Energy Resources, Economics and Environment
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Lecture 09
Non-Renewable Resources Economics- Part 1

We have already looked at fossil fuels and renewable resources. Today in today's module we are going to start looking at resource economics. So, we will start with some of the earliest studies and the earliest research which was done on non-renewable or fossil resource economics.

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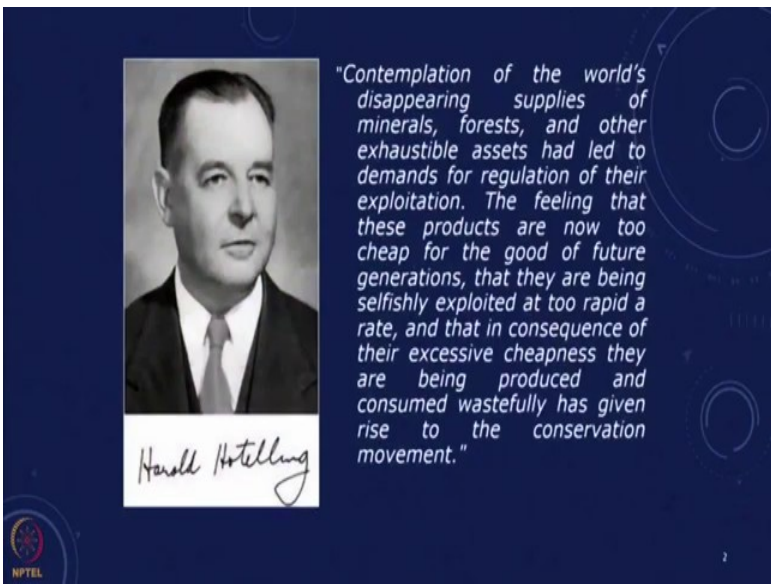

So, we start with way back in the 1930s Herald Hotelling, was an economist who was concerned about resources, published a paper and we will upload the original paper for you to look at. But in the introduction of the paper he talks about contemplation of the worlds disappearing supplies of minerals, forests and other exhaustible assets has let to demands for regulation of their exploitation.

And the feeling that these products are now too cheap for the good of human future generations that they have been selfishly exploited at too rapid a rate and that in consequence of their excessive cheapness they are being produced and consumed wastefully has given rise to the conservation movements.

So this was the initiation of his study, remember this is 1930s an era where we were thinking that we have abundant fossil fuels, abundant material and the idea is to try and use them at a fast pace, so that the world develops and the economy is developed and this was the first paper which talked about a fundamental theory to look at how much should be produce in a particular year given the fact that the overall resources are finite and that is also called in literature as the mine mangers problems, so we will start with his analysis and then we will built on that and see how to work that.

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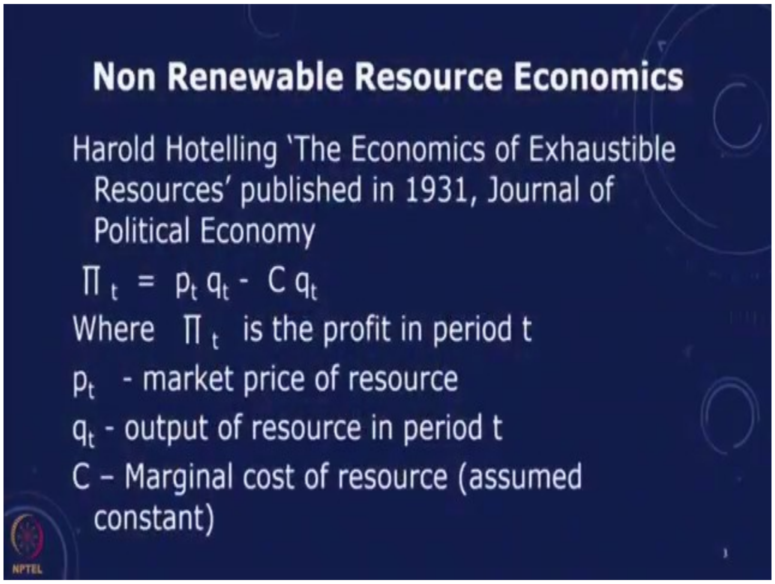

So, this is the paper which was published in the economics of, this is the paper title was Economics of Exhaustible Resources, it was published in 1931.

