

Energy Resources, Economics and Environment
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Lecture 25
Revision Paper – Part 2

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3.
 The inverse demand function for a fossil fuel is:
 $P_t = 5 - 0.6 q_t$, Assume that the costs of extraction are zero. The initial reserves are $R_0 = 50$ and $d = 10\%$.

- What is the price elasticity of demand for this function when $q_t = 4$?
- Determine the time path of extraction for a mining industry under pure competition.
- When does the resource get exhausted?
- Would the time path of extraction for a monopolistic mining industry be different?

Explain your answer qualitatively

- What is the effect of a higher discount rate on the path of extraction

[1+2+2+1+1]



Now, let us look at the third question, this question is the inverse demand curve which we have solved earlier and you can see in this case, if we look at this question the inverse demand curve for a fossil fuel is given as P_t is 5 minus 0.6 q_t , assume that the costs of extraction are 0, initial reserves R_0 is 50 and d is 10 %. What is the price elasticity of the demand for this function when q_t is equal to 4?

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$$\begin{aligned}
 q_t &= 4 & P_t &= 5 - 0.6 \times 4 \\
 & & &= 2.6 \\
 \eta &= \frac{\frac{dq}{q}}{\frac{dp}{p}} & \frac{dp}{dq} &= -0.6 \\
 &= \frac{p}{q} \left[\frac{dq}{dp} \right] & &= \frac{2.6}{4 \times (-0.6)} \\
 & & &= \frac{-2.6}{2.4} = \underline{\underline{-1.083\%}}
 \end{aligned}$$

So, this is straightforward, q_t is equal to 4 we can find out what is P_t , P_t is 5 minus 0.6 into 4 is 2.6 and dp by dq is minus 0.6, so the elasticity is dq by q by dp by p , p by q . and just substitute the values, this is going to be 2.6, q is 4 and this is minus 0.6. So, it is minus 2.6 by 2.4 minus 1.083%. Negative is obvious because if the price increases, the quantity will decrease, of course, if you will get the absolute value it will be 1.08.

So, this is the first part of the question, the price elasticity of the demand a), then determine the time part of extraction for a mining industry under pure competition.

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$$\begin{aligned} \text{at } P_T &= a, q_T = 0 \\ P_T &= 5 = P_0 (1.1)^T \\ P_0 &= \frac{5}{(1.1)^T} \\ P_t &= 5 (1.1)^{t-T} \\ P_t &= 5 - 0.6q_t \\ 5(1.1)^{t-T} &= 5 - 0.6q_t \\ q_t &= \frac{5}{0.6} - \frac{5}{0.6} (1.1)^{t-T} \end{aligned}$$

So, we can we have derived this in the module. So, at P_t is equal to a , q_t is equal to 0. So, P_t is equal to 5 is P_0 1 plus d raise 2 t 1.1 raise to T , where T is the time when the reserve gets exhausted. So that means P_0 will be 5 by 1.1 raise to T , P_t is 5 into 1.1 t minus T that is P_t . So, q_t will be a minus b p_t . So, q_t will be 5 minus 0.6, P_t is 5 minus 0.6 q_t .

So, 5 into 1.1 raise to T minus t is 5 minus 0.6 q_t . So, q_t is 5 by 0.6 minus 5 by 0.6, 1.1 raise to t minus T . Now only thing we need to determine is the value of capital T and that is there in the next question when does the resource get exhausted once we do that we can actually solve for this.

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$$T = \frac{bR_0}{a} + \frac{1}{d} \left[1 - \frac{1}{(1+d)^T} \right]$$
$$T = \frac{0.6 \times 50}{5} + \frac{1}{0.1} \left[1 - \frac{1}{(1.1)^T} \right]$$
$$= 6 + 10 \left[1 - \frac{1}{(1.1)^T} \right]$$

$T \sim 13.1 \text{ year.}$

$$q_t = 8.33 \left[1 - (1.1)^{t-13} \right]$$

We had derived based on the geometric progression that we had we had derived this expression T is equal to $b R_0$ by a by 1 plus d 1 minus 1 plus d raise to T . So, what we will get is T is equal to 0.6 into 50 by 5 plus 1 by 0.1 into 1 minus 1.1 raise to T , 6 plus 10 1 minus 1.1 raise to T . So, you can do this iteratively, start with a value of t then calculate it, get the next value until it gets converge and you will get T approximately equal to 13.1 year.

