

Game Theory and Economics
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Module No. # 03
Illustrations of Nash Equilibrium
Lecture No. # 02
Different Aspects of Cournot Model

Welcome to the second lecture of module 3 of the course called game theory and economics. Before we start this lecture let me just take you through what we have covered in this module so far in the first lecture; we have been discussing the application of game theory an application of Nash equilibrium in particular in various situations.

So, we have started out with application of Nash equilibrium in case of markets - in the oligopoly markets - and this markets that we are discussing right now is called the Cournot model; so, in Cournot model the number of firms that there are in the market, they are not large number of firms however the number of firms is not one also, it is a few number of firms are there, and that is called an oligopoly market. What these firms are doing is that, they are trying to maximize their profits and the decision variables that are the variables that they are choosing are their respective output levels, so by choosing their output levels they are trying to maximize their profit.

Now, the point is that, how is the price determine in such market; what happens is that after all the firms have chosen their quantities that is the output levels they are sold to the market, and if they have to be sold then the price has to be of a certain amount, and that price will guaranty that all the goods produced by the firms are getting sold, it is not that the amount that they are producing is more than what the consumers are demanding, because there is other side to the market which is form by the consumers.

So, the price in the market is determent where the demand is just equal to the supply, so that is how price is determent, price is not determent by the producers, it is determent by the market condition at the point where demand is equal to supply, that is the story more or less the basic story; we have started out with how to apply the concept of Nash equilibrium in such a market.

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$$\begin{aligned} & 2 \text{ firms} \\ & Q = \alpha - P, \quad P \leq \alpha \\ & \quad \quad \quad = 0, \quad P > \alpha \\ & C_i(q_i) = c q_i \\ & \quad \quad \quad i = 1, 2 \\ & P = \alpha - Q \quad \text{if } \alpha \geq Q \\ & \quad \quad \quad = 0 \quad \text{if } \alpha < Q \end{aligned}$$

We said that for the time being let us take a simple case where there are two firms only, and these 2 firms face the demand function in the market given by Q is equal to α minus P , if P is less than equal to α is equal to 0, if P is greater than α , and this is the direct demand function; and from this we can get the inverse demand function which is this, and **this** so these are the inverse demand functions what about the cost function of the producers, we have assume that cost of a producer is given by the unit cost is small q small p multiplied by q i , and this is for both the firms.

So, both the firms have the same unit cost of production; from this we have try to find out what are the best response functions, because in Nash equilibrium one way to find out the Nash equilibrium is to try to find the best response functions, and from the best response functions what is the Nash equilibrium? Because one property that we have seen before is that at the point where the best response functions intersect with each other, that is also the point of the Nash equilibrium.

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Best Response Function of 1,
 $q_1 = B_1(q_2) = \frac{\alpha - c - q_2}{2}$ if $q_2 \leq \alpha - c$
 $= 0$ if $q_2 > \alpha - c$

Max $\pi_2(q_1, q_2)$
 q_2
Best Response Function of 2, $q_2 = B_2(q_1) = \frac{\alpha - c - q_1}{2}$

So, here also we are going to do the same, we are going to find the best response functions and try to find the point of intersection. And the last lecture that is what we have been doing, we have found that the best response function of alpha firm 1 is $q_1 = \alpha - c - q_2$ divided by 2 if q_2 is less than equal to $\alpha - c$ is equal to 0 if q_2 is strictly greater than $\alpha - c$, this is what we have seen before; and by applying the same method that this we got by maximizing the profit function of firm 1.

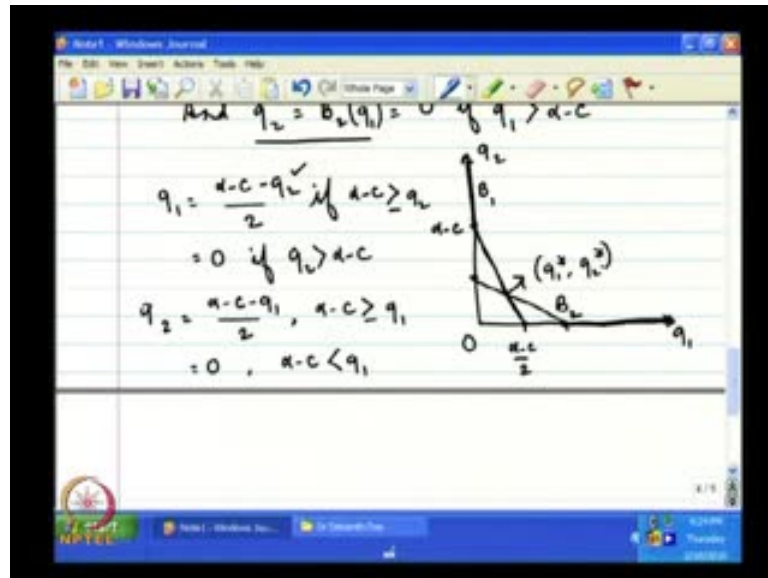
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q_2
Best Response Function of 2, $q_2 = B_2(q_1) = \frac{\alpha - c - q_1}{2}$
if $q_1 \leq \alpha - c$

