Energy Resources, Economics and Environment.
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Lecture 18.
Input/Output Analysis-Part 2.

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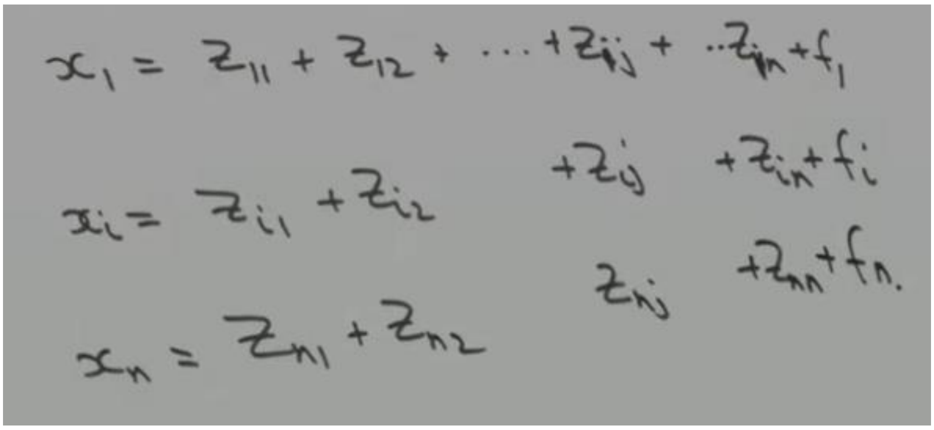

So, let us look at, let us derive this, move forward with this. We are looking at x1 as Z11, Z12 plus Z1j Z1n plus f1. Then we have for the ith row, this will be Zi1, Zi2, Zij, this is Z1n Zin plus fi, and xn would be Zn1 Zn2 Znj, Znn plus fn. So, this can be written in matrix form, whatever we have written so far this can be written in matrix form.

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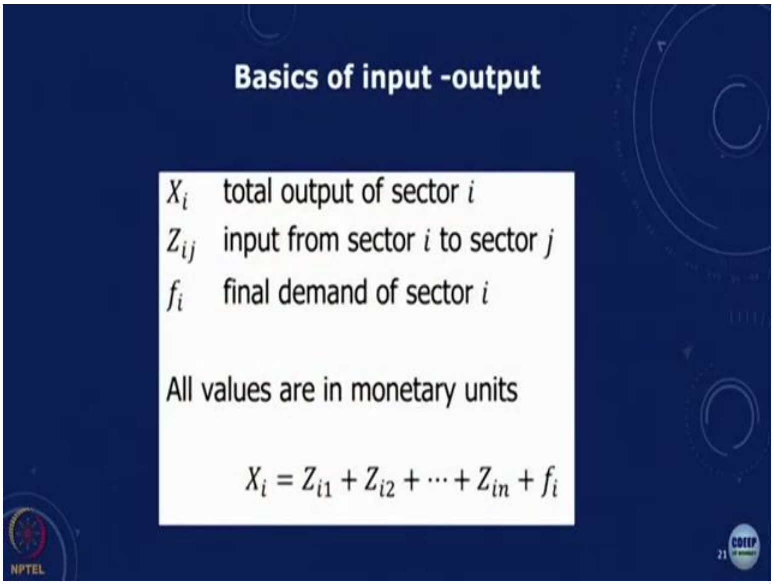

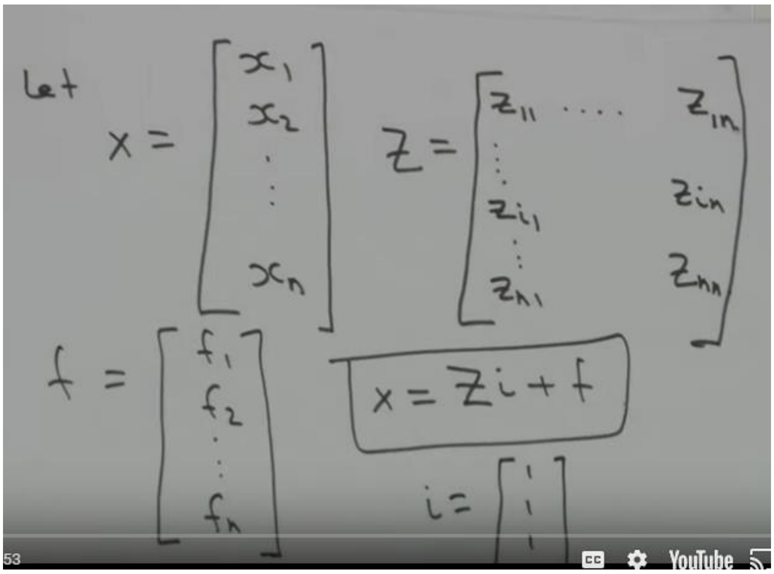

So, that we have these following matrices. X is x1, x2 and so on to xn. Z matrix is Z11 to Z1n and so on, Zi1 to Zin and then f is equal to f1, f2 and so on to fn. This can be written as X is equal to Zi plus f, where i is a column matrix with 1,1,1,1,1, all the identity 1 value.

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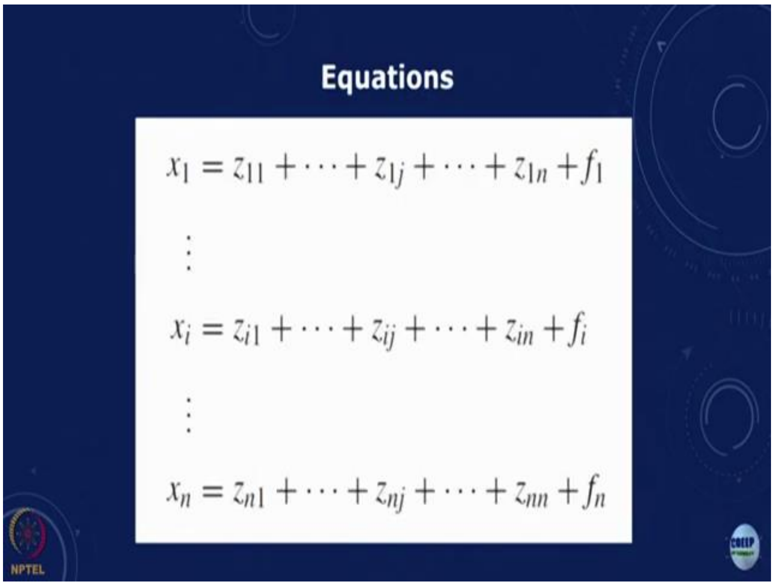

