

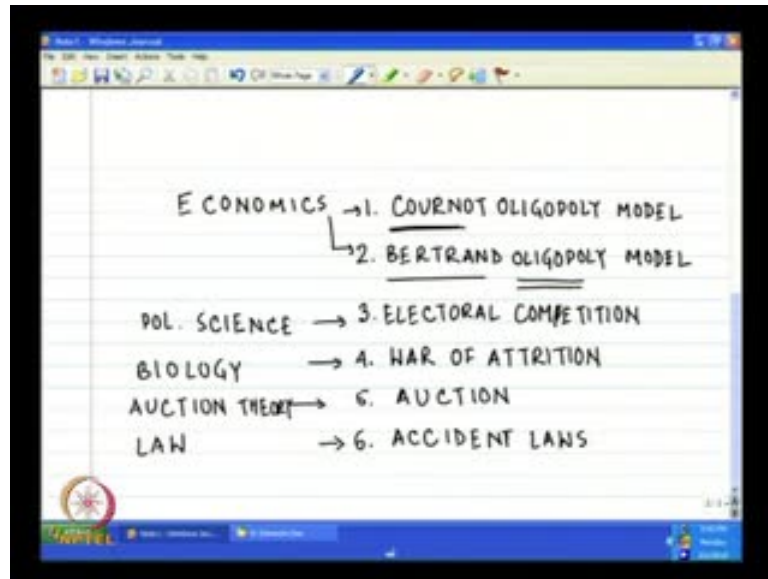
Game Theory and Economics
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Module No. # 03
Illustrations of Nash Equilibrium
Lecture No. # 01
Cournot Model of Oligopoly

Welcome, to the first lecture of module three of Game Theory and Economics. What we have done so far, let me recapitulate, what we have done. We have introduced the idea of Game Theory; what it means and what are the basic assumption of this whole premise of Game Theory. And, we have also in module two discussed, what is known as Nash equilibrium and what the assumptions, conditions of Nash equilibrium are. We have also talked about equilibria like symmetric equilibria, when they are applicable, symmetric Nash equilibrium.

We have also talked about things like dominated actions, strictly domination, quickly domination and what are the examples of such domination. And, we have also discussed various examples how to find out the Nash equilibrium in a particular game. And, we have talked about best response functions also. We have seen that best response functions can be useful in finding out Nash equilibria, especially when the number of actions is more than one. And, in fact if it is close to infinity, then we do not have any other way, but to depend on best response functions to find out the Nash equilibria of a particular game.

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What you propose to do in module three? In this module, is that we are going to discuss various applications of Game Theory. In particular, we shall be talking about six applications. The first applications will be from the domain of Economics. And, these are: number one is Cournot Oligopoly Model, second is Bertrand Oligopoly Model. So, these will be from the subject of Economics. Then, we shall be talking about what are known as Electoral Competition. So, this is from Political Science. And, of course in Economics also, there is a sub discipline of Economics called Political Economy. There also, these modules of Electoral Competition are used. Then, we shall talk about what is known as War of Attrition. We shall discuss Auctions and finally, Accident Laws.

So, this is from Economics, this is from Economics, this is from Political Science, this is mainly from Biology, this is Auction Theory. Auction Theory is studied as a part of Economics itself and finally, this is law.

So, what we are trying to do here is, to look into different subjects like Economics, Political Science, Biology and etcetera. And, try to see whether the idea of Nash equilibria gives us any hint or does it give us any fruitful prediction, as to what outcome will occur in each of these six cases. They can be more than what one outcome, of course.

So, just, to briefly talk about these six cases. In first and two, we are going to talk about Oligopoly markets. What are Oligopoly markets in Economics? When you study

markets, we divide markets generally depending on the number of producers. We assume that a number of producers are same as a number of sellers.

So, there is no difference between retail seller or whole sale seller and a producer. We assume that, just the producers are producing their goods and selling it in the market, without any intermediate or separate layer of sellers, as such.

So, in Economics, the markets are divided according to the number of sellers. In Monopoly, there is a single seller. In perfect competition, that is, at the other extreme there are infinite sellers. Or, you can think about very large number of sellers that is called perfect competition. In between, we have this case of oligopoly.

So, oligopoly is the case, where the number of sellers is not one; more than one. But, it is not a very large number either. So, we can say it is a few number of sellers are there. And, they compete within themselves. But, the question is, how they compete. And, depending on the answer to that question, how they compete, we can divide the markets into two. Oligopoly markets can be divided into two kinds of market. One is the Cournot market or Cournot model, which we shall be studying here in Cournot oligopoly model.

And, the second is Bertrand market, which we shall also study. The difference is that, in Cournot market or in Cournot model, the producers are deciding on their production level. They are not deciding on the price. Price is determined in the market. So, price is outside the control of the sellers.

On the other hand, Bertrand market or in Bertrand model, the producers are deciding at what price they are going to sell their goods, but how much they are going to produce, that is going to be decided by the market condition.

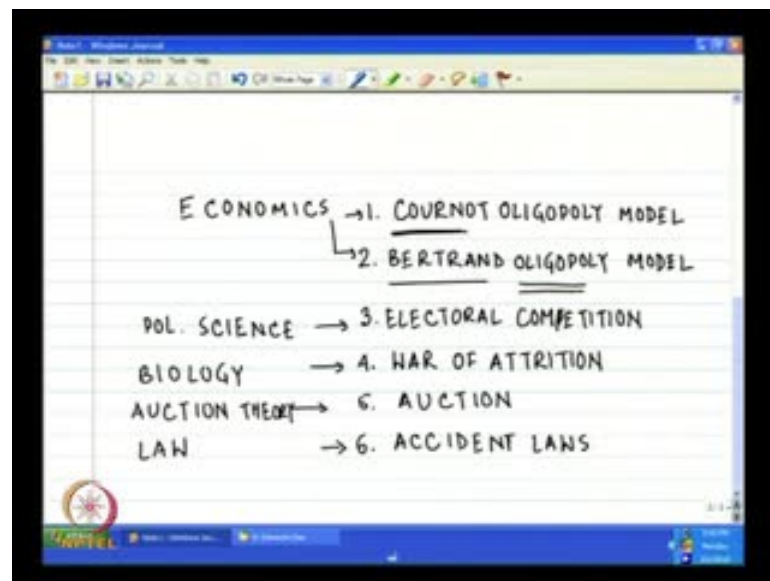
So, in first case, it is the quantity of goods that the producers are producing. That is in their control. In the second case, it is the price, which is in their control of the producers. But, both are the cases where the number of producers is few. Few, is a weak term. But, nevertheless we do not have any precise definition as far as the number of producers in the Oligopoly market is concerned.

Now, in the third topic, that is Electoral Competition, we are going to talk about how the candidates in a vote, in an election, decide on their agenda depending on the preferences

of the voters. And, can we predict what agenda are, they will pick up and what will be the equilibrium? In the fourth case, we are going to talk about War of Attrition.

Here, suppose there are two competing parties, who are competing over some common resource or common targets, for example, it might be two hunters targeting the same prey. Now, these two hunters will, then like to inflict damages on each other because only one of them will get the prey.

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So, this kind of competition between players was trying to get his common resource or common prey will be studied in War of Attrition. We can think of this case as not hunters and prey, but we can think of this as two companies, who are fighting over a very valuable employee, for example. And, they are trying to employ that particular employee by outsmarting the other player.