And $q_2 = B_2(q_1) = 0$ if $q_1 > \alpha - c$

We got this best response function of firm 1; similarly, if we maximize firm 2's best profit function, which will be again a function of q_1 and q_2 we can get a with respect to q_2 , we can get the best response function of player 2 that is firm 2 to be given by q_2 .

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So, these two are the best response functions of player 1 and player 2, that is firm 1 and firm 2, what is the Nash equilibrium? So, Nash equilibrium will be given by that point where these two functions intersect, can we represent these two functions in terms of diagram it terms that we can, suppose we draw this diagram, and we want to draw the best response function of firm 1 suppose.

Now, **firm 1's best response function is given by...**; now, and is equal to 0 if q_2 is greater than $\alpha - c$; so, let us suppose this is $\alpha - c$, and this is $\alpha - c$, and this is suppose $\alpha - c$ divided by 2; if q_2 is less than $\alpha - c$, then this becomes operative, this becomes operative, and how does this look like, if q_2 is equal to 0 q_1 is equal $\alpha - c$ divided by 2 which means this function this curve is going to start from here, and we can see that it is a straight line curve it is not a quadratic curve or a function of higher order.

So, the diagrammatic representation will be a straight line; if q_2 is equal to 0 q_1 is equal to $\alpha - c$ divided by 2; if q_1 is equal to 0 then q_2 is equal to $\alpha - c$ which means this point on q_2 axis, and this point on the q_1 axis are 2 points on the best response functions, so I can join them, so this is B_1 .

So, this part is for q_2 less than or equal to $a - c$; if q_2 is greater than $a - c$ then q_1 is 0 which means we are talking about the q_2 axis itself, **so this is my B 1...**; so, it consists of two parts one is the downwards sloping part which is after $a - c$, and another is vertical part which is over the point $a - c$; what about the representation of q_2 is equal to B_2 which is a function of q_1 , so let us first write it down.

So, again this is a straight line, and what are the points on the straight line if q_1 is equal to 0 then q_2 is equal to $a - c$ divided by 2, which means this point is on the curve or on the best response function; if q_2 is equal to 0 then q_1 is equal to $a - c$, so this point is again on the best response function, so I can join them; and if q_1 is greater than $a - c$ then q_2 is equal to 0, which means that I am getting this axis itself - the horizontal axis itself - which means that this B_2 is cutting this B_1 at only one point, this is the point, this is the point of intersection, let us call it q_1^* q_2^* , q_1^* q_2^* is the Nash equilibrium because that is the point of intersection.

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Nash Equilibrium = (q_1^*, q_2^*) .

$$q_1^* = \frac{a-c-q_2^*}{2}$$

$$q_2^* = \frac{a-c-q_1^*}{2}$$

$$q_1^* = \frac{a-c}{2} - \frac{1}{2} \left(\frac{a-c-q_1^*}{2} \right)$$

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The image shows a handwritten derivation on a digital whiteboard. The equations are as follows:

$$\begin{aligned} \therefore q_1^* &= \frac{\alpha - c}{2} - \frac{\alpha - c}{4} + \frac{1}{4} q_1^* \\ \therefore \frac{3}{4} q_1^* &= \frac{\alpha - c}{4} \\ \therefore q_1^* &= \frac{\alpha - c}{3} \\ \therefore q_2^* &= \frac{\alpha - c}{2} - \frac{1}{2} \frac{\alpha - c}{3} = \frac{1}{3} (\alpha - c) \\ \text{NE output levels} & \frac{\alpha - c}{3}, \frac{\alpha - c}{3} \end{aligned}$$

But we have to find out what is the value of q_1^* and q_2^* , we have to find out the coordinates, so to do that we have to solve these 2 equations, one is $q_1^* = \frac{\alpha - c}{2} - \frac{q_2^*}{2}$, the other is $q_2^* = \frac{\alpha - c}{2} - \frac{q_1^*}{2}$; these two equations we have to solve simultaneously, and if we do that then what we get is the following, so this is $\frac{3}{4} q_1^* = \frac{\alpha - c}{4}$, which means $q_1^* = \frac{\alpha - c}{3}$, which means q_2^* which is nothing but one-third of $\alpha - c$.