So, when you substitute this what we will get is q_t , q_t is 8.33 into 1 minus 1.1 t minus 30 and if you look at that, I have just plotted it for you, you can see that it starts, extraction part starts from here and it closes down at about the 13^{th} year and the total area total sum of all of this is 50 million tons R_0 .

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3.
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- Would the time path of extraction for a monopolistic mining industry be different?

Explain your answer qualitatively

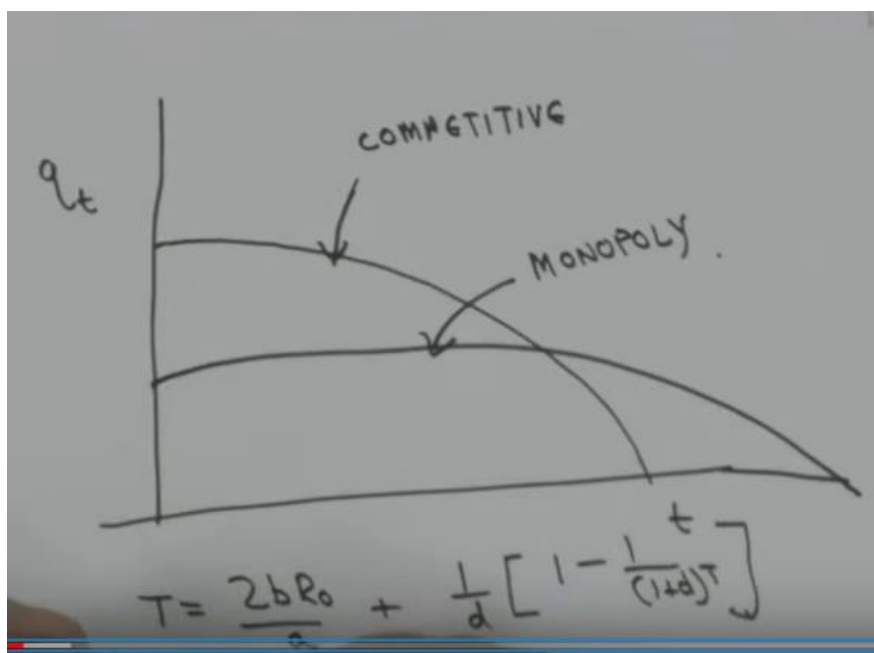
- What is the effect of a higher discount rate on the path of extraction

[1+2+2+1+1]



The next question is, would the time part of extraction of a monopolistic mining industry be different? The answer is obviously yes and we have seen the reason for this. The here the since it is a monopolistic mining industry, if it controls the quantity, the price will get changed. So, if it releases less amount, it will have a higher price. So that means that it would try to maximize its revenue, so that we have the total marginal revenue in different intervals increasing by the discount rate.

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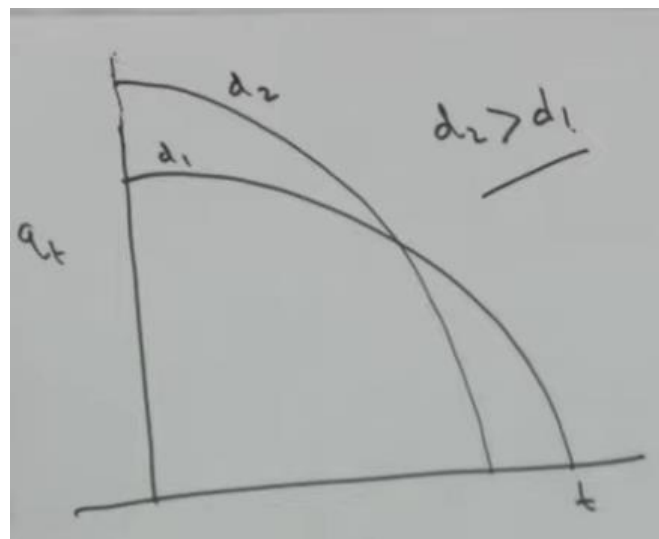
And as a result of it qualitatively what would happen is that if you look at this, this is the competition q_t versus time competitive market and if you had a monopoly, this is you would it

