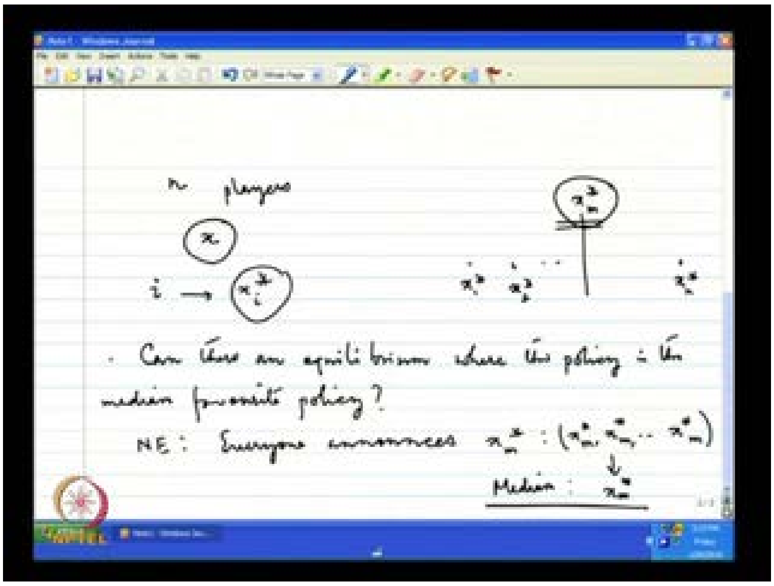
Game Theory and Economics
Prof. Dr. Debarshi Das
Department of Humanities and Social Sciences
Indian Institute of Technology, Guwahati

Module No. # 02
Strategic Games and Nash Equilibrium
Lecture No. # 10
Symmetric Game and Symmetric Equilibrium

Welcome to this 10 th lecture; the last lecture of this module - of this course - is called game theory and economics. Before we start, let us recapitulate what we have done in the previous class or previous lecture.

(Refer Slide Time: 01:00)



We have been doing some applications of weak domination. The last lecture is deliberated on one particular case of collective decision making, the story is the following. Suppose there are n numbers of players, so n players, who are trying to take a common decision, a common policy. This policy can be the expenditure of this community; it can be represented by a number. Suppose that number is x, now this community is trying to take this decision as to what x to choose. Every one of these n players has a favorite policy, suppose for player i, favorite policy of that player is x i star.

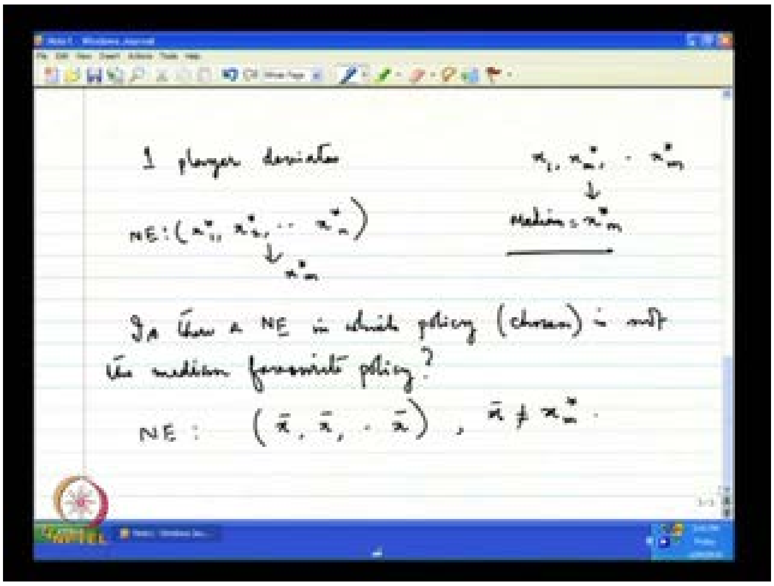
Now, the question is, how this community arrives at this common policy x? The mechanism here is the following that everyone is asked to announce a particular number, the median of all these numbers is taken to be the policy of the community. That is the mechanism through which the design players or this community arrives at the common decision or collective decision. The result of this exercise is that if everybody is asked to give his or her number, then giving x i star weakly dominates over any other action.

In the sense that any player can announce any number, but if that person announces his actual - the real favorite policy, then that action is weakly better or weakly dominant over other possible numbers that he can announce. That is the proof that we have done in the previous lecture. Related to this exercise, we can take others short exercises. For example, can there be equilibrium, where the policy is the median favorite policy? In other words, in this group, everyone has a favorite policy, suppose x 1 star, x 2 star etcetera and the last suppose is x n star. This x 1 star, x 2 star, etcetera, they have been arranged in an ascending order. In the middle, there is 1 x m star, which is the middle most x or middle most x star and this is what we are calling as the median favorite policy.

Now, question is, can there be an equilibrium, where in the equilibrium, the choice of the community or the collective decision of the community is exactly this x m star? Yes, there can be such equilibrium. One is that one Nash equilibrium, example of one Nash equilibrium is, suppose everyone announces x m star, it is like this, the actions of the following; x m star, x m star, dot, dot, dot, x m star.

Now, in this case, everyone is announcing this x m star, so the median of all this numbers is in fact x m star, this is the median. This will be the chosen policy of the community, question is, is this Nash equilibrium? To check whether is this Nash equilibrium, what I have to do? What I have to check is that if someone deviates, can he or she be better off.