So, both this output levels, that is Nash equilibrium output levels $\frac{\alpha - c}{3}$, $\frac{\alpha - c}{3}$, they are producing the same level of output, and which is not surprising, because their cost functions are same, and they are facing the same demand function, so there is no difference between these two firms conditions that is why in equilibrium their producing the same level of output.

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Handwritten notes on a digital whiteboard showing the derivation of equilibrium output and price:

$$Q^* = \text{total market output in equilibrium}$$
$$= q_1^* + q_2^* = \frac{2}{3}(\alpha - c)$$
$$P^* = \text{equilibrium price} = \alpha - \frac{2}{3}(\alpha - c)$$
$$= \frac{1}{3}(3\alpha - 2\alpha + 2c)$$
$$= \frac{1}{3}(\alpha + 2c)$$

What is the total output? Suppose, Q^* is the total market output in equilibrium, so this is q_1^* plus q_2^* which means $\frac{2}{3}\alpha - c$, and **price** the **equilibrium price** price formula is that it is $\alpha - \frac{2}{3}(\alpha - c)$.

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Handwritten notes on a digital whiteboard showing the derivation of equilibrium profit for each firm:

Equilibrium profit of each firm

$$\pi_1^* = \pi_2^* = q_1^*(\alpha - c - q_1^* - q_2^*) - q_1^*c$$
$$= q_1^*(\alpha - c - q_1^* - q_2^*) - q_1^*c$$
$$= \frac{\alpha - c}{3}(\alpha - c - \frac{2}{3}(\alpha - c))$$
$$= \frac{\alpha - c}{3} \frac{\alpha - c}{3} = \frac{(\alpha - c)^2}{9}$$

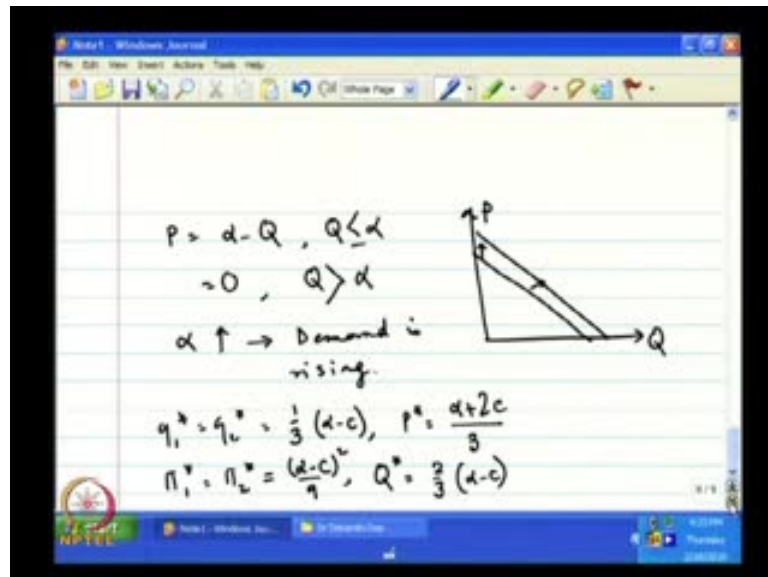
So, $\alpha - \frac{2}{3}(\alpha - c)$, and whatever the **profit** equilibrium profit which is given by suppose q_1^* this profit will be the same, because they are producing the same level of output, it is given by the price in equilibrium or we can use the result that

we have to derived before the profit function which is q star minus multiplied by alpha minus c multi minus q 2 star minus q 1 star square.

So, this was the profit function that we have derived before; so, I can take q 1 star common, so alpha minus c minus q 1 minus q 2, and this is alpha minus c divided by 3 alpha minus c minus 2 by 3 alpha minus c , because there are 2 alpha minus c 's, so this is nothing but alpha minus c divided by 3, so this is alpha minus c whole square divided by 9, so this is the equilibrium profit of each firm.

Now, m by 1 is started out with this analysis of Cournot equilibrium, we said what are the main purposes of studying Cournot equilibrium or any market, for example, we want to find out what is the equilibrium quantity and price that we have found out, but what we get from here is what will be the effects of various parameters on the equilibrium price and output; for example, suppose in the market demand conditions improved, demand curve shifts to the right, what is meant by demand curve shift in to the right I can show it in terms of diagram.