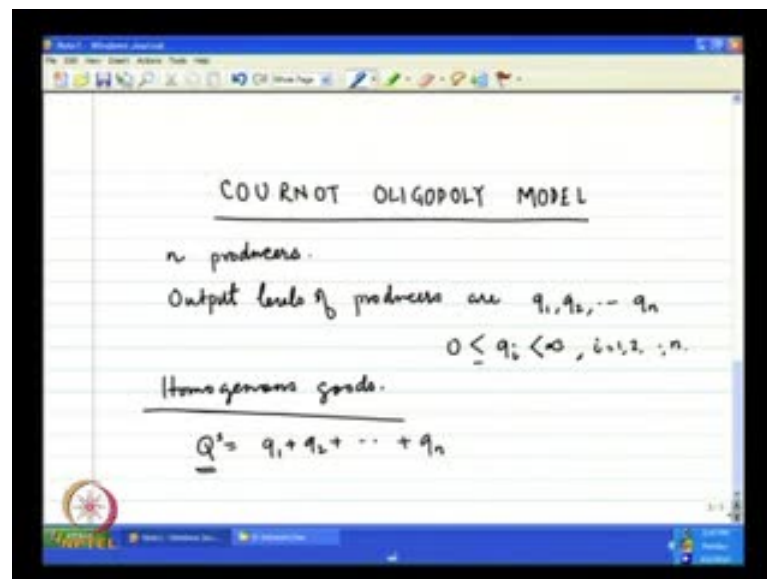
If we are going to talk about Auctions; where Auctions is very commonly used term. The people, the players are trying to get same resource or same commodity. And, they are trying to get that commodity by bidding. There will be different kinds of bidding. That we are going to talk about.

You can see that there is some similarity between War of Attrition and Auction. Because in both cases, the object that, the players are after is the same and the players are basically fighting within themselves to get that object.

And finally, in say as topic six, we are going to talk about Accident Laws. So, if an accident happens, there is one victim and one person, who have caused the accident. You just call that person to be the injurer. Now, it is easy to figure out that, if an accident happens, may be some blame of the accident lies with the victim and a part of the accident also lies with the injurer. For example, if a pedestrian is crossing a road and a car runs in down, then the blame may lie in either side. It may be lying in both sides also.

For example, the pedestrian might have been careless while crossing the street or the car might have been very rash. So, in such cases, how shall the State, how shall the Government frame the laws? Such that, these kinds of accidents when they happen, the damages that have to be paid by each or borne by each or fairly distributed. So, we are going to talk about those laws; which laws are most efficient and which are not. So, in all these cases, we are going to use the idea of Nash equilibrium and Game Theory to see whether we can predict the equilibrium; the outcome in the real world is, through our tools of Nash equilibrium and Game Theory.

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So, let us start with this Cournot Oligopoly Model. As, I just say, it is going to be a model of market. So, in a market, basically there are two sides in a market. One is the side of the sellers; where producing the good and selling it to the market. And then, there is the side of the buyers; the people were demanding the goods. Before we go into the

details of this Cournot model, let us, clear about the general structure of this Oligopoly model as such and in specifically, Cournot model.

What we are going to assume that, suppose there are n producers, n is any number greater than 1, but not very large obviously. And, suppose, the outputs or let us say output levels of producers are suppose q_1, q_2, \dots, q_n ; where q_i is any number between 0 and infinity. i go from 1 to n . And, we are going to assume, that the goods that they are producing are homogeneous. What does it mean?

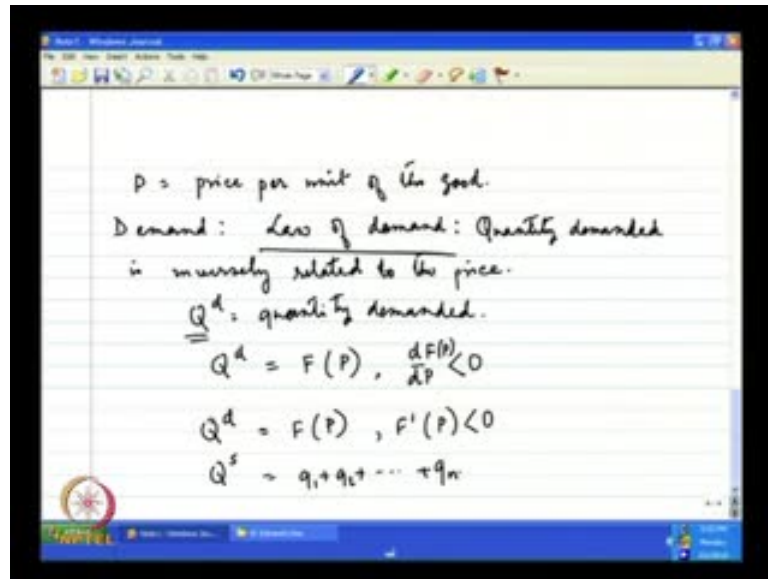
It means that producer one, the output that he is producing looks exactly the same way as the output of producer two, as the output of the producer three, etcetera. So, it means that, there is no brand name as such; there is no level over the products that the producers are producing. So, it is not, like you have the cold drinks market. And, in the cold drinks market, you see different brands of cold drinks produced by different companies.

So, you can make the difference between Coca-Cola and Pepsi, for example. But, here we are going to assume that, does not matter, where the cold drinks is coming from. The producers are making the cold drinks, in such a way that, thus the buyers cannot distinguish between the product of one producer from the product of the other producers.

So, that is why, we have said that the goods are homogeneous. They are similar. Now, suppose Q is the summation of all these output levels produced by all the producers, so, producer one is producing q_1 amount of outputs. So, q_1 might be 30. And, q_2 is the output produced by producer two. Suppose, q_2 is equal to 15 and let us, suppose there are only two producers q_1, q_2 . So, there are q_1 and q_2 only. There are no q_3, q_4 and etcetera. So, here the total supply in the market, which is given by q .

Let us, put s here to denote supply. Q_s is the supply in the market. This Q_s is equal to q_1 plus q_2 , that is, 30 plus 15, which is equal to 45. So, this is the total supply in the market. Now, once we have seen the supply side, now we must also talk about the demand side. When these goods are being sold, these 45 minutes of goods are being sold in the market; they must be getting sold at a particular price.

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Now, how is this price going to be determined? Let us, suppose P is the price per unit of the goods, now remember that P . That producer one gets must be equal to P that producer two gets. Because their goods are exactly the same, they look the same way. If they look the same way, the quality is the same. Then, price also must be the same because the consumers are not able to make the difference. So, they are going to pay the same price.

Now, if price is P , then one may wonder how this price in the market is being determined. At what price this Q^s is being sold in the market? To understand the answer to that question, we must look at the demand side. Now, so demand, to make things clear, what we say that in the market there is a law of demand. What does it mean? That, quantity demanded is inversely effect to the market price. So, suppose Q is quantity demanded, now when we say Q is quantity demanded; Q^d is the quantity demanded in the entire market.

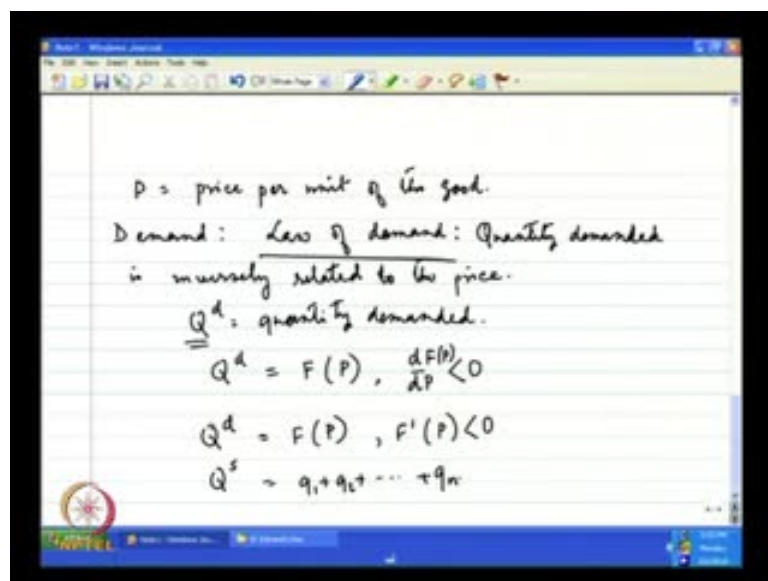
So, think about this, that there is a rice market and the numbers of producers are there and they are selling the rice in the market. And, from the side of the consumers, the customers, they have their demand for rice. Suppose at a particular price level 20, suppose price level is 20, I am demanding 5 kilogram of rice, the other people are demanding in total 100 kilogram of rice.