(Refer Slide Time: 05:30)



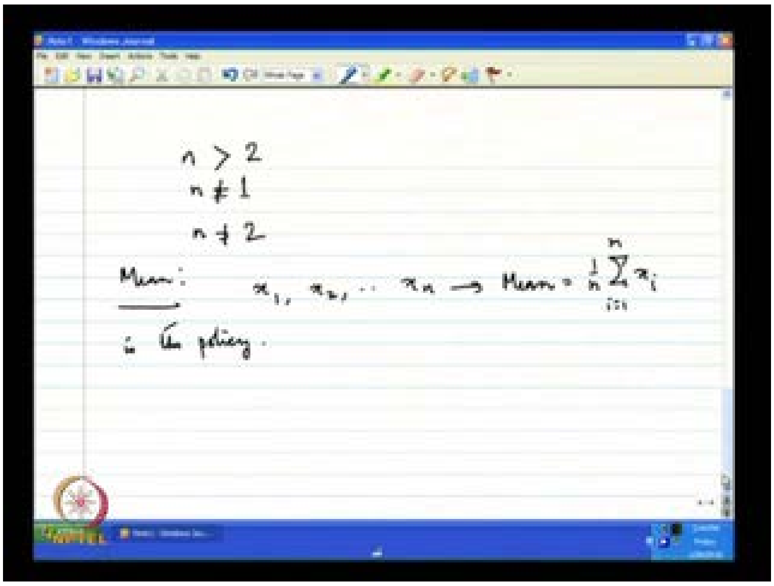
Now, for example, player 1 deviates, now even if he deviates, he cannot change the policy, because there are other players in the group. If he is taking some other, he is announcing something else, suppose x 1, but the other things remain as they are. The median of all this numbers is again x m star. So, by announcing some other number, other than x m star, no player can change the outcome, the policy of the group. If he cannot change the policy of the group, his payoff cannot be better, it is same as it was when he was announcing x m star.

So by changing my behavior, changing my action, I am not better off, I am as happy as when I was announcing x m star; this is therefore Nash equilibrium. One can think of other in Nash equilibrium also at the policy of the community, is in fact x m star. For example, suppose everyone is announcing his or her favorite policy, this is x 1 star, x 2 star, x n star. In this case also, you see the median is again x m star, no players can be better off by changing his or her policy, because as we have just seen that announcing your favorite policy is in fact the weakly dominant action, it dominates over all other actions.

So, by changing my policy, I cannot be better off. So this is again Nash equilibrium, the answer is again yes. I can give the following example; suppose everyone is announcing a particular number, which are not x m star, then that is Nash equilibrium. For example, suppose everyone is announcing x bar, suppose where x bar is not equal to x m star, then this is Nash equilibrium. Why this is Nash equilibrium? Like as you have seen, now that by changing the action, no player can change the outcome of this game, because other players are taking the action x bar.

If other players are taking the action x bar, I do something else, the median remains at x bar. So, if the median remains at x bar, my payoff remains the same as I was getting by announcing x bar. Therefore, this is again Nash equilibrium.

(Refer Slide Time: 09:55)

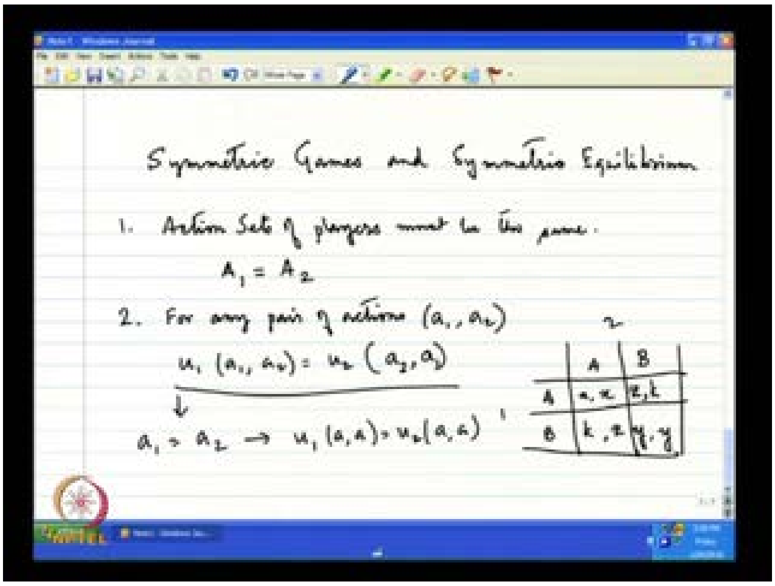


Now, notice here the fact that I am changing my action, other players are taking the same action and therefore the median remains the same, is crucially dependent on the fact that n - the number of players is greater than 2, how do I know that n is greater than 2? Firstly, I definitely know that n is not equal to 1, because had it been a one person game, it would not have been a game. The game has to be a case, where there is interaction between players.

So, n is 1 is not a game, n cannot be 2 either, because if n is equal to 2, then n is even, which we have seen it is not the case; in our case, n is odd. Then, three people are announcing the same number, if someone announces something else the median remains the same, so that is crucial.

Now, this was one case of collective decision making, where people were making their announcement, their x i's. The median of these x i's were taken to be the policy. What about mean? Suppose people make their announcement x 1, x 2, x n and the mean of them that is, this is the policy. So, we have a separate mechanism, now which is not the mechanism that we had before, in this case, is it still true that people will find announcing their favorite policy and weakly dominates over announcing something else? So that is more or less about this section called dominant and dominated actions.

(Refer Slide Time: 12:08)



The next topic that we shall cover today is symmetric games and symmetric equilibrium. Now, before we introduce this topic, let us remember how we have visualized games as such. We have visualized, suppose we are talking about two player game, we have visualized two player games as the following. That there are two populations - two distinct populations; in a particular play of the game, one person from each group is randomly picked up. These two people randomly picked up play a game with each other. Since, these people have some previous knowledge about how the game is being played, they have a belief regarding other people's action, the other players' action from that belief they play their best action - best possible action.

So, that is the story; that is how we visualize this idea of games. In symmetric games, it is not the case that people come from two different populations, so they come from the single population. Now, if they come from the single population, then the specification has to be modified accordingly. In particular, there will be two changes. One, the action sets must be the same, so the players were playing with each other - we shall assume that there are two players, these two players were playing with each other, they have the same action set. Suppose A 1 is the action set of the first player, it must be equal to A 2. Numbers of actions are at the same, the name of the actions is also the same. Secondly, remember, they come from the same population, so it must happen that if I take a particular action, other player take some other action, then the payoff that I get should be the same.