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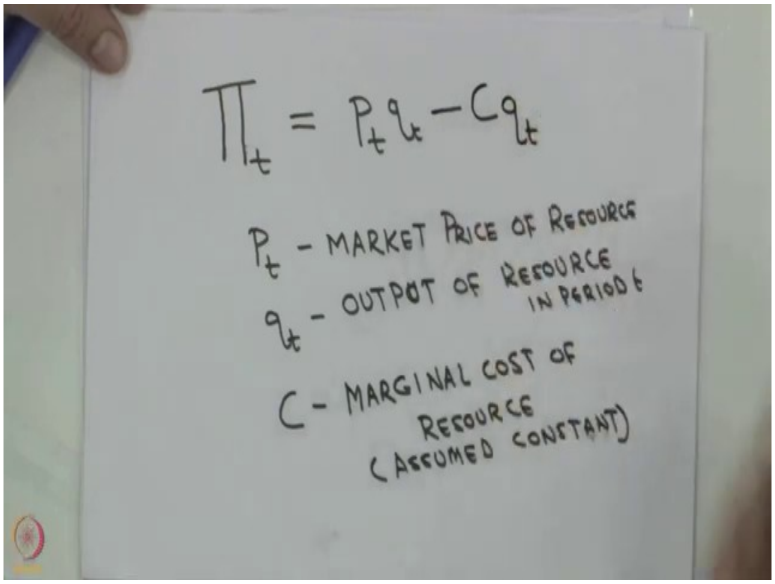

$\prod_{t}$ ¿ $P_t q_t - C q_t$

$P_t = MARKET\ PRICE\ OF\ RESOURCE$

$q_t = OUTPUT\ OF\ RESOURCE \in PERIOD$

$C = MARGINAL\ COST\ OF\ RESOURCE\ (ASSUMED\ CONSTANT)$

And it starts with the idea that we have a function in the Tth year the profit that a mine manger has is Pt where Pt is the market price of the resource, let us say coal or oil market price of the resource per unit of course and qt is the production or the output of that resource in the Tth year, output of resource in the Tth year, in period t.

So, this will be Pt into Qt minus, minus the cost that means there is a marginal cost of the resource for extraction of the resource there is an amount that we have to pay to extract the resource marginal cost of the resource, which in our simplest model we will assume that to be constant, assumed to be constant. So, from the Pt qt which is the revenue we subtract minus C into qt, this is the profit in the Tth year.

Now for a mine, the decisions that we have to take are how much should we produce in each of the years and then you are going to have a production till the time when the entire resource gets utilized and that is where it gets exhausted.

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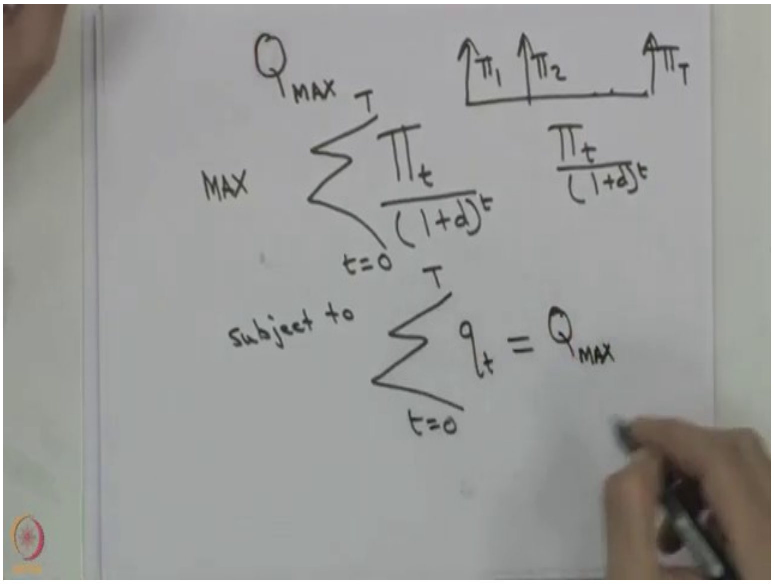

So if we look at this we are looking at the mine owner having a fixed stock of homogeneous resource let us call that Q max and that is the total amount of coal or oil whatever we have in the mind and we are saying that the fix cost of extraction which is C, so we are trying to get the discounted value of the profits, so we want to maximize T equal to 0 to T we want to maximize the profit and this profit remember just we discuss in the economics portion when we looked at a project. Each year we will have a profit so you will have like profit Pie 1, Pie 2 and so on till the resource gets utilized.