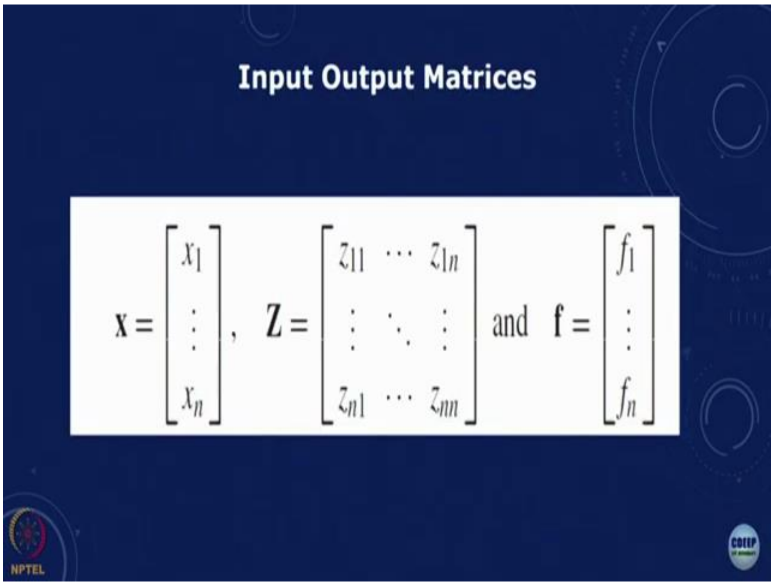

So, this is the way in which we can write this, we can also see, this is the equation which are there and this is the values.

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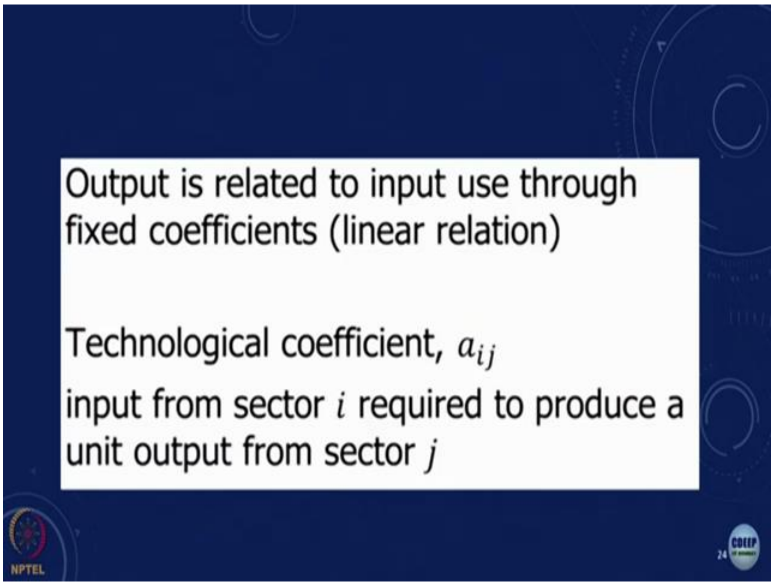

Now, the basis of this method, the fi, if you see the basis of this method is that we are going to write this in the form of the amount that we need from each of these. Finally, which we are looking at is going to be dependent.

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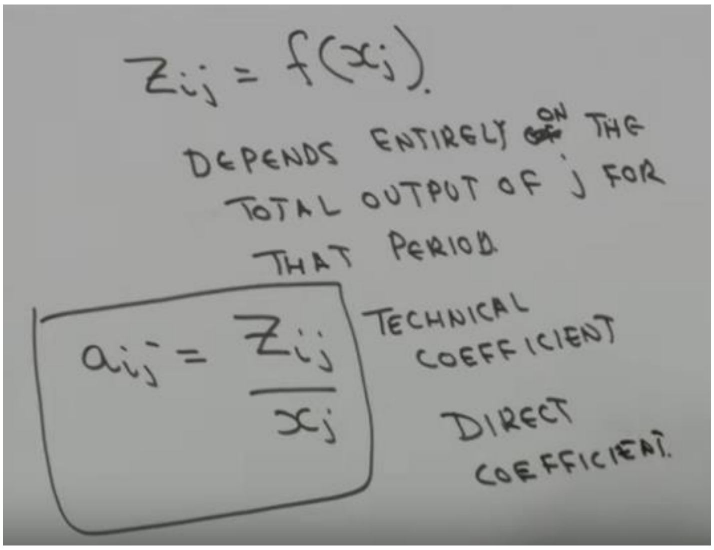

The Zij will be a function of xj. That means the amount that we need from the ith sector to the jth sector will depend on the total output that we have from the jth sector and one way the input output method, the fundamental assumption is that the inter industry flow from the ith sector to the jth sector depends entirely on the total output, depends entirely on the total output of the jth sector, entirely on the total output of j for that period.

So, which will mean that we say that aij, this is the coefficient that we will define, is Zi j by x j. In the input output method this coefficient is assumed to be constant, this is a technical coefficient, this is also known as a direct coefficient or the direct coefficient.

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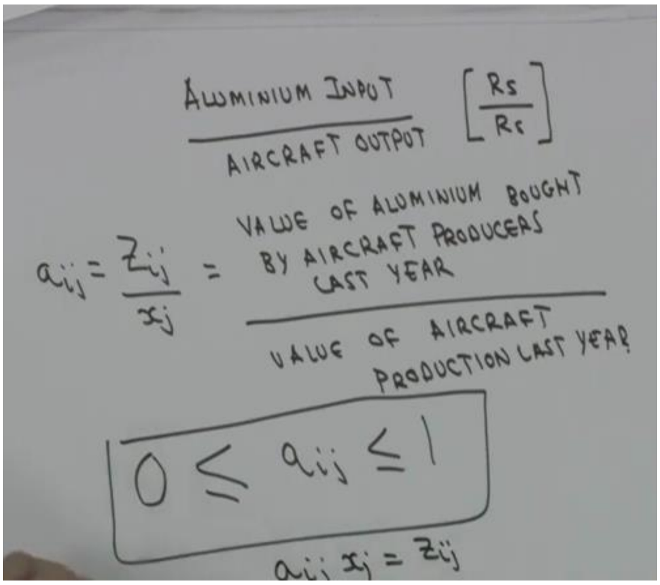

So, for instance, if we are looking at aluminum being used for aircraft production, so this will be aluminum input by aircraft output. Now what will be the units, this will be in millions of rupees, crores of rupees, so, it is going to be in the monetary units rupees per rupees. So, it is a ratio and so this will be, aij will be defined here as value of aluminum bought by the aircraft producers in the last year, in the year we are looking at, divided by the value of air craft production.

