Game Theory and Economics

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Module No.# 02
Strategic Games and Nash Equilibrium
Lecture No. # 08
Strictly and weakly Dominated Actions

Welcome to lecture 8 of module 2 of the course called game theory and economics. Before we start this lecture, let me take you through what we have discussed in the previous lecture. We have basically introduced the idea of best response functions; this was done because in many cases the actions of the players may not be discrete and they might be continuous variables. If there continuous variables there is no way in which you can construct payoff matrices and seek for Nash equilibria.

To find out the Nash equilibria in this case, where the actions of players might be continuous variables - an infinite number of actions, we need to use the idea of best response functions. What we have seen is that if we construct the best response functions of players then the Nash equilibria or Nash equilibrium will be situated at the intersection point of this best response functions.

In particular, if we have two players and if you construct the best response functions for them then, the point at which they intersect - this best response functions intersect - will be the points of Nash equilibrium. So, that is what we have done and we have done some exercises also for applying this idea. Today, we shall do another exercise and move on to the next topic and this is not exactly an exercise, it is an example and this idea is the following; this is called Contributing to Public Good.

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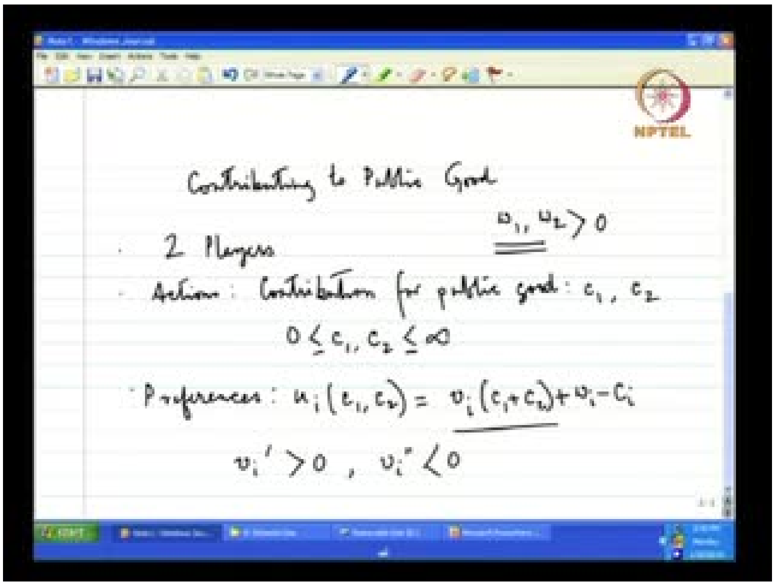

If you remember we have already done one or two examples of public good. Public good are cases where people derive some utility from the goods from that public good but I cannot exclude from anyone enjoying that public good.

For example, a government road may be I am deriving some benefit out of that government road but I cannot exclude someone else from using that and so there is a problem of whether that public good will actually be constructed because, you cannot exclude anyone from using that good then, you will not be able to make them pay also, because if I can use that good without having to pay for it, I will not pay for it.

So, there is an inherent problem of provision of public good which cannot be solved from standard private good frame work and this is another example of that. Here, the question is not whether people will contribute or not, it is not a zero one kind of decision; it is a decision of if people contribute then they may like to contribute more or less also.

I mean, I can choose any number between 0 and infinity, denoting the amount of money that I am ready to pay for the public good. Here, the question is not 0 or 1 or contributing or not contributing, but the question is not contributing and if contributing then how much to contribute, so I have infinite number of actions here. So, let me tell you the setup first, to make the problem simple there are 2 players, these are the players.

Actions, they are contribution for the public good and we shall denote this by c 1, c 2 what is the nature of this c 1 and c 2? c 1, c 2 can lie between zero and infinity. It can take any non-negative value and what are the preferences? Preferences of any player are given by the utility function of that player or the payoff function of that player. So, let me denote payoff function for player i as u i; it is a function of 2 variables both his contribution and the contribution by the other player. Also, I assume that to begin with players have the endowment of w 1 and w 2 this is, their initial wealth which are positive.

Out of this w 1 and w 2 they decide how much to contribute or whether to contribute at all, if they decide to contribute 0 that means, they are not contributing. How does this payoff function look? This is given by v i c 1 plus c 2 plus w i minus c i, to interpret this function little bit, this first part - that is v i part. If I look at it more carefully, I will see that it is a function of c 1 plus c 2. Now, what is c 1 plus c 2? c 1 plus c 2 is the total contribution made by two players put together. So, this is at their total contribution for the public good c 1 plus c 2 and depending on the value of c 1 plus c 2 some amount of public good is being produced; v i tells me what is the benefit that player i is getting out of that total amount of public good.

In particular, if c 1 plus c 2 high then, more public good will be created and v i will be high, so I can safely assume that v i is an increasing function. So, v i dashed is positive that is a reason this assumption to make. Another assumption that we shall need in understanding this problem of a public good is that v i double dashed is I am assuming that this is negative.

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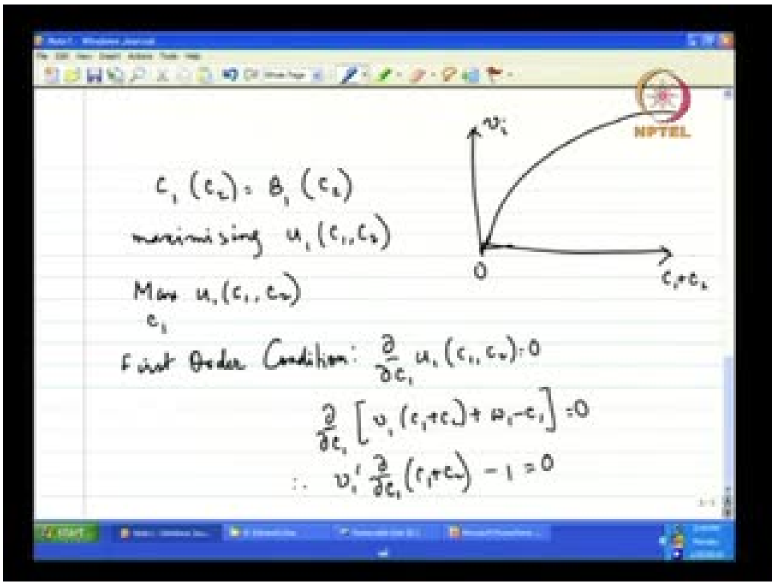

This essentially means that the v i function will be function like this, so this is v i and this is c 1 plus c 2, as such c 1 plus c 2 raises v i raises, so I have positive slope of this function. At the same time the slope goes on declining which means that the second derivative is negative that is what I have written here, but the function is positively sloped, suppose there it is starts from 0 to avoid complications.

