Energy Resources, Economics and Environment
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Lecture 09
Non-Renewable Resources Economics- Part 2

So we have in the last module we have just seen the basics of Hotelling's model in which we saw the wide mangers problems reduces to a situation where we essentially keep the profit in different intervals divided by the discount, the discounted profit in different years should be constant and we also saw that when we talk in terms of the demand curve, we can look at different kinds of elasticity.

And we will now take the result that we had got where we said that the price of the commodity net the extraction cost, should increase at the discount rate for different time horizons, we will take that and derive for a given demand curve which is known, we would like to see how long will the resource last.

So, will consider a situation where there is a competitive mining industry which has a known facing will first start with taking a linear inverse demand curve and then we will take constant elasticity demand curve and we will derive how much is the time for which a given resource will last in a mine. So, that is the problem that we are looking at and as we saw this is a kind of inverse demand curve that we are focusing on, we would like to see first of all we start with a competitive mining industry which is facing a linear inverse demand curve.

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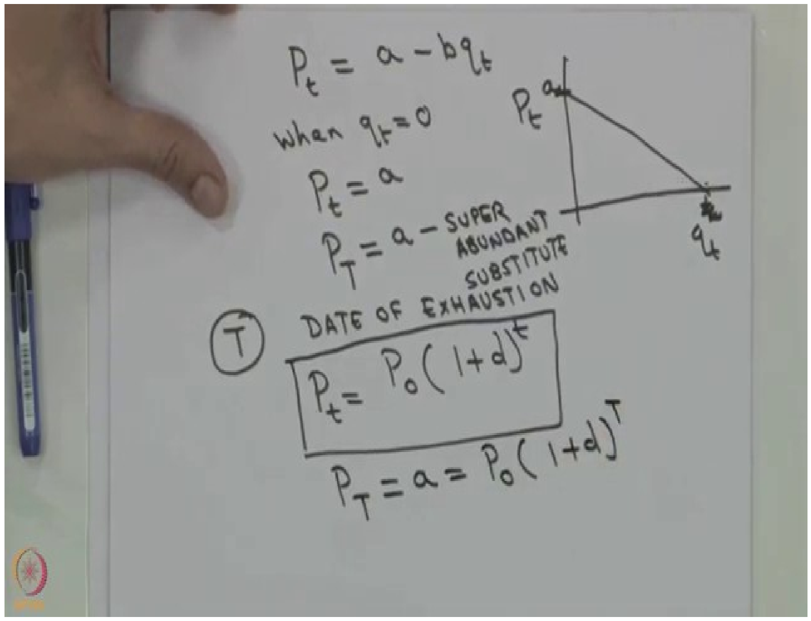

Let us say that when Pt is a minus bqt, so at that is given as the demand curve. Now at the time when qt will be equal to 0, we talked about, so we were looking at a situation when we will have the qt will be equal to 0 at some time interval.

And this is at a time where essentially we will have the price when qt is equal to 0, the price Pt let us take linear when qt is equal to 0, Pt turns out to be a, so let us call this time interval as t when the entire this is where we talk of Pt and qt at the time when the qt is equal to 0 that pt becomes equal to a, this is a and this is at a time when over a period of time when the resource has got completely utilized it is like we have something like there is a super abundant substitute.

So that no one continues to use this let us say coal, you have something which is much bet preferred and then this is and then there are no additional resources. Now the question for us is to determine this t, what is the date of exhaustion t and we would like to determine it using the fact that the, it is a competitive economy and the mine manger is trying to maximize the profit.

So we have, we know that the optimal strategy is where Pt will be P0 into 1 plus d raise to t, for this please remember that we have taken a constant extraction cost and we have subtracted, so we can take essentially we are taking P0 minus C, we are neglecting the extraction cost or we taking that the extraction cost is constant. So having said this, if this is the equation that we have got we can now substitute and get Pt is a, is equal to P0 1 plus d raise to t.

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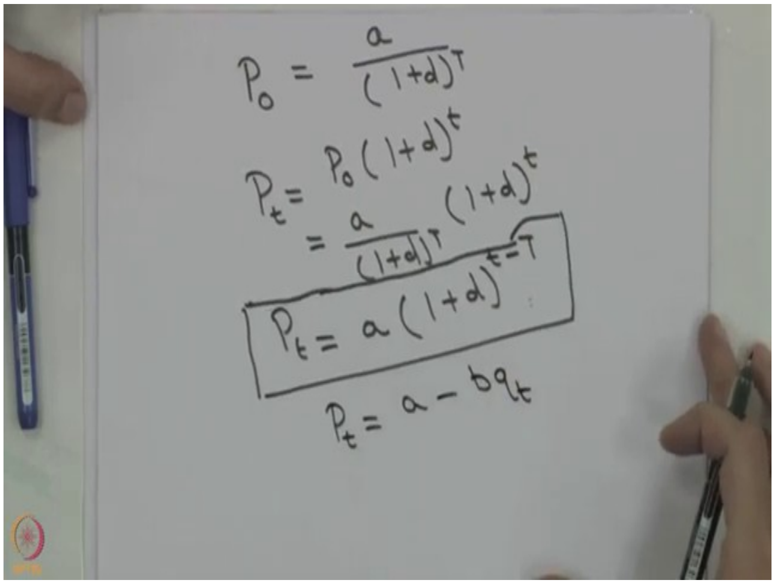