Now, can we say anything about aij? So, aij has to be between 0 and 1, it cannot be negative, which is a physical amount of quantity that is required. It cannot be greater than 1 because finally, the total value that is there in that sector has to be a combination of all the value adds of different components. And since all the, since none of them can be negative when we add it up, this is going to be there. So, this aij xj is equal to Zij, this is the basis. These coefficients are constant, which means that economies of scale are ignored and this operates under constant returns to scale. In the Leontief system, the entire basis is that the production operates under constant returns to scale.

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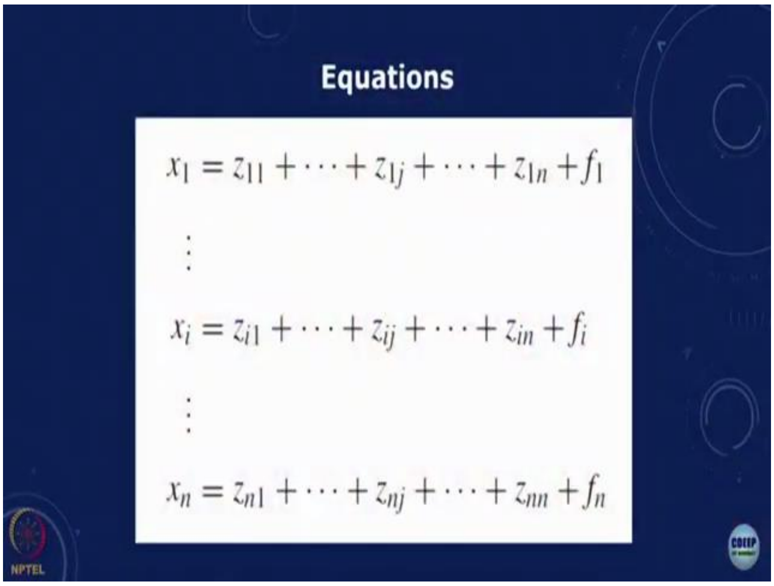

So, we can now write this as, if we look at this matrix a11, a12, a1n, Zij, if you remember the Zij's which we had, we can write the expression, let us start from the… Let us write down let us look at the expression, that we had got earlier, which was in terms of x, xi being equal to that Zi plus fi. We can write the Zij, as we said is going to be a combination of a into x.

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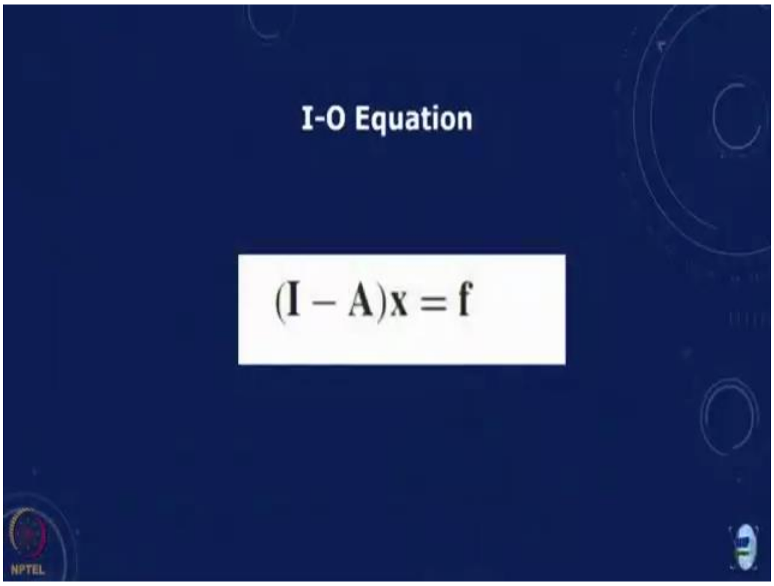

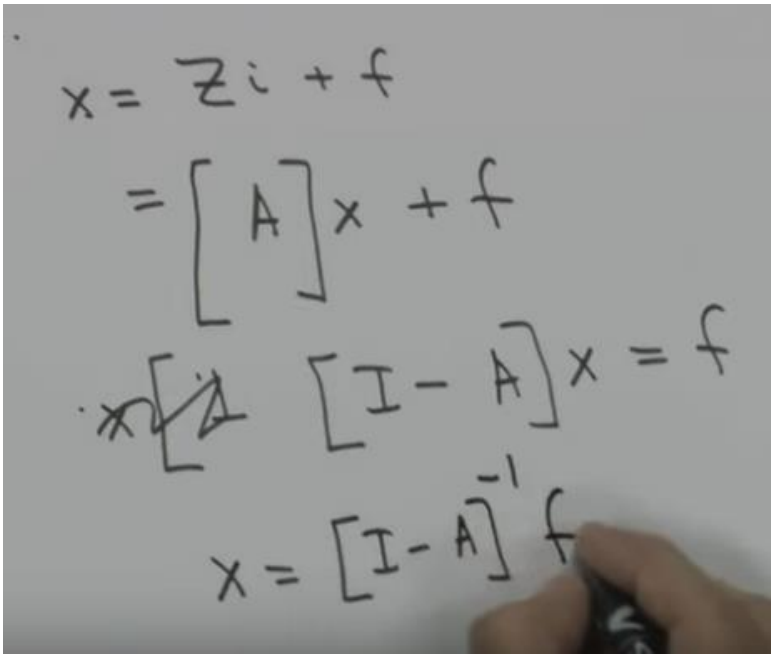

So, we are going to have X is equal to the Zij, Zi plus f is what we had and this is going to be nothing but A into your X. So, this is a X plus f. So, we can take X into the identity matrix, we have identity matrix I minus A, once we take it on this side into X is equal to f and now we can get X, what we want to do is if we know the final demand, what will be the values of X that we will get. So, X will be, we can take the inverse of this I minus A inverse into f.

