Energy Resources, Economics and Environment
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Lecture 09
Non-Renewable Resources Economics- Part 3

We continue looking at Non-renewable Resource Economics and in the previous module we have looked at a situation where there is perfect competition and we found out what is the optimal strategy for the mine manager. We found out what is the price trajectory, we found out how much quantity would be taken in different years and the time for which the resource will lasts.

In this condition what happens is the demand or the inverse demand curve is given and any company or an owner of a mine has no way of influencing that demand. Let us now look at the question what happens if all the mines are controlled by one company or one individual? That means what happens if there is a monopoly? And of course as you would expect the rules would be changed because then the monopolists can actually influence the total quantity which is being release and because the total quantity being released is being influence the price would change.

And so the monopolists has a way to dictate a price and can then decide so in this case the optimization changes, the monopolist tries to maximize the revenue and this in a similar fashion like the analysis we have done in the last section where we said that the cost are constant and we can take the price minus the cost or we can neglect the cost.

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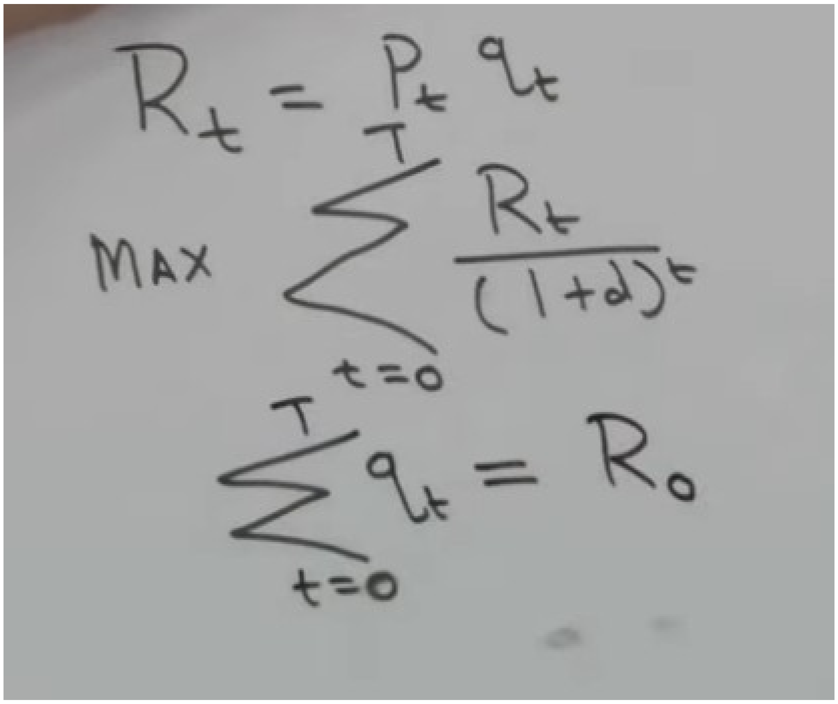

So the revenue that we will have Rt will be Pt into qt, now in actual practice we have seen this case of a monopolist effecting the prices, in some cases it may be one individual which is a monopoly or it could be a cartel of producers, for instance Opaque is a cartel of oil producing and exporting countries and in the 1970s Opaque decided that it was going to control the quantity the oil that it is going to release.

And with the result you could see a sudden spurt in the oil prices, this is called the oil shock and that is the point at which all countries started looking at energy independence, looking at energy efficiency and this was the start of the whole movement to look at energy conservation and energy efficiency.

So, we would like to now look at from a monopolist point of view if you have the revenue and we want to maximize the discounted sum of the revenue Rt by 1 plus d raise to t, t is equal to 0 to T. So, it is very similar to the earlier situation that we had, only thing is that in the case of Rt the monopolist is able to influence the quantity that is being released overall and hence is also able to influence the price and this will be subject to the constrain that sigma qt t is equal to 0 to T is equal to the total reserve we had which is R0.

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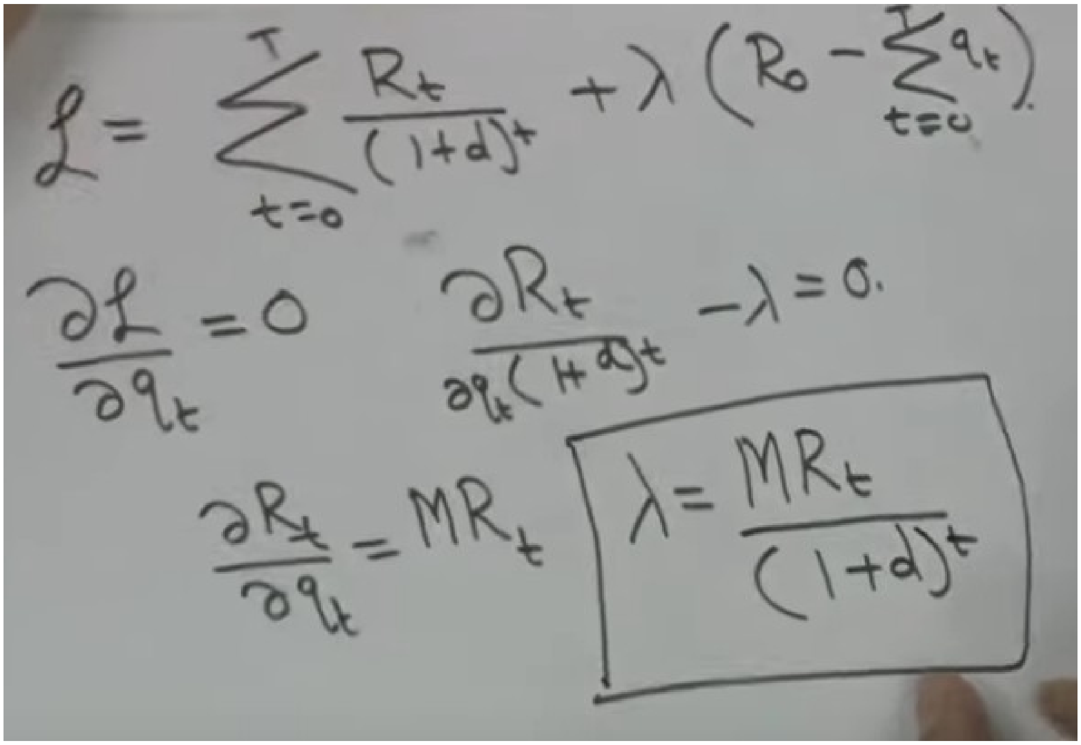