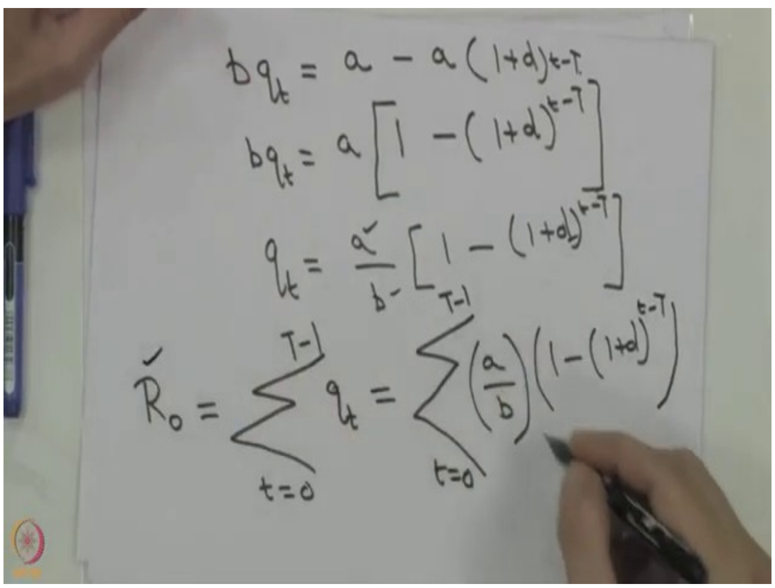

If that is the case then from this equation we can substitute and we will get P0, a is known P0 is 1 a divided by 1 plus d raise to t. We can substitute back in that equation, so that we get Pt and any time interval will be P0 into 1 plus d raise to t and substitute P0, so we get a into 1 plus d raise to capital T, capital T is a constant which we do not know which we want to find out and this is at any time interval t is equal to 1, 2, 3, 4 etc.

So, then this can be written as a 1 plus d raise to t minus capital T, this is now our expression for Pt, we also know from the equation that Pt is a minus b qt this was our original inverse demand curve, so now we can equate these two and what do we get? We will get that bqt is equal to bqt will be equal to a into a minus, that's the what we equate this, that mean b qt is equal to a 1 minus 1 plus d raise t minus T that means qt we have now got an expression for qt.

Generic expression for qt in terms of known coefficients a, b and d only unknown is capital T, now what we can do is that we will take that the total sum t equal to 0 to T minus 1, why do we say T minus 1? Because q capital T is equal to 0, so production is there from the 0th year to T minus 1 here, qt this will be equal to the total reserve which is R0, or in the earlier case what we has said q max and this is we can write the sigma notation t is equal to 0 to T minus 1 a by b into 1 minus 1 plus d raise to t minus T.

Now look at this we have got one equation we know R0, we know a, we know b, we know d the only unknown that we have is capital T, we have a sigma notation, this is a geometric progression, we can derive an expression for capital T in terms of these coefficients a, b and R0. In which case we have completely solved the problem we found the time for which the resource will lasts when it gets exhausted and we can substitute back and then we can get essentially qt as a function of time or Pt as a function of time and was our objectives. So, let us just do the simple calculations which are the, so when we look at this we get.

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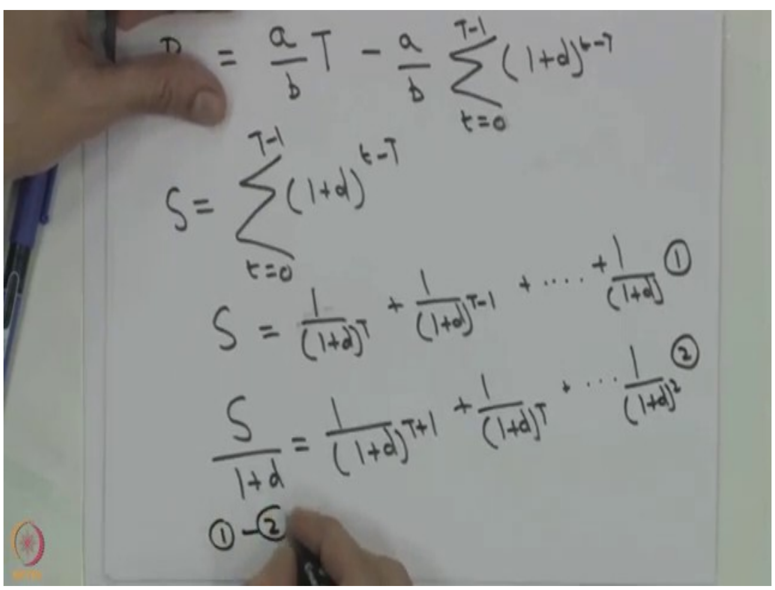