Now, the question is if this is the setup, if there are 2 players each player can contribute towards the construction of public good and that contributions are called c 1 and c 2 and their utility functions are given preference or payoff functions are given by this then what is the equilibrium or what are the equlibria. In particular, will there be any equilibrium where people contribute towards the public good or can there be any equilibrium where people just do not contribute and no public good is constructed.

We have seen such examples before, but there was equilibrium where nobody was contributing towards the construction of public good, so that is the problem here. We want to solve this problem by using the idea of best response functions and how to do that? We know that best response function for player 1 will give me that c 1 as a function of c 2 which maximizes player 1’s payoff.

So, I have to find out that c 1 as a function of c 2, this is given by also called b 1 c 2 that maximizes u 1 because u 1 is the payoff function for player 1, so I have to maximize u 1 with respect to c 1 because on c 1 player 1 has control, on c 2 player 1 does not have any control. Max with respect to c 1, this is the familiar problem of maximization and minimization, what we need to do is to set the first derivative with respect to c 1 of this function equal to 0 that will be my first order condition. I know what is u 1? u 1 is this much, so this is what? This is v 1 c 1 plus c 2 plus w 1 minus c 1, this has to be equal to 0.

So, this is nothing but v 1 dashed I am applying the chain rule here, what is v 1 dash? v 1 dashed is the derivative of v 1 with respect to c 1 plus c 2.

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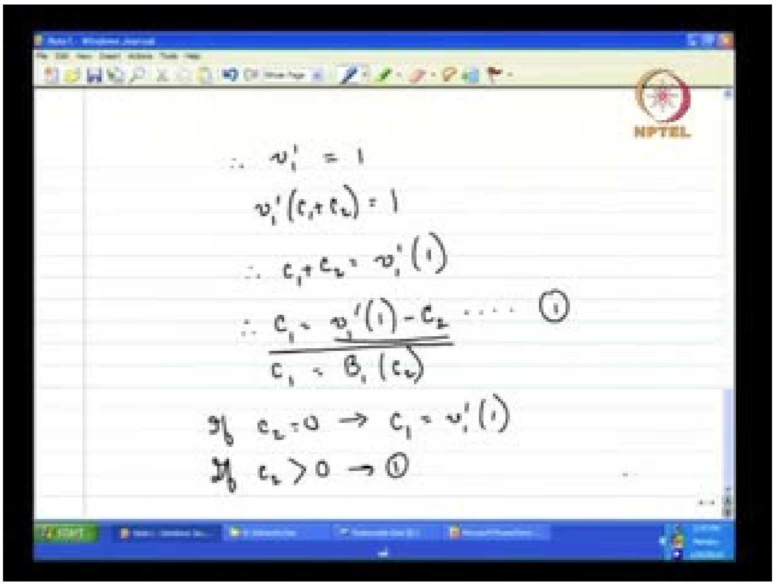

From here, what I get is v 1 dashed is equal to 1, remember what is v 1 dashed? When I think about v 1 and v 1 dashed v 1 is a function of c 1 plus c 2, so v 1 dashed is also a function of c 1 c 2. So, I can write it as v 1 dashed c 1 plus c 2 is equal to 1 and from this I can take the inverse, v 1 dashed inverse of 1. So, c 1 is equal to v 1 dashed 1 minus c 2 and this is the best response function of player 1. So, it expresses c 1 as a function of c 2, so I have c 1 as a function of c 2, this is my best response function.

Before, I going to the best response function of player 2 and try to find the equilibrium or equilibria for this problem, let us have a closer look at this one. What essentially it is saying that if c 2 changes, the change in c 1 will be in the opposite direction. If c 2 rises, this right hand side is declining that means, c 1 is declining which means that if player 2 contributes more, player 1 will contribute less, so there is an inverse relation between them, in fact the slope is minus 1.

So, 1 percent rise in c 2 will lead to 1 percent fall in c 1 how do I interpret v 1 dashed 1? If c 2 is greater than 0 then, it is given by - the relationship between c 1 and c 2 is given by - equation 1 and if c 2 raises so much that it becomes greater than v 1 dashed 1.

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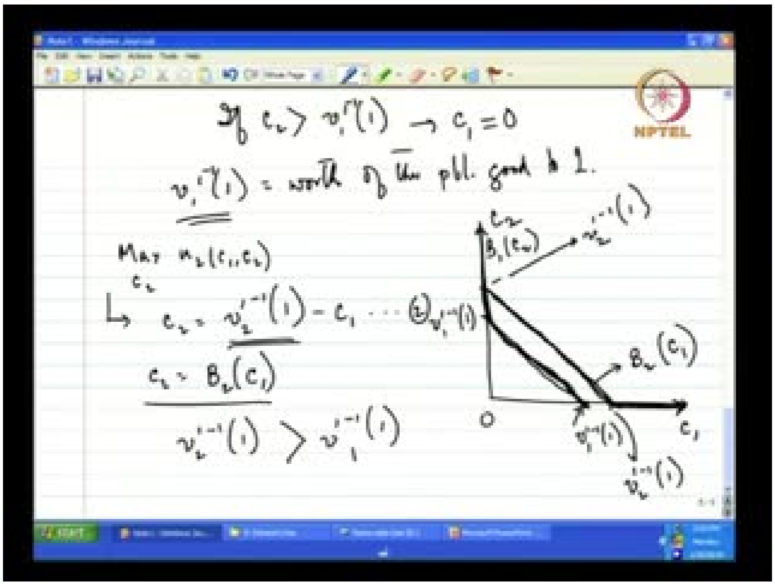

Well, if c 2 exceeds v 1 dashed 1 then equation 1 cannot give the solution for c 1, because this will tell me that this has to be negative but c 1 cannot be negative - I have this restriction before - c 1 cannot be negative, it has to be positive or 0. So, if c 2 exceeds v 1 dashed 1, what is the minimum value c 1 can take? The range of values for c 1 is such that it varies from 0 to v 1 dashed, the maximum value it can take is v 1 dash. This value of c 1 is dictated by the best response function of player 1 which means that player 1 if he is trying to maximize his payoff will never contribute more than v 1 dashed.

If the other player is not contributing at all then player 1 will contribute v 1 dashed which means, v 1 dashed is the worth of the public good to player 1, because if player 2 is not contributing at all then player 1 is alone to contribute, so that is the worth then, we are getting this v 1 dashed as a major or as an indication how much player 1 values the public good.

If he values the public good less, he would be contributing less. If he values the public good more he would have contributed more than v 1 dashed, sorry, this should be inverse, so v 1 dashed inverse of one that is what of public good for player 1.