So when we take this we can take the Lagrange which is the very similar to the last analysis that we did but R0 minus sigma t is equal to 0 to T qt and this Lagrange divided by the differentiated with respect to del qt set this equal to 0, what we get is we will get the, we are differentiating the total revenue that we have with respect to qt.

So, what we will get is the marginal revenue, we can differentiate this and we will get delta Rt by del qt 1 plus d raise to t minus lambda is equal to 0. So, essentially what we get is delta Rt by del qt is also known as the marginal revenue, that means the revenue per unit of q and the what we would get then is that the lambda value is going to be equal to marginal revenue divided by 1 plus d raise to t. So, this is that in each time interval just like we had in the earlier case we had the price increasing at the discount rate.

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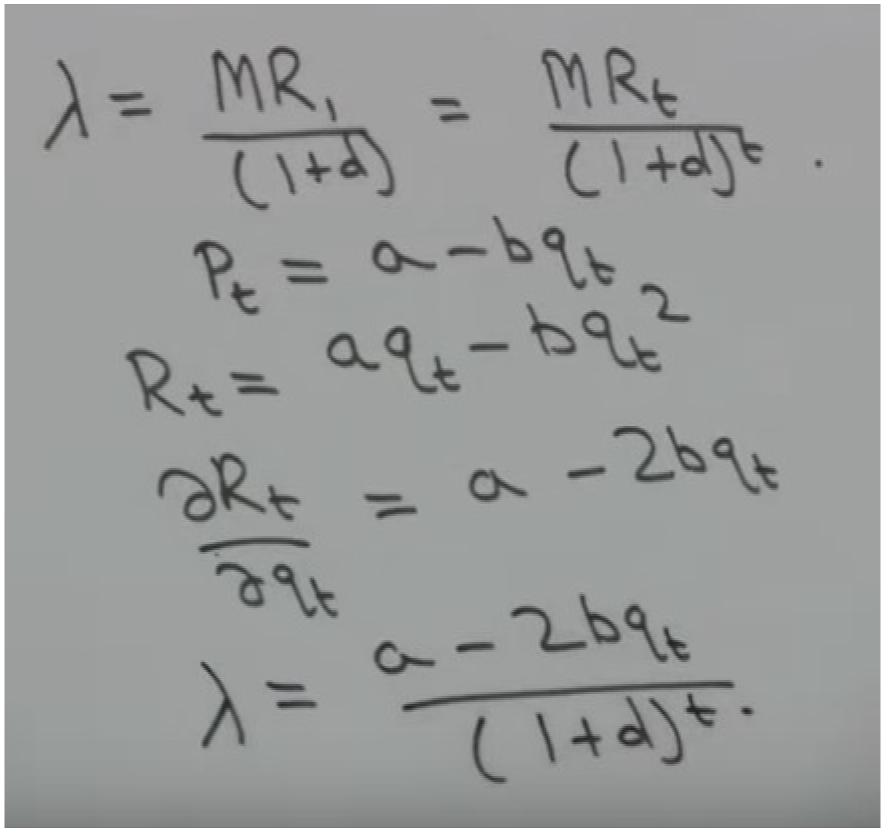

Now we are having the marginal revenue. Marginal revenue 1 by 1 plus d and so on marginal revenue t by 1 plus t raise to t. Now let us take a situation where we have linear inverse demand curve, so we have Pt is a minus b qt so then Rt becomes a qt minus b qt square, so del Rt by del qt is a minus 2 bqt. So, once we plug this in the value of lambda which we get is a minus 2 b qt by 1 plus d raise to t.