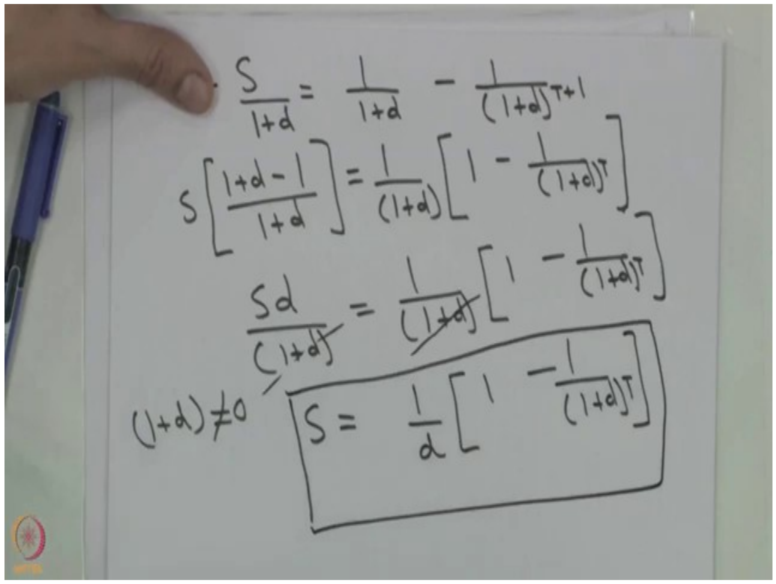

We open out the bracket and you will get a by b into sigma t is equal to 0 to t minus 1 there are t terms, so this will be a by b into t minus a by b into sigma t is equal to 0 to t minus 1, 1 plus d raise to t minus t. Now this is geometric progression let us just write this down in a separate way, let us write this as S is equal to sigma t is equal to 0 to T minus 1, 1 plus d raise to t minus T.

Let us expand it, let us write it down, this is going to be the t is equal to 0 this will become 1 plus d raise to minus t, so that is capital T and then it will be 1 plus d raise to t minus 1 and it gets the denominator changes till you get 1 plus t. There are t terms and you can see that it goes 1 plus d raise to T, 1 plus d raise to T minus 1, T minus 2 and so on till 1 plus T. So if you take this as one equation which is S, I can just take S by 1 plus d, this will be 1 plus d raise to T plus 1 to and then we can take 1 minus 2.

We can just do 1 minus 2 and if you do this you will find that we get S minus S by 1 plus d and if you see in this case what remains is that we are going to have the all of these terms will get cancel you will get 1 by 1 plus d over here minus 1 by 1 plus d raise to d plus 1 these are the 2 terms which will be remaining. So this is going to be 1 by 1 plus d minus 1 by 1 plus d raise to t plus 1.

So, we can take this as with simple algebra, this take 1 by 1plus d common this is actually just you could have just used the geometry progression formula but we just in one step we just derive it and this turns out to be Sd by 1 plus d equal to 1 by 1 plus d now 1 plus d not equal to 0, so we can cancel this and we get S is equal to 1 by d into 1 minus 1 by 1 plus d raise to t.

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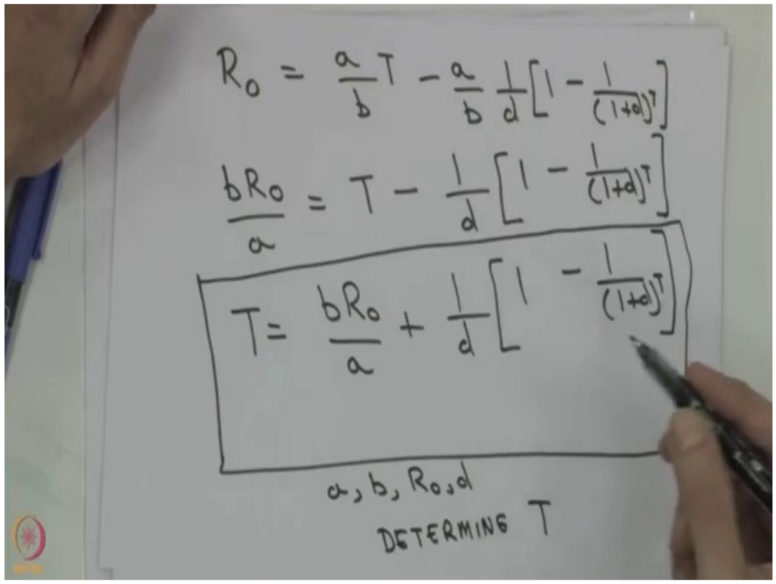

So, we can substitute back in the equation that we had earlier which was R0 is equal to a by b capital T minus a by b into S and S is we just derived the S as 1 by d into 1 minus 1 plus d raise to T. So, now we can simplify this, we can take b on b by, a by b is common here, I can take it on the other side, so that I get b R0 by a as T minus 1 by d into 1 minus 1 plus d raise to T.