So, the total market demand at 20 rupees per kilogram will be 5 plus 100 is equal to 105 kilo of rice.

So, that is the total demand in the market, which we are denoting by Q^d . But, the point is that, this Q^d is not constant. It is not fixed. Q^d is going to be affected by the market price in this example. When their price was 20 rupees, I was demanding 5 kilo of rice, but suppose market price rises from 20 rupees, it jumps to 40 rupees. Then, I might find it economical to consume something else, may be wheat. I may shift to wheat and then, I will not demand 5 rupees of rice.

Instead of 5, I will demand 3 kilo of rice. And, other people who are demanding 100 kilo of rice, if the price goes up to the 40 rupees per kilo, they might demand less. They might demand 60 kilo of rice. So, if the price in the market for any good rises, the demand for that has a tendency, to fall and vice versa. That is, if the price falls, then the demand might increase.

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So, there is an inverse relationship between price in the market and the quantity demanded. So, this is written as the following Q^d is a function of P ; where dF by dP is negative. Now, the question may arise that, if the commodity that I am demanding is extremely necessary, I cannot live without it, then if the price goes up; will I be able to cut down its consumption? Will I be able to demand less?

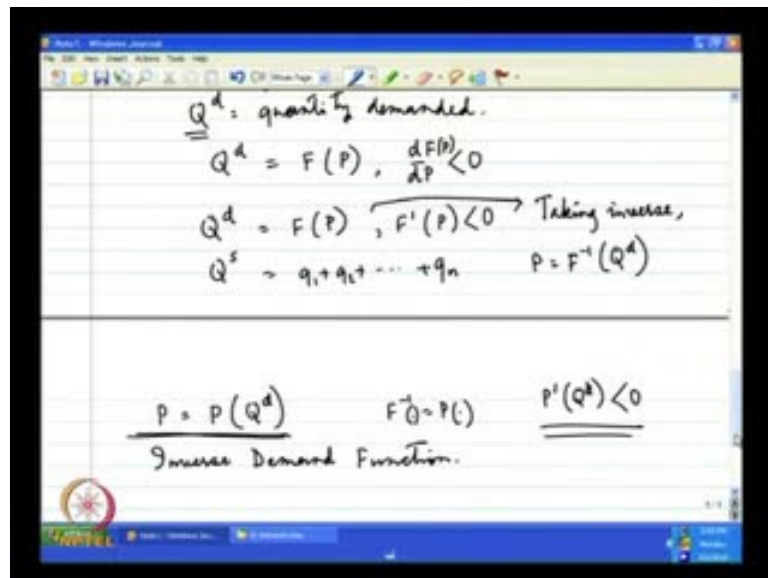
Well, it is true that in some cases, this law of demand, this inverse relationship between quantity demanded and price, may not hold. People will not be able to cut down their consumption, even if the price rises.

Or, it may happen in the other way also. Suppose there is a good, which I am demanding, but I do not like to consume lot of it. So, I have reached the saturation point. So, has to stay of that good. And, in that case, if the price falls, even then, I will not be buying more of that good. Nevertheless, so, there are these exceptional cases, where quantity demanded may not respond to the price in the expected way.

But, overall if we look at everybody's consumption, everybody's demand, sum them up. Then, there is a tendency, not always true of course, but there is a tendency that, quantity demanded varies inversely to prices. So, that is, what is known by this Law of Demand.

So, we have two things. Now, one is quantity demanded, which is given by $F(P)$; where $F'(P)$ is less than 0 and the other is quantity supplied, which is supplied by all these people, all these producers.

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Now, before we go further from here, I can also write this. Taking the inverse of this P is equal to F inverse of Q^d that I can write and in a very simple way, instead of F inverse. So, P can be written as a function of Q . And, this is known as Inverse Demand Function.

In demand function, what we had is quantity demanded as a function of price; if P was rising, quantity demanded was falling and vice versa. In inverse demand function, it is just the opposite. Opposite in the sense that, we have taken the inverse; we are expressing P as the dependent variable on the quantity demanded, which is Q d. And, here also the same thing will happen.

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The image shows a digital whiteboard with handwritten text. At the top, it defines the inverse demand function as $P = P(Q^d)$ and $F^{-1}(Q) = P(Q)$, with the derivative $P'(Q^d) < 0$. Below this, it states the equilibrium condition: Quantity Demanded = Quantity Supplied. At the bottom, it provides an example: $P = 20$, $Q^d = 100$, and $Q^s = 50$, with a downward arrow next to P indicating a price decrease.

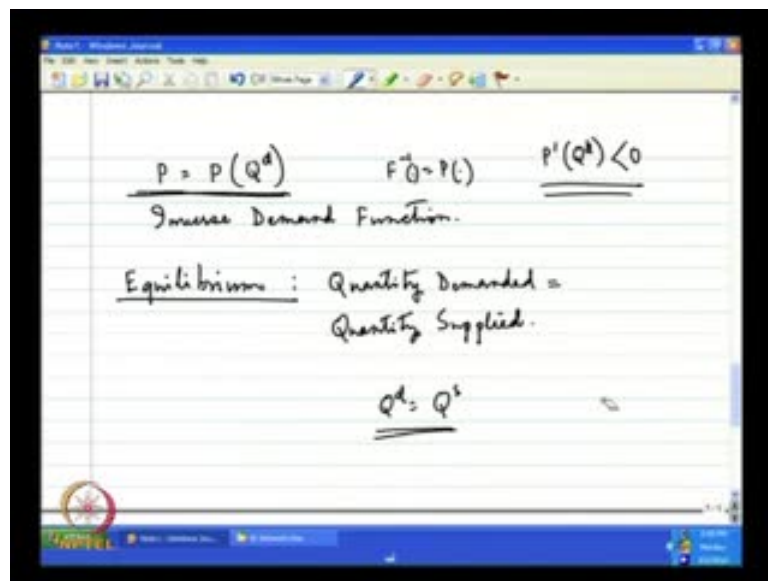
Because if there is an inverse function, if there is a function, which is downwards sloping and monotonic and if I take the inverse, the inverse remain a downward sloping function and monotonic. So, this is called an Inverse Demand Function. But, what is required in the market is that, what is known as equilibrium? Equilibrium demands that, quantity demanded must be equal to quantity supplied. What does it mean? Is that, if quantity demanded is not equal to quantity supplied, then there will be movement in the market and the market is not in the state of rest. The equilibrium, the notion of equilibrium, that it is a state of rest, in this context. Of course, Nash equilibrium has been used, but it is in a different state, it was in a different context. In that case, a notion of Nash equilibrium, it was a steady state. So, the steadiness or stability is, inherent in the concept of equilibrium.

So, in this case, if quantity demanded is not equal to quantity supplied, take this case, that in the market, at the current price, suppose again the price is 20 rupees, quantity demanded is 100 and the quantity supplied is 50, now, can this situation go on forever?

Is this a state of rest? The answer is no because if the quantity demanded is higher than quantity supplied, what will happen is that, people who are demanding goods, all of them will not be able to get the goods. So, people are demanding 100 kilograms and the supplied is 50 kilograms.

So, what will happen? In the market, demanders or the people who are demanding goods, they are going to push up the price in the market. So, P here is going to rise, it is not going to stay at 20 rupees. And so, since the P is changing here, it means that this is not a state of rest. And, similarly, if just the opposite thing happens, suppose, this is, what is the case that demand is less than the supply, and then what will happen? that the sellers who are taking this, all these 100 kilograms to the market, will not be able to sell this 100 kilograms; because the demand in the market is only for 50.