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So, demand function is given rather in the inverse demand function is given by this, this is for Q less than alpha, so demand curve is given by this downwards sloping line; when demand curve shifts to the right, that means, this intercept is rising, and what is the intercept? Intercept is alpha, so alpha rising means that demand is rising, so this alpha

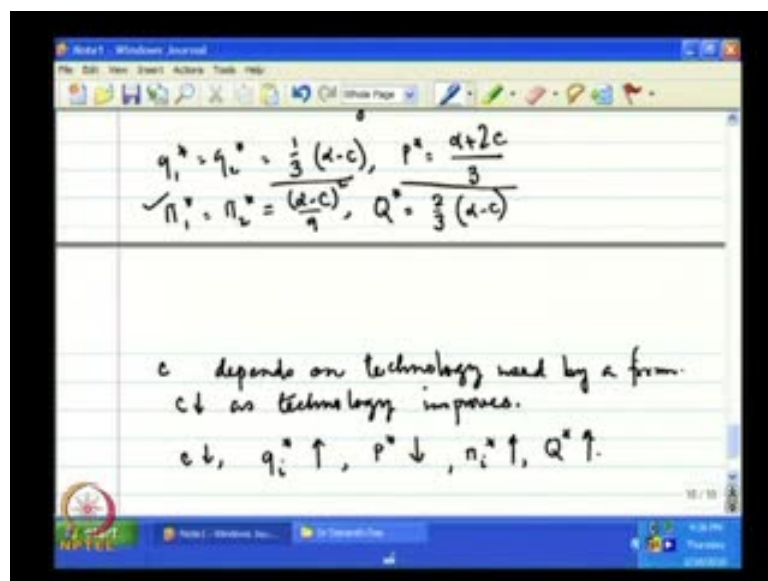
parameter captures the position of the demand curve, if consumers are willing to buy more goods and that will be represented in terms of rising alpha.

So, can we look at how alpha is affecting each of these equilibrium variables and the answer is yes; we have the following results that, q_1^* is equal to q_2^* is equal to $\frac{1}{3}(\alpha - c)$, P^* is given by $\frac{\alpha + 2c}{3}$, and the equilibrium profit is given by $\frac{\alpha - c}{9}$ this whole square, and capital Q^* that is total output is $\frac{2}{3}(\alpha - c)$.

Now, this we know, so it means that if alpha rises all these equilibrium variables that is q_1^* , q_2^* , capital Q^* , P^* , profit that is π_1^* , everything is going to rise; why it is happening is that, if people are trying to buy more goods demand is rising then the all the firms will start producing more good, which means q_1^* and q_2^* will rise in equilibrium, which you push up the total market output, which is capital Q^* that will rise; and what will be effect on price? Well, price we see that P^* is increasing in alpha, which means that if more demand is generated in the market price is going to rise which is not very surprising.

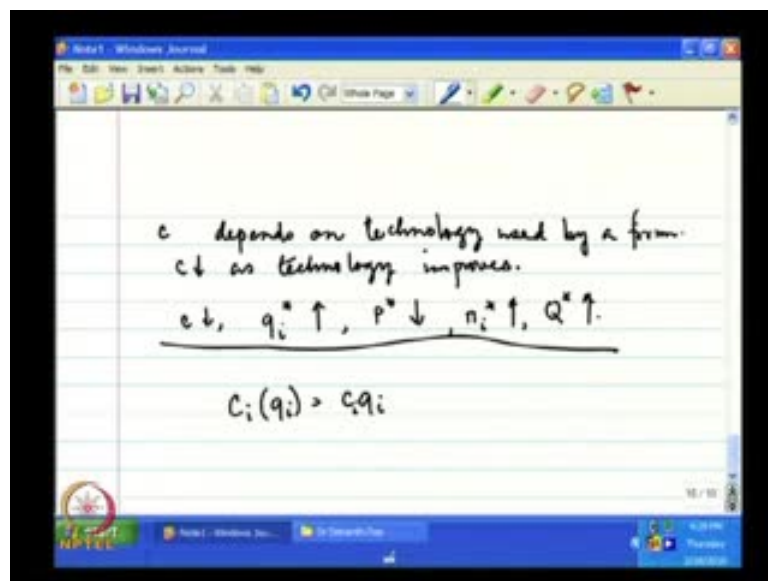
If people are ready to pay more in the market the price are likely to rise, and profit earn by the firms that also shows improvement, because π_1^* and π_2^* are rising in alpha, so all these things are rising.

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What about c , the effect of c , now c is a representation of the unit cost of production; now, c depends on technology used by a firm, because why I am saying this because as technology improves the unit cost of production of that firm goes down. So, c goes down as technology improves, how does that effect each of these variables, we can see that as c declines q_i^* 's, q that is called q_i^* that improves, because q_i^* is a declining function of c , what about P^* ? P^* declines, p_i^* rises, Q^* rises; and again the rational is not difficult to find as technology improves the firms are ready to produce are able to produce goods at a cheaper rate, they can produce goods at lesser cost.