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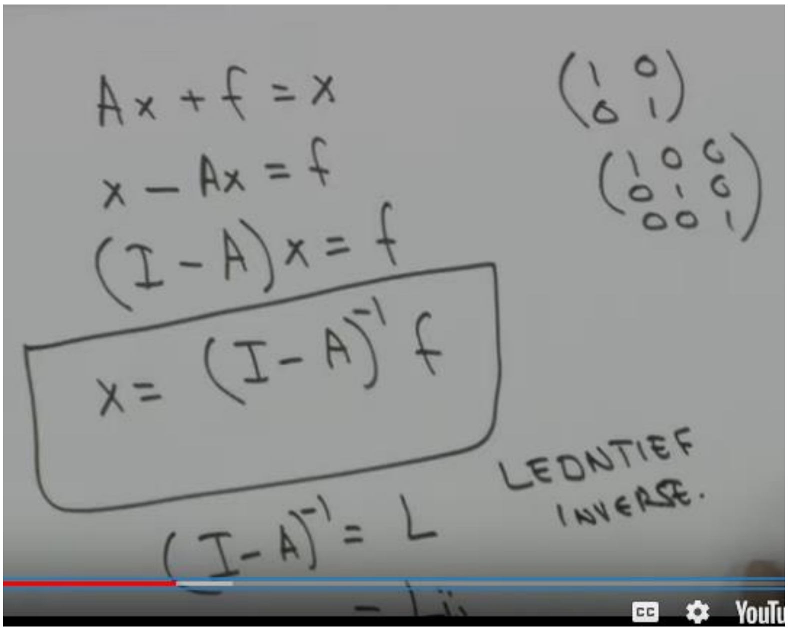

So, essentially what we had is we started off with Ax plus f is equal to x, x minus Ax is equal to f. So, this x is the identity matrix, identity matrix will be for 2 by 2, it is 1,0,0,1, it will be 1,0,0,0,1,0,0,0,1 for 3 by 3, this is I minus A into x is equal to f and so x is equal to I minus A inverse into f. This value I minus A inverse is called the Leontief inverse and this can be written as Li j.

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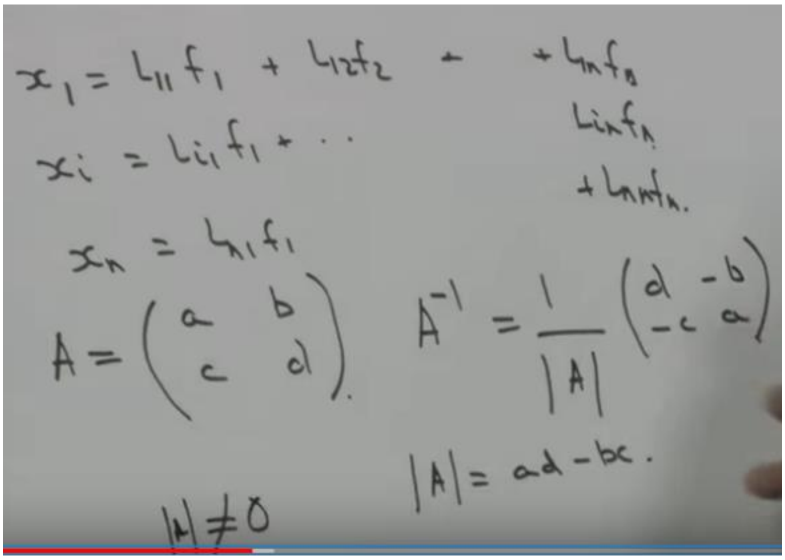

So, we can write this as x is equal to x1 is equal to L11 f1 plus L12 f2, L1n fn, and so on xi is equal to Li1. Similarly xn, so we have to calculate in each case the Leontief inverse and so, by essentially for instance, you may remember, if you look at 2 by 2 matrix, if you have A is equal to a, b, c, d, we can calculate A inverse will be nothing but 1 over module determinant of A and this is d minus b minus c, a and we have determinant of A as ad minus bc and of course, determinant of A should not be equal to 0.

With the 2 by 2 is something we can do by hand, but if we want to calculate this for 3 by 3, 4 by 4, 5 by 5, n by n we can use any, you can use MATLAB or you can use Excel and you can do the matrix inversion and get the Leontief.

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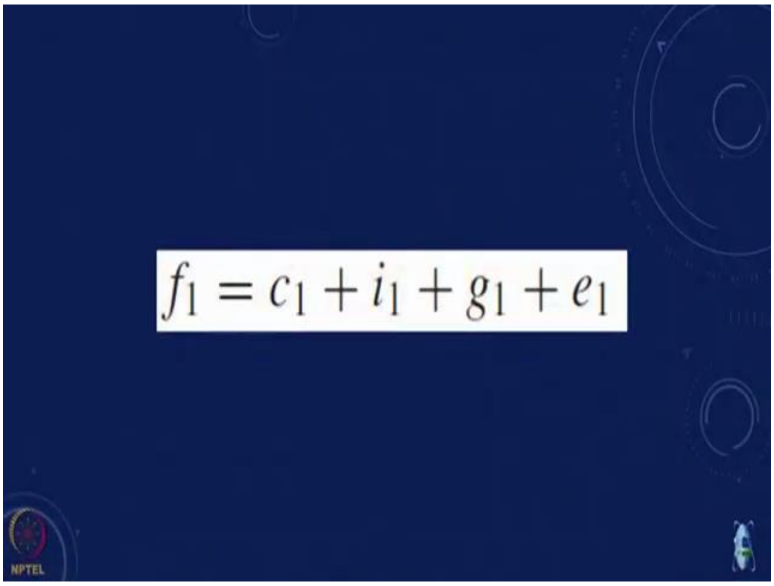

So, essentially what happens is we can take this and we would have the different sectors, the buying and selling sectors and then we will get, we also said that the final demand is a combination of the different sectors, the consumption, the government consumption, the exports.