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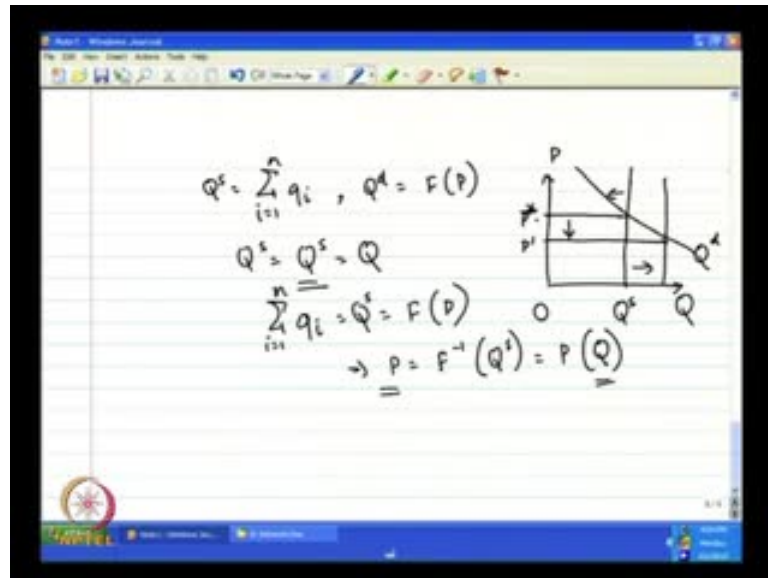


So, what they will do is to compete among themselves and they are going to reduce the price; so that, they are able to sell their goods. So, as a result, price will fall and so, again it is not a state of rest. There is a movement here. So, the moral of the story is that, if and only if Q^d is equal to Q^s , we are going to have a state of rest. Prices will not move and that is known in Economics as an equilibrium situation.

So, in equilibrium, people who are trying to buy goods are able to buy goods, all of them at the prevailing price. People who are trying to sell goods are able to sell goods. There is

no dissatisfaction, as such. If there is no dissatisfaction, then there is no reason for price to move. So, this is the basic idea of market equilibrium.

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Now, here in our model, what we had is the following. Q^s was the summation of all this q_i s. And, Q^d was a function of P . If I now impose this condition, equilibrium condition, Q^s is equal to Q^d . Let us, suppose it is equal to Q , then we must have them summation. Summation q_i equal to Q is equal to $F(P)$. And, which means that, P must be equal to, we can write Q^s also. And, this is what we have denoted as P . So, what is happening here is that, if I have to know what is the price, the equilibrium price in the market; given that, all these producers are producing q_1, q_2, q_n amount of output, what I need to do is to sum all these q_i , get this Q^s . And, put that Q^s in this P function. And, if I put that Q^s , which is equal to Q , the equilibrium quantity into this P function, I will get the equilibrium price. So, that is the cracks of the entire thing.

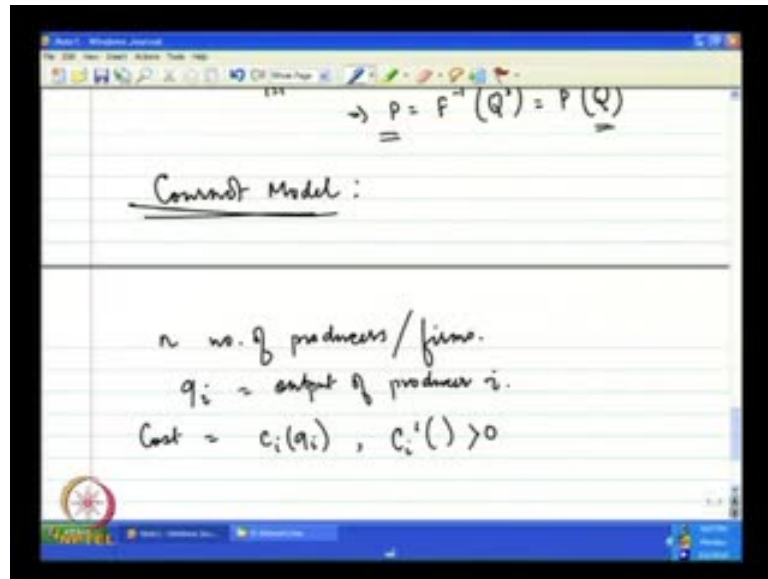
This gives me the equilibrium price in the market. This can be also represented in terms of a diagram. So, this along the vertical axis, we are representing price, price per unit; along the horizontal axis, we are representing the quantity, total amount of quantity that all these producers are producing together. I know the demand function will look like this. This is a demand function, downward sloping. And, this is, what is known as supply Q^s . This is the demand.

And, where they are equal? Is at the intersection point; so, this is the intersection point and therefore, let us call this P^* is the equilibrium price. And, you can see that, as this produces, changing their Q s, this equilibrium price is going to change. For example, if they decide to produce more output, then this equilibrium price is going down to P^d . And, it will happen in the other way also. If they decide to produce less output, then it is going to shift this way and equilibrium price will go up. And, what is happening is very simple that all the producers taking together. They produce certain level of output. If they have to sell that level of output that quantity of output, then there has to be a particular given price. That price is the equilibrium price, at which they will just be able to sell that level of output.

If they now decide to produce more, then to make the buyers buy that extra amount of output, higher level of output price must go down. Otherwise, the buyers are not going to buy. On the other hand, if they decide to produce less amount of output, then for all these outputs to get sold. And, for equilibrium to happen, it must happen that, price must go up because at the previous level of prices, if that output goes down, then there will be unsatisfied buyers, who are not getting their goods, even if they are ready to pay that P^* amount of price.

So, now the price must go up; so that, there is no unsatisfied buyers and there are no unsatisfied sellers as well. So, that is what is happening here. This was the general story of market equilibrium. This was just to introduce, the basic underlying things to students, who are not familiar with the market equilibrium mode.

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So, in this Cournot model, what is our purpose of studying this model? Our purpose is the following. Firstly, we are going to find out, how the outcome in this market; by outcome we mean, what is the quantity, which are being bought and sold? And, at what price they are bought and sold? How this quantity and price are going to be affected firstly, by the demand conditions. For example, suppose the consumers are demanding more goods, then how the price and quantity are going to be affected? Also, the profits of the producers, how are they going to be affected?

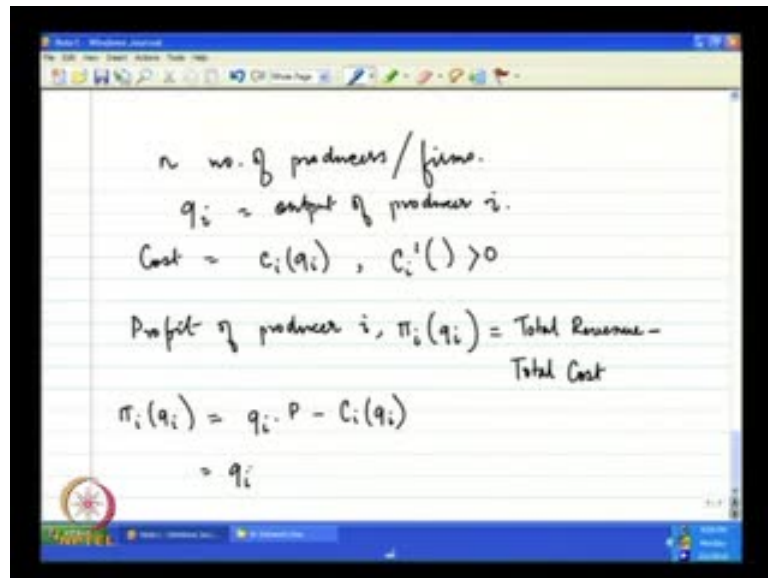
We are also going to look at the question, if suppose there is some technological advancement, for example, if the producers are now able to produce goods at a lower cost. Then, how it is going to affect the market equilibrium? Will the quantity of goods produced in the market going to go up, if the price going to go down, What is going to happen to the profits? We are also going to look at the number of firms.