So, how does it look in the diagram, this is c 1 and this is c 2, these are the 2 axis and how to represent this equation 1. If c 2 is 0 c 1 is v 1 dashed inverse of 1 and if c 1 is 0 then c 2 is of the same value v 1 dashed inverse of 1. So, we have a downward sloping line representing the inverse relationship. If c 2 exceeds v 1 dashed inverse then, c 1 is 0 which means, this best response function is this - thick line - it coincides with the vertical axis above v 1 dashed inverse of 1 and then, it coincides with this downwards sloping line and it stops here, so this is my b 1 c 2.

Similarly, I have to find out the best response function of player 2 which I can find by maximizing u 2 which is a function of c 1 and c 2 with respect to c 2 and we shall find the same kind of best response function which is c 2 will be a function of c 1, it will be v 2 dashed inverse of 1 minus c 1, so this will be my 2. So, this is best response function of player 1a player 2.

Now, how to represent this? To represent this I have to have some idea whether v 2 dash inverse of 1, is it higher or lower than v 1 dashed inverse of 1. Suppose, it is the case that this is higher than this, so v 2 dashed inverse of 1 is higher than v 1 dashed inverse of 1.

Suppose, this is v 2 dashed inverse of 1 and this is again v 2 dashed inverse of 1, then how does this b 2 look like, b 2 as a function of c 1? Here, in b 2 c 1, c 1 is the independent variable. So, for each value of c 1 I have to find out what is the value of c 2 which maximizes player 2's payoff and this is given by this line - am thickening it - and this part of the horizontal axis also, because if c 1 exceeds this value v w then, c 2 can take the minimum value of 0. So, we are getting this horizontal portion and then this downwards sloping part, so this is my b 2 c 1.

So, I have constructed the two best response functions, one for player 1 and one for player 2. From the diagram it is very clear that there is one and only one equilibrium, only one nash equilibrium which is at 0 and v 2 dashed inverse of 1, this is the only equilibrium that we can get.

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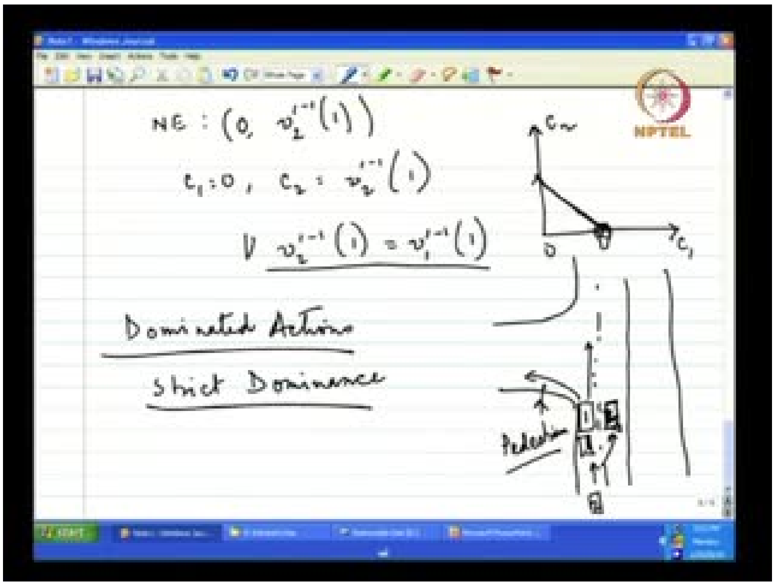

What does it mean that player 1 does not contribute the c 1 is equal to 0 and c 2 is equal to v 2 dashed inverse of 1 but this result, the player 1 is not contributing at all and player 2 is contributing. The entire thing is crucially dependent on this assumption that v 2 dashed inverse of 1 greater than v 1 dash inverse of 1. If it had been the other way round, then you can draw a diagram and see, in that case it will be player 1 who will contribute the entire amount and player 2 will not contribute anything.

Why is this happening? This is happening precisely because, if we take this case that is v 2 dashed inverse of 1 greater than v 1 dashed inverse of the 1 that means, the worth of the public good to player 2 is more than the worth of the public good to player 1. If you remember we have just couple of minutes ago we have shown that v 1 dashed inverse of 1 is worth of the public good to player 1.

Similarly, we can show that v 2 dashed inverse of 1 will be the worth of the public good to player 2. So, if the worth is more for player 2, this model is telling us that it will be player 2 who will contribute the entire amount and the person who does not value that public good that much will not contribute anything, he will contribute 0.

However, these two cases that is, one is v 2 dashed inverse of 1 greater than v 1 dashed inverse of 1 and the other way that is, v 1 dash inverse of 1 greater than v 2 dash inverse of 1, do not exhaust all the possibilities because we might have instead of greater than, we may have this case also.

This is also possible in that case; there will be infinite number of Nash equilibria will be there, because what will happen? The 2 best response functions will coincide on this downwards sloping portion and there are infinite numbers of such points here, so all of them are Nash equilibrium. Here, we are having situations where both the players are contributing for the public good but obviously, this is surely coincidence that my worth for the public good is same as your worth for the public good.

In exceptional situations only both the players will contribute and both the players may contribute. Why I am saying this because, in this case also there might be equilibrium at this corner point, where only one player is contributing but of course, there is a possibility that both the players will contribute, this was one application of best response functions for public good games.

Let us now introduce some new concepts or new ideas in game theory which are widely used. This is the idea of Dominated Actions, to explain the idea let me give you an illustration what it means. First, we shall talk about strict dominance, the idea is the following. Suppose, there is a road and on the road there is an intersection - to make it simple - let us say that this is a t intersection, so the road looks like this. Suppose, it is a two lane road and in this intersection there is a red light obviously and a car from here is coming and approaching to the intersection.

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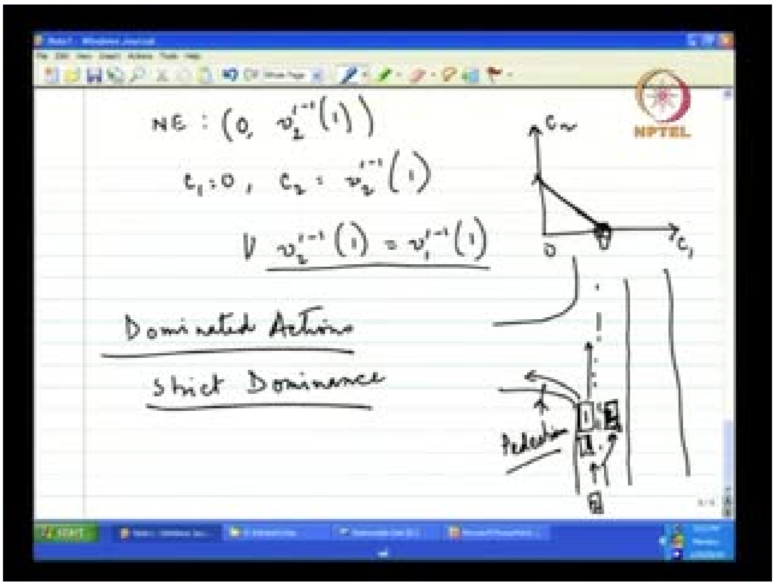

In the intersection right now there is a red light and since, there is a red light one car is waiting here - waiting for the green light to come on. Now, for this car it can choose to either go and wait behind this car here or it can go this way and wait here. So, there are two actions here, what are the preferences? Any car does not like to be behind another car, because if you are behind another car it restricts your movement. So, ideally I like to have no car in front of me and it so happens that if this car which is waiting here at the intersection - let us call this as the first car and this is the second car.