So each one will be the different values which you have will be defined as Pie t divided by, so will take the discounted sum of this, so we will take this as, we are maximizing this, subject to the constraint t is equal to 0 to t qt is equal to Q max. Naturally this constraint in the when you get the maximum profit the entire resource will be utilized, so the decision is we have a total amount of resource we want to decide how much should be mine each year till the resource gets over.

And in each year based on this we are getting a certain amount of profit that is discount, that is summed up and we get the net present value of all the profits that is what we want to maximize. So, this is a classic optimization problem where we can see the time horizon T is exogenously determine we have to find out what is that time horizon T. we can form the Lagrange and then differentiate it to get the optimum.

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$$\sum_{t=0}^{T} \frac{\pi_t}{(1+d)^t}$$

$$MAX \sum_{t=0}^{T} ¿¿¿$$

$$\sum_{t=0}^{T} q_t = Q_{MAX}$$

So, the problem which is being defines is simply sigma t is equal to t Pie t by 1 plus d raise to t, d is the discount rate and in Hoteling's original formulation he talked about this as the interest rate but we now know that when we are looking at this it is the discount rate that we should really be talking about. So, this Pie t we can write this down in this form t is equal to 1, 0 to t Pt qt minus C qt 1 plus d raise to t, subject to sigma t is equal to 0 to T qt is equal to Q max.

This is an equality constraint and we are maximizing an objective function subject to an equality constraint we can use the method Lagrange multipliers, we can create the Lagrangian and then differentiate it with respect to the variables.

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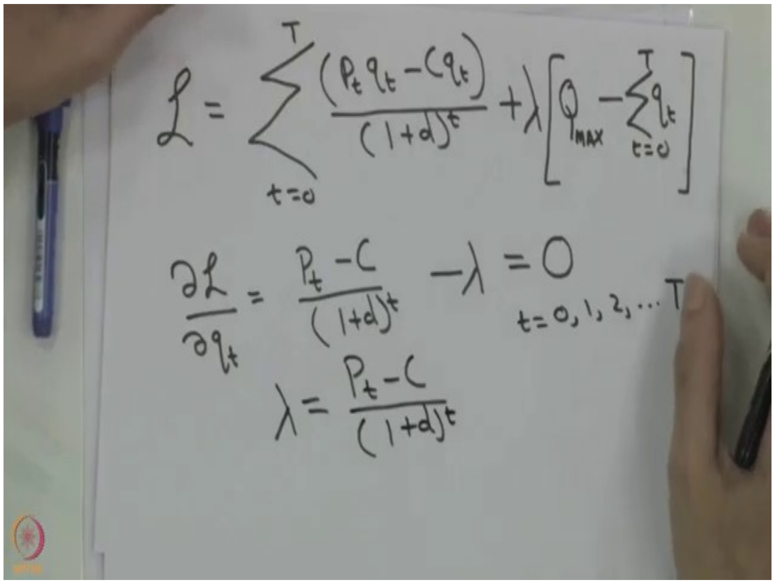

So, the Lagrange that we will get will be, Lagrange will be sigma t is equal to 0 to t, Pt qt minus C qt by 1 plus d raise to t, this is your original objective function, plus lambda there is only one constraint lambda. You associate a Lagrange multiplier with respect to the constraint and we take that constraint as Q max minus sigma qt this is the Lagrangian that we have constructed.

Remember this is the objective function, this is the constraint which we have multiplied by the Lagrange multiplier and added it to the modified objective function. Now we can differentiate this with respect to the variables.

What are the decision variables, what are the variables that we have? The variables are qt that means q1, q2, q3 and so on till it gets depleted, so this we can differentiate till Lagrangian by del q and this will come to Pt minus C and then when you differentiate this Q max is constant, this will have a term minus lambda is equal to 0. We have equations so we will have this as the equations that we have you will have the dell by dell qt we can essentially, we will have this as t is equal to 0, 1, 2 and so on to t. So, if you look at this what will be, there is the lambda will be Pt minus C by 1 plus d raise to t that is the Lagrange multiplier and this is constant across the time intervals.