I mean, if the number of producers goes up or if it goes down, how is it going to affect the competition, in the sense that is the total quantity in the market going to go up, is it going to go down, what is happening to prices, what is happening to profits, etcetera.

So, these are some of the issues that we are going to address in this model. So, there are in general n number of producers, we shall also call them firms. And, when they produce something, output of a particular producer, this is the output of producer i . But, when you are producing an output, at level of output, then there must be some cost that you have to

incur. So, cost we are going to say as c_i , which is also a function of q_i . so, c_i is the cost, what more can I say about c_i ? What kind of function is it? It is going to be increasing function. It is going to be increasing function because if the producer is producing more, if q_i rises, then obviously cost also rises.

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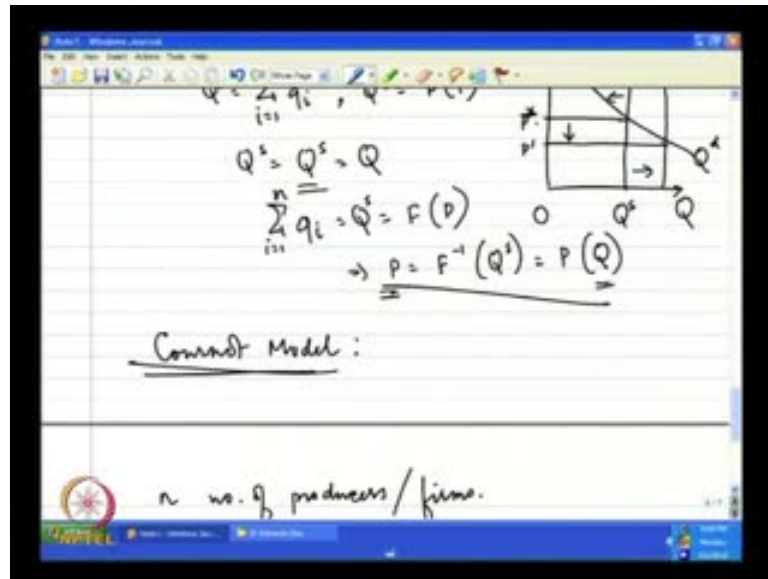
The image shows a digital whiteboard with handwritten notes. The notes define the number of producers/firms as n , the output of producer i as q_i , and the cost function as $C_i(q_i)$ with the condition $C_i'(\cdot) > 0$. It then defines the profit of producer i , $\pi_i(q_i)$, as Total Revenue minus Total Cost, and provides the equation $\pi_i(q_i) = q_i \cdot P - C_i(q_i)$.

$$n \text{ no. of producers/firms.}$$
$$q_i = \text{output of producer } i.$$
$$\text{Cost} = C_i(q_i), C_i'(\cdot) > 0$$
$$\text{Profit of producer } i, \pi_i(q_i) = \text{Total Revenue} - \text{Total Cost}$$
$$\pi_i(q_i) = q_i \cdot P - C_i(q_i)$$
$$= q_i$$

So, c_i prime is positive. Now, let us look at the profit of this producer. Let us call it π_i . Now, what is profit after all? Profit for any producer is the total amount of money that he is getting by selling his goods to the market. That is what, is known as total revenue. The total amount of money a producer gets by selling his goods in the market minus the total cost; the cost that he bears to produce that level of output. And, this entire thing must be a function of q_i .

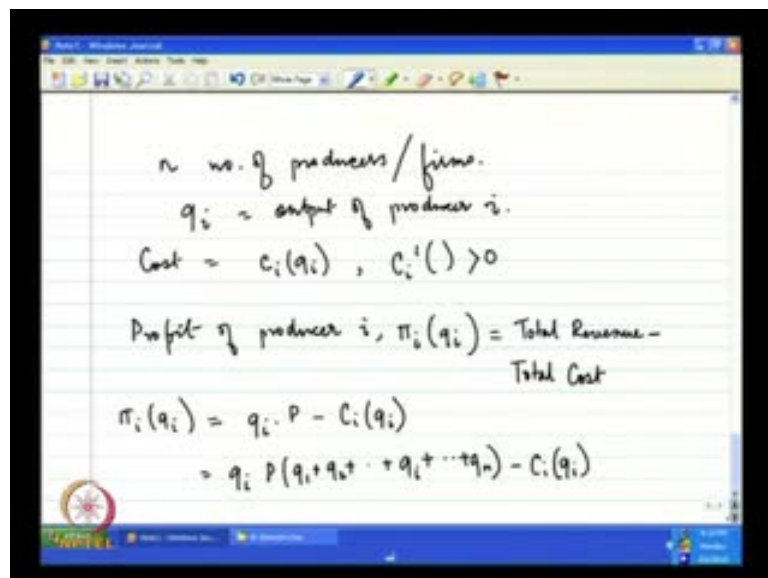
So, what is total revenue? Total revenue is the quantity I am selling, which is q_i multiplied by the price that I am getting per unit, which is P . So, this is the total amount of money I am getting by selling my goods in the market minus $c_i q_i$. This is the cost, total cost that I incur to produce q_i level of output.

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Now what is P ? P , now if you remember is this. This is the equilibrium price in the market. This is the price which will prevail in the market, if I produce q_i and other producers are producing their q .

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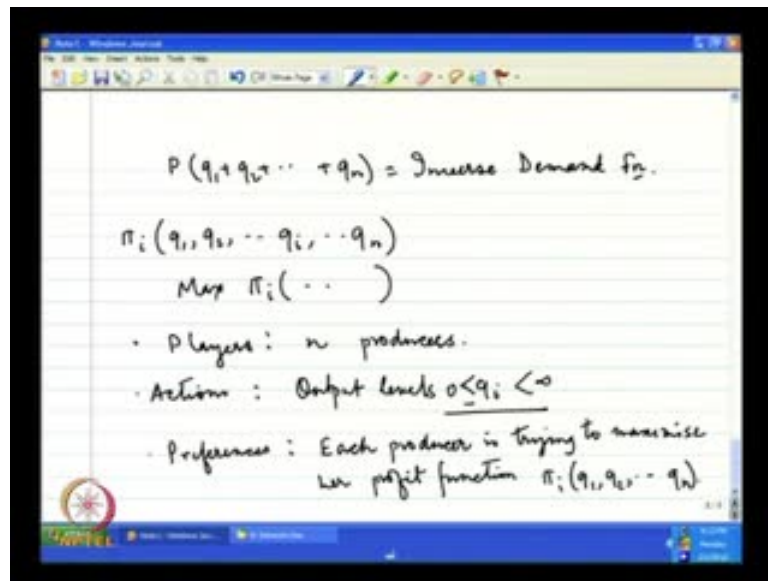


So, it is a function of... There is a q_i here and q_n minus $c_i q_i$. So, this is the profit of this producer in general. So, here this, remember this P as a function of q_1 plus q_2 plus q_n is, nothing but the inverse demand function. And, inside the inverse demand function

not only q_i is there, but all the q s are there. And, here we have this inkling as to why Game Theory is important here.

Now, we can figure out that the π_i is not only a function of q_i , but it is a function of so many q 's. And, which is the basic cracks of Game Theory? That, my preference or what I am getting out of this entire thing, it depends not only what I am doing, but it also depends on what other players are doing.