Now you want to actually solve for t so we can just write this now as T is equal to take the terms on the other side, we have now solved for capital T which is what we wanted to do, we wanted to find the date of exhaustion which is unknown, we do this in terms of the coefficients a, b, R0 and d, a, b, R0 and d are known please remember this is a an equation where you have T on both sides, so you can do it in an iterative fashion, you can start by assuming a certain value of T get the next value and then so on till we get a converge value.

And so with the result that we right now we can now determine T and we will do a tutorial example where we will plug in the values and then see how this can be done. So, once we do this then we know the T, once we know T we now know the price trajectory as well as we know the qt, so we know Pt as a function of T and we know Qt as a function of T.

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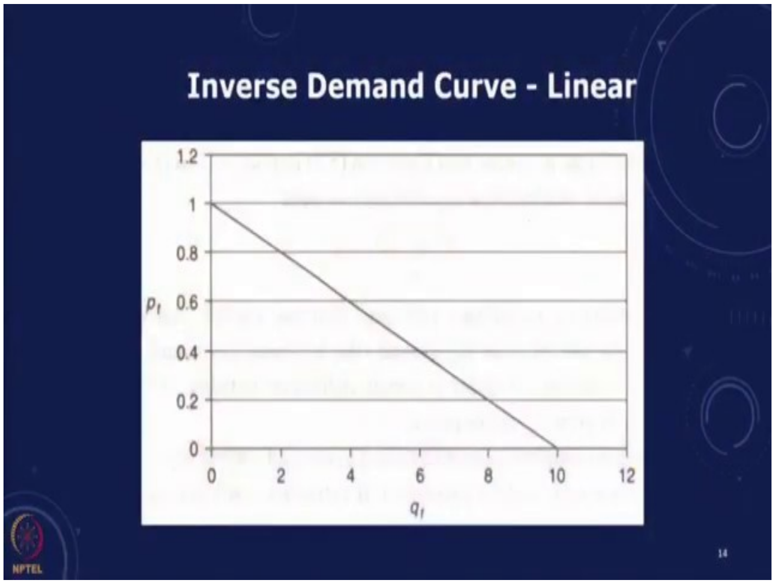

And if you just see for a linear model typically for a, this is a example from Conrad book on resource economics and we will also solve particular example with some values, so that we can make our own calculations. This is a linear inverse demand curve and corresponding to this you see that the price starts from a certain value and the price as a function of time increases at the discount rate.

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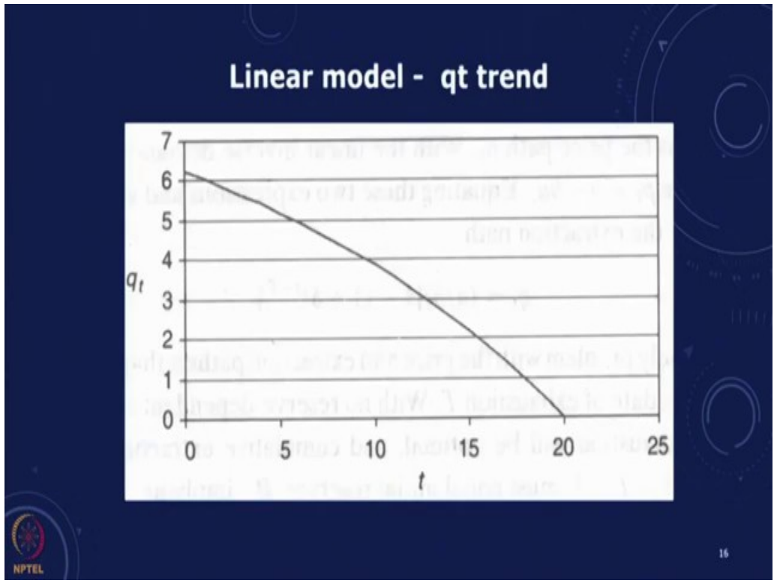

And when we look at the qt versus time we will see that the qt starts from some value and it decreases till about the 20th year where it becomes equal to 0, so this is a example to show you how the whole mine model problem is done for a competitive market. Now what happens if instead of the linear inverse demand curve we had a different demand curve which is we had the constant elasticity demand curve.