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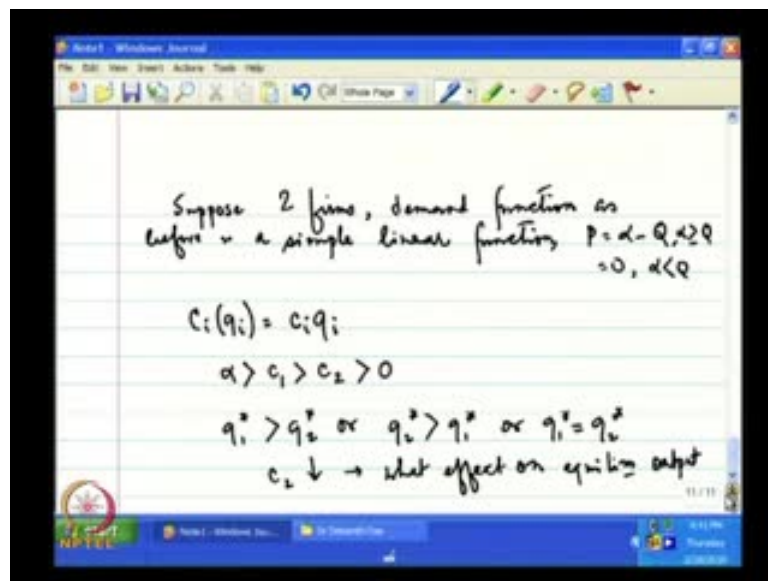
If they produce goods at a lesser cost then they are going to produce more, because the profit margin the profit that is can be obtain by producing **one more good** one more unit of output that is high; in that case they are going to produce more, that is why q_i^* is rising; P is declining that is profit equilibrium price in the market is declining, and this is happening because the firms are ready to produce goods at a higher quantity, because their cost has gone down, and since in the market more supplies has been created without any change in demand because α is remaining constant here price goes down, that is why P^* has gone down.

So, overall effect on profit is positive, which means that the firms are earning high profits now than then they were earning before, because the cost of production has gone

down, so that is why P^* has gone up; so, these are the basic pieces that we can draw from this skeletal frame that we have so far constructed.

Now, we can let talk about some applications or further extensions of the model that we have just seen, because if we remember that this model is a very elementary kind of model, it assumes that the cost of production of both the firms are the same; now, in real life that is not the case, the firms may be using different technologies, so they are unit cost of production, this we have to assumed to be C_1, C_2 .

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But it may happen that instead of this it can be like this if that is the case, if cost of production differs across firms thus the same result that we have seen before hold the answer is no, and that is what we are going to investigate in the next illustration; suppose, two firms are there like before, just to keep the story simple, the demand function as before is a simple linear function that is the inverse demand function α minus Q and $0 < \alpha < Q$.

But here the cost functions are such that the unit cost of production is different for different firms, and let us suppose c_1 is greater than c_2 and also α is greater than c_1 , because we have seen that α has to be greater than the cost of production otherwise the firms may not produce anything, and c_2 is greater than 0 , so the cost is positive and the unit cost differs for firm 1 and firm 2; in this case what is the

equilibrium? And more particularly suppose if this is the case then which firm produces more output in equilibrium.

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price, total output?

$$\text{Max}_{q_1} \pi_1(q_1, q_2) = \text{Max}_{q_1} q_1(\alpha - c_1 - q_1 - q_2)$$

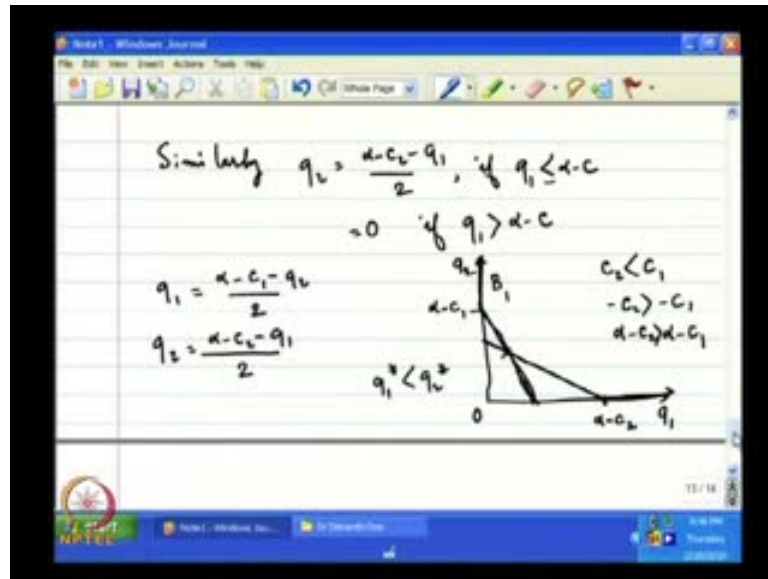
$$\rightarrow q_1 = \frac{\alpha - c_1 - q_2}{2} \text{ if } q_2 \leq \alpha - c_1$$