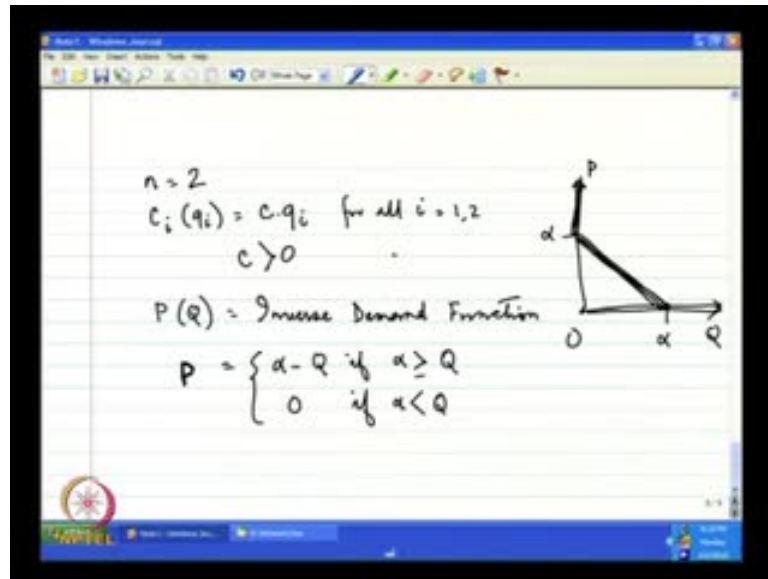
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So, what the producers are going to do here is, they are trying to get as high profit as possible. So, each of them is trying to maximize this one. But, this π_i , which they are trying to maximize, is not only a function of what they are doing individually. But, it also depends on what other people are doing, what other producers are producing. And, there is coming the element of interdependence. And, since we have the idea of interdependence, people's payoffs are getting affected by the actions of other producers; we can profitably use the idea of Game Theory here and Nash equilibrium.

So, let us write it down, in terms of game theoretical model. Then, first is who the players are? n producers, actions output levels because that is, what they are deciding as output levels? How much they are going to produce? And, this output levels can be any number between 0 and infinity. And preferences, profit function π_i . This is, what they try to maximize.

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Now, what we are going to do in order to look and discern the equilibrium in this model, in this oligopoly model. We are going to simplify it a little bit. In fact, considerably we are going to assume that, n is 2; that the number of producers in this market is just 2. This is just to simplify. We are going to then generalize this model by taking n as any number greater than 2. Secondly, we have to assume that c_i , which is q_i is c multiplied by q_i for all i , there are only two. So, I do not want to bother about.

So, it basically means that the cost functions for players are same; in the sense, that their cost functions are represented by c , which is a constant. c is a number greater than 0 and the total cost is just that, c multiplied by the quantity that they are producing.

So, in a sense, that this c is the unit cost of production; so, if they want to produce 1 unit of output, the total cost is c . if they want to produce n units of output, the total cost is n multiplied by c . And, we are going to assume that P , this is the inverse demand function. It takes a very simple linear form. This is represented by suppose is equal to 0, if α is less than Q .

So, this is a very simple linear representation of the demand function. You can see that is, of course, a downwards sloping line. And, as Q rises, this $\alpha - Q$ obviously falls. But, that is true, only when α is greater than equal to Q . This is important because if Q is greater than this α , then this first formula gives me a negative. The first line gives me a negative P and price can never be negative.

So, the minimum value that P can take is 0. And, which will happen if Q is greater than α or Q is just equal to α . So, this is represented by a diagram. Suppose this is P , this is Q and then I have this downward sloping line. It is, in fact of line, which creates a 45 degree angle with this axis, q dashed axis. And this, intercept is α , here also it is α .

And, this demand functions is in fact this part plus this vertical part. So, this is the demand function in the market. So, this is again simplified approach towards this oligopoly model. We have simplified a lot of things. Firstly, we have assumed that a number of producers are not in general, n it is 2. We have assumed the cost function, which is again a constant term plus the constant term multiplied by the output level q_i .

And, we have assumed the linear demand function. Now, after doing this, our model becomes simplified. And then, how do I write the profit function? Now, before we write profit function and try to find out the Nash equilibrium, let us be clear as to what we are trying to do. We are going to find the Nash equilibrium in this model. Now, how we are going to do this? Here, what is happening is that the number of actions is infinite. If you remember q_i is the action and q_i can take any value between 0 and infinity.

And, if there are infinite numbers of actions, obviously we cannot just construct a P of matrix and find out what are the Nash equilibria. So, what we are going to do is to find out the best response functions and from the best response function of these two players, we are going to apply the idea that the Nash equilibria or equilibrium lies in the intersection point of the best response functions.

So, our task is to find out the best response functions and then try to see at what point the best response functions intersect. So, to do that, what we need to do is that, we are going to look at the preference or the payoff function of the players. And, try to maximize that payoff function because that is what the players are doing. And then, we shall get the best response functions because what are the best response functions is that, it gives me for a player, gives me that action of that player, which is best for him, given what the other players are doing.

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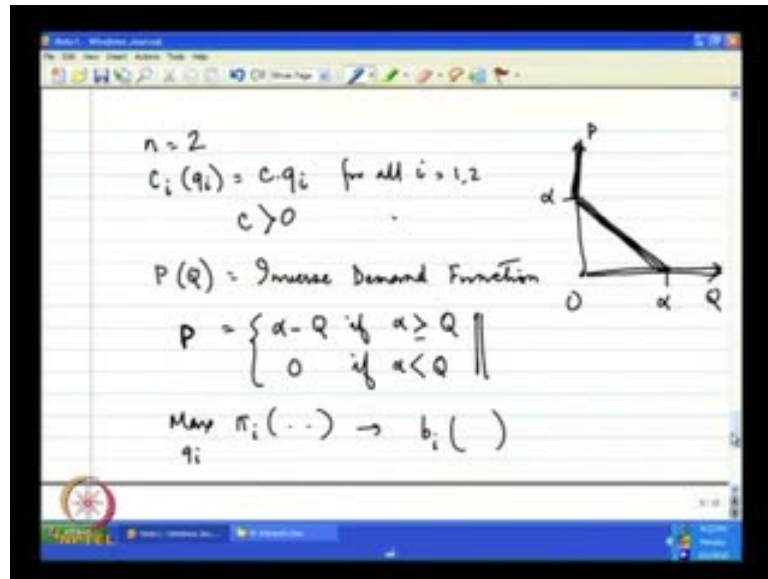
$$\text{Max}_{q_i} \pi_i(\dots) \rightarrow b_i(\dots)$$

$$\begin{aligned} \pi_i(q_1, q_2) &= q_1 P(q_1, q_2) - cq_1 \\ &= q_1 [\alpha - (q_1 + q_2)] - cq_1 \\ &= q_1 (\alpha - c - q_1 - q_2) \quad \text{if } q_1 + q_2 \leq \alpha \end{aligned}$$

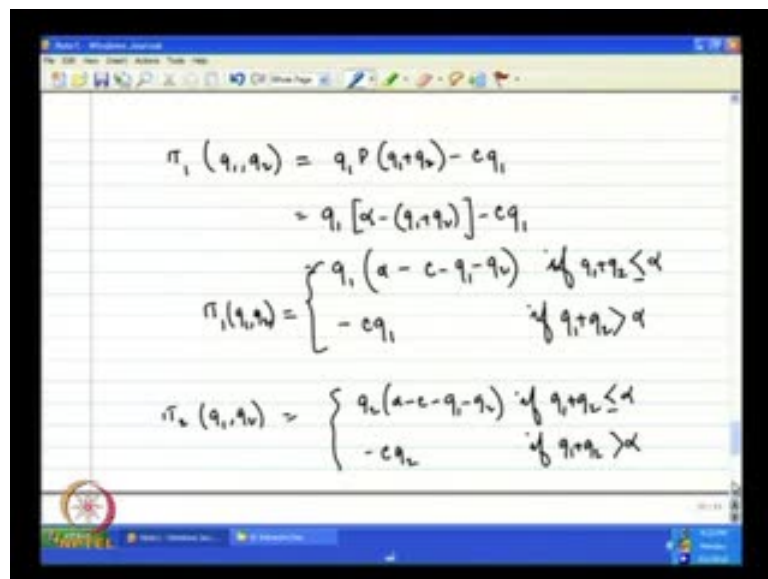
So, if we maximize this with respect to q_1 , then we shall get the best response function of this player i . And so, we get the best response functions of these two players here. There are only 2 players here. And, try to find out what is the equilibrium? So, that is the strategy of approaching this problem.