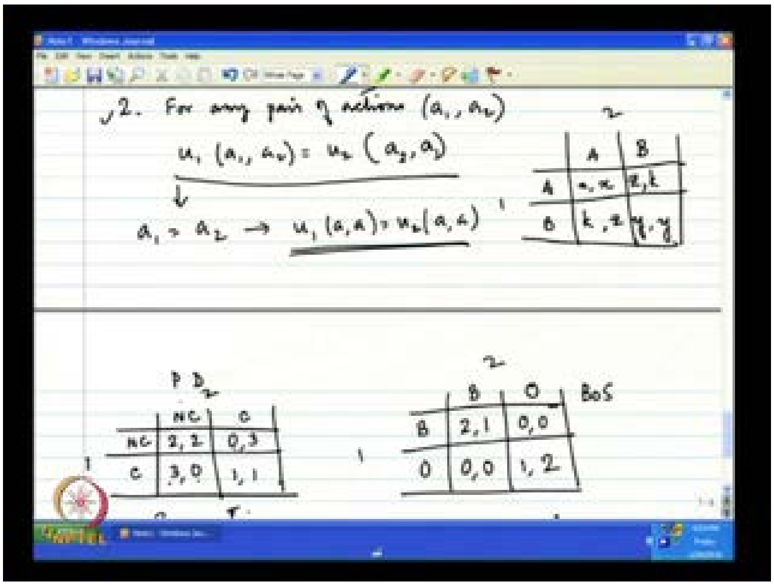
If the other player takes my action, I take his action, what is the payoff that he is getting out of it; these two payoffs must be the same. So, it is written as the following. On the left hand side, I have u 1, u 1, a 1, a 2, which is the payoff player 1 gets, if player 1 takes action a 1 and player 2 takes action a 2. Now, since the players are indistinguishable, they come from the same population, they have the same preferences, same actions that payoff must be the same as player 2 gets.

Player 2 gets - if player 2 plays a 1, player 1 plays a 2, so that is what it must also happen. This second characteristic is basically emphasizing the fact that the identity of players do not matter, the combination of the actions if they are the same, then the payoffs to the two players must also be the same. Now, from this, we get the following. That suppose a 1 is equal to a 2, which means that player 1 and player 2 are taking the same action, then it must happen that (Refer Slide Time: 17:00). That means if the actions of the two players are the same, then the payoffs that they are getting must also be the same.

I can combine these two characteristics in the following way. Suppose this is action A, this is action B; player 2 must also have the same actions. Suppose this is x, then this also must be x, because here the actions are same. If this is y, then this also must be y. Now, if suppose this is z, if player 1 is taking action A, player 2 is taking action B, then player 1 is getting set that payoff should accrue to player 2, if he takes action A and player 2 takes action B.

Similarly, this number also must be the same number, the payoff that player 1 gets by playing B when player 2 is playing A must be same as the payoff that player 2 gets when he plays B and one plays A. So, this is the general structure of a symmetric game, not a general structure, but an example. Why I say that this is not a general structure, because there can be more than two actions for each player.

(Refer Slide Time: 19:06)



Now, from this, it is obvious that in many of the games that we have basically studied, we have basically studied symmetric games. For example, Prisoner’s dilemma, if you remember the structure, so this is the structure of a Prisoner’s dilemma game. We can easily verify that this is basically symmetric game, because when NC NC is played, the payoffs that players are getting are equal.

If c c is being played, again the same thing is happening, the payoffs that the players are getting one each, they are again equal. When the actions are different, for example, player 1 is playing in c, player 2 is playing in c, then player 1 is getting 0 that 0 must accrue to player 2, when player 2 is playing NC - this and player 1 is playing c. This number is equal to this number; this number is equal to this number.

This is Prisoner’s dilemma, it is a symmetric game. What is not a symmetric game that we have studied? Well, there were many. This implies that I can think of, is suppose the battle of sexes going to boxing match, going to opera, this was a case of a game, which is not a symmetric game, a non-symmetric game; the battle of sexes game. Player 1 has two actions, B and O, going to boxing match, going to opera. Likewise, player 2 also has two actions B and O, the payoffs were 2, 1, 0, 0, 1, 2; these were the payoffs.

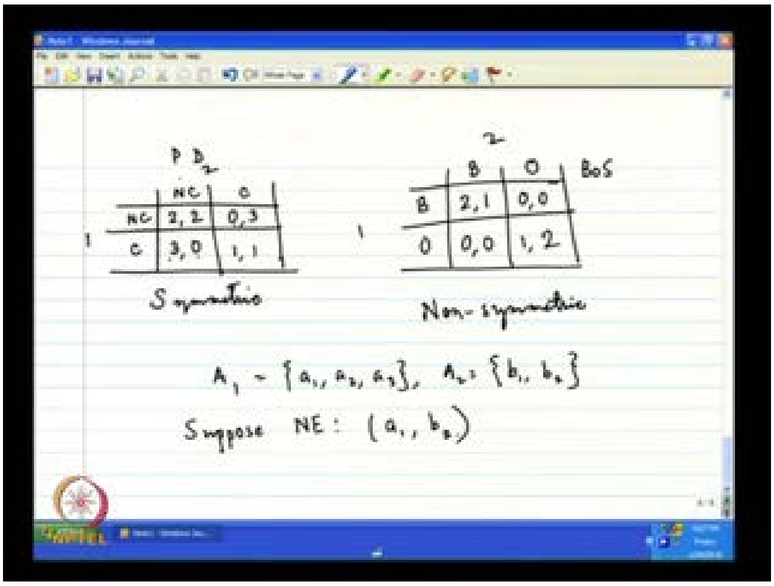
Now, notice that as far as comparing B O and O B is concerned; that is when player 1 is playing boxing match, player 2 is playing opera, the players are getting 0 0. If I compare that with O B that requirement is satisfied, the second requirement is satisfied. First requirement is also satisfied, because they have the same action pairs. But, if I consider this B B and O O, then this requirement is not satisfied that u 1 of a a is equal to u 2 of a a that is not been satisfied, so that is why it is not a symmetric game. That is more or less a definition of a symmetric game.