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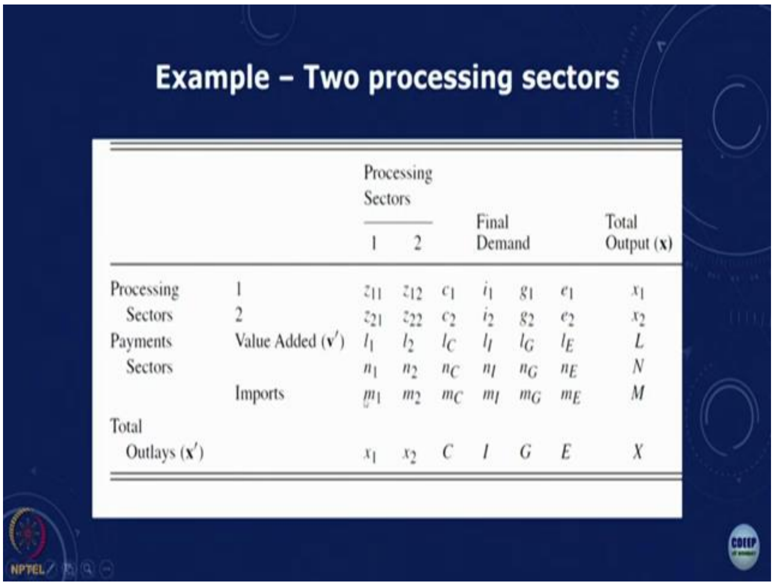

		Processing Sectors 1	Processing Sectors 2	Final Demand				Total Output (**x**)
Processing Sectors	1	z_{11}	z_{12}	c_1	i_1	g_1	e_1	x_1
	2	z_{21}	z_{22}	c_2	i_2	g_2	e_2	x_2
Payments Sectors	Value Added ($\mathbf{v}'$)	l_1	l_2	l_C	l_I	l_G	l_E	L
		n_1	n_2	n_C	n_I	n_G	n_E	N
	Imports	m_1	m_2	m_C	m_I	m_G	m_E	M
Total Outlays ($\mathbf{x}'$)		x_1	x_2	C	I	G	E	X

And then there are these payment sectors which we talk of, which is, in the columns, these are the additional sectors that we are talking of where we are paying the amount which is going to the other like wages and to government and any other services, labour, government services and inputs. And this will add up to the 2 outlays, the rows will add up and so will the columns. And so, typically, if we are talking of 2 processing sectors and some payment sectors, this is what we will get and this is finally the kind of input output table.

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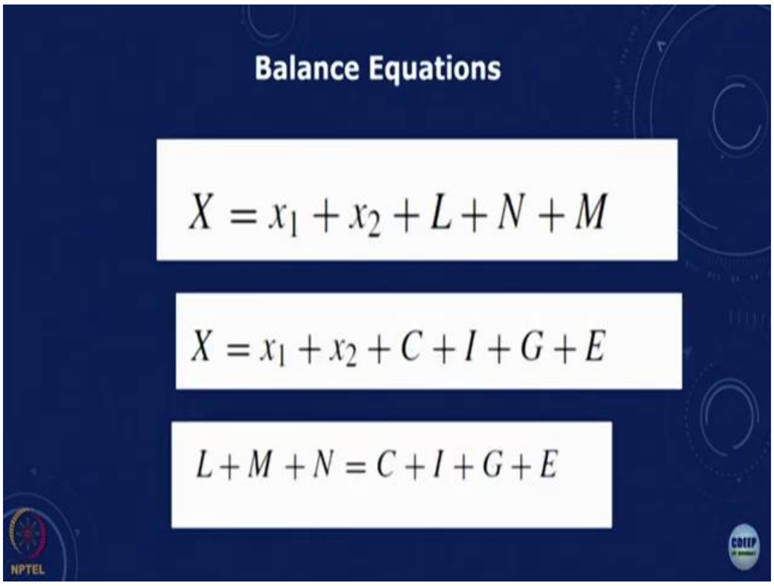

These are the balance equations that we already talked of for the 2 sectors that is X is equal to x1 plus x2 plus L, this is a balance equation for the row and the balance equation for the column and with this, this you will get this kind of calculation.

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So, if we look at these sectors, L is the labor services employment, N all other value added and M are imports, all of these will come under each sector in the column. And in the row if you look at it, there are additional final demands, which will come in terms of consumption, investment goods, government and export. So, F will be equal to C plus I plus G plus E. And this is the payment sector, which is the additional thing which will come in the column.

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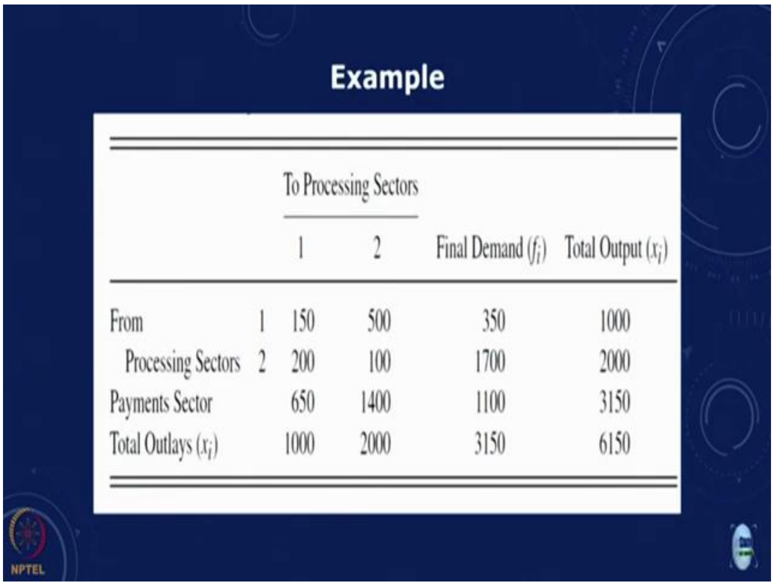

		To Processing Sectors 1	2	Final Demand (f_i)	Total Output (x_i)
From	1	150	500	350	1000
Processing Sectors	2	200	100	1700	2000
Payments Sector		650	1400	1100	3150
Total Outlays (x_j)		1000	2000	3150	6150

With this, we will take an example, which is from the book by Blair and Miller. It is a 2 sector example and we will take that example and then process it and then see what will happen when we make, what are the coefficients, how do we take Leontief inverse, what is the implication of the Leontief inverse and how can we use it to see what if in case there is a growth or there is some change in the sectoral demand.