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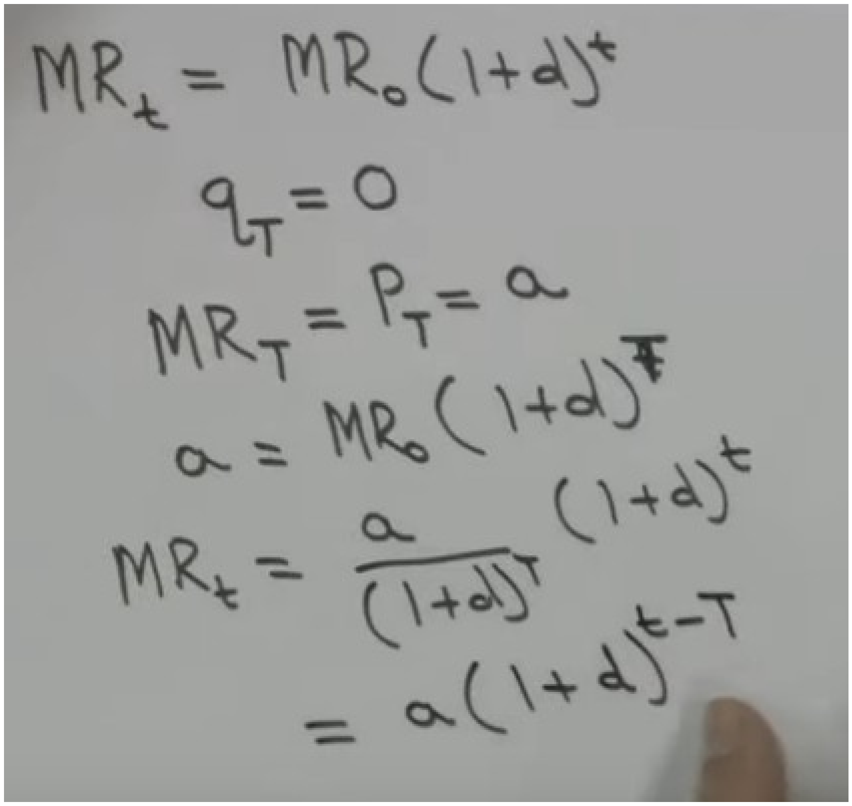

And we said that MRt increase, so M R marginal revenue in time horizon t will be equal to marginal revenue 0 first year into 1 plus d raise to t. Now let us consider the linear inverse demand curve and take the situation when the resource is completely exhausted. When the resource is completely exhausted at that point Qt capital T will be equal to 0. At this point what will be happening will be the marginal revenue which we have must equal to the price per unit that will be equal to the price and that is when the monopolist will not want to produce any more the marginal revenue will be equal to the price then that is equal to we said a minus b qt.

So this is going to be equal to a, so a is going to be equal to MR0 1 plus d raise to t and then we can substitute this in this expression, so that we get MR t is equal to this is capital T. When it gets exhausted capital T this is going to be a by 1 plus d raise to capital T multiplied by 1 plus d, so this is marginal revenue is going to be 1 plus d raise to t minus T.

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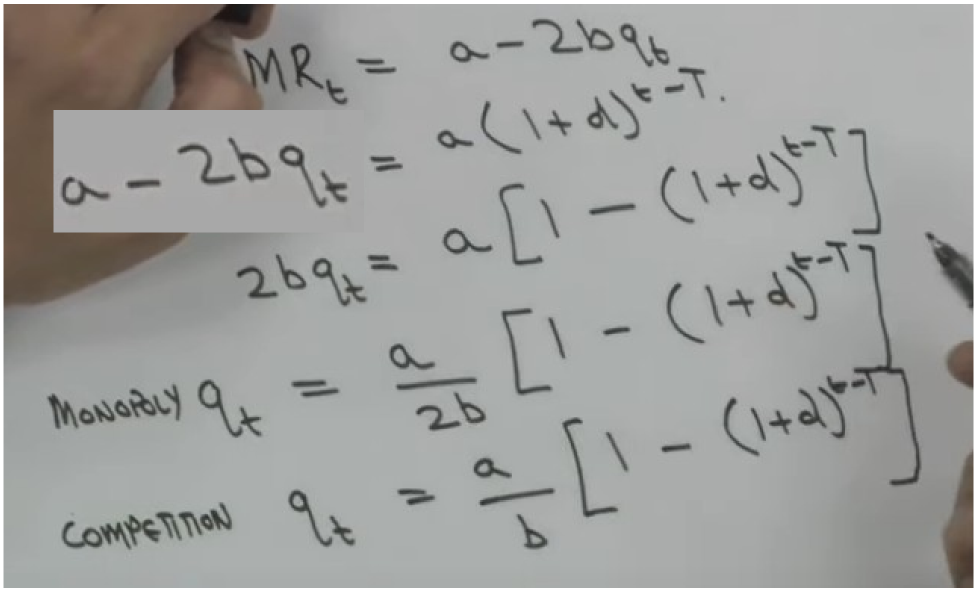

And we have already derived that the marginal revenue for the linear inverse demand curve case is a minus 2 bqt, so we can put this as a minus we can equate these 2 terms bqt is a 1 plus d raise to t minus T. We can now get from this we can put this as 2 bqt is equal to a into 1 minus, looks very similar to the competition case but with a difference, we have now this is qt is a by 2b into 1 minus 1 plus d raise to t minus t.

If you remember you can look back at the earlier derivation that we had done. In the case of the, this is for the monopoly and for perfect competition for a competitive market we got qt is equal to a by b. So, if you look at this of course in the case of the monopoly the capital the value of exhaustion the number of years T would be different but in general what you would find this that the monopolist would, in a particular year release less amount of q so that the price increases and the overall revenue increases with the result that as we would expect the resource is going to lasts for a longer period under a monopolist case.