So, here if I look at, suppose player I , his payoff function is what? It is q_1 multiplied by P , which is a function of q_1 plus q_2 minus c of q_1 . And, this is equal to α multiplied by q_1 minus q_1 plus q_2 minus c of q_1 . Now, from here, I can take q_1 common. And, what I get is, $\alpha - c - q_1 - q_2$. But, remember this is going to happen if $q_1 + q_2 \leq \alpha$.

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So, here it means that, $q_1 + q_2 \leq \alpha$. And, if $q_1 + q_2 > \alpha$, then this part, the first part become 0. And, then the second part is minus q_1 . So, it is minus $c q_1$; if $q_1 + q_2$ is, strictly greater than α . So, this is the profit function of player 1.

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Max $\pi_1(q_1, q_2)$
 q_1
First Order Condition: $\frac{\partial}{\partial q_1} \pi_1(q_1, q_2) = 0$
 $\therefore \frac{\partial}{\partial q_1} [q_1(a - c - q_1 - q_2)] = 0$
 $\therefore a - c - q_1 - q_2 - q_1 = 0$
 $\therefore q_1 = \frac{a - c - q_2}{2}$

So, similarly for player 2, it will be q_2 multiplied by... So, the next task is to maximize either of these functions and find out the best response function. So, we are going to maximize, subject to q_1 . Now, if I do that, what we need to find out? What are the first order and the second order conditions?

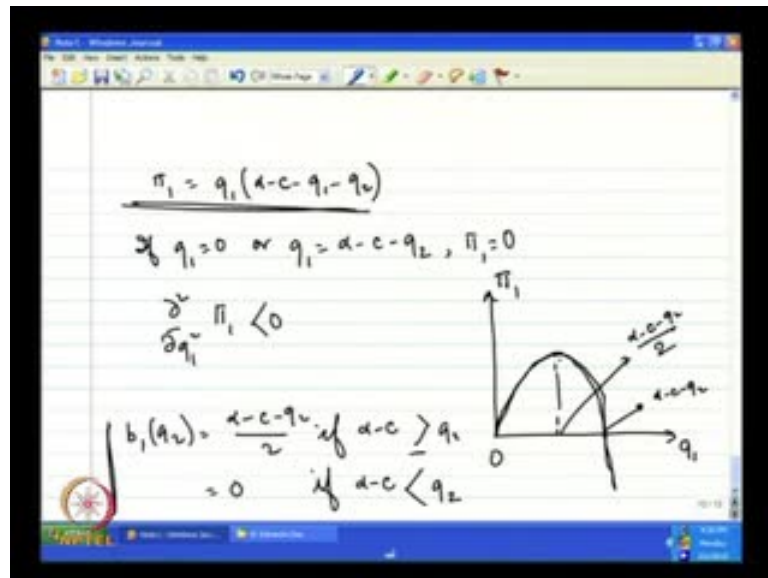
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$\frac{\partial^2}{\partial q_1^2} (q_1(a - c - q_1 - q_2))$
 $= \frac{\partial}{\partial q_1} (a - c - q_2 - 2q_1)$
 $= -2 < 0$
Best Response Function of producer 1 is
 $q_1 = \frac{a - c - q_2}{2}$

So, what is the first order condition? The derivative of this is 0. So, if I take the derivative of... this has to be set to 0. And so, alpha, I am just using the product rule. And, I have to express q_1 as a function of q_2 . So, this becomes $a - c - q_2$

divided by 2. Is the second order condition satisfied? What I need to check is that, this has to be less than 0. And, is this less than 0? This is, nothing but alpha minus c minus q 2 minus 2 q 1. This is coming from here, second line. And, this is nothing but minus 2, which is less than 0.

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So, the second order condition is satisfied. So, the first order condition, which is giving me this, is the best response function. So, the best response function of producer 1 is q_1 . Now, one has to be careful as to what is the range, for which this is true because if you remember here, there was some range, but does the same range apply here also? Let us check out that. Remember the profit function, if I draw the profit function how will it look? The profit function is this.

Now, if q_1 is equal to 0 or q_1 is equal to alpha minus c minus q_2 , profit is equal to 0. That we have seen. That is clear from this profit function itself. If I put q_1 is equal to 0 the first element is 0. So, the entire thing becomes 0. If I put q_1 is equal to alpha minus c minus q_2 , the second term becomes 0, which means again π_1 is equal to 0.

So, if I have to draw this curve, how will it look? Suppose, I replace in this q_1 along the horizontal axis and π_1 along the vertical axis and I also know that, this thing is true; which means this curve, π_1 curve is a concave curve. And, it is a concave curve and it is cutting this horizontal axis at 0. And, at this point, suppose this point is, alpha minus c

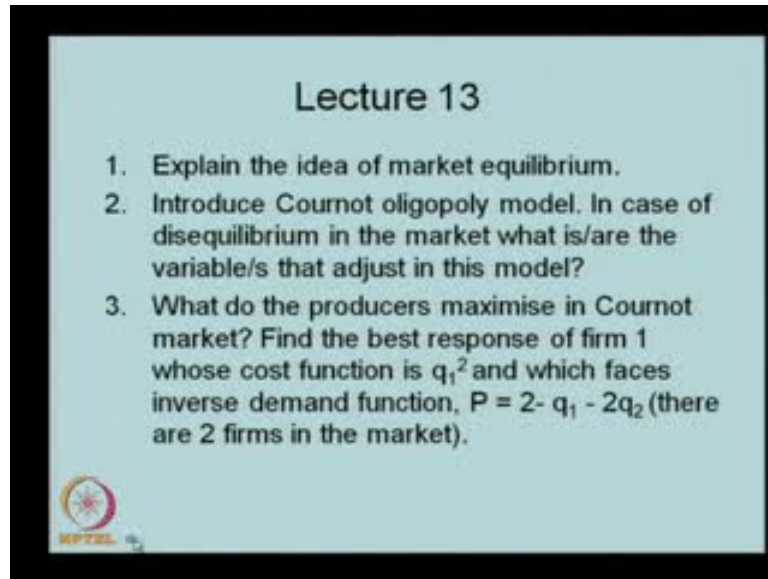
minus q^2 , so, it is going to be like this. Concave curve, which is cutting the horizontal axis at two points.

And, there is a point at which, it is reaching its maximum value. And, that maximum value is, $\alpha - c - q^2$ divided by 2. But, I know that if $q > 1$ exists in $\alpha - c - q^2$, then this profit is negative. And, in that case the firm of one will not produce anything.

So, this best response function that I found out is true only if, $\alpha - c - q^2$ is either 0 or positive. If it is negative, then the firm is not producing anything. So, best response function will be $\alpha - c - q^2$ divided by 2; if $\alpha - c - q^2$ is a greater than q^2 or equal to 0, if $\alpha - c$ is less than q^2 .


So, this is what, the best response function of player 1 is. We should proceed from here in the next class. So, before we finish this lecture, what we have done here is the following. We have introduced the idea of Cournot model of Oligopoly. We have tried to find out what are the main aims of studying the Cournot model of Oligopoly, one to study the equilibrium in this module. And, how this equilibrium is affected by changing demand condition, changing cost conditions, changing level of competition, all of this we are going to study here. And, in doing so, we have seen that, what we are going to do here is to find out the best response function. And then, try to see what the Nash equilibrium is. So far, we have found the best response functions. In the next class we are going to look at the Nash equilibrium. Thank you.