The symmetric games are cases where the players are coming from the same population. Now, if players are coming from the same population, then what is the notion of equilibrium that there can be? Remember, when we talk about Nash equilibrium, the idea of Nash equilibrium that we have is that it is a vector of actions, which is a steady state. In the sense that given that the other players are playing their actions, I am not going to deviate and play something else; I am going to stick to my equilibrium action.

Now, if it is the case that these players are coming from two different sets of populations, then it is easy to distinguish between them. For example, the usual in the usual game, there were two populations and people are coming from these two populations, two players were chosen. Now, since, I know that I belong to population one, the other player is belonging to population 2, the action that population 2 people have been taking before that action is going to be repeated now. Given, with that action, I play my best action, so that is the idea that I have believed that what this population two people have been doing, they will continue to do.

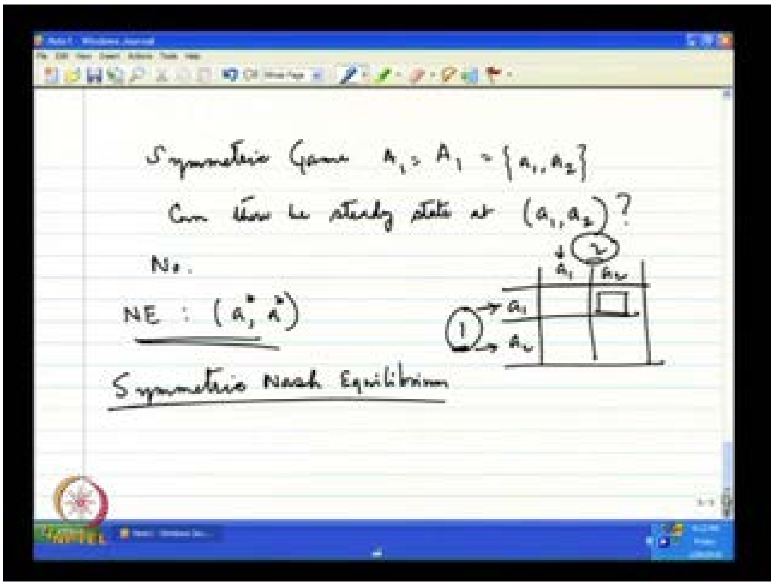
Similarly, if I belong to population 2, I will have a belief that whatever action population one people have been taking, they will take the same action now. Depending on that belief, I will take my action. These two actions will again support each other. But, if it is the case that suppose they come from the same population, then there will be some problems here, as per as the equilibria are concerned.

(Refer Slide Time: 24:24)



For example, let us take the case of two populations first. Suppose A 1 that is the action set of player 1 and suppose Nash equilibrium is a 1 b 2, which means that if b 2 is split, a 1 is the best action for player 1. If a 1 is split, b 2 is the best action for the player 2, but if it is the case that instead of b 1, b 2, you see here, I have taken a 1, a 2, a 3, b 1 and b 2.

(Refer Slide Time: 25:18)



a and b are two different letters and I can clearly distinguish between them, but if I have a symmetric game, then I cannot say that these actions are a 1, a 2, a 3 and b 1 and b 2, then the action sets are same.

Let us suppose the action set is A 1 and suppose A 2, to make it simple and then can there be steady state at a 1 a 2? What is meant by can there be steady state at a 1 a 2? In the sense that can a 1 a 2 be Nash equilibrium in a steady state sense? The answer is no. Because a 1 and a 2 are two different actions, these two players are coming from the same population. If they are coming from the same population, so it will look like this.

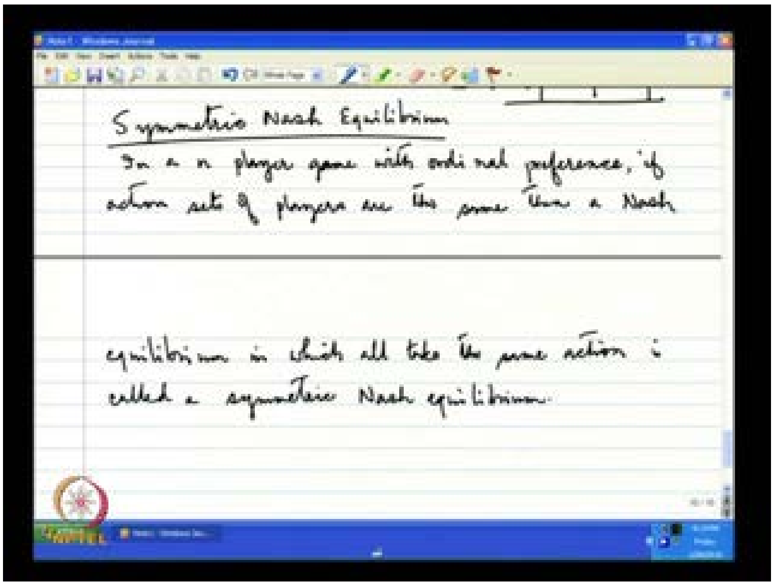
This is the case we are trying to figure out, whether a 1, a 2 - this combination is Nash equilibrium. Now, the problem is that a 1 and a 2 are such that these are the actions by player 1 and 2; these 1 and 2 are coming from the same population. Now, if they are coming from the same population, they cannot distinguish what action the other player will take. For example, when 1 goes to play the game, he does not know whether 2 is going to play a 2. It may happen that 2 is going to play a 1, thinking that 1 is going to play a 2.

In that case, it will be foolish for 1 to play a 1, it will be best for him to play a 2. On the other hand, if 2 indeed play a 2, it will be best for 1 to play a 1. So, there is a problem of identity here, because they are coming from the same population, their identities are not distinguishable. If the identity is not distinguishable, then in a steady state, they cannot take separate actions. If they take separate actions, then that will not be sustainable, because they will be confusion as to what action a particular player will take, in the next play of the game, like here.