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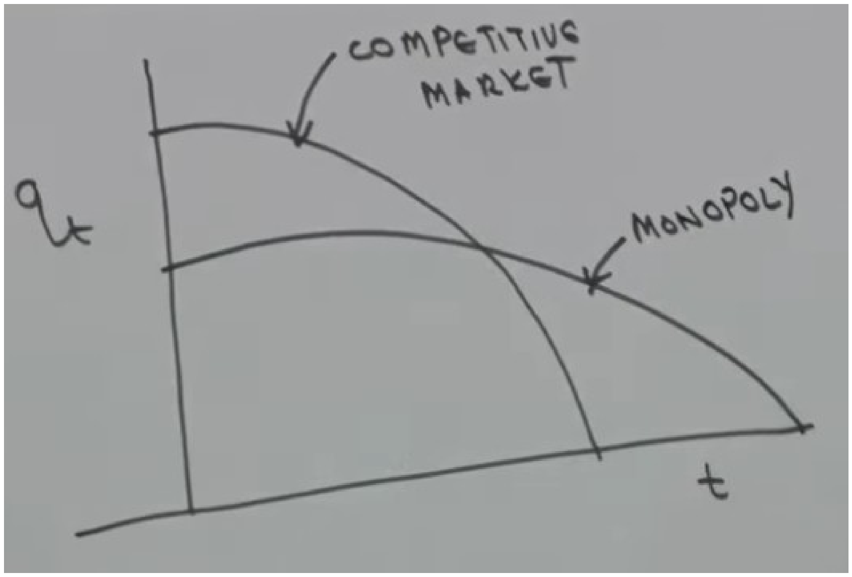

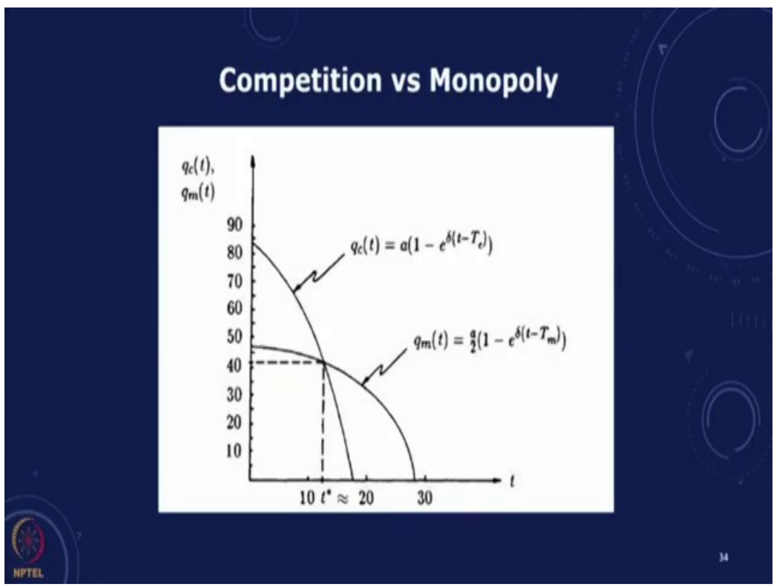

So, if we look at this we would expect qualitatively a curve like this where you have qt and t this is for if this is the shape of our competition, competitive market then the monopolist would be this is how the monopoly would look like. So, you can look at the book by Conrad on non-renewable resource, on resource economics and there is a chapter on renewable resource economics which shows some of this.

If we then take this the same thing you can see the this is the actual this is for a this is a plot which his shown from Conrad which shows similar kind of trend for a particular example.

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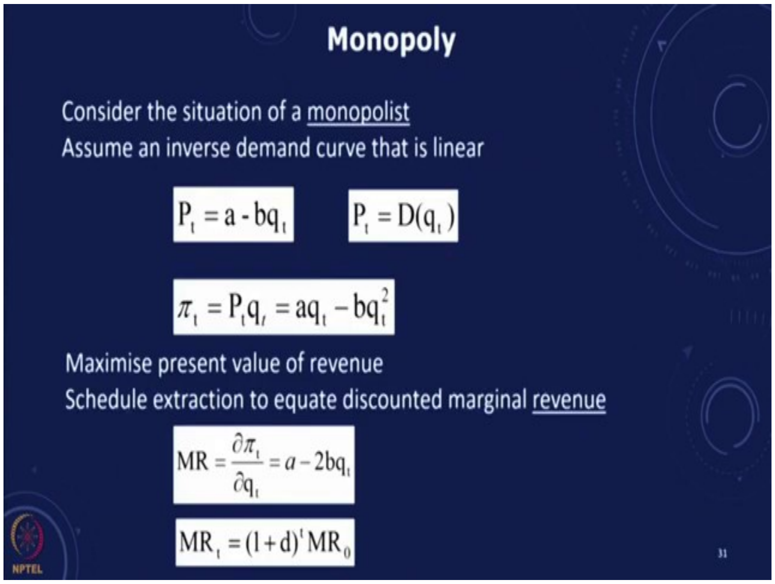

So now what we would like to do is we would like to look at this take that expression and just like we did for the competition case we would like to derive how much time the resource is going to last for.