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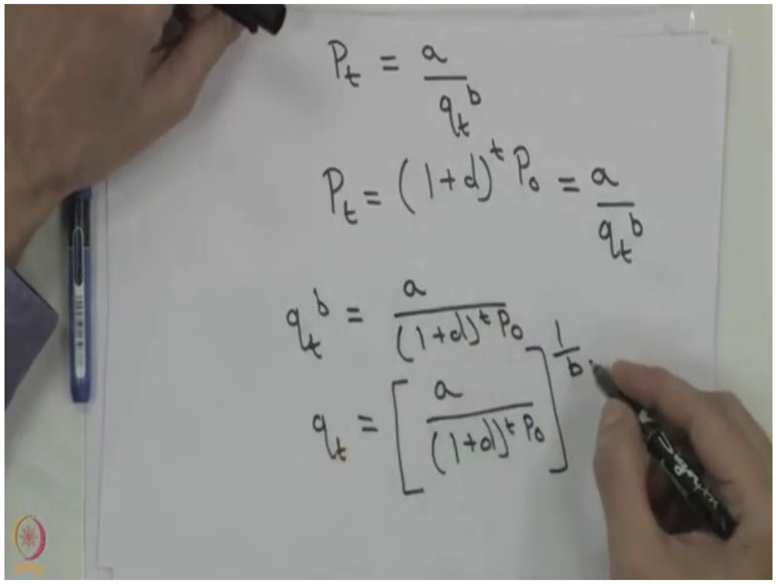

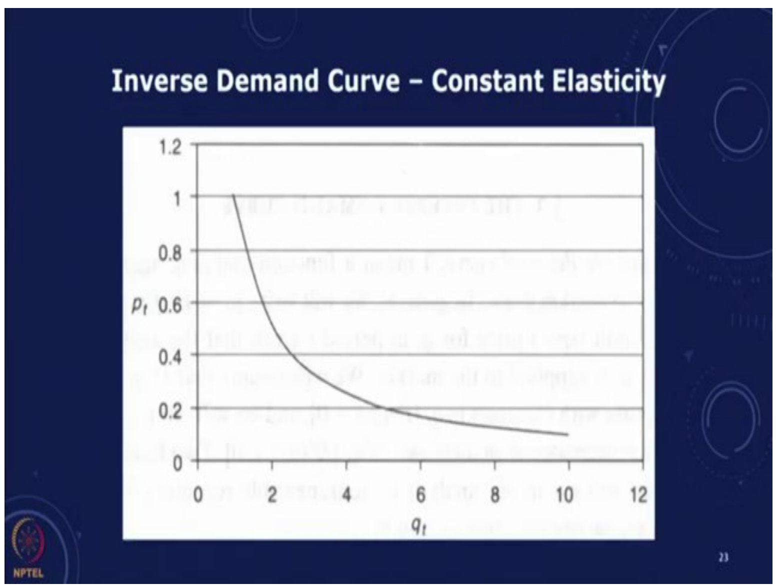

Let us derive for that. Pt is equal to a by qt raise to b, so in this case what would happen is you would see that typically if we have this curve, you will find that the qt would not become 0 and qt would tend to 0 only as it goes to infinity.

So, what we can do is we have the same term which we saw in the earlier Hotelling's analysis where we said pt will be 1 plus d raise to T P0, so in this case we do not have a choke price or a price which where we find that, that will be equal to a in this earlier case where qt is equal to 0, Pt was equal to a, we just have this expression and we can use this expression and substitute back and we will get qt, we can write in this Pt is a by qt raise to b.

So, qt raise to b is going to be a by 1 plus d raise to T P0 and what we need to do is we need to find out, so qt if I write this can be written as a by 1 plus d raise to t P0 raise to 1 by b, so what we know is this is an expression that we have now for qt we know a, b, and d we do not know P0.

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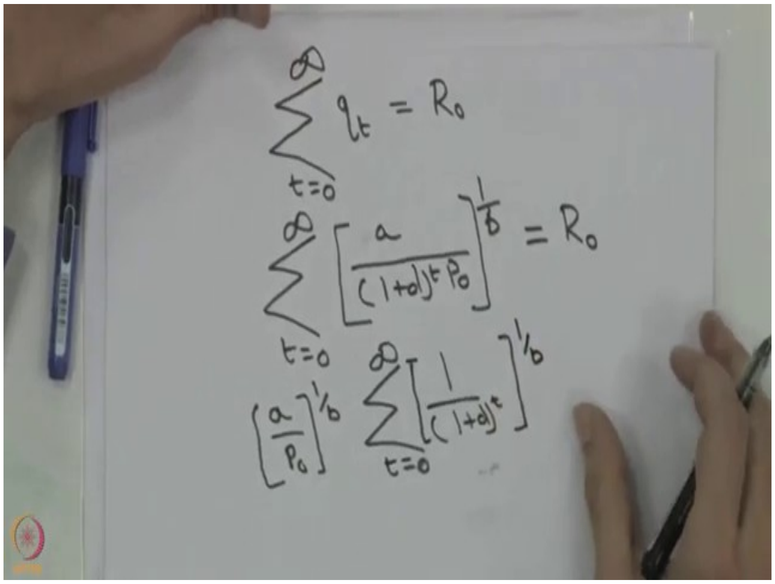

And in order to find P0 we can use the, fact that sigma t is equal to 0 to infinity qt will be equal to R0 or sigma t is equal to 0 to infinity, we plugin this value, our problem what we want to do is we want to determine this value of P0.