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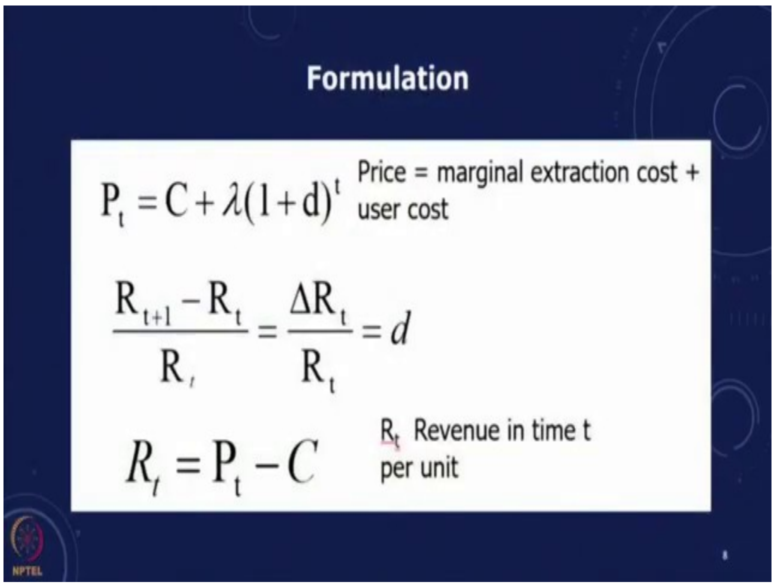
$$\lambda = P_0 - C = \frac{P_1 - C}{(1+d)} = \frac{P_t - C}{(1+d)^2} \ldots..$$

$$P_t = C + \lambda(1+d)^t$$

$$\frac{R_{t+1} - R_t}{R_t} = \frac{\Delta R_t}{R_t} = d$$

$$R_t = P_t - C$$



So, if we write this down we will find that P0 minus C by 1 plus d raise to 0 which is 1 is equal to P1 minus C by 1 plus d equal to P t minus C by 1 plus d square and so on. So, if you look at this, this is essentially what we get is Pt is constant plus lambda into 1 plus d raise to t.

So, the price is equal to the marginal cost of extraction plus a user cost which increases with the time. So, if we look at what is this difference between Pt minus C which is profit that we are getting if we take return Rt plus 1 minus Rt by Rt we will get this as per unit profit that we are getting or return is increasing has to increase year by year, the revenue in time t, this is Rt is Pt minus C.

Revenue per unit in time t has to increase on a year to year basis at the discount rates, so that is what we have shown. That would mean that in general what will happen is that the price if the c is constant price will increase with time and the rate of increase would be at the rate of the discount rate.

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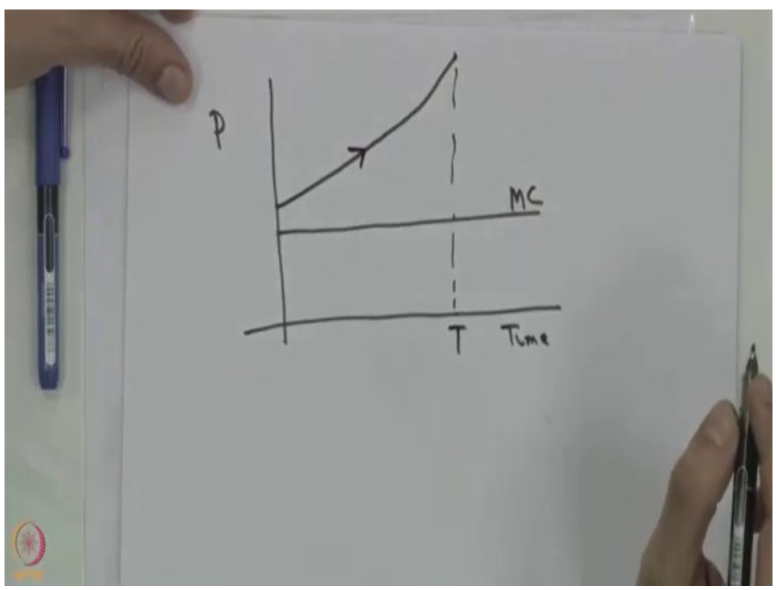

So, if you look at it in terms of a graph, what we will get is, you will get that, if this is the marginal cost which is C which is constant and we start with price and time till the time when we will actually, T when it becomes so costly that qt becomes equal to 0. Now this can be put in the form of what is known as a demand curve but before we see the demands curve we will look at what the result which Hotelling put in his paper.