$$= 0 \text{ if } q_2 > \alpha - c_1$$

So, q_1 star is greater than q_2 star or q_2 star greater than q_1 star or equal; we had the case of equality before, now will that remains same or now it will change, so this is what we are going to investigate, also we are investigate suppose q_2 which is already less than q_1 falls then what is the effect? What effect on equilibrium output for example, and **price** how the price is going to change or total output; so, these are the some of the questions that we are going to answer in this little bit more realistic case where the cost of production might differ.

So, like before the way to approach this problem is not going to be different from what we have already done; so, we are going to maximize this function, profit function of firm 1, and if we do that then what we are maximizing is the following where maximizing q_1 alpha minus c_1 minus q_1 minus q_2 , this is what we are maximizing; so, if we do that then with respect to q_1 , and then with the result that we shall get is this is going to be maximized at q_1 equal to alpha minus c_1 divided by 2.

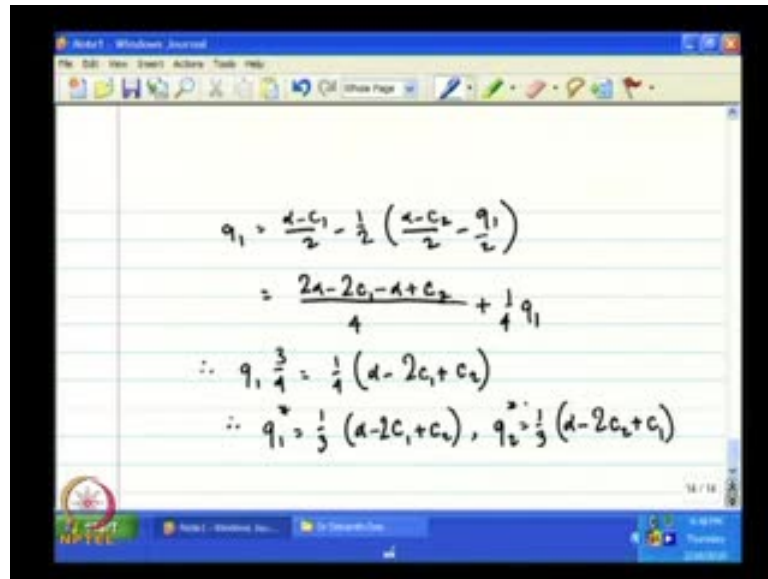
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What we have done is just instead of c which was the common cost of production before we are writing c_1 ; and similarly, this is the best response function, q_2 that is a best response function of firm 2 will be $\alpha - c_2 - q_1$ divided by 2, if q_1 is less than equal to $\alpha - c_2$ is equal to 0, so these are the best response function; and let us see what is the equilibrium in terms of a diagram.

So, if I have to draw firm 1's best response function, it is this B_1 , and this intercept is $\alpha - c_1$, what about firm 2's best response function, it is going to be something like this, where this intercept is $\alpha - c_2$; now, intentionally I have drawn this intercept $\alpha - c_2$ to be higher than $\alpha - c_1$, the reason is that c_2 is less than c_1 , which means $-c_2$ is going to be greater than $-c_1$, which means $\alpha - c_2$ is greater than $\alpha - c_1$, so this intercept on the horizontal axis is higher than the intercept on the vertical axis, and if that is the case then obviously from this illustration itself it is found that in equilibrium that is at the point of interception q_1^* is going to be less than q_2^* , so this is one result which we can at least gauge from this diagram, but is that true mathematically that we can verify, why? Because we know that the equilibrium can be found out by solving by these two equations.