Similarly, it will happen for player 2 also. If a 1, a 2 has been played in that previous game, then player 2 does not really know whether one will play - one will think himself as player 1 or one will think himself as player 2. If one thinks himself as player 1, he should have played a 1. If one thinks himself as player 2, he should play a 2, but here I am just using 1 and 2, for the sake of convenience of drawing this matrix. In reality, players cannot be pigeon hold into player 1 and player 2, because they belong to the same population.

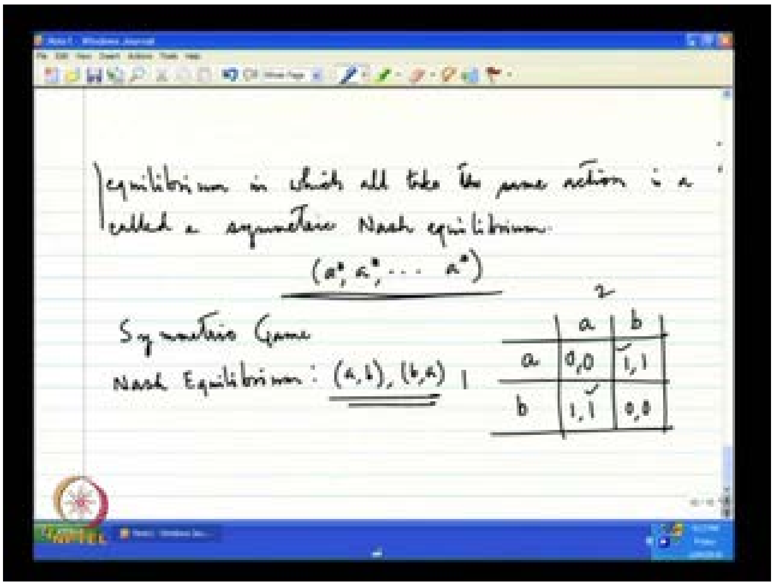
That is why in case of symmetric games, such cases of different actions by different players cannot be a steady state. The only case of steady state, there can be is that the actions are same. If the actions are same, it does not matter as long as the other player is playing the same action. This will be true for every player, every player will think that the other player will be going to play the same action a, so I should also play a and this becomes a Nash equilibrium; so that is it. But, this kind of Nash equilibrium, where each player is playing the same action in the Nash equilibrium is often called symmetric Nash equilibrium with some additional qualification.

(Refer Slide Time: 30:27)



So, the qualification is that in a n player game with ordinal preference, if action sets of the players are the same, then a Nash equilibrium in which all take the same action is called a symmetric Nash equilibrium.

(Refer Slide Time: 32:00)



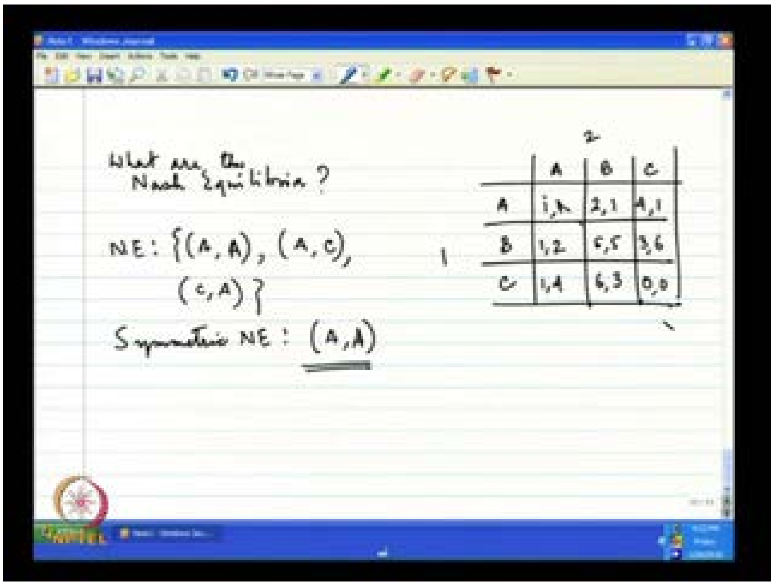
So, it looks like this a star, a star, dot, dot, dot, a star, so this is the general form of a symmetric Nash equilibrium, but notice when I have defined this symmetric Nash equilibrium, it is not necessary that all the symmetric Nash equilibria occur only in symmetric games. Because, symmetric games also satisfy the second criteria, we have talked about the second criteria.

In this definition, I have said nothing about this second criterion, so a game can be non-symmetric, but it can have a symmetric Nash equilibrium; this is one thing. At the same time, it is also true that any symmetric game may have no symmetric Nash equilibrium; it may have Nash equilibrium, which is not a symmetric Nash equilibrium.

So that Nash equilibrium in that symmetric game cannot be a steady state kind of pair of actions. For example, let me take one game of the following form, so he has the actions a b, similarly for him. Is this game, is it a symmetric game? The answer is yes, it is a symmetric game.

How do I know because, the action sets are same along this diagonal cells that is a, a, b, b, the payoffs of the players are matching with each other. Thirdly, the payoff that player 1 gets by playing a b that is one, is the payoff the player 2 gets when b a is played, this one. This number is equal to this number, this number is equal to this number and we are having a symmetric game. However, which are the Nash equilibria here? There are two Nash equilibria here, one is a b, other is b a. One can easily see that none of them is a symmetric Nash equilibrium, in symmetric Nash equilibrium, the actions of the players must be the same actions.

(Refer Slide Time: 35:15)



Let me do one exercise, the exercise is the following. Suppose I have a game as follows; this is the game, question is what the Nash equilibria of the game are? Secondly, which of the equilibria - if any correspond to a steady state, if the game models sphere wise interaction between the members of a single population? So, if there is any Nash equilibrium in this game, if there is more than one, then of those Nash equilibria, can any Nash equilibrium be thought of as a steady state outcome of players, who belong to the same population?