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And the result shows that the market price of a resource net the extraction cost must rise at a rate equal to the rate of interest and this is what was there in his paper but we know subsequently that when we are looking at this we are basically talking of the market price of a resource, net its extraction cost will be rising at the discount rate and that is when we talk of an exhaustible resource we will try to get the optimum strategy is to see that the price minus the extraction cost divided by 1 plus d raise to t is constant across different time intervals and that gives us basically an optimal extraction strategy. Now, let us move forward and look at what we understand as a demand curve.

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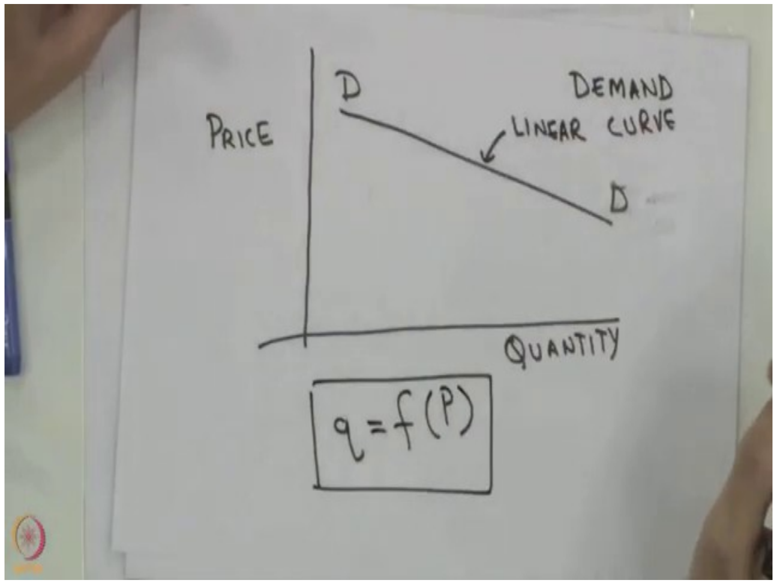

So, typically what happens is for any commodity when we look at price and quantity, typically what happens is that as the price of quantity increases the quantity demand decreases, so you, we have something like this which is, this is a something called a demand curve, this is a linear demand curve and typically what we will do is we will say quantity is a function of the price that is typically what we expect, this is a sort of linear correlation. Now, one of the things when we want to understand in economics when we talk in terms of the demand we define something called the elasticity.

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Price Elasticity

$$\eta = \left| \frac{\frac{d q_t}{q_t}}{\frac{d p_t}{p_t}} \right|$$

And it is called the price you would have studied this in your basic economics, we talk about elasticity and we will talk in terms of a price elasticity of demand and that typically is that if we say that suppose the price of a commodity or a good increase by 1 percent what happens, what is the percentage of demand for a unit percentage change in the price?

So, the way we define this is we define an elasticity which will be the delta dqt by qt divided by dPt by Pt, so this is in general what will happen is as the price increases the quantity demanded will be decreasing, so this could, this would generally be a negative term. Of course, in the elasticity we may take the elasticity and we may take the absolute value of the elasticity, so this is what we can define as an elasticity of price.

And of course, remember that as prices increase in the short term depending upon the kind of good and service sometimes if prices increase for instance the prices of onions in seasons sometimes increase very drastically. Sometimes you can substitute onions by something else at some point in some cases for instance if you are looking at using a certain amount of electricity and the price increases, in the short run you may still require that electricity and you have no option but to pay that price.

So, then the demand may be considered you might have a relatively a demand where there is it is in elastic, so even if the price increases it may not change much. On the other hand you might have possible substitute and you might be able to instead of electricity you may be able to use some other source of energy.

So then there would be an elasticity, so there in all these cases we have, we define two terms. One is the short-term elasticity and then there is a long term elasticity. Normally long-term elasticity is higher because you can actually putting investments so that you can have different kinds of substitutes and there are different studies by which we estimate these long terms and short-term elasticity.