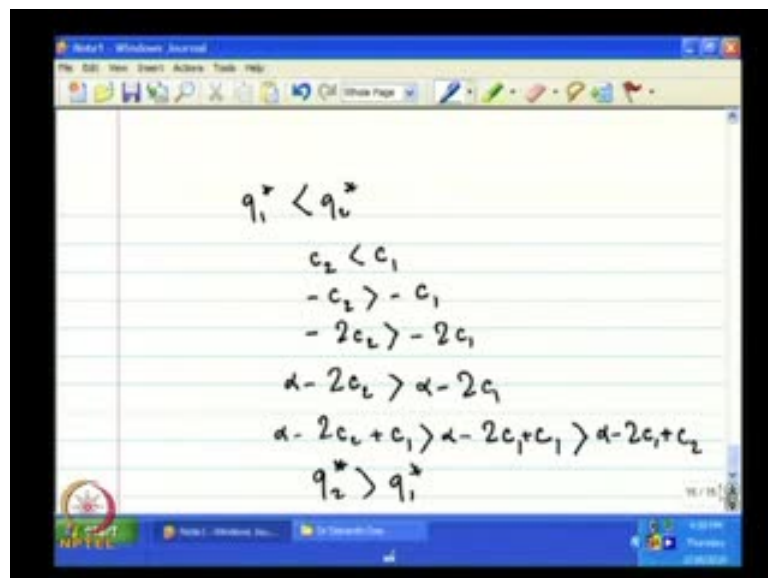
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A screenshot of a digital whiteboard showing the derivation of the best response function for firm 1. The equations are written in black ink on a white background with horizontal lines. The derivation starts with the equation $q_1 = \frac{\alpha - c_1}{2} - \frac{1}{2} \left(\frac{\alpha - c_2}{2} - \frac{q_1}{2} \right)$. This is then simplified to $= \frac{2\alpha - 2c_1 - \alpha + c_2}{4} + \frac{1}{4} q_1$. From there, it follows that $\therefore q_1 \frac{3}{4} = \frac{1}{4} (\alpha - 2c_1 + c_2)$, and finally $\therefore q_1^* = \frac{1}{3} (\alpha - 2c_1 + c_2)$, with $q_2^* = \frac{1}{3} (\alpha - 2c_2 + c_1)$ also indicated.

If we solve these two equations simultaneously we get the Nash equilibrium, and solving them we get the following that q_1 is equal to α minus c_1 divided by 2 minus half of α minus c_2 .

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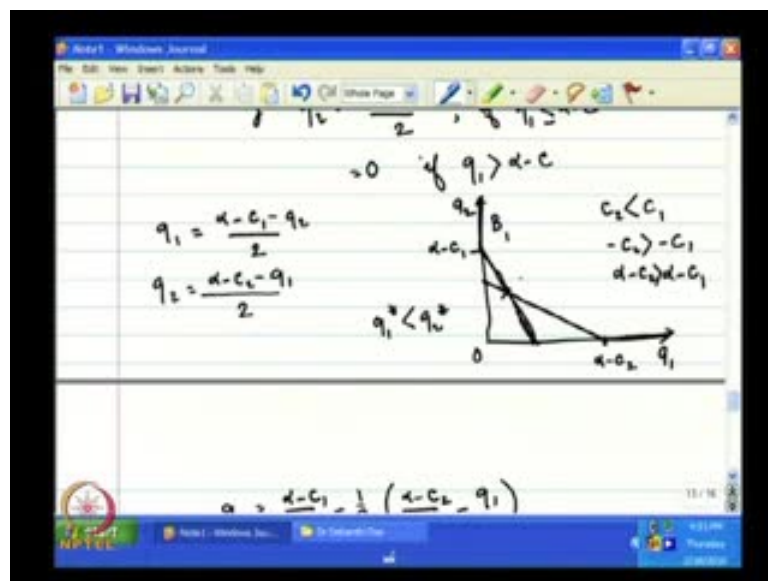


A screenshot of a digital whiteboard showing a series of inequalities. It starts with $q_1^* < q_2^*$. This is followed by $c_2 < c_1$, $-c_2 > -c_1$, $-2c_2 > -2c_1$, $\alpha - 2c_2 > \alpha - 2c_1$, and $\alpha - 2c_2 + c_1 > \alpha - 2c_1 + c_1 > \alpha - 2c_1 + c_2$. The final result is $q_2^* > q_1^*$.