will go for a larger amount of time. So, essentially what happens is that because it has a incentive to this is a monopolistic case, monopoly. So, qualitatively the time taken would be more, the time actually comes out to be $2 b R_0$ by a as compared to the earlier case.

Well, here we are not asking for this to be, we have not asked for this to be calculated you could calculate it and see that you would get to instead of 13 years you would get something like 20 years, it would last and so, that the idea is that, because it has an incentive, it can get a higher price initially by releasing less into the market, the overall revenue that it gets is more and so the monopoly actually tries to increase its total returns.

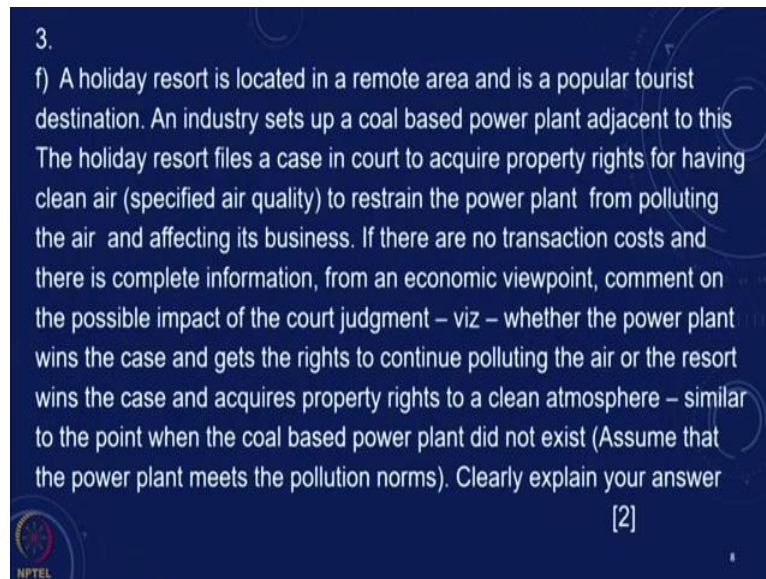
And in the process, of course, the utility of the consumers is affected. What is the effect of a higher discount rate on the path of extraction? Well, if you have higher discount rate, if you look at it what would happen is that the price would increase at the discount rate and we would try to if you the discount rates are higher we will try to extract more initially as compared to in the future, and so that that is what would happen.

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You would have and if we had a higher discount rate this would last less. So, this is d_1 , d_2 so with this week, this is very similar. This is exactly what we had covered in the portion where we were looking at non-renewable resource economics.

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3.
f) A holiday resort is located in a remote area and is a popular tourist destination. An industry sets up a coal based power plant adjacent to this. The holiday resort files a case in court to acquire property rights for having clean air (specified air quality) to restrain the power plant from polluting the air and affecting its business. If there are no transaction costs and there is complete information, from an economic viewpoint, comment on the possible impact of the court judgment – viz – whether the power plant wins the case and gets the rights to continue polluting the air or the resort wins the case and acquires property rights to a clean atmosphere – similar to the point when the coal based power plant did not exist (Assume that the power plant meets the pollution norms). Clearly explain your answer [2]

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Let us move ahead to the next question, and this question is this is question number. The question number in is this is 3 f) A holiday resort is located in a remote area and it is a popular tourist destination, industry sets up a coal based power plant adjacent to it. This holiday resort files a case in court to acquire property rights for having clean air specified, air quality to restrain the power plant from polluting the air and affecting its business.