In the earlier case we had the value of P0, so we want to find P0 in terms of the known coefficients a, b and R0 and d and now when you look at this you will see that this is basically again this is a geometric progression when we look at this sigma if we take this out you will see that this is we can take a by P0 raise to 1 by b and you get sigma t is equal to 0 to infinity.

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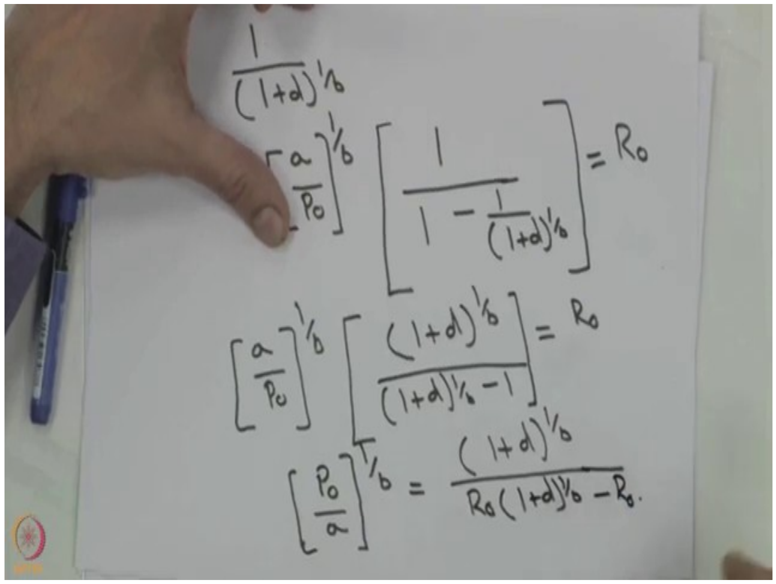

So, this is actually a geometric progression were we have this is of the form you know if we have a geometric progression and we are multiplying the constant factor the multiplicant that we are doing is 1 plus d raise t by b, so 1 plus 1 by 1 plus d raise to 1 by b and as the limit as we are looking at term where this is of course going to be less than 1 in the limit as we take it to infinity we take this sum to infinity, this is going to be equal to the initial term divide by 1 minus r.

So, this sum is going to be equal to so the what we get is that expression that we got a by P0 raise to 1 by b into 1 by 1 minus r and the r is nothing but 1 plus d raise to 1 by b and this term will be equal to r0 you can convince yourself that this is true. So, what this would mean is that we can take this as a by P0 raise to 1 by b and we can take this as, we can multiply by 1 plus d raise to 1 by b so that you get 1 plus d raise to 1 by b 1 plus d raise to 1 by b minus 1 equal to R0.

So, we can now simplify and write this in terms of the basically we can get we want to, what is our objective, we know d, we know b, we know a, we need to know P0. So, if you look at this and you want to take this as a by P0 we want to find P0, P0 by a raise to 1 by b is what we have here and this will be equal to 1 plus d raise to 1 by b raise to R0 into this is going to come in a0 will come in here and this will come there. So, you get R0 1 plus d raise to 1 by b minus R0.

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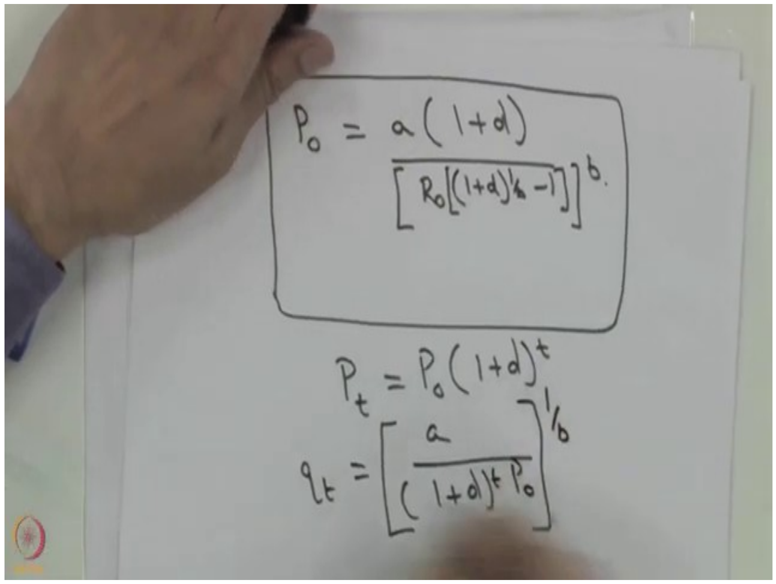

So, when we simplify this will get an expression which is going to be nothing but P0 a into 1 plus d raise to b, so once we know P0 we know that Pt is P0 1 plus d raise to t we know what it is Pt and we if you remember we have already calculated what was qt, qt was, so we can calculate at any point of time qt. Now, what about the time of exhaustion, capital T? In this particular case since there is a constant elasticity it will go asymptotically to 0, it will never become equal to 0.