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$$q_t = f(p_t)$$

Inverse Demand Curve

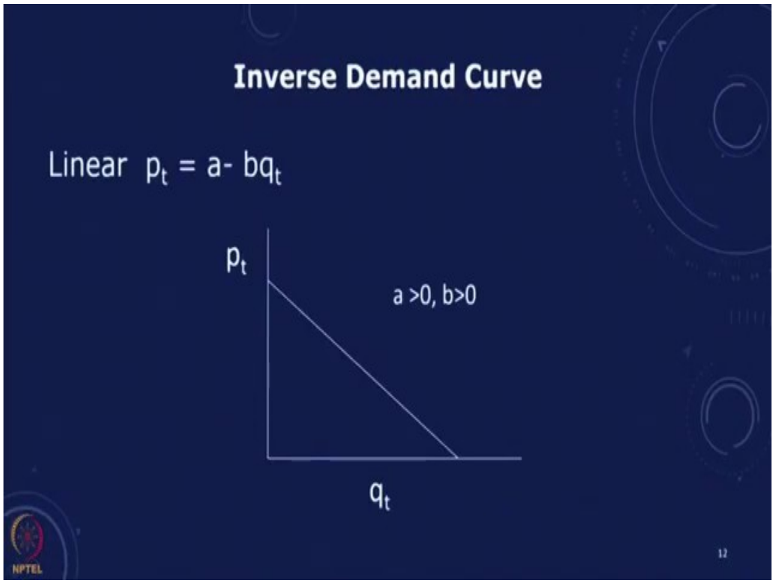
$$P_t = a - b\,q_t \qquad a>0$$

$$b>0$$

$$\eta_t = \left|\frac{d\,q_t\,P_t}{d\,p_t q_t}\right|$$

$$\frac{dp_t}{dq_t} = -b$$

$$¿\frac{a - bq_t}{-bq_t} = \frac{-a}{bq_t} + 1$$



So, we will take the demand curve that we have and then define this in term of an inverse. So, we said demand curve is where you have qt as a function of Pt, we will talk about an inverse demand curve and here will say Pt is a minus bqt, so if you look at the figure here you will see that this corresponds and we are saying a greater than 0, b greater than 0 then you have a straight line where you as there is a certain point at which at a particular Pt where the Pt goes very high then the qt is equal to 0 and so on.

So, this is the kind of inverse demand curve, let us now calculate for this what is the elasticity of this demand curve, we want to calculate what is delta t dqt by dpt and to pt by qt, so if you see this, this is going to be, dpt by dqt is equal to minus b and so the elasticity is we look at dpt by dqt, pt by minus dqt and pt is a minus bqt minus pqt. So, you will see that essentially the, at different points the elasticity will change at different points on this curve. So, you look at this and you will see that this is minus a by b qt plus 1, 1 minus a by bqt and so this is the kind of calculation that you can do.

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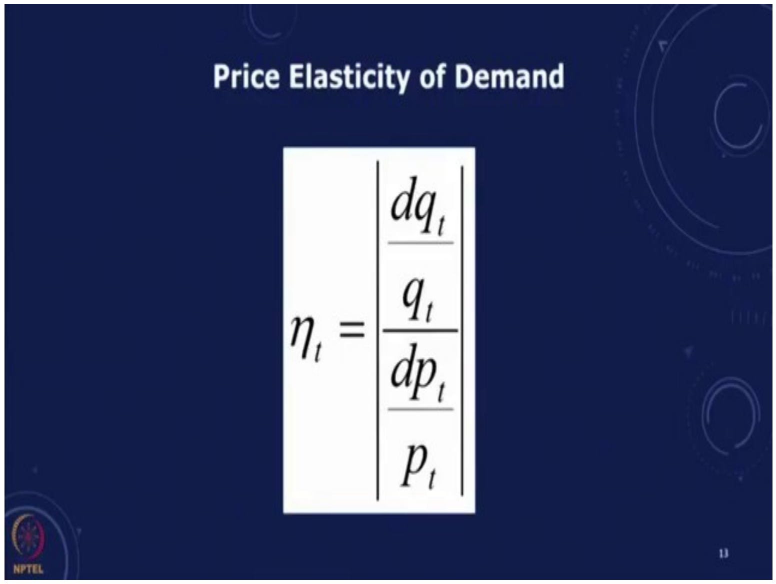