If there are no transaction costs and there is complete information from an economic viewpoint, comment on the possible impact of the court judgment viz whether the plant power plant wins the case gets the right to continue polluting the air or the resort wins the case acquires property rights, so clean atmosphere, similar to the point when the whole plan power plant did not exist. Assume that the power plant meets the pollution norms clearly explain your answer.

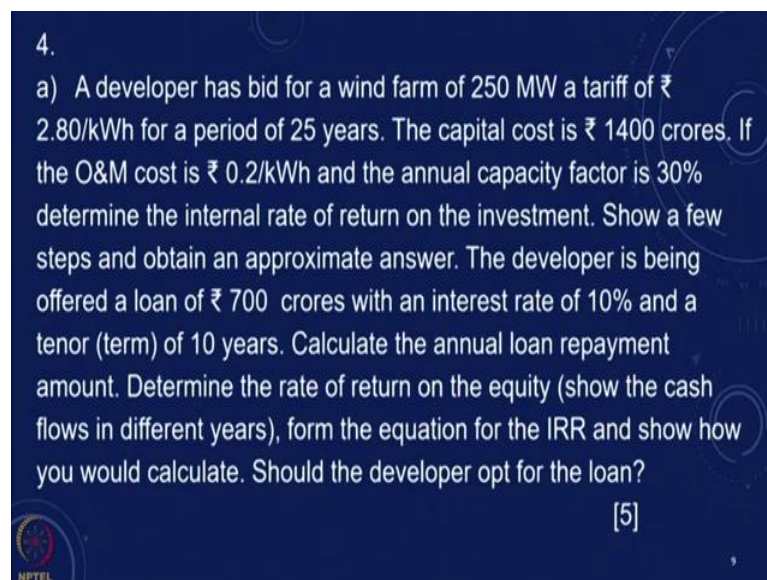
So, in this case, this is the classic course theorem. There are two options, one is the power plant obtains the right that it meeting the polluting, it is meeting the norms of environmental norms and it can continue to pollute. Even so, in such a case, what would happen according to this is that if the tourist destination, the holiday resort has a loss in terms of the foregone revenue.

And if there is a certain cost in terms of cleaning up the pollution of the power plant, and the cost of the total lost revenue is more than the cost of the holiday resort could actually pay the power plant to make the controls pollution controls and have a cleaner atmosphere so that that would be the kind of thing. But in case the power plant wins the case, in case the holiday resort wins the case and the power plant has a loss because it has to shut down then the power plant can compensate the holiday resort for the loss and continue to generate its electricity.

So, it will depend on what is the loss, whichever economically whichever is more based on that the calculation of who is winning the case will not affect the economic judgment and will not affect the viability. The final solution will be from an economic viewpoint, if there are no transaction costs is independent of the judgment which is happening, because if we look at the same industry owning the holiday resort and the power plant an optimal solution will be that, which will give you the maximum benefit.

Of course, from an equity point of view, there may be different issues but this is the classic case of the Coase theorem, if there are no transaction costs, the property rights, whoever owns the property rights, the economic decision would be the same, whether the property rights are owned by the holiday resort, or by the power plant, so this is a revision of the Coase theorem.

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4.
a) A developer has bid for a wind farm of 250 MW a tariff of ₹ 2.80/kWh for a period of 25 years. The capital cost is ₹ 1400 crores. If the O&M cost is ₹ 0.2/kWh and the annual capacity factor is 30% determine the internal rate of return on the investment. Show a few steps and obtain an approximate answer. The developer is being offered a loan of ₹ 700 crores with an interest rate of 10% and a tenor (term) of 10 years. Calculate the annual loan repayment amount. Determine the rate of return on the equity (show the cash flows in different years), form the equation for the IRR and show how you would calculate. Should the developer opt for the loan? [5]