If there is green light then this car can go either this way that is, it takes a turn to the left or it can decide to move ahead. So, player 2 has two actions either to wait here or to wait here; player 1 also has two actions, it can either turn to the left or go straight at. However, if he decides to turn to the left, it has to wait for some time because there is a pedestrian here who is going to cross the road.

So, since this pedestrian is going to cross this road, this car if it decides to turn to the left we have to wait for some time which means, the player 2 - that is car 2 - if it decides to wait behind the first car it will also have to wait there when the green light comes. Now here, waiting on the right lane is better in any case irrespective of the action of player 1 than waiting behind player 1 because, if player 1 decides to go straight ahead, first car decides to go straight ahead what are the payoffs? If player 2 was behind player 1, it still remains behind player 1 which is worse than waiting here on the right lane, because if it had waited on the right lane they would not have been any car in front of it.

So, in this case player 2 is better off parking his car on the right lane. On the other hand, if player 1 decides to take the left turn player 2 is definitely better off waiting on the right lane, because if player 1 decides to turn to the right then it will have to wait and which will make player 2 also wait here because it is behind player 1. So, it means that no matter what player 1 does player 2 is worse off by waiting behind player 1 is instead of pulling up on this right lane.

So, pulling up on the right lane will be strictly dominating the action of pulling up behind player 1, we call this strict domination. The idea is that no matter what the other player does one action is better than the other action and the action which is better will be called a strictly dominating action and the action which is worse will be called as strictly dominated action.

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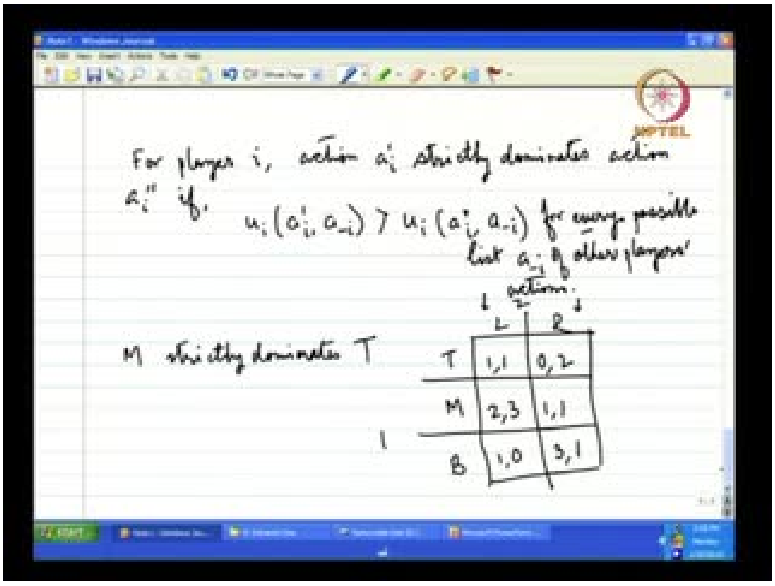

Let me give formal definition, for player i action a i dashed strictly dominates action, a i doubled dashed, if both these a i dashed and a i doubled dashed are (( )) action player i's action. This is the definition that if there is one action a i dashed and another action a i double dashed. If it so happens that no matter what the other players are doing here, this is coming for every possible list a naught i of other players action. So, I consider every possible set of actions or vector of actions by other players and for each possible vector of actions of other player, it is so that by taking the action a i dashed I get better payoff than taking the action a i double dashed. In that case I call a i dashed strictly dominating over a i double dashed.

Let me show this in terms of payoff matrix, so that it becomes more clear let us consider the following payoff of matrix. Suppose, this is player 2 and this is player 1 left and right top, middle, bottom and the payoff numbers are following. Now, let us look at this game from the point of view of player 1, from the point of view of player 1 I can see that no matter whether player 2 takes the action L or takes the action R, M is always better than t because if player 2 takes the action L, M is better than T because 2 is greater than 1.

If player two takes the action R, 1 is greater than 0 that is why M is again better than T, so I say that M for player 1 strictly dominates T. However, it is also clear from this that if there are any two actions it is not necessary that what strictly dominates over the other. Take the case of M and B, if player 2 takes the action L, M is better than B because 2 is greater than 1. If player 2 takes the action R, B is better than M because 3 is greater than 1, so it is not necessary that is strictly dominated actions or strictly dominating actions will exist.

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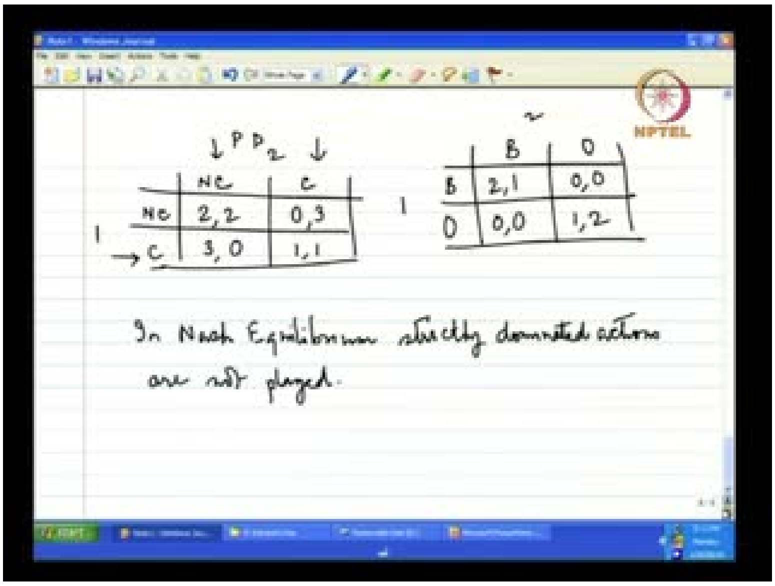

For example, if you remember the games that we have discussed before, I take the case of battle of sexes. In battle of sexes there is no strictly dominated action, this was the structure, here there is no strictly dominating or strictly dominated action, because if player 2 plays B for player 1 it is better to play B, but if player 2 plays O B no longer remains the best action, it becomes O, but if you remember the prisoner's dilemma game there where as strictly dominated actions - this is prisoner's dilemma.