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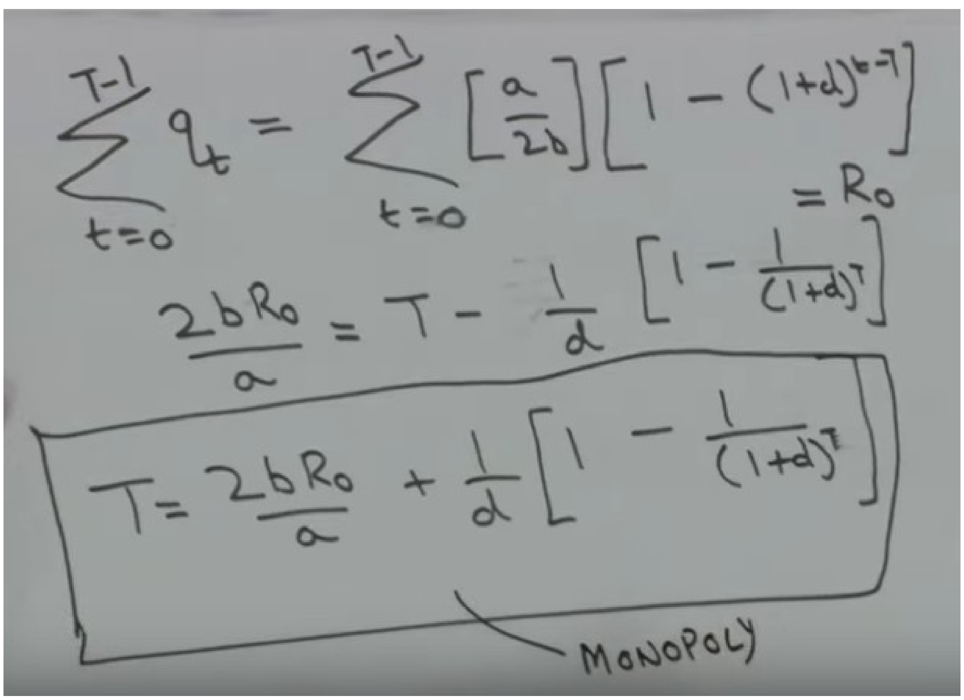

So in a similar fashion we take t is equal to 0 to T minus 1, remember that in the last year q capital T is equal to 0 so that will not be added q t will be equal to 0 to T minus 1 a by 2 b into 1 minus 1 plus d raise to, and if we use this in the same fashion as we did derive for the competition case this becomes a geometric progression and finally we get an expression which is we get 2 bR0 this sum will be equal to R0.

So, 2 b R0 by a is T minus 1 by d 1 minus 1 plus d raise to t and the final expression that we get is t is equal to 2 bR0 by a plus 1 by d 1 minus 1 plus d raise to T. This is for the time for a monopoly and you would remember that we have a similar expression when we had the competition and only difference was that in this case, this was b R0 by a and so what you would find is that the time taken would increase and now the question is of course thus that mean that a monopoly is better from a resource point of view?

From a resource point of view the monopolists conserves the resource because the monopolists is looking at the overall maximization of revenue but in the process given the discount rate that is there the population and the consumers are exposed to much higher prices and because of that the overall utility of society is less under a monopolist case even though the resources gets conserved for a longer time.

So now let us do one thing let us take the same whatever we have learnt for competition and for monopoly let us now solve one particular example a numerical which is there in your tutorial sheet.

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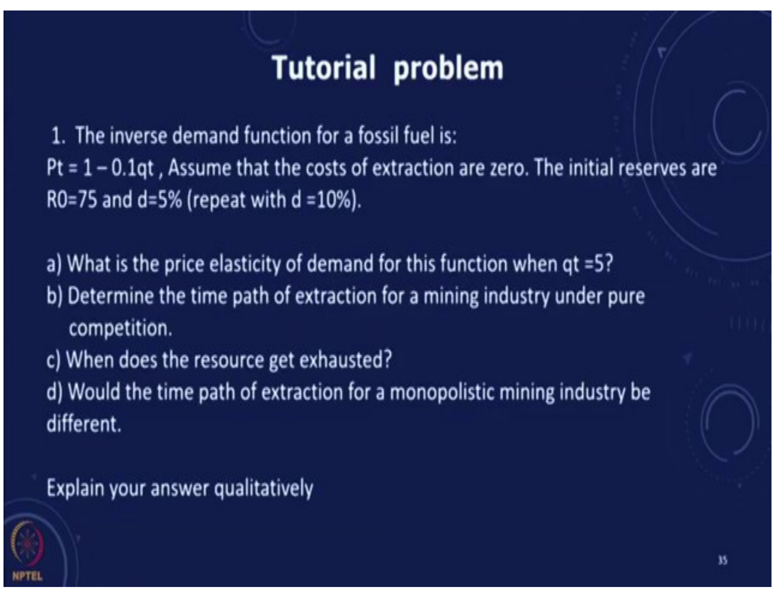