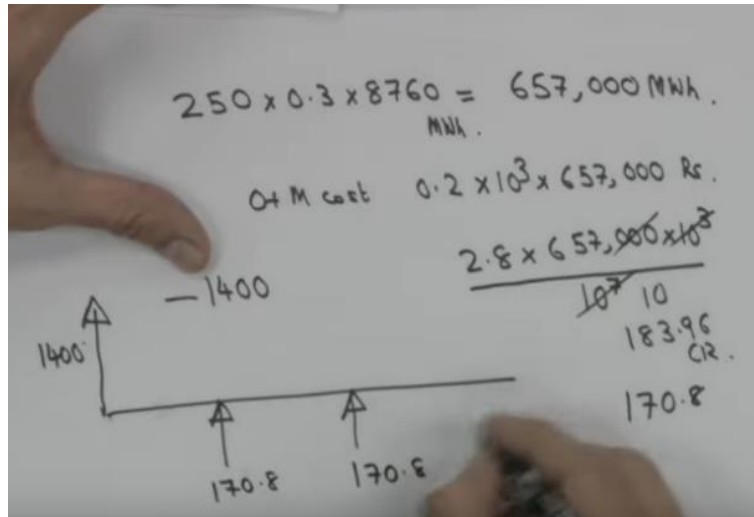
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Let us look at the 4th question, 4th question is, a developer has bid for a wind farm of 250 megawatts, a tariff of rupees 2.8 kilowatt hour for a period of 25 years, capital cost is 1400 crores. If the O and M cost is 0.2 rupees per kilowatt hour and the annual capacity factor is 30 %, determine the internal rate of return on the investment, show a few steps and obtain an approximate answer.

Developer is being offered a loan of 700 crores with an interest rate of 10 % and a tenure term of 10 years, calculate the annual loan repayment amount, determine the rate of return on the equity, show the cash flows, form the equation for the IRR and show how you would calculate, should the developer opt for the loan? So, if you look at this, this is very similar to the question

that we had solved in the class. So, if we look at this the 250 megawatts annual capacity factor is 30 %.

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$$-1400 + \sum_{k=1}^{25} \frac{170.8}{(1+r)^k} = 0$$

$$\text{Annual loan repayment} = \frac{i(1+i)^{N_L}}{(1+i)^{N_L} - 1}$$

$$= \left[\frac{0.1(1.1)^{10}}{(1.1)^{10} - 1} \right] 700$$

So, 250 megawatts into 0.3 into 8760 is the total amount of generation in megawatt hours and if the O and M cost is, so this is 250 into 0.3 into 8760, if we just calculate this, it is going to be this is 657000 megawatt hour. And if the O and M cost is 0.2 rupees per kilowatt hour, then the O and M cost is going to be 0.12 into 10 raise to 3 into 657000 rupees. Then when we look at the,

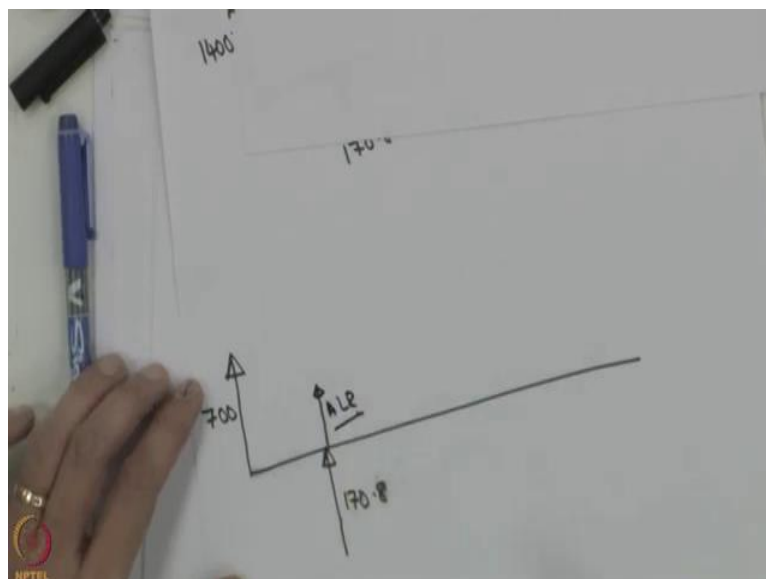
So, the developer is being showed few steps, we have asked determine the internal rate of return. So if the entire payment is being made by the developer, then it is going to be minus 1400 crores is the, this is the payment, C_0 is 1400 and annually we are getting 2.8 rupees into 657000 into 10 raise to 3, this is rupees if you are dividing it by if you want to get it in crores this will be divided by 10 raise to 7.