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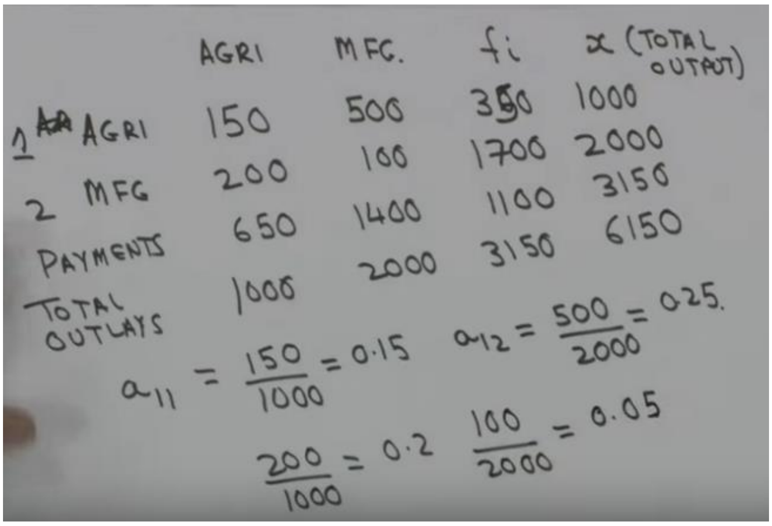

So, if we look at this sector, let me just write down this. We have essentially, let us say there is an agricultural sector and a manufacturing sector. And so, if this is the agricultural, you have an agricultural sector, and then you have a manufacturing sector, and in this case, this is 1 and 2 and then here also we have agriculture and manufacturing and this is some 150 some units, billion rupees, million dollars, the financial units, manufacturing is 500 and the total final demand fi for this is 300. So, the total which is there, the xi which is there to total final demand is going to be 150 plus 500 plus, this is 350.

So, this is total output, total output. This is the transaction which we have noticed that means, from agriculture of the output of agriculture 150 million rupees is being used in agriculture, 500 million rupees is used in manufacturing and 350 is the final demand. So, total output of agriculture is 1000 million rupees and from the case of manufacturing, 200 is going here and 100 going to the manufacturing sector. The final demand for all the manufacturing products is 1700. So, if you add this up this comes out to be 2000.

And then there is this payment sector as we said wages, taxes, profits whatever else we are looking at. So, remember this has to balance out, so the total outlays which are xi's, total outlays must balance out, so this must be equal to 1000 which will mean that this is 650. And this is again this will be equal to 2000 and this will be 1400. The final demand in this case is for the payments 1100. So, if you add this up this is 28 and this is 3150, this is 3150 total, which is the total value add in the economy is 6150 appropriate financial units.

Now, let us look at how do we calculate the aij's. So, if we look at a11, a11 is what is the amount per unit of agriculture what is the amount of agriculture use. So, this will be equal to 150 by 1000 which is 0.15. That means per unit of agricultural output 0.15 times of that is the ratio of what is used within this sector itself. a12 is the percentage of the agriculture which is used in the agriculture… If you look at the transaction from agriculture to manufacturing is 500 units and this will depend on the output per unit of manufacturing.

So, here this is going to be 500 divided by 2000. This will be divided by xj in this case x2, so this is going to be 0.25. Similarly, this is going to be 200 divided by 1000 which is 0.2 and this is 100 divided by 2000 which is 0.05.

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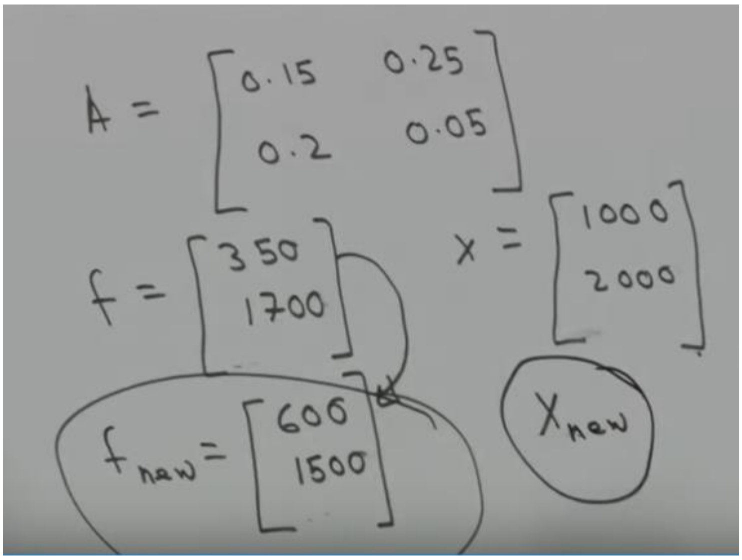

These are the technical coefficients, we have the A matrix which is point 0.15, 0.25, 0.2 and 0.05, this is the A matrix and we can then put down, if you see this is the A matrix, the f matrix is 350 and 1700 and the value of x is 1000 and 2000. Now, the question is that what if instead of this kind of output we had a change, where the agricultural output decreased and both of these…

If the agricultural output suppose instead of the final demand for agriculture instead of 350 if that increases to f new, if we say that instead of this we are converting, we increase it to 600 and the industrial demand decreases. So, suppose we move from here to here, we want to know how, what will be the changes in the economy and how much of each of these products would be calculated. So, this is what we want to do in terms of x new.

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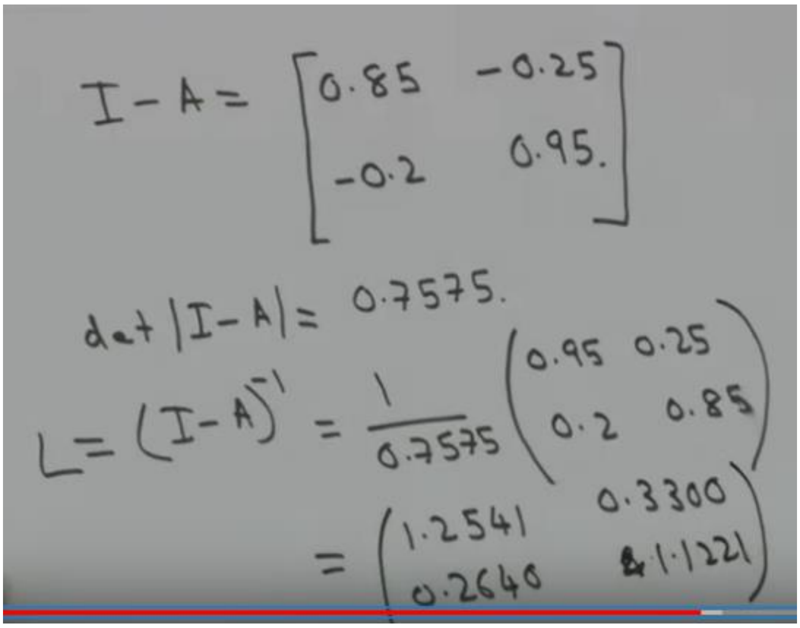

So, in doing this, first thing which we can do is calculate I minus A, if you remember this is A, so 1 minus point 0.85 and here is 0 minus 0.25, so it is minus 0.25. This is 0 minus 0.2. So, this point 0.2, 1 minus 0.05, so it is 0.95. This is I minus A, and we want to calculate the inverse of this. And you can see the determinant of this I minus A.