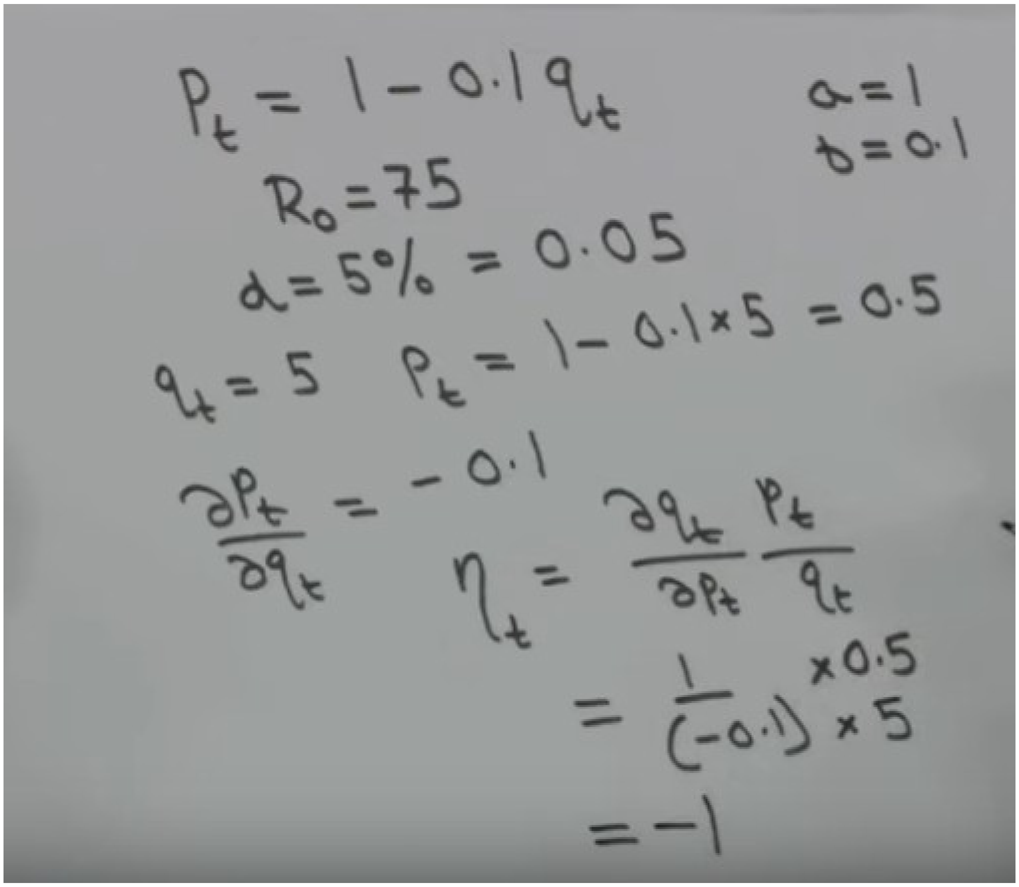

I will just show you this number and this is the tutorial sheet. This shows that we have a tutorial problem the inverse demand function for a fossil fuel is given to you as Pt is equal to 1 minus 0.1 qt which means that a is equal to 1, b is equal to 0.1 and we have also the value of discount rate R0 is given to you as 75, R0 is 75 and d is 5 percent which is nothing but 0.05.

So, the first part of the question is, what is the price of elasticity of demand for this function and qt is equal to 5 units, so when qt is equal to 5, let us just substitute qt is equal to 5, so what is the value of Pt? Just 1 minus 0.1 into 5 this is 0.5, so the answer is 0.5. So, the differentiate this del Pt by del qt this is minus 1, so if we look at the elasticity that is going to be del qt by del Pt into Pt by qt.

Which is this is del qt by del Pt this is 1 by minus 0.1 Pt we said is 0.5 and qt is 5. So, you will find that the elasticity is minus 1 which implies that if we have a 1 percent increase in the price there will be a 1 percent decrease in the quantity and that is the elasticity, so we have solve the first part of the question. The second part b says determine the time value of extraction for a mining industry under pure competition.

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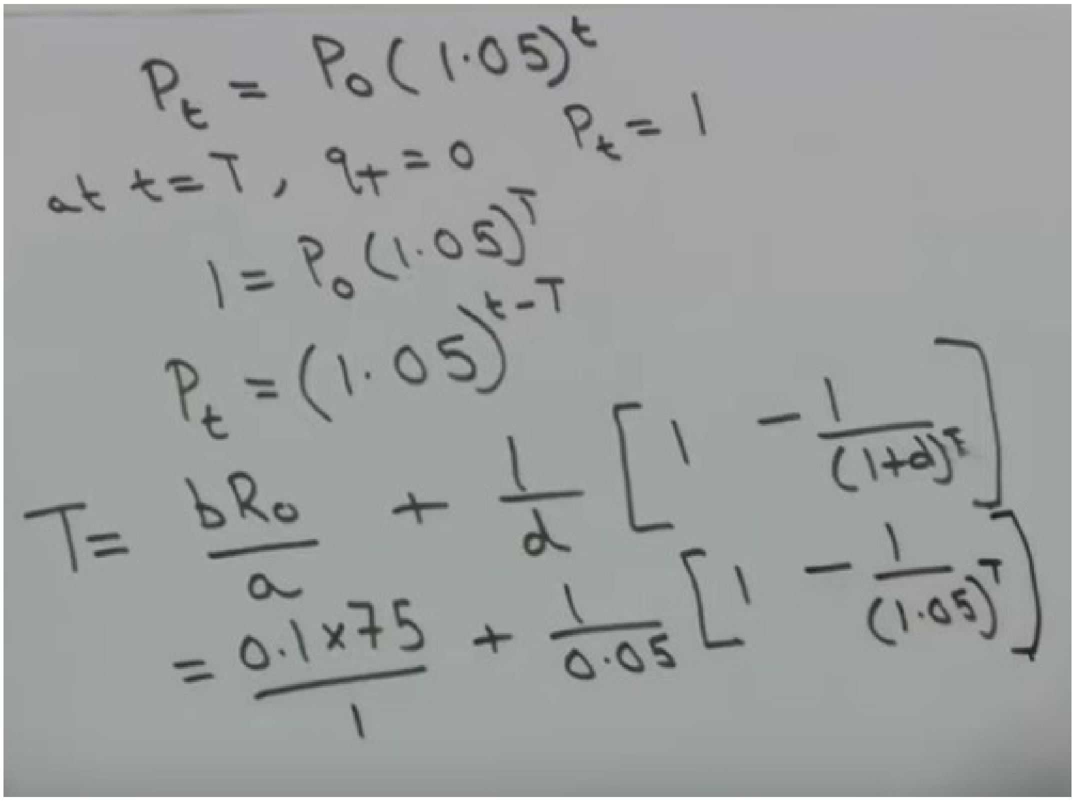