So, this is the amount that we will get and if we look at this, 10 raise to 3, 2.8 into 657, 2.8 into 657 divided 10, get this as 183.96 crores. We have also, actually we will be paying an O and M cost of 0.2. So, instead of this we can take this as multiplied by 2.8, so 2.6 is what we are going to get.

So, we are going to get 170.8 crores each year so, this is 170.8 and the IRR that we will get will be will set the present value 1400 plus 170 .8, $1 + R$ raise to k and this is for 25 years, set this is equal to 0 and then we can solve this for the IRR.

So, in a similar way, we can get the value of the internal rate of return for the loan of 700 crores without interest rate of 10 % and tenure term of 10 years, we will just get the annual loan repayment is going to be $I \cdot 1 + I \text{ least } 2 \cdot N \cdot L$, $1 + I$ raise to $N \cdot L$ minus 1, interest rate is 0.1, 1.1 raise to 10. So, you can calculate this and then this will be multiplied by 700 to get the annual loan repayment.

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So, in the case of in this case the value is going to become instead of 1400, we will get now we are only paying 700 crores and here we are going to get back 170.8 and instead and we are paying out the annual loan repayment. So, this is very similar to the problem that we have

solved in the module when we did this. I will upload for you the excel sheet which gives you the complete solution and you can cross check your numbers with this.


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4.

b) The transactions in a state economy using electricity and industry are shown below (in Million Rupees)

	Electricity	Industry	Final Demand
Electricity	800	900	1500
Industry	1000	1200	3000

i) Calculate the total sectoral outputs and the direct technical coefficients- A matrix. What do $A(1,2)$ and $A(2,2)$ signify? [2]



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Let us go to the last question, and this is a simple input-output calculation that we had done. So, the transactions in a state economy using electricity and industry so there are two sectors, there is an electricity sector and there is industry sector. So, if we look at it, the electricity to electricity is 800 million rupees, electricity to industry is 900 million rupees and the final demand is 1500 million rupees, industry to electricity is 1000 and industry to industry 1200, the final demand for the industry is 3000.

So, if there is a society with this 2-sector economy, we want to calculate the total sectoral outputs and the direct technical coefficient that is the A matrix.

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	E	I	FINAL DEMAND	TOTAL OUTPUT
E	800	900	1500	3200
I	1000	1200	3000	5200
PAYMENT SECTOR	1400	3100	1500	6000
TOTAL	3200	5200		14400

$\begin{bmatrix} \frac{800}{3200} \\ \frac{1000}{3200} \end{bmatrix}$	$\begin{bmatrix} \frac{900}{5200} \\ \frac{1200}{5200} \end{bmatrix}$	A	$\begin{bmatrix} 0.25 & 0.1731 \\ 0.3125 & 0.2308 \end{bmatrix}$
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So, let us start by writing down the sectors as E and I. E, I and the final demand and then total output. And here we can have another payment sector and then we have the total output. So here we have given that this is 800, this is all in million rupees, this is 900 and this is 1500 right. So, we can add this up and we get 1700 plus 1500, this is 3200, this is given to you as 800 plus 1000.

Now this total must add up to 3200, so 1800, 3200 minus 1800 will be 1400. Now in this case this is 1000, 1200 3000 that means 4200, 5200 we add this up and we get 5200 and this comes to 5200, so this is 2100 and this is 3100. Now this remaining, final demand for the payment sector, this value should be given to you if you want to complete the table, let us say that this was given as 1500.

Then when we add this up, this is let us look at this, we can now add this up 1400, 4500, 6000 is this total, so, this total comes out to be 14400. Now, the question which has been asked is total sectoral outputs, total sectoral outputs of E is 3200 million tons, this is the first answer, and for the industry is 5200 million tons. Then let us calculate the A matrix. So, when we talk about the A matrix we are going to just divide.