The firstly, which are the Nash equilibria? By examining this game, it is easy to figure out that A A is a Nash equilibrium, because if player 2 is taking the action A, then player 1 by taking some other action other than A cannot be better off. Similarly, for player 2, A is a Nash equilibrium. Which of the other Nash equilibria? If there is any other Nash equilibria, A C is a candidate, because if player 2 is playing C, player 1 is deviating by playing B, he is getting 3 by C, he is getting 0.

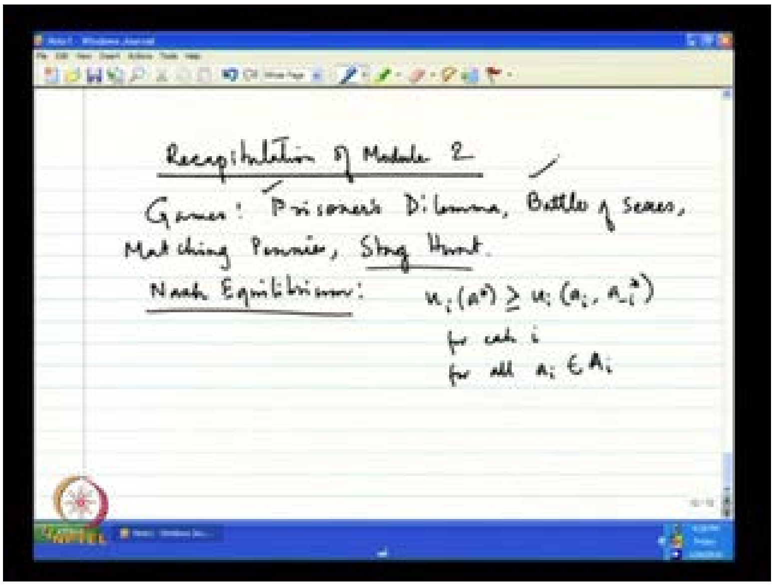
Whereas, by playing A, he is getting 4. From player 2's point of view, if player 1 is playing A, by playing A, player 2 gets 1, by playing B, player 2 gets 1, which is not greater than 1. So, A A is a Nash equilibrium, similarly C A is also a Nash equilibrium, we can check.

In fact this game is a symmetric game, we can see it very clearly, because along this diagonal, along this A, A, B, B, C, C diagonal the payoffs of the two players are the same. For the elements which are not on the diagonal or the cells which are not on the diagonal, the payoff of a player 1 from playing B A is equal to the payoff that player 2 gets if A B is split. This applies to all these cells, so these are the Nash equilibria. Now, which of them are symmetric? Obviously, the symmetric Nash equilibrium is one which is A A, because in symmetric Nash equilibrium, we know that the actions of the players must be the same, they must match with each other.

That is what it is been asked that though they have not use the word symmetric Nash equilibrium, they have said that which of the equilibria, if any correspond to a steady state, if the game models pair wise interaction between the members of a single population. We have already figured out that if it has to be a steady state between the members of a single population that steady state has to be symmetric equilibrium that is how it is defined.

In a symmetric equilibrium the actions must be the same, so A A is the equilibrium. A C and C A are not symmetric equilibrium and therefore is not a steady state. So that is more or less this module of Nash equilibrium.

(Refer Slide Time: 40:40)



What we have discussed in this module let me go over it; I am trying to be brief, so this is a recapitulation of this module 2. We have started out by giving the definition of a game; we have seen that there are three elements of a game. One is the set of players; two is the action set of each player and third is the preferences of each player. So that has to be mentioned; that has to be specified. Secondly, then we have given many examples of games, we have talked about the games called Prisoner's dilemma. We have talked about battle of sexes matching pennies, stag hunt. Though these names are quite exotic and each of them has a story behind it, but it is not as such, this particular story that interest us to games. Basically, these modules - this Prisoner's dilemma or battle of sexes, they are generic situations, in the sense that they can be applied to a lot of real life cases. After showing these different sorts of usual games that are studied, we have then defined what Nash equilibrium is.

We have said that Nash equilibrium is an action profile, where given the actions of other players, no player can deviate and be better off. What is known as unilateral deviation is unprofitable. A player can be as happy as he was in Nash equilibrium, if he deviates, but he can never be better off. So it is like this; this must be satisfied.

After defining Nash equilibrium, we have tried to figure out what is the Nash equilibrium of the games that we have discussed. For example, in Prisoner's dilemma, we have seen that the Nash equilibrium is at the case where both the prisoners are confessing. That we have said is Nash equilibria, is a situation, which is not a very efficient situation. In the sense, both of them could have not confessed and both of them could have earned better payoff. Which basically means, if we have an a Nash equilibrium, it is not necessarily an efficient situation, both of them - both the players can be better off by doing something else, but that something else may not be a Nash equilibrium.

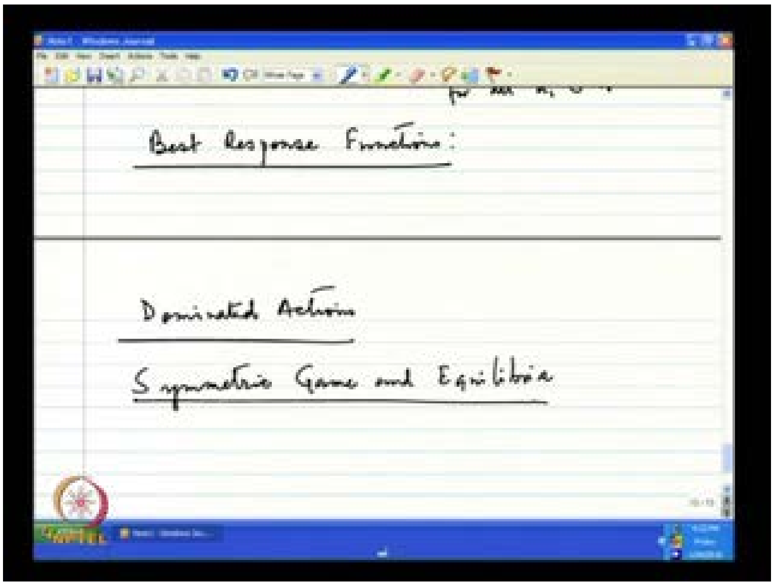
So, Prisoner's dilemma, basically a generic case for that the equilibrium is not efficient. Stag hunt if I talk about; here, we have a situation where there are two Nash equilibriums. Nash equilibrium, it is not necessary that a game only has one Nash equilibrium, there might be two Nash equilibria. One of the Nash equilibria could be better for both the players than the other, so by the very fact that we are talking about equilibrium, again we are not reaching the best situation for the players.