This was the game here if player 2 plays N C then, it is better for player 1 to play C. If player 2 plays C, again it is better for player 1 to play C, so does not matter what player 2 does, it is best for him to play C. A similar logic applies for player 2 also does not matter what player 1 does, it is best for player 2 to play C.

If you play something else he guesses strictly less than what he is getting at C. This illustration of prisoner's dilemma should give us inkling that if we have a Nash equilibrium then, in that Nash equilibrium strictly dominated actions are never going to be played. Because in this prisoner's dilemma, we have found that C N C both are strictly dominating actions and N C N C are strictly dominated actions and this N C N C strictly dominated actions are not being played in the Nash equilibrium, because in the

Nash equilibrium only C N C are being played. So, let me write it down, in Nash equilibrium strictly dominated actions and the logic is very simple I can prove it in 23 steps.

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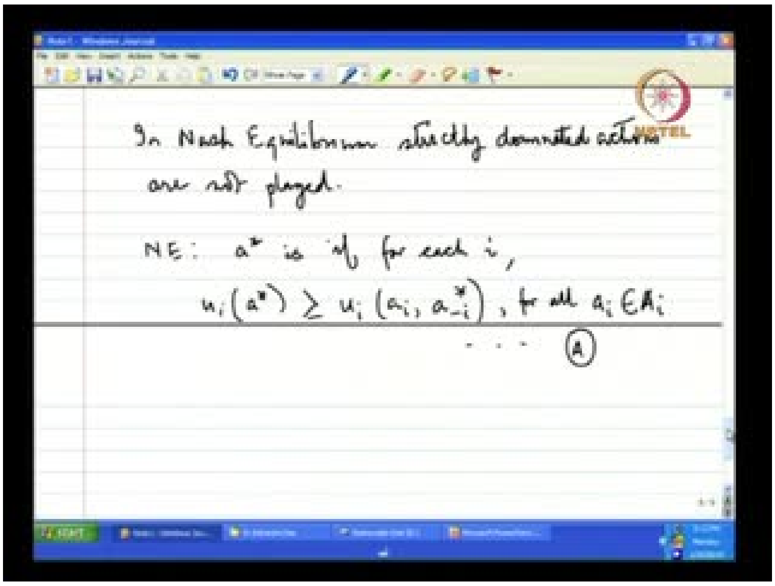

Firstly, what happens in Nash equilibrium? For each i must have this u i, this must happen in Nash equilibrium - let us call this condition A.

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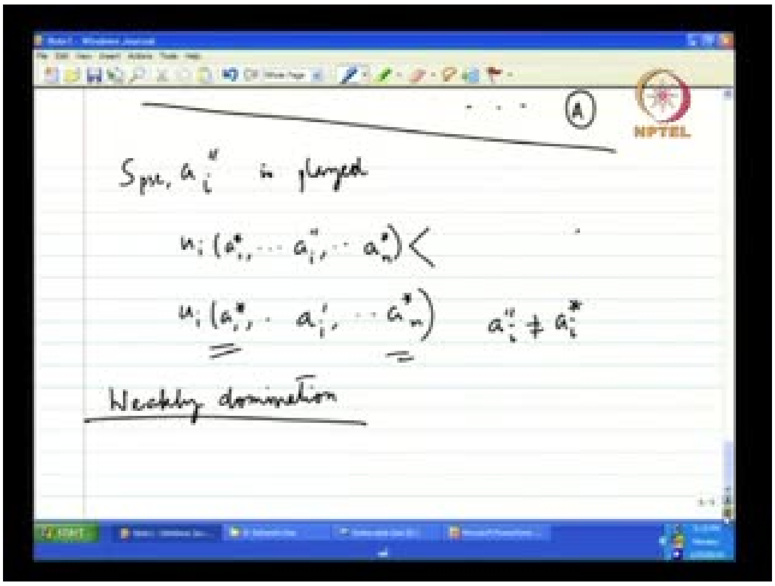

Suppose, there is an action a i double dashed which is strictly dominated and suppose, it is being played in Nash equilibrium then there is a contradiction and we shall see that there is a contradiction. Suppose, a i double dash is played then, what does player i get? Keeping the actions of other players constant, he get this much. Now, if this is what he gets suppose (( )) switches over to a i dashed that is the dominating action.

Now, I know that this must be true by the definition of dominating action which means that this condition is going to be violated, if I consider that a i double dashed is equal to a i star. So, if i instead of a i star I put a i double dash then this condition a is going to be violated which means that I am never going to be playing the strictly dominated action in Nash equilibrium. From that action I can always deviate to a i dashed and that will improve my payoff because if you remember in strictly dominated action for every possible list of action it is better to play the strictly dominating action.

So, this must be true for the star actions as well and hence, the a i double dashed is not going to be played in Nash equilibrium. Now, come to the other concept of domination which is called weakly domination and to motivate the idea let me go back to that example of cars and intersection.

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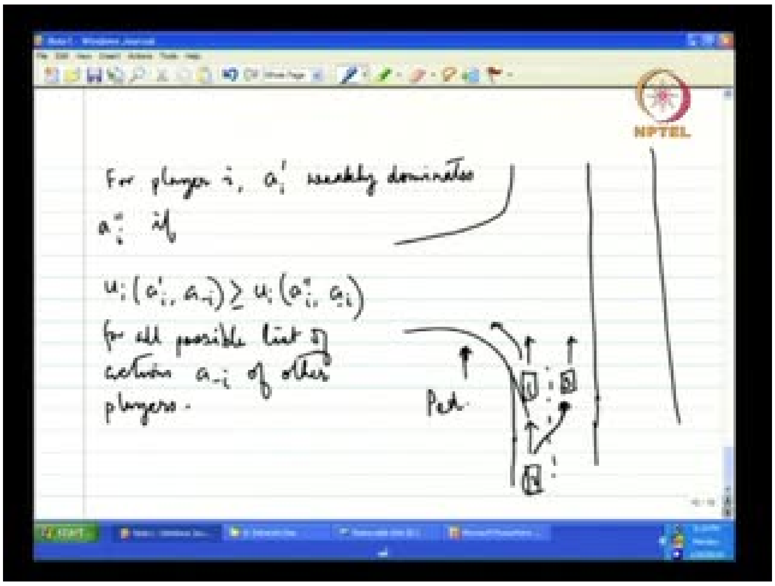

Once again, we have this traffic intersection and there is a car waiting here at the red light car 1 and here, car 2 is coming it can either park it is car here or it can pull up here these are the two actions where should it go.