So, when we talk about solving this for pure competition we will have this as, we have Pt will be for pure competition this will be Pt into 1 plus d raise to t, so 1 point d is 0.05, 1.05 t. Now t is equal to T, qt is equal to 0 and Pt is equal to a which is 1. So, 1 is equal to P0 1.05 raise to T we can substitute for P0, so that we get Pt is equal to 1.05 raise to t minus T.

So, you remember the formula that we had derived for the time that this will last, we want to this is the only unknown is capital T we need to determine capital T then we can plug it back and we can get the equations for Pt and qt in which case we would determine the time path of extraction. So, once we do this we check b R0 by a plus 1 by d into 1 minus 1 plus d raise to T. Just substitute the values this is going to be 0.1 into 75 by 1 plus 1 by 0.05 into 1 minus 1 by 1.05 raise to T.

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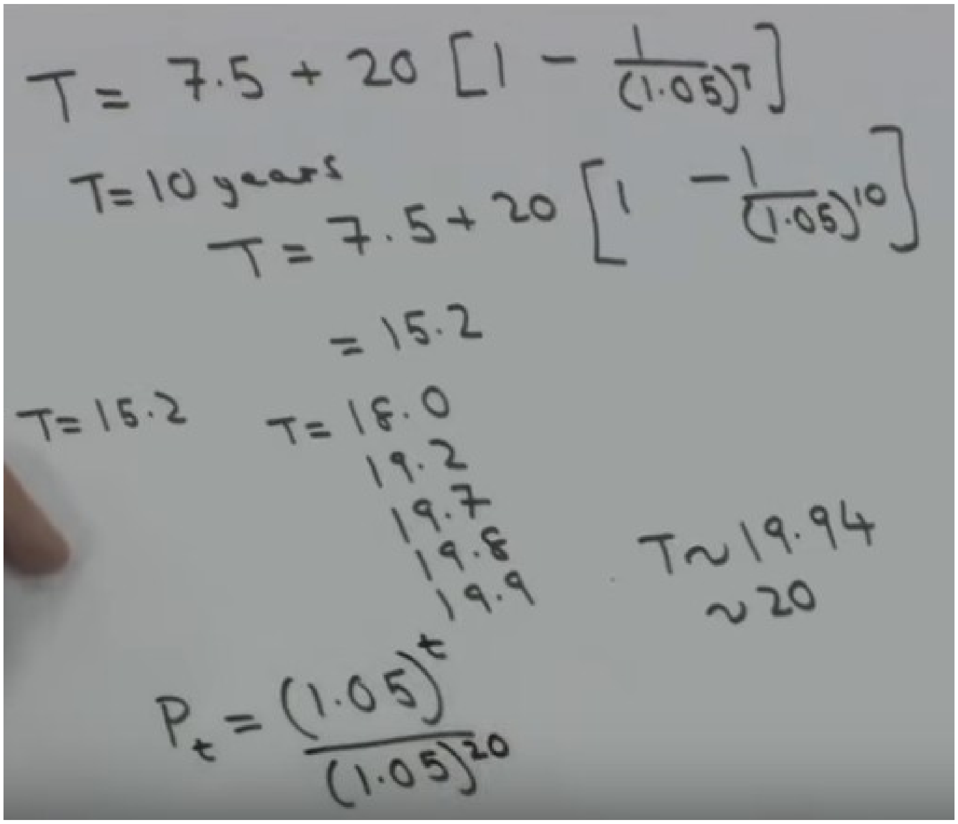

So, if we simplify this we will get T is equal to 7.5 plus 20 into 1 minus 1 by 1.05 raise to T. Now when we look at this we will this is as we said this is an equation where we have to iteratively solve for T. So, let us assume a certain value of T, let us say suppose T is equal to 10 years we can substitute T and get T is equal to 7.5 plus 20 into 1 minus 1 by 1.05 raise to 10.