One can have very efficient equilibrium and one can have equilibrium which is not efficient that is what we saw in stag hunt. Battle of sexes tells us the story of the players, see that they can cooperate and be better off. For example, both of them taking the same action is good for both of them, but they differ whether which combination of actions is better.

It might happen that one of the players likes the action A A, over the action B B, whereas the second player is liking B B over the action A A; that is the case of battle of sexes. Again, in battle of sexes also, there can be multiple Nash equilibrium. Then, we talked about matching pennies, where there is no Nash equilibrium at all, in the sense that we have defined Nash equilibrium that Nash equilibrium may not exist in many cases.

Then, we said that in each of these games that we have discussed, one characteristic is that the action set of the players are consisted of finite number of actions, in particular, we had only talked about 2 or 3 actions for each player. But that is not a general case obviously, it may happen that actions are infinite in number, there can be continuous actions. In those cases, the technique of just looking at an action profile try to see whether people can deviate be better off may not work.

(Refer Slide Time: 47:31)



For continuous actions, one can, one should have a better way of finding out Nash equilibrium that is what we have done by taking the help of best response functions. We have defined what is known a best response function. Best response function of a player is the function relationship between what action maximizes his payoff, given the actions of other players. So, this is how best response functions are defined.

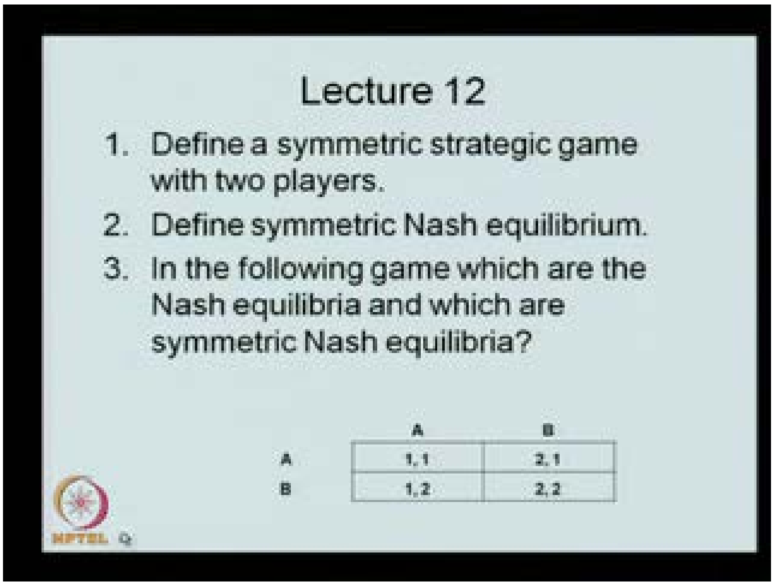
We have shown that Nash equilibrium is a situation where the actions belong to the best response functions of each player, so if I have that result, then it becomes easy to find

Nash equilibrium or Nash equilibria. If I have two players, I construct their best response functions and I can have a continuous action that is not a problem.

By constructing these two best response functions, I get two equations, I find out what is the intersection point or points. At the intersection point or points I have Nash equilibrium. Then, we have talked about dominated actions, we have said that there are two sorts of domination, one is weak domination and the other is the strong domination. There are some properties that they satisfy; for example, in Nash equilibrium, strong dominated actions are not played. If we talk about weak dominated action - weakly dominated actions, well they are played in Nash equilibrium, but those Nash equilibria where they are played should not be strict Nash equilibria.

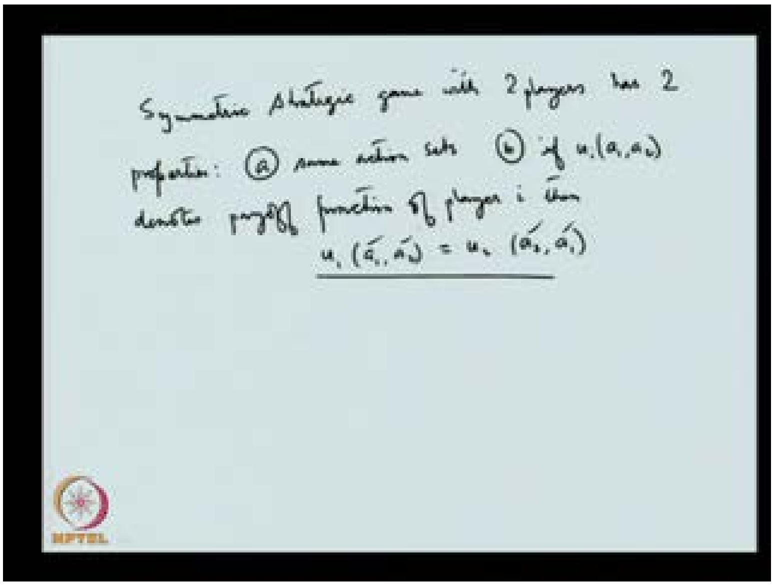
In strict Nash equilibria, weakly dominated actions are not played. Finally, we talked about symmetric games and equilibria, which is the case where people come from the single population. In general, when we talk about games and Nash equilibria, the players that we considered, they come from different population groups. They are randomly picked up from different population groups and they play their games. But, in symmetric games, the players come from the single population group, which means that their preferences are similar, their action sets are similar. We have also looked at those games how they are defined. What is known as symmetric Nash equilibria? that also we have defined. In the next lecture, we shall be starting with module 3, see you there; thank you.