In case of weakly dominated action if I have to show the example, in this case there is another car waiting here - let us call it car number 3. Now, if there is a car number 3 waiting on the right lane as well and a car 1 which is waiting at the left lane both are waiting for in other traffic intersection because there is a red light then which is the better action for player 2.

Here, it is not very clear whether 1 is definitely better than the other because it can so happen that player or the car number 1 also go straight ahead and car number 3 also go straight ahead in that case player 2 is indifferent he can pull up either behind car number 1 or behind car number 3 it is all the same.

However, it may so happen that player 1 that is car number 1 turns to the left and like before there is a pedestrian here who is going to cross the road in which case, this car 1 will have to wait for some time and wait for this pedestrian to cross and only then it will go to the left which means, there is a loss of time for car number 2, which means that depending on the actions of other players, one action can be strictly better and there might be cases where both these actions are same for player 2 - they are giving in the same payoff. In this case we say that this action, action of going and pulling up behind car number 3 is weakly dominating the action of waiting behind player 1 that is car number 1.

So, weakly dominating is a situation where it may happen that for some action of other players these two actions are giving me the same payoff which is case when both this car 1 and 3 are going straight ahead in that case player 2 is indifferent, but there is at least one case where pulling behind car number 3 is better that is the case where player 1 decides to turn to the left.

One important thing is that there is no indicator for player 1 - I mean - there is no indicator for player 1’s car, so player 2 cannot make out before hand whether car number 1 is going to go straight ahead or going to turn to the left.

If I have to formalize this it will look like the following, for player i a i dashed - action a i dashed weakly dominates action a i double dashed. So, for every possible action profile of other players it must be the case that the weakly dominating action is giving player i at least same or more then the payoff he is getting from the weakly dominated action but there is more.

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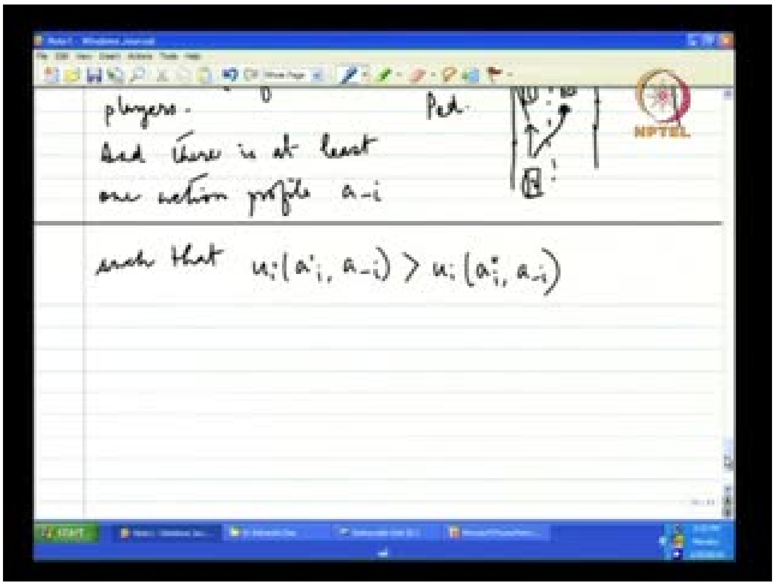

At least one action profile which means, there must be at least one action profile by the other players - coming from the other players - such that the weakly dominating action that is a i dashed should be giving player i strictly more pay off than the action profile of a i double dashed that is the weakly dominated action, so this second thing is also important.

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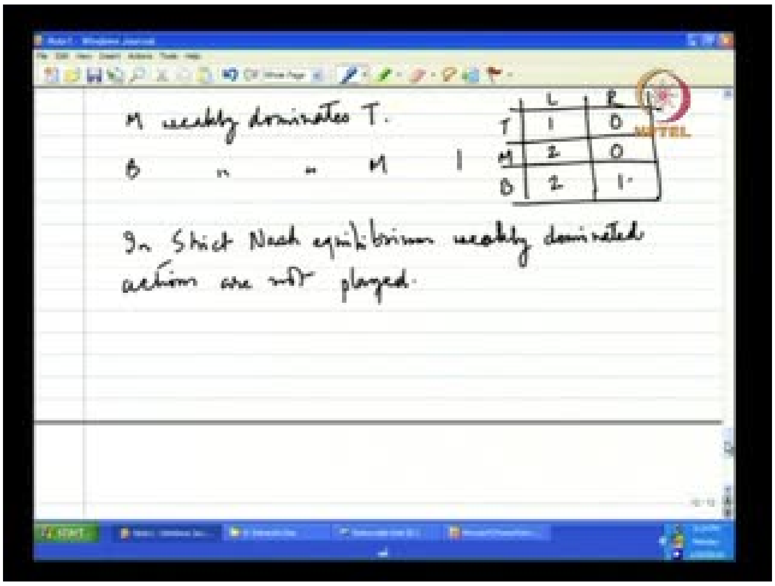

Examples: let me give you one example here to save effort and time, let me write down the payoffs of player 1 only. So, these numbers 1, 0, 2, 0, 2, 1 are just payoffs which player 1 gets not player 2's payoffs, suppose I have to compare between T and M.

Now, I can see that if player 2 plays R, player 1 is indifferent between T and M however, if player 2 plays L then, M is better than T. So, here I have this satisfaction of weakly dominating and dominated actions and I say that M weakly dominates T, because M is better than T if L is played. M is giving the player 1 same amount as T is giving if R is played. What about M and B? Again, I see that B weakly dominates M because if L is played M and B are same they are giving player 1 same payoff if R is played B is giving player 1 better payoff.