You can plug in this values and you will see you get T comes out to be 15.2. Now we take 15.2 as a starting point and solve t get the next value of T then you get T is equal to 18.0 and then the next iteration we get 19.2 you can solve this and check 19.7, 19.8 and it converges to about 19 point, you get T approximately 19.94. We can round this off to about 20 years. So, we solve this part C when does the resource get exhausted.

The resource get exhausted in at 20 years and then what happens is that if we now substitute back we get the Pt which we had already solve we got this as Pt is equal to 1.05 into t divide by 1.05 raise to 20 and if you see this value of 1.05 raise to 20 turns out to, so you can get this so we got an expression for Pt, we also now can substitute this and get the expression for qt and with that we will get essentially the value of qt in different years.

Now let us look at for the same situation the part d, would the time path of extraction, so once we have this we can plot it for different years and we have got the plot of Pt versus time and q t versus time. Now the question is would the time path of extraction for a monopolistic mining industry be different.

So, if you look at this as we have seen earlier what happens in a monopoly is that you are able to effect the quantity supplied and hence the price and because of that you release less then it perfect competition it is better for you to have less quantity mine and with the result that we expected to last longer.

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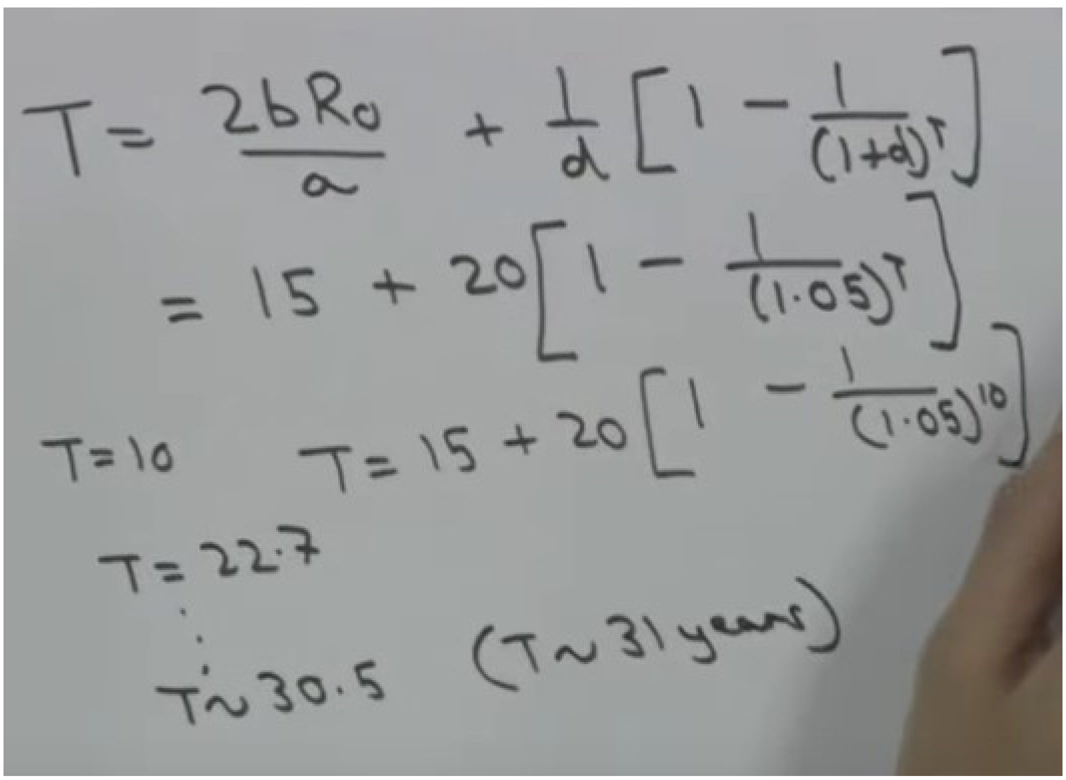

If we take this, if you remember we had derived now for the monopolists that this is going to be 2 b R0 by a plus 1 by d 1 minus 1 plus d raise to T more or less things the equation looks very similar expect instead of 7.5 this is now 15. So, once we do this we will get obviously a different converge solution.

So if we start with T is equal to 10, you will get T is equal to 15 plus 20 into 1 minus 1.05 raise to 10 and you get the next value becomes T is equal to 22.7 and as we go ahead you will find that it converges to about 30.5 years. So t approximately 31 years in the first case we found T is 20 years and in this case it is 31 years.

So, qualitatively we realize that essentially the monopolists wants to maximize revenue and because of that we produce less from the mine in the initial years as compared to the competition case and with the result that overall the revenue increases. Now the question is that what happens we also saw repeat this with d is equal to 10 percent if the discount rate is higher what would happen? If the discount rate is higher it means that we are counting future cash flows and discounting it by a larger amount.

So, we would prefer to have a profit or a revenue today as compare to in the future with the result that what would happen is that we would actually mine qt at the initial period would be higher and then the mine would get exhausted in a shorter time period. You can repeat this on your own and cross check.

So, with this we have completed the portion on non-renewable resource economics we have of course done this with simplistic set of assumption, you can relax all of these assumptions you could have a situation where the cost of extraction change, you could have situation where there are different kinds of demand inverse demand curve and but this gives us a way in which from first principles we can identify how an optimal mine manager would think and what is the way in which the resources would be used subject to the fact that of course the total amount of resources are finite.