So we can just take the point where a certain amount 90 percent of the resource is used or 80 percent but it will never get completely used because the price is going to keep increasing but you will always use a certain amount of it and this is the constant elasticity case.

So, let us just take stock what we have done is we have solve the mine managers problem and we first started with Hotelling’s original paper and we said that when we talk about a non-renewable resource we want and if the we are looking at a mine manager whose is trying to maximize the profit.

Given the constraint in terms of the known total resource which is there in the mine we want to decide how much to mine at different points of time and based on that we saw that the quantity which will be mined will be such that the price will increase at the rate of the discount rate and of course this is assuming a constant extraction cost we can have more complicated models where if the extraction cost are also function of time and one can have various additional complications.

But the principle by which we will do the calculation will be exactly the same and with that we found that there will be a point at which the price increases to the point where there is a super abundant substitute and the mine gets completely exhausted and that was the choke point we used that and then said that suppose we know we have a perfect competition case and we know what is the demand curve and we know the inverse demand curve we show that for a linear inverse demand curve we can calculate what is the time for which the resource the mine would last and this is under perfect competition

In the second case, we said that suppose we have another situation where there is a constant elasticity demand curve, mean the constant elasticity demand curve we never get to a choke point but the quantity keeps decreasing and beyond the particular point this will go asymptotically to the price will keep increasing and the quantity goes asymptotically to 0 and in such a case we identified all the parameters so that we could get qt as a function of time and we got Pt also as a function of time, so this is the constant elasticity price trend.

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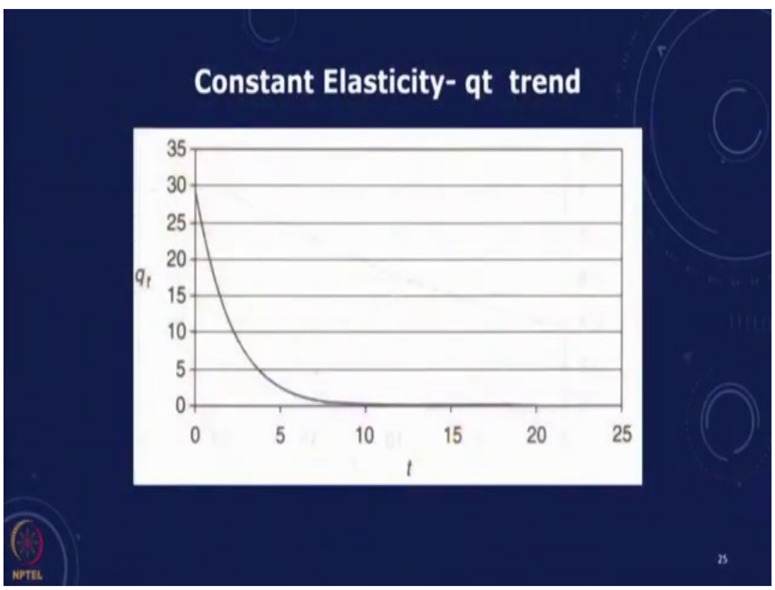

And constant elasticity qt trend you can see that beyond a point after certain point of time it goes asymptotically it is not exactly 0 but its keep getting going down.

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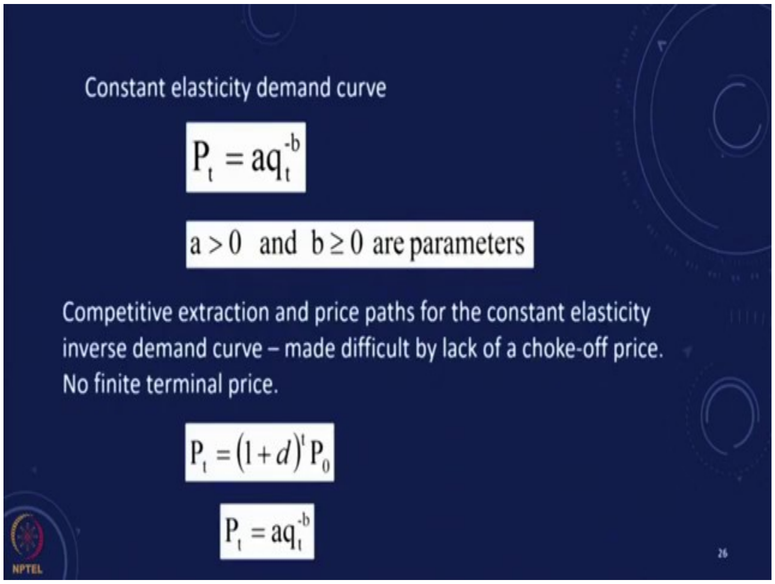

And this is we had derived this where we are looking at the competitive extraction and price path, there is no choke point but we get the price.