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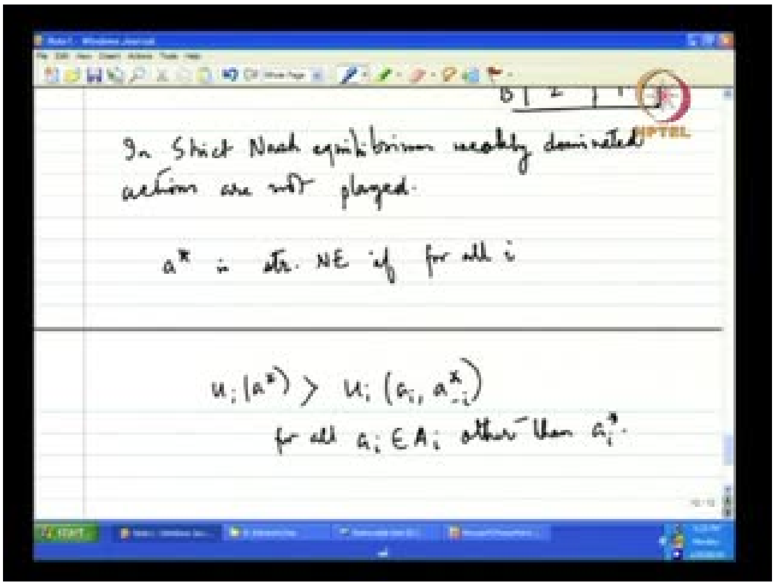

So, this is weakly dominating and weakly dominated actions, R weakly dominated actions played in Nash equilibrium. Well, there are two results here, one is in strict Nash equilibrium weakly dominated actions are not played and how do I know that, if you remember what was strict Nash equilibrium? Strict Nash equilibrium was such that suppose a star is strictly Nash equilibrium. If for all i what must happen is that u i a star is strictly greater than this; this was the definition of strict Nash equilibrium, that for each player if he plays some other action other than the Nash equilibrium action his payoff should go down.

Now, from this one can conclude that in strict Nash equilibrium weakly dominated actions are not going to be played because if you are playing weakly dominated actions suppose, player i is playing a weakly dominated actions a i double dashed then, there must be some action which is weakly dominating this action which is a i dashed

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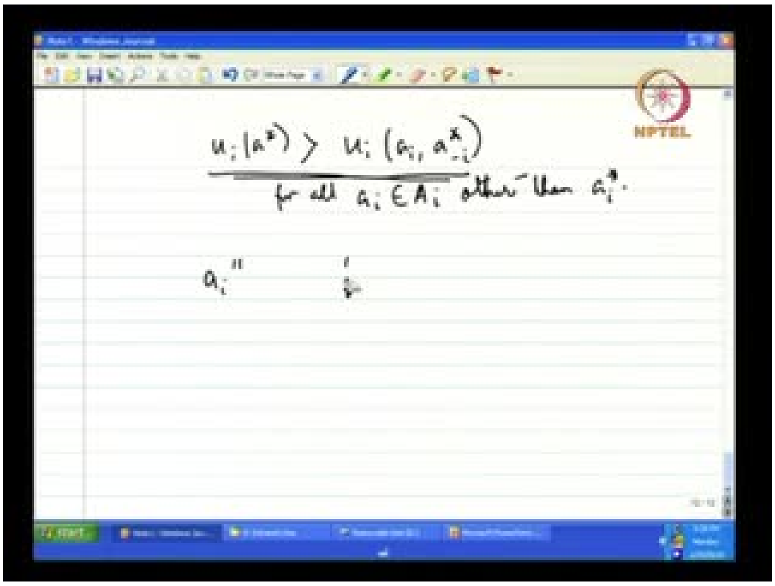

Now, if player i changes his action from a i double dash to a i dashed then, how is this payoff going to be affected? Either it is going to remain constant depending on the action profile of other players or it will go up because a i dashed is strictly dominating a i double dashed. So, if shifts from the double dashed to the dashed, it can either go up or it will remain constant, but whatever it is not going to satisfy this condition because this condition demands that if you shift then your payoff must go down.

Here, it is remaining constant or going up, so any action which is weakly dominated is never going to be played in strict Nash equilibrium. Can weakly dominated actions be played in non-strict Nash equilibrium? The answer is yes.

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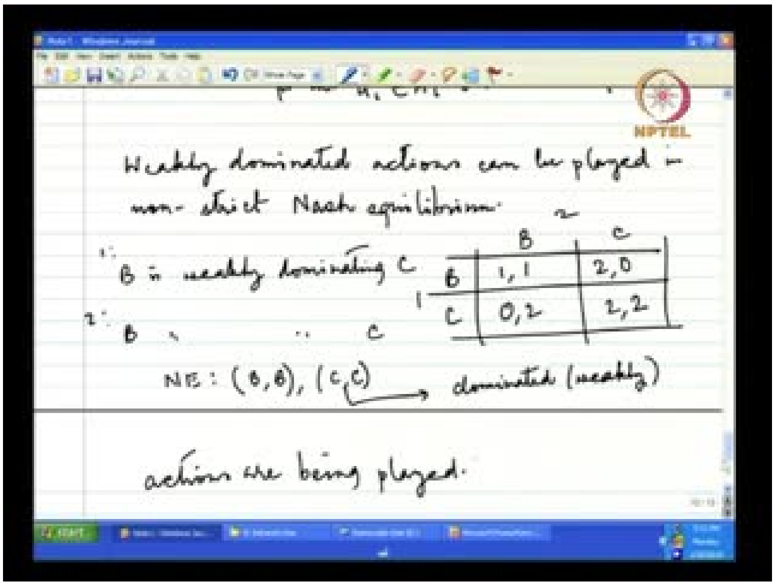

So, weakly dominated actions can be played in non-strict Nash equilibrium and the idea is not very difficult to grasp. In non-strict Nash equilibrium if people deviate there is unilateral deviation of their actions, it may happen that their payoff is remaining constant only thing we need is that it should not go up, it can go down or it may remains constant in non-strict Nash equilibrium.

In weakly dominated actions also if you shift to a weakly dominating action, from the weakly dominated action the payoff not necessarily goes up, it may also remain constant. So, there is a bit of consistency between weakly dominated actions and a Nash equilibrium action in a non-strict Nash equilibrium. So, that is why it can be possible that a weakly dominated action is being played in non-strict Nash equilibrium, I shall give you one example where it happens. So, this is the example this is the payoff matrix and there are two players and two actions for each player.

Now, which is the weakly dominating action here, I can see clearly that for player 1 for example, B is weakly dominating C because if player 2 plays B it is better to play B, if player 2 plays C player 1 is indifferent between B and C, which means the B is weakly dominating C. The same thing will happen for player 2 also; B is weakly dominating C because if player 1 is playing B then, it is better for player 2 to play B if player 1 is playing C player 2 is indifferent, so for player 2 also B is weakly dominating C.

However, what are the Nash equilibria in this game? There are two Nash equilibria one is B B and other is C C, you can check it out that if the other player is playing B, no player is going to shift from B because that is going to reduce the payoff.

What about C C? Well, if the other player is playing C and if you deviate then, your payoff is not going to go down, it is going to remain constant but that is included in the definition of Nash equilibrium. It may happen that you deviate and your payoff remain constant, so that invalidates the Nash equilibrium.

