we have to apply here to find this leakage components. Then magnetizing inductance depends upon the stator mmf and area flux density. So, the magnetizing inductance Lms is equals to 3 by 2 into 4 by pi that is constants and number of turns square by number of poles and k 1 is nothing, but winding factor that is the product of different factors like pitch factor, distribution factor, skew factor and slot opening factors and pi D by P is nothing, but pole pitch le is the length of the stator core and 2 by pi into B g 1 is nothing, but average area flux density F s 1 is nothing, but stator peak fundamental mmf.

Then rotor resistance per mesh that is R r the rotor resistance with respect to one particular mesh is equals to this one that is 2 into rotor resistance with respect to a single bar rotor resistance with respect to one end ring divided by 2 sin square into 2 sin square of slot angle with respect to rotor pi P pi into P by Q r is the slot angle alpha by 2 we have to consider here.

Then rotor resistance with respect to single bar based upon the conventional resistance equation rho l by a we can see here that is R b is equals to resistivity of the bar material and length of the each bar and area of cross section of the bar length of the bar how to calculate if without skewing is there then length of the bar is equals to le. If we will skew the bar the bar length will increase right that is skewing angle is nothing, but pi into P by Q s. So, based upon the skewing angle alpha skew we have to find the new length of the bar. So, that is le by cos skew angle and area of the area of cross section of the bar is equals to I bar current rms value divided by current density of the bar. So, with respect to the aluminum we can consider the current density and bar current also we know then area of cross section of the bar we can calculate it.

Similarly, the rotor resistance with respect to each end ring this is the end ring portion each end ring like the two end rings are there right. So, based upon these two end rings we have to calculate the end ring resistance again the same formula rho l by a the length of end ring we have to calculate the mean length the inner diameter of the end ring as well as outer diameter of the end ring we have to consider. Then the perimeter with respect to these two diameters will be pi into D naught r minus 2 d e by 2 is nothing, but perimeter with respect to the inner diameter of the end ring and pi D naught r is nothing, but perimeter with respect to the outer diameter by 2 will give the mean length of the end ring that is final equation will be pi into outer diameter of the rotor minus depth of the each end ring that is d e value and cross sectional area of each end ring we can calculate it by I by J.

Then rotor leakage inductance per rotor mesh we can calculate similar to the resistance thing that is 2 into bar leakage inductance and bar harmonic leakage inductance. So, L b e is nothing, but effective bar inductance that consist of two components one is bar inductance other is end ring inductance and L b harmonic is nothing, but harmonic leakage inductance.

So, we have to calculate the bar leakage inductance depends upon the n square into permeance equation and here permeance with respect to the each bar will be in this manner depends upon the slot dimensions like d 3 d 1 r b s r and d naught r are the slot dimensions we have discussed already in the earlier lecture and length of the bar we can calculate it with respect to the skewing angle.

And the reference for this equation we can go through this text book and end ring leakage inductance we can calculate it by utilizing this formula and here d b is nothing, but rotor inner diameter measured at the middle of the end ring. So, that is nothing, but D naught r is the outer diameter of the rotor minus 2 d s r that is slot depth into depth of the each end ring mean depth I can say d e. So, based upon that thing we have to calculate the diameter and we have to substitute d b value in the end ring leakage inductance that is equals to mu naught into rotor inner diameter measured from the middle of end ring divided by 2 into number of rotor slots into the remaining equations we can see here.

And then rotor harmonic leakage inductance is nothing, but L b harmonic we have to calculate based upon the permeability and pole pitch with respect to the rotor and length of the core and then number of slots at the rotor side and number of poles.

So, this is the harmonic sum that is 2 into N into number of slots by P plus 1. So, how it will vary this term with respect to the different harmonics we have seen the curve already based upon that thing we can calculate the harmonic leakage inductance at the rotor side. Once we have calculated the rotor resistance and rotor leakage inductance how to transform the these variables to stator side. So, we have to consider the transformation ratio or turns ratio right. So, the turns ratio for multiple winding based ideal magnetic system will be like this N 2 by N 1 N2 is the number of turns at the secondary side N 1 is number of turns at the stator side voltage and currents voltage at the secondary side will be V 2 voltage at the primary side will be V 1.

So, the MMF turn N2 I 2 is equals to N 1 I 1. So, by utilizing this MMF equation we will derive the turns ratio first then transformation ratio to transfer the rotor side resistance and reactance to the stator side. First MMF term we can consider. So, the stator side MMF that is ms is the number of phases at the stator side, Ns is the number of turns per phase at stator side and k 1 s is nothing, but winding factor with respect to stator winding and Ir dash is the rotor current referred to the stator side. Secondary side we have mr is the number of phases at the rotor side and number of turns per phase at the rotor side that is number of turns per phase will be 1 right if I will consider 1 bar per phase.

If I will consider Nr by 3 per phase then number of phases at the rotor side you have to consider 3. If you are considering number of phases at the rotor side that is Qr equals to Nr then here number of turns per phase should be 1. In both cases you will see Nr only the product of mr into Nr will give the Nr value only and then rotor current is Ir and winding factor with respect to the rotor is k 1r. So, here k 1r depends upon the skewing the winding factor at the rotor side the winding factor at the stator side depends upon the pitch factor distribution factor and skewing factor and slot opening factor. From this equation we can see the transformation ratio or turns ratio Ir by Ir dash is equals to ms number of phases at the stator side Ns and k 1s divided by mr into Nr into k 1r.