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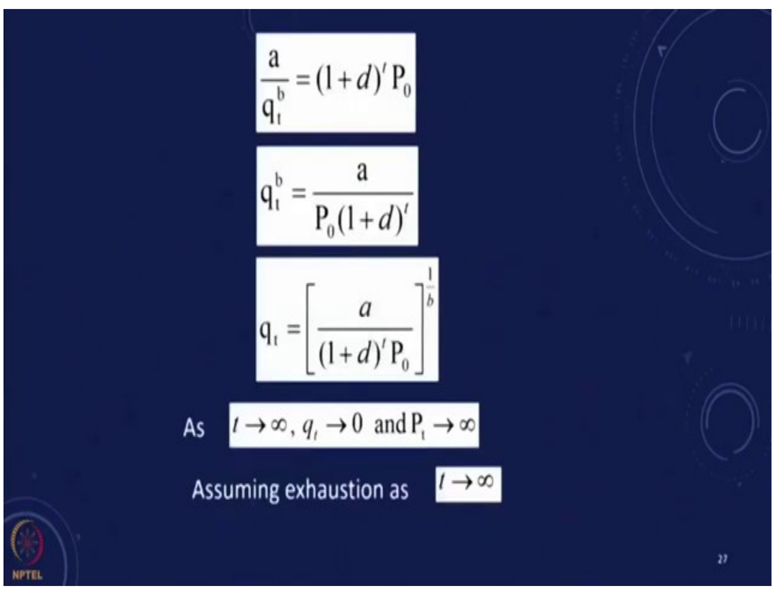

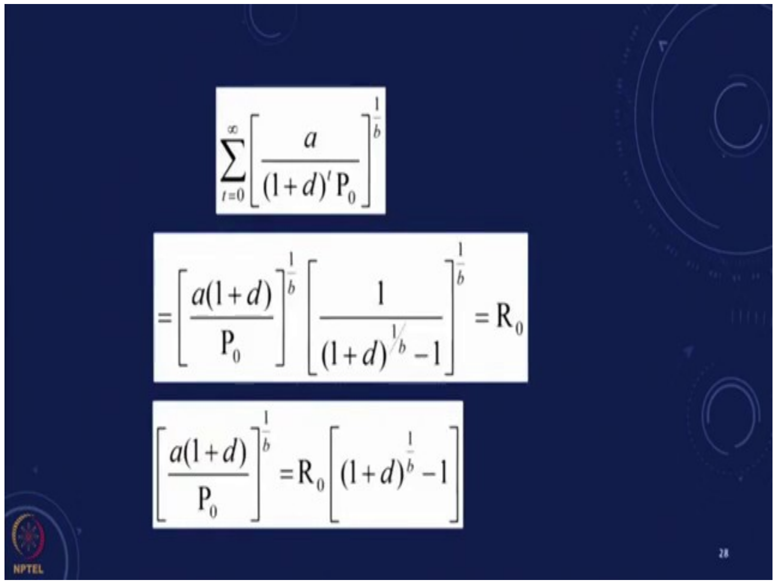

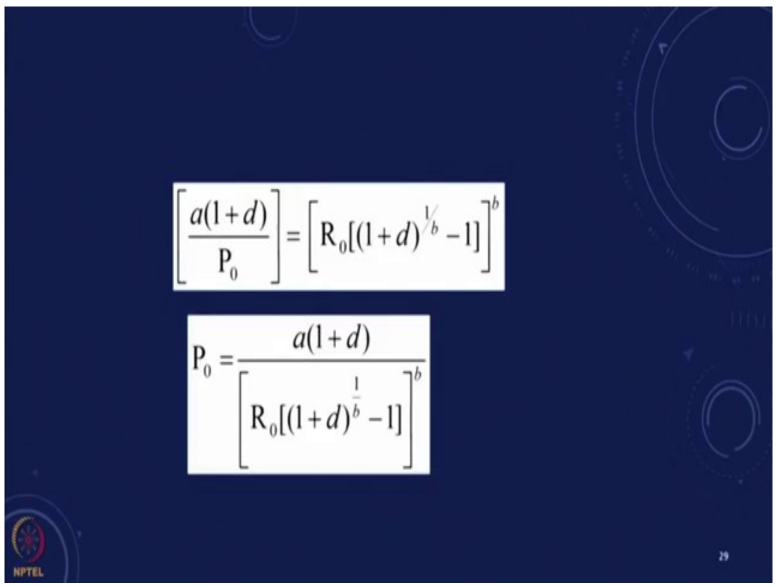

And this is how we say that we can derive this and as t tends to infinity qt tends to 0, Pt tends to infinity and assuming an exhaustion we got this and we got the expression for the P0.

So, with this we have now derived all the expressions for the price trajectory and the quantity trajectory for perfect competition. Now the question is what if there is a monopoly? What if the mine manager controls all the mines? And the mine manager can then, will the strategy remain the same or will it be different? We will look at this in the next module.