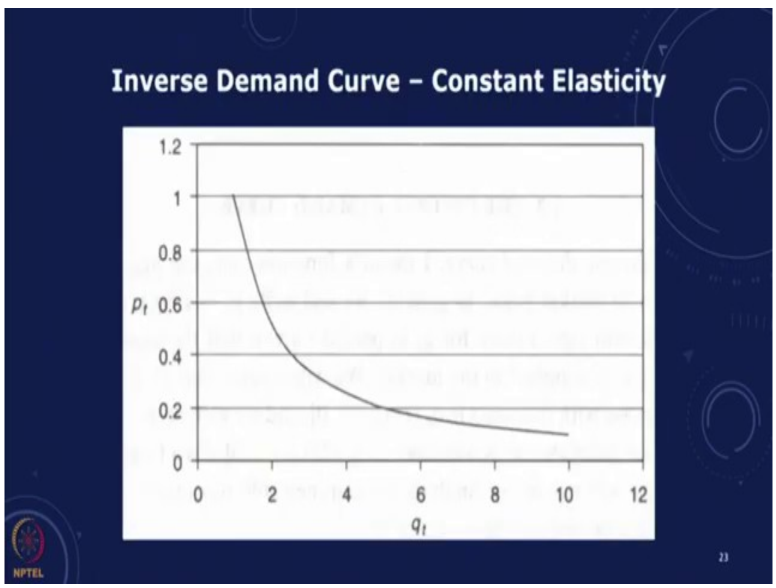

You will see that in these kinds of curve what will happen is that you will be able to calculate and you will have the values will be different at different points, it is not a constant elasticity curve. There is we can also look at an equation where we talk of a different kind of curve where you have a constant elasticity curve and then in the constant elasticity curve you will see that we can look at this, this is the kind of curve which we will have.

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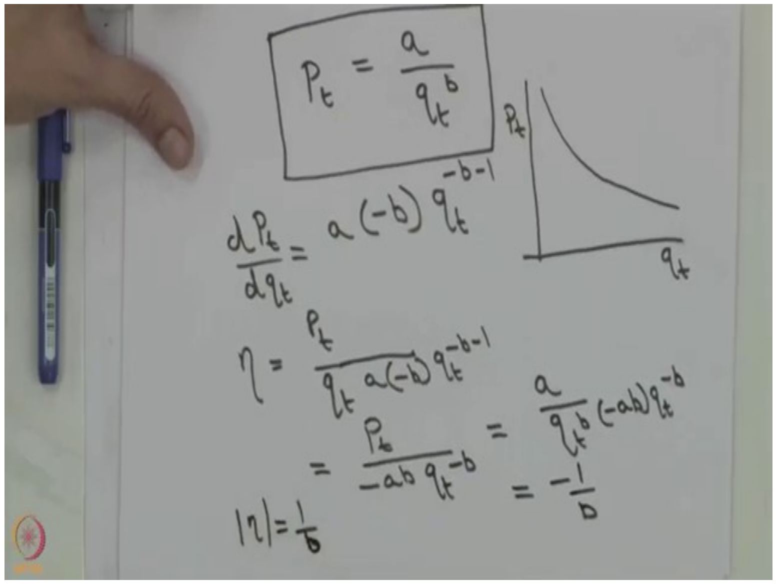

In this case, Pt we can write down an expression for Pt which is a by qt raise to b this is another inverse demand curve where essentially. So, at t tends to infinity the qt tends to 0. So, in general this is a constant and we look at, let us look at the calculating the elasticity of this curve, so if we look at the Pt by dqt this is going to be a minus b qt raise to minus b minus 1 and if we look at the elasticity which we had, if we look at the elasticity this is going to be now we are going to calculate this as Pt by qt into dpt by dqt which is we have a minus b qt raise to minus b minus 1.

And if we see this this becomes qt raise to minus 1 plus 1, 0. So, this qt raise to minus b and Pt is nothing but a by qt raise to b, so this is minus ab qt raise to minus b, this cancels and so you get minus 1 by b. If it is the modulus value then this is 1 by b. So, if you see this a and b are constants so this curve essentially represents a constant elasticity case.