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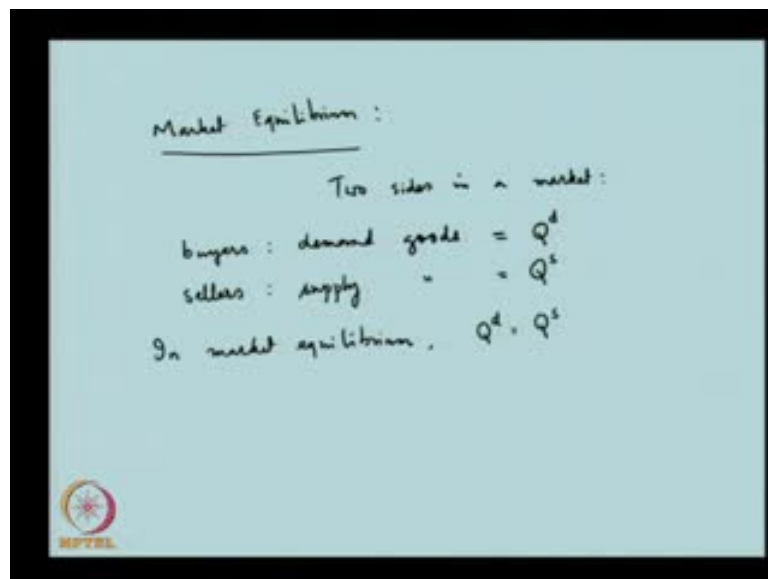


Lecture 13

1. Explain the idea of market equilibrium.
2. Introduce Cournot oligopoly model. In case of disequilibrium in the market what is/are the variable/s that adjust in this model?
3. What do the producers maximise in Cournot market? Find the best response of firm 1 whose cost function is q_1^2 and which faces inverse demand function, $P = 2 - q_1 - 2q_2$ (there are 2 firms in the market).



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Market Equilibrium :

Two sides in a market :

buyers : demand goods = Q^d
sellers : supply " = Q^s

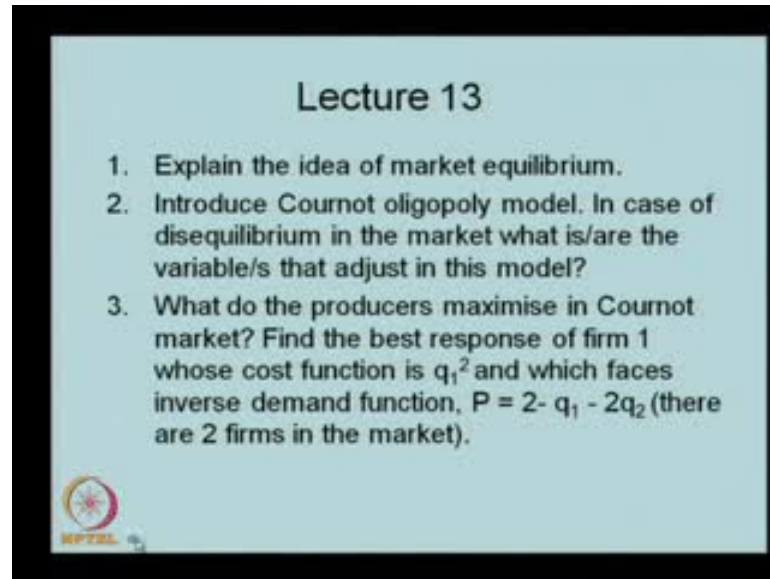
In market equilibrium, $Q^d = Q^s$



Explain the idea of market equilibrium. So, what happens in market equilibrium? If we have a market, then there are basically two sides in the market. Buyers demand goods. So, we shall call it the amount of goods that they are demanding will be called Q^d . And, there are sellers who supply goods and this we shall call as Q^s .


Now in equilibrium, these two must be equal; that is Q^d , the amount of goods that is being demanded in the market at a particular price, must be equal to the amount of goods that is being supplied in the market at a particular price.

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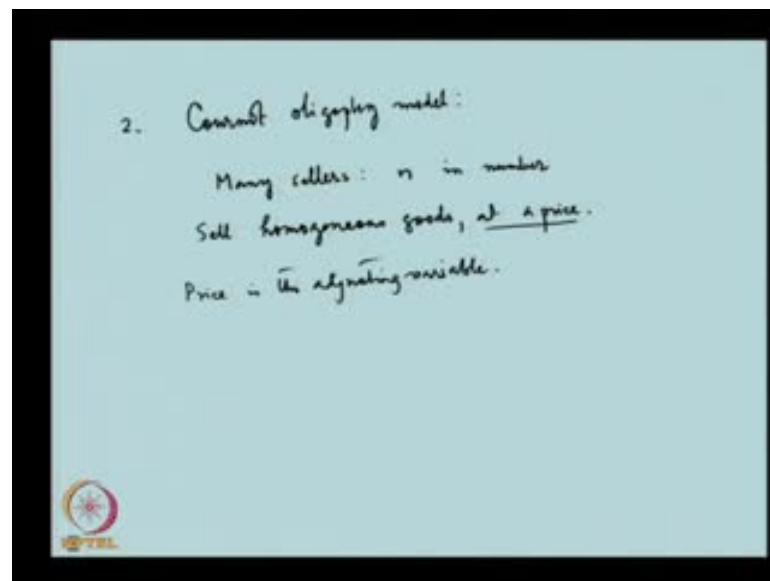


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2. Cournot oligopoly model:

- Many sellers: n in number
- Sell homogeneous goods, at a price.
- Price is the adjusting variable.



If that happens, then we shall call that situation as an equilibrium situation because that is situation of state of rest. Second, Introduce Cournot oligopoly model, in case of disequilibrium in the market, what is or are the variables or variable that adjust in this model? So, Cournot model, Cournot oligopoly model so, there are basically many sellers, but not infinite. Let us say n in number. n can vary from 2 to any number, which is less than infinity, but not a large number also. There are n sellers in the market. And, they sell similar goods.

Let us say, sell homogeneous goods. And, at a price, since the good is homogeneous the buyer cannot make out whether the good is coming from seller 1 or producer 1 or producer 2. So, the price for each of these producers, the price that they will get or each unit of a good is going to be the same. That is why we are saying that, it is at a particular price. What adjust is just disequilibrium; if suppose demand is not equal to supply, then what adjust in this market, is the price. Price is the adjusting variable.

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3. Firms maximize profits.

$$C_1 = q_1^2$$

$$P = 2 - q_1 - 2q_2$$

$$\pi_1 = Pq_1 - C_1$$

$$= q_1(2 - q_1 - 2q_2) - q_1^2$$

FOC $\frac{\partial \pi_1}{\partial q_1} = 0 \Rightarrow 2 - q_1 - 2q_2 - q_1 - 2q_1 = 0$

$$\therefore q_1 = \frac{1}{2}(2 - 2q_2) \quad \begin{cases} \text{if } q_2 \leq 1 \\ = 0 \quad \text{if } q_2 > 1 \end{cases}$$

Last question is what do the producers maximize in Cournot market? Find the best response of firm 1 whose cost function is q_1 square and which faces inverse demand function; P is equal to 2 minus q_1 minus 2 q_2 . So, what do the firms do in Cournot competition? Firms maximize profits. This is our kind of bench mark assumption. Though, there could be variations in this. we can sometime say that they maximize not profits, but output or may be maximizing their market share. But, this is the kind of a bench mark model that they want to maximize profits.

Now, let us take this particular exercise, if a firm 1 has cost function q_1 square and the inverse demand function is given by $2q_1$ minus $2q_2$. This is the inverse demand function. Now, let us suppose that π_1 is firm one's profit, this is nothing but P multiplied by q_1 minus the cost function and this is 2 minus q_1 minus $2q_2$ minus q_1 square. Now, this is supposed to be maximized by firm 1. So, the first order condition is

this. From this, we shall get this. And, if we simplify this, then what we get is q_1 is equal to $1 - q_2$. Now this is true.

If this thing within the bracket is positive or 0, that is, if q_2 is less than or equal to 1, if q_2 is more than that, then obviously this is going to be negative, which is not possible. So, we are going to assume in that case q_1 is equal to 0.

Thank you.