(Refer Slide Time: 50:33)



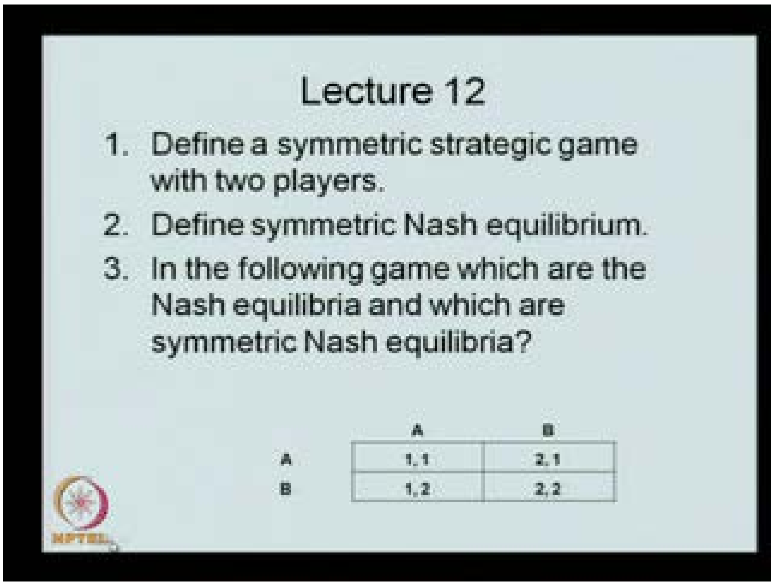
Define a symmetric strategic game with two players.

(Refer Slide Time: 50:38)

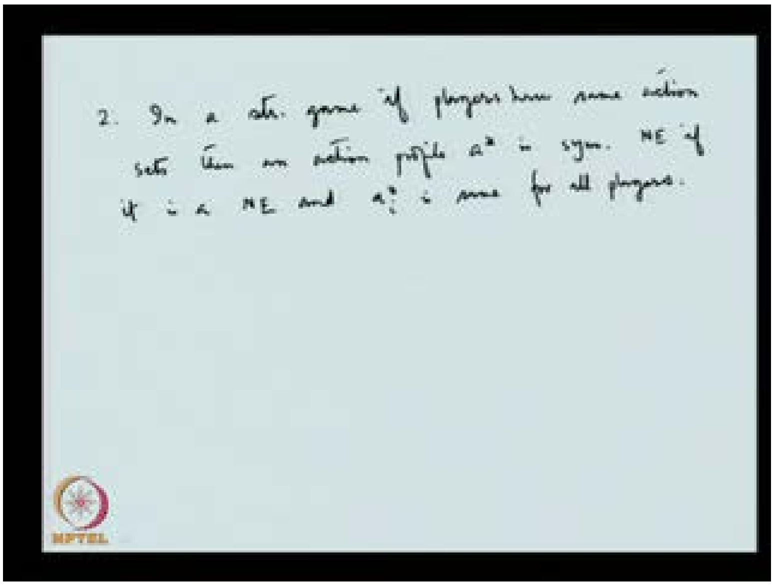


Two properties it has to satisfy; a, action sets of players must be same; same action sets for players b, this must also hold. That for player 1, if he plays a 1 and other player plays a 2 that will give him the payoff, which will be the payoff two players they will get, if player 2 plays a 1 and player 1 plays a 2.

(Refer Slide Time: 52:14)

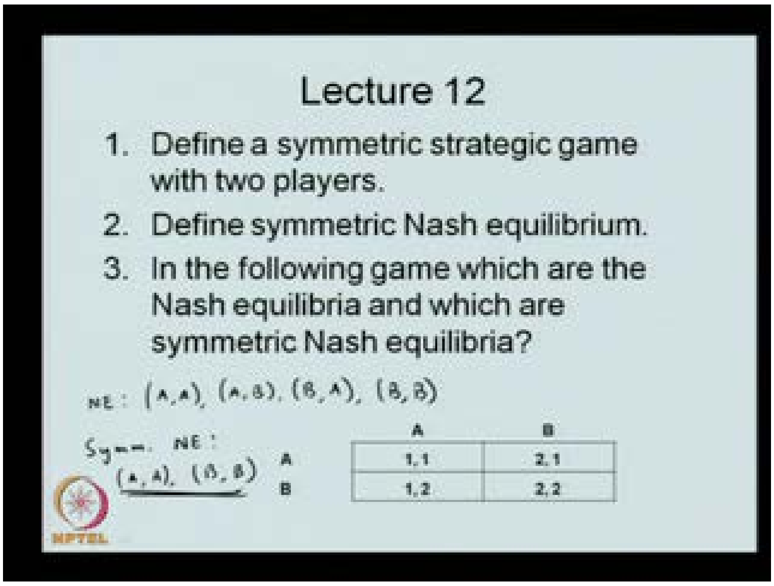


(Refer Slide Time: 52:17)



Second question, define symmetric Nash equilibrium. Symmetric Nash equilibrium, in a strategic game, if players have same action sets, then an action profile is symmetric Nash equilibrium, if it is Nash equilibrium and a i star is same for all players. For all players, the action that they are taking is the same action. Remember, the action sets are same, so the actions should be common to everyone. Now, if they are taking this exactly same action, if that action profile is Nash equilibrium, then we call that Nash equilibrium action profile as a symmetric Nash equilibrium.

(Refer Slide Time: 53:45)



Third question; in the following game, which are the Nash equlibria and which are the symmetric Nash equilibria? Now, we can see from this game itself, what are the Nash equilibria, in this game. In fact, all action profiles here are Nash equilibrium action profiles.

For example, A, A, A, B, B, A, B, B; all four are Nash equilibria, because from each of the profiles, if any player deviates and take some other action that player cannot be better off; but, what about the symmetric Nash equilibria? Well, by applying these criterions that you have said there are only two symmetric Nash equilibria, this and this; thank you.