A matrix is going to be electricity. This is the this is the destination sector per unit of electricity output that means 800 by 3200 and this is going to be 900 by 5200, 1000 divided by 3200 and this is 1200 divided by 5200. So, these values, the A matrix turns out to be 0.25, 0.1731, 0.3125, 0.2308. What do A₁₂ and A₂₂ signify? So, A₁₂ is the amount of electricity used per unit of

industrial output that means, rupees per rupee of industrial value and electricity intensity of the industrial sector.

And the A 22 is the industrial output which is being used for the industrial sector itself and that is the amount of industrial output which is used in this so that that is what it means.

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4.
b)
ii) Compute the Leontief inverse matrix L. If the final demand for industry increases by 5 % and the final demand for electricity remains the same, compute the changes in the total output of both the sectors [2]
iii) If the average price of electricity is 4 ₹/kWh, what is the electricity intensity in industry in MWh/ Million ₹? What are the limitations of the Input –Output method? [2]
c) What is Net Energy Analysis? How would you compute the Net energy required for a bio fuel? [2]

Let us look at the next question is to compute the Leontief inverse matrix L.

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$$I - A = \begin{bmatrix} 0.75 & -0.1731 \\ -0.3125 & 0.7692 \end{bmatrix}$$
$$L = (I - A)^{-1} = \begin{bmatrix} 1.471 & 0.331 \\ 0.598 & 1.434 \end{bmatrix}$$
$$\begin{bmatrix} 1500 \\ 3000 \end{bmatrix} \rightarrow \begin{bmatrix} 1500 \\ 3150 \end{bmatrix}$$

So, that is we can do I minus A and you find that this is 0.75 minus 0.1731 minus 0.3125 and L is I minus A inverse, you can just use the formula, which we had done and then you will get this as 1.471, 0.331, 0.598, you can see the diagonal elements are greater than 1. So, now the

question which was asked is if the final demand increases by 5 %, final demand for electricity remain the same, compute the changes in the total output of both the sectors.

So, if the final demand for industry increases by 5 %, final demand was 1500. The final demand for industry increases by 5 %, and the final demand for, this is the final demand for the electricity sector, 1500 remains unchanged, so, the initial thing was 1500 and 3,000, this will now change to final demand will now change to 1,500 and 5 % 1.05 of 3000 will become 3150 right, 10 % is 300 so this is. So, if this is the case now, we want to find out what is what will be the change in the overall.

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$$X = \begin{bmatrix} 1.471 & 0.331 \\ 0.598 & 1.434 \end{bmatrix} \begin{bmatrix} 1500 \\ 3150 \end{bmatrix}$$

So, what we can do is that we have already calculated the Leontief inverse and if we now multiply the Leontief inverse with the final demands, we can then get what will be the values for, we can multiply this 1.471, 0.331, 0.598, 1.434 into 1500, 3150 so that we can get what will be the values of x. And once you do this, then, you can see that the final table that we will get will be of this.

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		937.2	1500	3249.7
E	812.4			
		1249.7	3150	5415.2
I	1015.5			

You will have electricity, industry, get this and then we can multiply the totals, we can get the totals and multiply them and 937.2, 1500, 3150, you can cross check this answer. You get the values of x and then once you get the values of x we can multiply by the A matrix to get these coefficients and then we can get the values which is there. So, the changes in total output are calculator in this form 3249.7 and 5415.2 that is the value which is there.

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iii) If the average price of electricity is 4 ₹/kWh, what is the electricity intensity in industry in MWh/ Million ₹? What are the limitations of the Input –Output method? [2]
c) What is Net Energy Analysis? How would you compute the Net energy required for a bio fuel? [2]

Then the question which is asked is if the average price of electricity is 4 rupees per kilowatt hour what is the electricity intensity in industry? Let us take the original table that we had.