So, the final equation with respect to the turns ratio we will get this one. Now, if we will see the equivalent circuit of the induction machine before transferring the rotor side variables or parameters to the stator side and after transferring the rotor side variable to the stator side the loss with respect to the rotor resistance before and after referring the with respect to the transformation terms should match. So, that ratio we have to maintain. So, the loss with respect to the resistance before referring and after referring has to be match. So, based upon that analogy we can see here with respect to the single bar resistance here Rb is the single bar resistance Ir is the rotor current and mr is the number of phases at the rotor side and ms is nothing, but stator side number of phases and Ir dash is nothing, but rotor current referred to the stator side and Rb dash is nothing, but rotor resistance bar resistance referred to the stator side.

So, the copper loss equations we have to match. So, based on this equation Rb dash rotor resistance referred to the stator side divided by rotor resistance of the bar is equals to Qr into Ir square. Qr is the number of bars or number of slots at the rotor side divided by number of phases at the stator side into Ir dash square. So, from this equation if we will substitute Ir divided by Ir dash terms ratio term here square means 2 times we have to substitute this is 1 and this is 2 substitute those things here Ir by Ir dash term 2 times here then we will get the Rb dash rotor bar resistance referred to the stator side. The turns ratio or transformation ratio will be this one that is 4 into ms is the number of phases at the stator side and N phase is nothing, but number of turns per phase square into winding factor with respect to the stator square divided by number of slots or number of bars at the rotor side into winding factor at the rotor side square.

Now, if we will consider the complete resistance per mesh, Rr is the resistance per mesh, right rotor mesh. So, if I will consider complete mesh, so we have to multiply the I square R loss with respect to the how many number of meshes are there, right. So, m is m mesh is nothing, but number of meshes at the rotor side that into Ir square into Rr the resistance of the each mesh that should be equals to the number of phases at the stator side that is 3 if number of phases are 3 then 3 Ir square that is reflected referred to the stator side current into rotor resistance referred to the stator side.

So, based on this equation also we will get we will arrive at the same transformation ratio Rr dash divided by Rr is equals to 4 into ms is the number of phases at the stator side and turns per phase and winding factor at the stator side divided by rotor number of slots or rotor number of bars into winding factor at the rotor side square. So, this transformation ratio will not change whether you are transferring one bar resistance or complete rotor bar resistance that ratio will not change that same ratio will be applicable to refer the leakage reactance component to the stator side that we can see here the bar resistance and rotor resistance per mesh both are having the same transformation ratio.

And this rho TR is nothing, but transformation ratio this transformation ratio term will be applicable for reactance also. So, the leakage reactance of the rotor referred to the stator side will be this thing Llr dash divided by Llr is equals to the rho TR that is the transformation ratio this one Xlr is nothing, but reactance of the rotor referred to the stator side here the frequency is the stator side frequency fs into Llr dash.

So, the final equivalent circuit parameters we have we know how to calculate the resistance at the stator side reactances and resistance of the rotor resistance of the sorry reactance of the rotor referred to the stator side and magnetizing inductance and mechanical load component if you know the slip then we have calculated the all impedance terms of this equivalent circuit then we can analyze the stator current equations and power factor equations right. So, the impedance with respect to the rotor side is nothing, but Rr dash by S plus J into Xlr dash just addition of these two impedances and then Zf is nothing, but parallel or equivalent impedance seen at these two terminals Jxm parallel to the Rr plus JXlr dash that is the Zf. So, equivalent impedance with respect to these two terminals A and B is nothing, but this one then total impedance seen with respect to the supply terminals will be Zs that is sorry Zs is the stator impedance the total impedance seen at the stator input terminals is nothing, but Z equivalent Z is equivalent is equals to impedance with respect to the stator that is Zs plus the impedance with respect to the points A and B that is Zf then we can calculate the stator side current is nothing, but voltage divided by impedance then magnitude and angle both we will get it from this equation that angle is nothing, but power factor angle.

So, cos of that particular angle will give the power factor of the machine under the loaded condition then the rotor equivalent equations as per this equivalent circuit we can calculate based upon the current division rule the stator current we know that is the Is into opposite impedance divided by total impedance Z opposite plus Z same thing. So, here Z opposite is nothing, but magnetizing reactance and Zr is nothing, but Rr dash plus J Xlr dash based upon that thing we can calculate the rotor current referred to the stator side.

Then how to find the rotor short circuit current equations under blocked rotor condition generally the blocked rotor test we will perform with the reduced voltage and reduced frequency if we will operate at the rated voltage because of the lesser resistance as well as impedance huge currents will flow because slip is equals to 0 because of that reason we have to reduce the phase voltage as well as frequency with a factor Xsc voltage per phase reduced value and frequency reduced value we can get it the factor of reduction with respect to the short circuit condition into the phase voltage and factor of reduction into frequency. So, with respect to this reduced phase voltage and equivalent impedance we can calculate it based upon the equivalent impedance then the short circuit current under the blocked rotor condition is equals to V divided by Z equivalent and that equation with respect to the resistances and inductance term we can see in this equation this one. So, this is the rotor short circuit current equation.

So, with this I am concluding this lecture. In this lecture we have discussed the summary of the equivalent circuit parameters like resistances, inductances or leakage reactance components and how to find the power factor and how to find the different currents with respect to the equivalent circuit analysis we have discussed. Thank you.