So, C C is an equilibrium where you have dominated that is weakly dominated actions is being played. So, these are some of the properties of strict Nash equilibrium and its relationship with weakly dominated or strongly dominated actions. Before we end this lecture here, let me just recapitulate which what we have discussed in this lecture.

We have discussed the game of public goods where people can take infinite number of actions; people can decide to contribute more towards the public good or less. We have found out what is the Nash equilibrium in that case depending on a certain particular payoff function. We have seen that in most of the cases there will be a single Nash equilibrium, in that Nash equilibrium a single player will contribute the entire amount the other player will not contribute at all.

Now, shear case of coincidence it may happen there are infinite number of Nash equilibria in which case both the players contribute that is what we have done. Then, we have introduced the concept of dominating actions, we have talked about dominating actions, strict dominance and weak dominance and what is the relationship between this dominated actions and strict Nash equilibrium. We have seen that in strict Nash equilibrium strictly dominated actions are not played. In non-strict Nash equilibrium weakly dominated actions might be played and in Nash equilibrium in general strictly dominated actions are not played, thank you.

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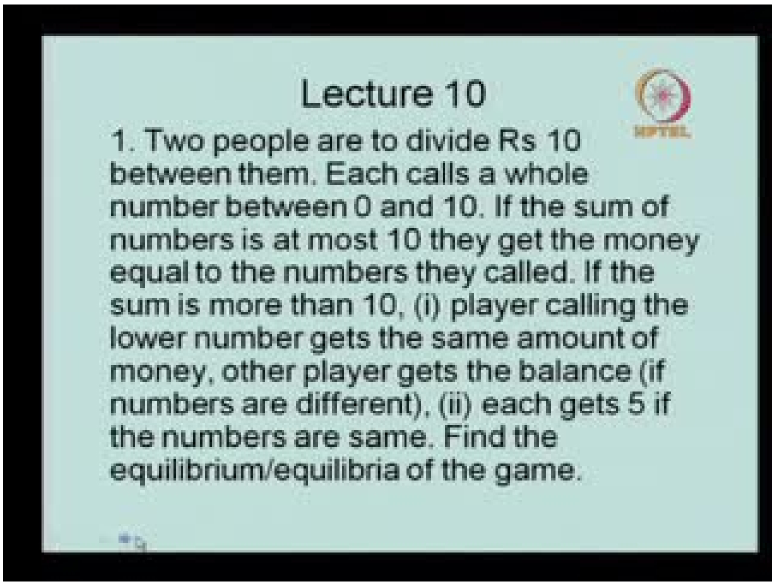

Two people are to divide 10 rupees between them. Each calls a whole number between 0 and 10. If the sum of the numbers is at most 10 they get the money equal to the numbers they called. If the sum is more than 10 firstly, player calling the lower number gets the same amount of the money, other player gets the balance if the numbers are different each gets 5 if the number are same, find the equilibrium or equilibria of the game. So, this question basically asked us to use the idea of best response functions to find out the equilibrium or equilibria of the game.

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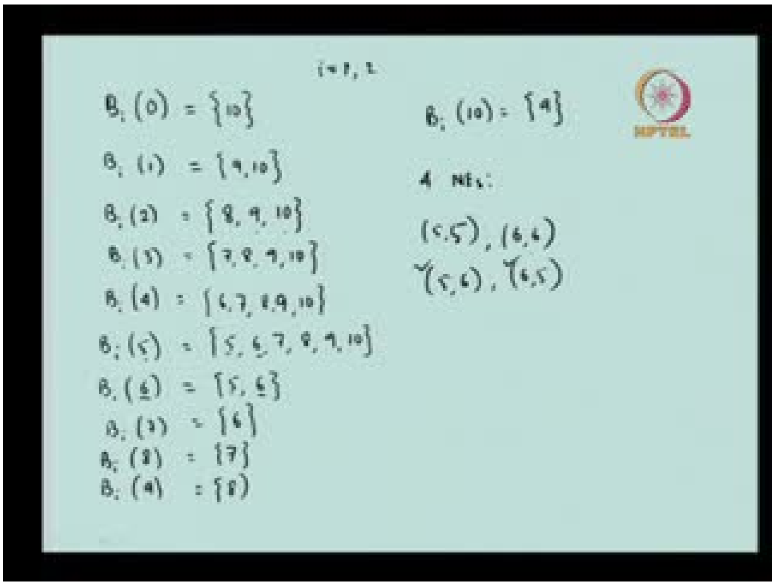

So, let us try to find out how the best response function of player i look like, i could be 1 or 2. Suppose, the other player is calling 0, what should be the best response for this player? I claim that the best response should be to call 10; i could be 1, i could be 2. Why because suppose, I am player 1 if player 2 has called 0 then, if I called 10 I get 10, there cannot be anything more than this, this is the maximum that I can get.

Now, for example the other player is calling 1 then, what is my best response? Here, if the other player is calling 1 I should call either 9 or 10 both are my best responses in other case of what I am getting is 9. Similarly, if the other player is calling 2, the maximum that I can get is 8 which I will get if I call it, if I call 9 again I will get 8, if call 10 also I will get 8, because in this last two cases the sum is exceeding 10. If the sum is exceeding 10 the rule is, that the person was calling the role number will get the money that is the other player will get 2 rupees, I will get the rest which is 8.

So, likewise it will go on if the other player is calling 3, I should call 7 or 8 or 9 or 10 in either case I am getting 7 rupees. So, this where it will go on if for example the other player is calling 5, let us write it for 4, for 4 these are the best responses. If the other player is calling 5, I can call 5 because here the maximum I can expect to get is 5, if I call more than 5 does not matter, I get 5 rupees.

Now, suppose the other player is calling 6, in this case however this rule we were following will not hold. If the other player is calling 6 the maximum that I can expect to get is 5 rupees which will happen if I call 5 I will get 5 or if I call 6 then, there are numbers are equal, so we shall divide this 10 rupees means, I will get 5 rupees.

If the other guy is calling 7 I will undercut him I will call 6 and I will get 6, so here it will be 7 then 8 and 9. Now, these best responses are true for each of these 2 players I could be 1, I could be 2 and looking at these best responses it is found that there are only 4 Nash equilibria, which are these 4 such that the best responses are matching with each other.

These are 5, 5 why 5, 5? Because, if the first player is calling 5 the second player will call 5 and if the second player calls 5 the first player will again call 5 because I could be 1, I could be 2.

Similarly, 6 6 is also a Nash equilibrium because you can see 6 is a best response to 6 for each of these two players. In these two cases, there are 2 Nash equilibria, but I also claim that these two are also Nash equilibrium that if 5 is called 6 is the best response, if 6 is called 5 is a best response, so for the same reason this is Nash equilibrium, thank you.