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900 Million Rs of Electricity
5200 Million Rs of Industrial.

$$\frac{900}{4} = 225 \text{ Million kWh.}$$
$$\text{Rs/kWh} = 225 \times 10^3 \text{ MWh.}$$
$$\text{EI} = \frac{225 \times 10^3 \text{ MWh}}{5200} \text{ MWh/Million Rs.}$$

900 Million Rs of Electricity
5200 Million Rs of Industrial.

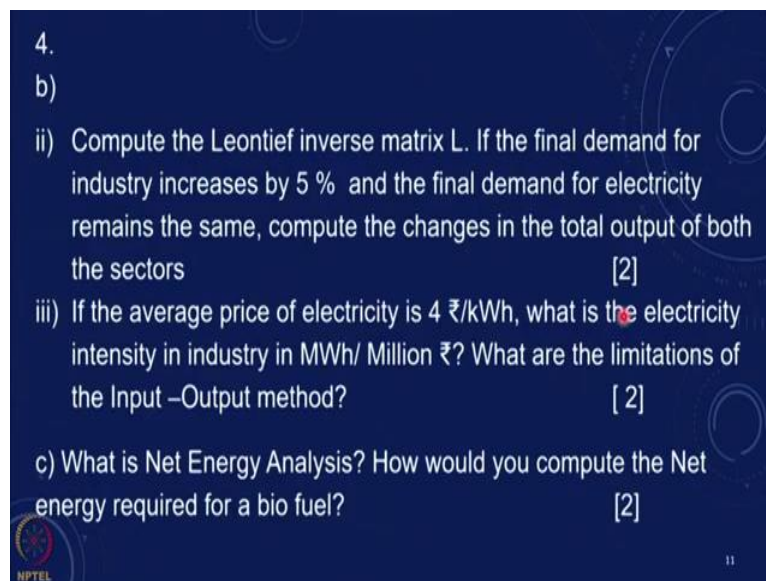
$$\frac{900}{4} = 225 \text{ Million kWh.}$$
$$\text{Rs/kWh} = 225 \times 10^3 \text{ MWh.}$$
$$\text{EI} = \frac{225 \times 10^3 \text{ MWh}}{5200} \text{ MWh/Million Rs.}$$
$$= 43.3 \text{ MWh/Million Rs.}$$

The original table we got a value as the industry, electricity intensity in industry that means the amount of electricity used in industry per unit, this was 900. We had the value of 900 million rupees of electricity for a total output of industry of 5200 million rupees, industrial output.

So if we take 900 and divide by 4 that is the price of electricity rupees per kilowatt hour, we get this as 225 million kilo watt hour or this will be to 225 into 10 raise to 3 megawatt hour so the intensity will be to 225 into 10 raise to 3 megawatt hour divided by 5200 million rupees, so this is megawatt hour per million rupees, 1000 can be cancelled, 2250 by 52 is the answer that we are going to get, 2250, it comes to 43.3.

And then what are the limitations with, main limitation is the fact that the relationships are linear, and the coefficients are static. As things change over a long period of time, these coefficients would also change and so that is the main limitation.

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4.
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ii) Compute the Leontief inverse matrix L. If the final demand for industry increases by 5 % and the final demand for electricity remains the same, compute the changes in the total output of both the sectors [2]
iii) If the average price of electricity is 4 ₹/kWh, what is the electricity intensity in industry in MWh/ Million ₹? What are the limitations of the Input –Output method? [2]
c) What is Net Energy Analysis? How would you compute the Net energy required for a bio fuel? [2]

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The last question that we have is what is net energy analysis? Net energy analysis essentially looks at different choices from an energy viewpoint. So, we try to see what is the total amount of energy required in creating a product including all the materials and the energy required for that. When we talk about net energy required for the biofuel, we have to start from the agriculture and write down all the steps in creation of the bio biofuels, right.

From the farm to the process where you have the biofuels, esterification and others and then to the use, and we write down the entire chain, if you look at your notes with that figure is available and in each case, depending on the yield, we would calculate how much energy per

unit of fuel and then make that comparison. So, with this, we complete the solution of this paper. You can compare your answers with these answers and then see how you have fared.