You can check this out, it comes out to be 0.7575. And so, the Leontief inverse is, I minus A inverse is 1 by 0.7575 and this is now 0.95, 0.25, 0.2, 0.85. You will see that this comes out to be 1.2541, 0.330 and 0.2640 and this is… So, this is what was told, we calculated this.

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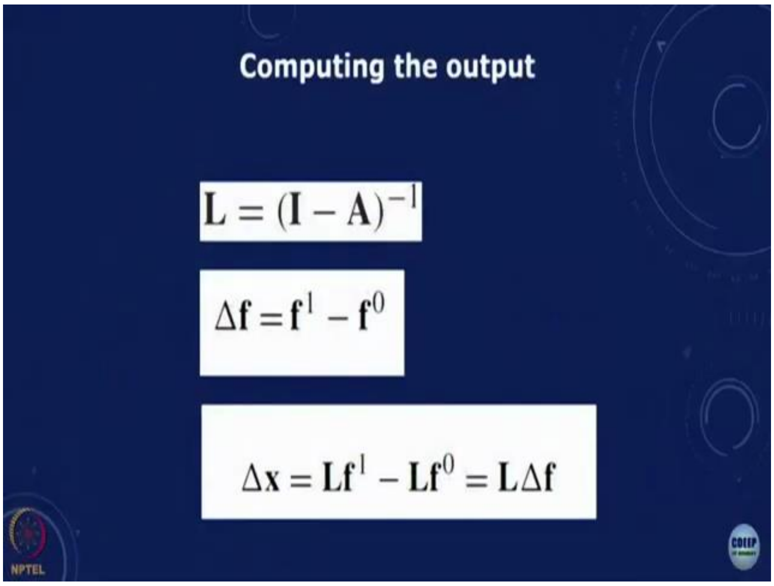

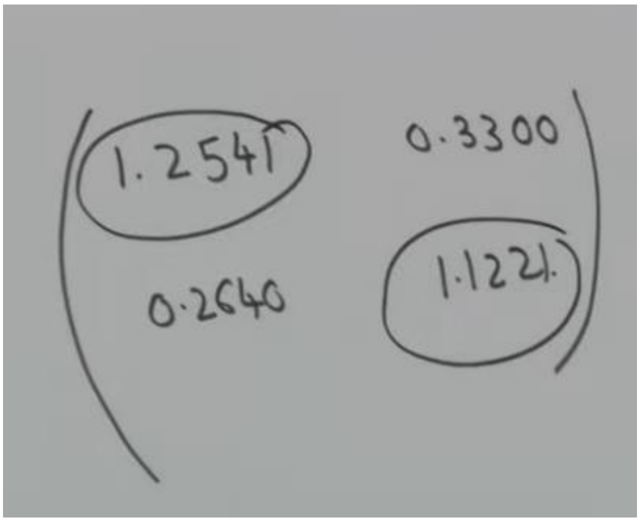

And so the interesting thing to see is if you look at this values that we have of the Leontief matrix 1.2541, 0.3300, 0.2640, 1.1221, you will notice that all the coefficients which are there in the diagonal these are greater than 1 and that is essentially which means that in the per unit of, we had said there is a direct coefficient which is what is the amount of agricultural output increase per unit of agriculture.

But if you look at for a particular value of output, if we look at x, how much is the total direct plus indirect requirement these values in the diagonal will always be greater than 1 and that is by the nature of this.

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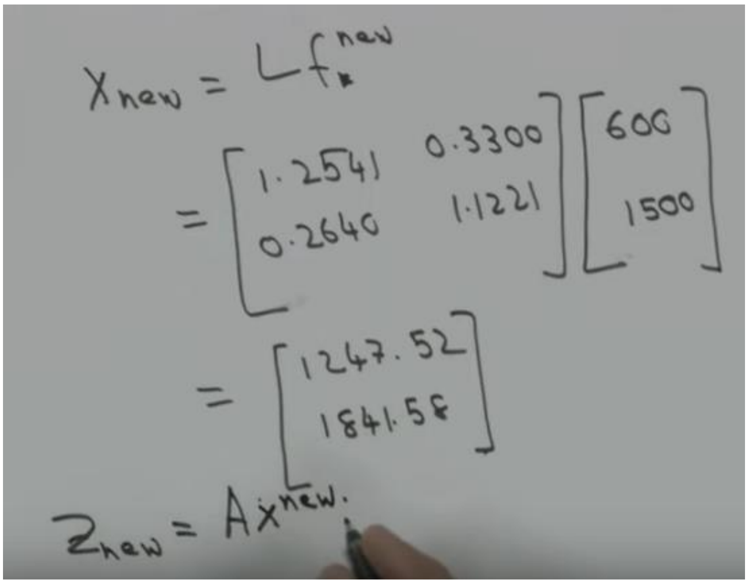

So, we can now take, if we want to calculate the value of x new, we can just take L into f new and this is 1.2541, 0.3300, 0.2640, you multiply the two matrices 600, 1500 we get 1247.52

1841.58 and what are the Z new? Z new can be the new inter industry transaction will just be A into X new.

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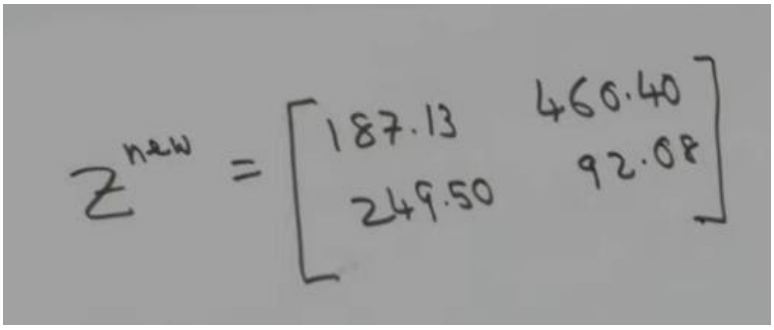

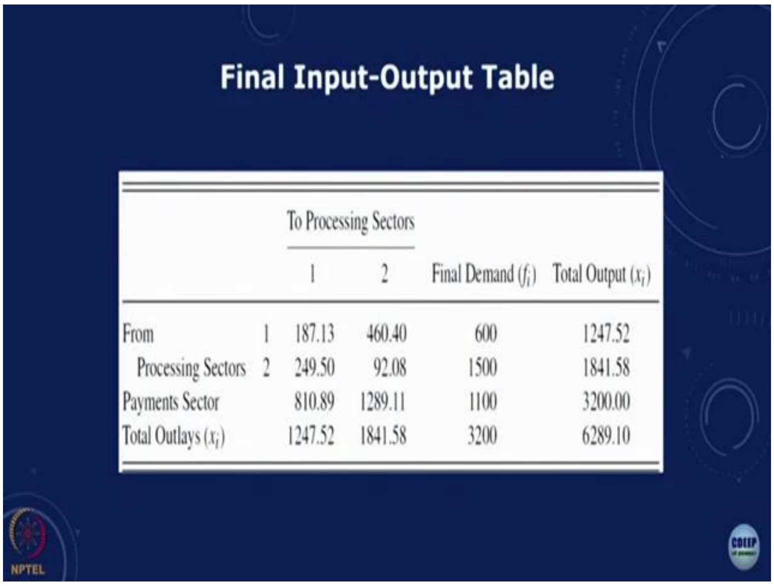

Final Input-Output Table

		To Processing Sectors 1	2	Final Demand (f_i)	Total Output (x_i)
From	1	187.13	460.40	600	1247.52
Processing Sectors	2	249.50	92.08	1500	1841.58
Payments Sector		810.89	1289.11	1100	3200.00
Total Outlays (x_j)		1247.52	1841.58	3200	6289.10

And if you do this you will find that Z new is 187.13, 460.40, 249.5 and then if we look at this, then we can we can get the new final output input output table.

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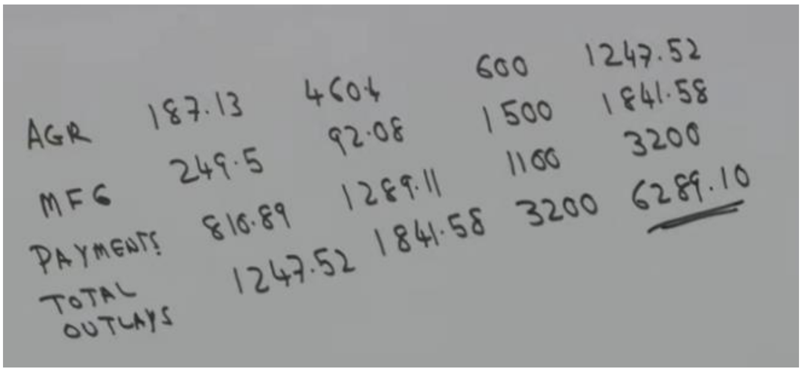

And that will now be agriculture, manufacturing 187.13, 460.4 fi is 600, 1247.52, we could round it off also, this is 249.5, 92… Payment sector. Now to calculate the payment sector, you will see that this total is 1247.52 subtract from that these two and you will get this as 810.89, this is 1289.11, this will remain unchanged, the total will be 3200, total outlays, I can add this will be 1247.52, 1841.58, 3200, 6289.10. Let us look at what it was earlier and then we can compare these two.

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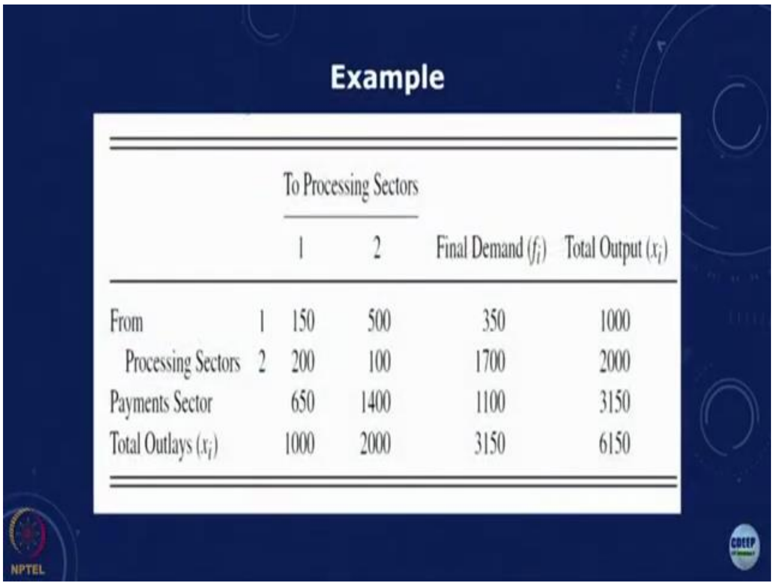

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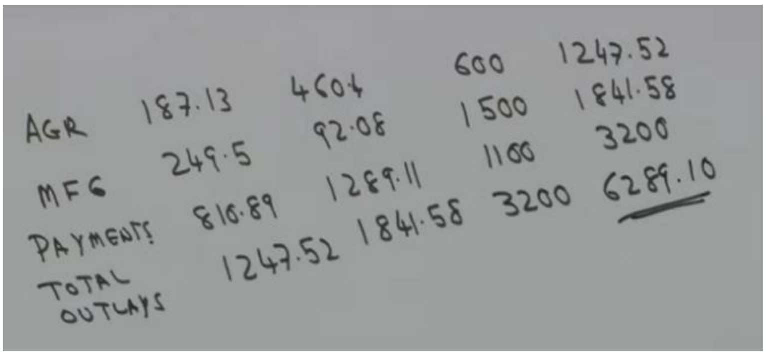

So, you see to what has happened here is that the total output in the agricultural sector even though the final demand of the agricultural, the final demand of the agricultural sector has increased from 350 to 600 with the result that the total output of the agriculture sector in order to meet this demand has increased from 1000 units to 1247 units.

And in the case of industry the demand has reduced final demand has reduced from 1700 to 1500, it has reduced by 200 units, but the total output has also reduced but not in the same amount. It has reduced, it has reduced to 1841.

And when we look at the addition of this, the earlier the total output of the economy, the 2 sector economy was 6150 overall now, the economy has increased to 6289. And so, the we can see that increase in agriculture decrease in industry, what is the impact. So, there are many different things that can be done with this and with this we can also look at some of the energy sectors as well as the manufacturing sector.

So, the impact of energy intensity, we can also, all of this can be done in, this was done in money units, we can also do it in hybrid units where some years, some of the terms, some of the sectors are represented in energy terms and the other sectors are represented in money terms and we will look at some of these examples and the applications in the next module.