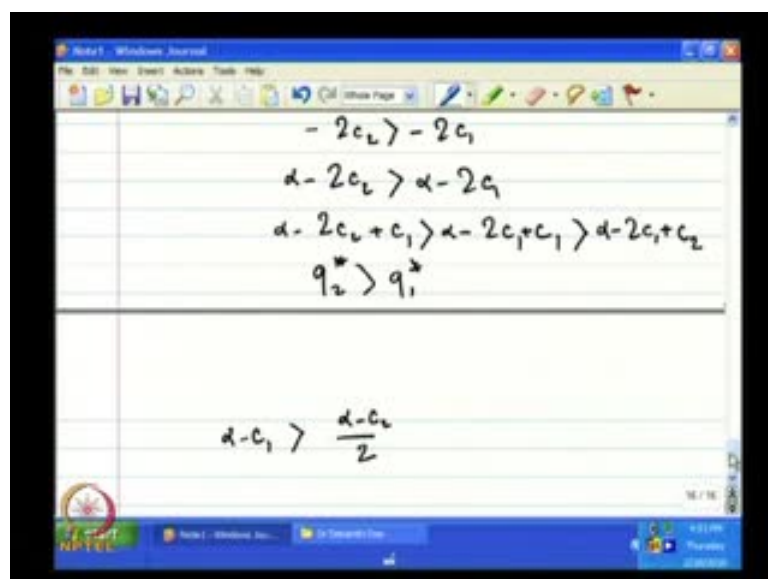
So, this is the equilibrium quantity for firm 1, if I can substitute this q_1 into the best response function of firm 2, and I can get q_2 equal to α minus $2c_2$ plus c_1 , so these are the equilibrium quantities; from this can I say that $q_1^* < q_2^*$ we can say that for a while, because we know that c_2 is less than c_1 , which means minus c

2 is greater than minus c 1, which means minus 2c 2 is going to be greater than minus 2c 1; so, **alpha minus 2c 2 is going to be greater than...** and which is going to be greater than **alpha minus...**, because what I am doing is that just replacing c 1 by c 2 and I know c 2 is less than c 1, so this must be true; so, which means that q 2 star, this is q 2 star is greater than q 1 star, so that is what we have found that in equilibrium the firm which has a lower unit cost of production is going to produce a high level of output than the other firm which has a high cost of production; **what happens** now this is the case where the firms best response functions are intersecting at a particular point.

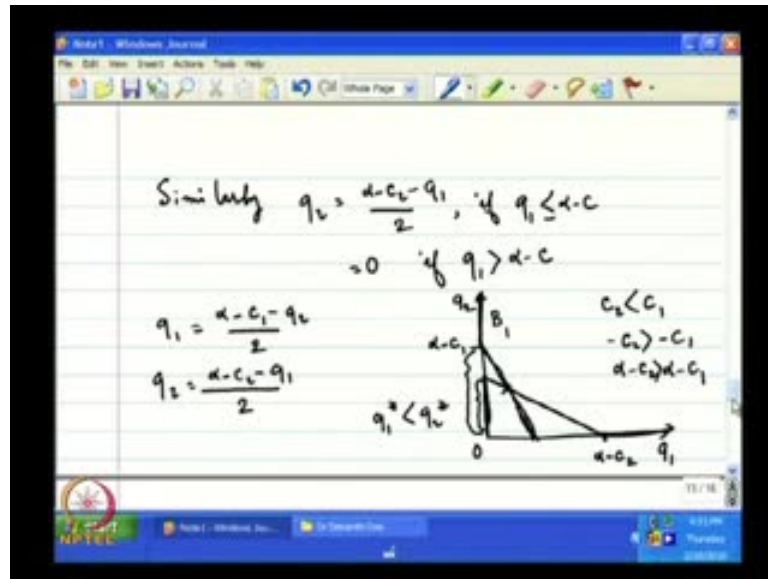
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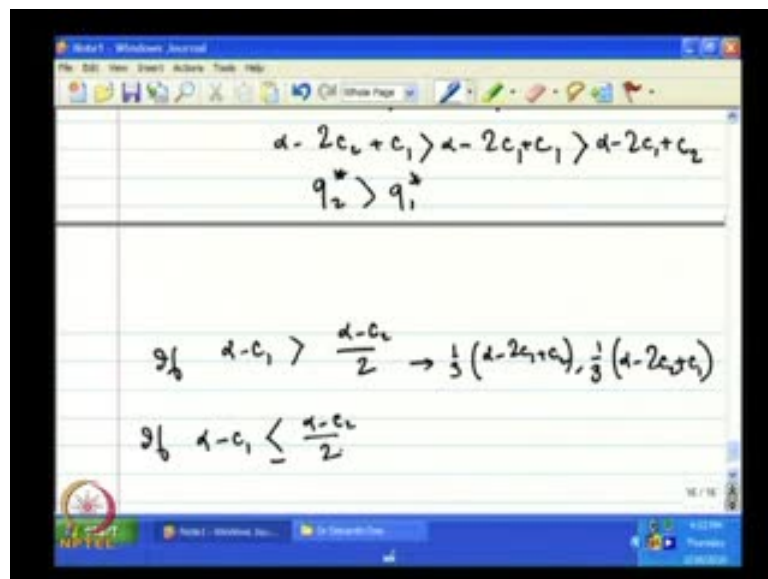
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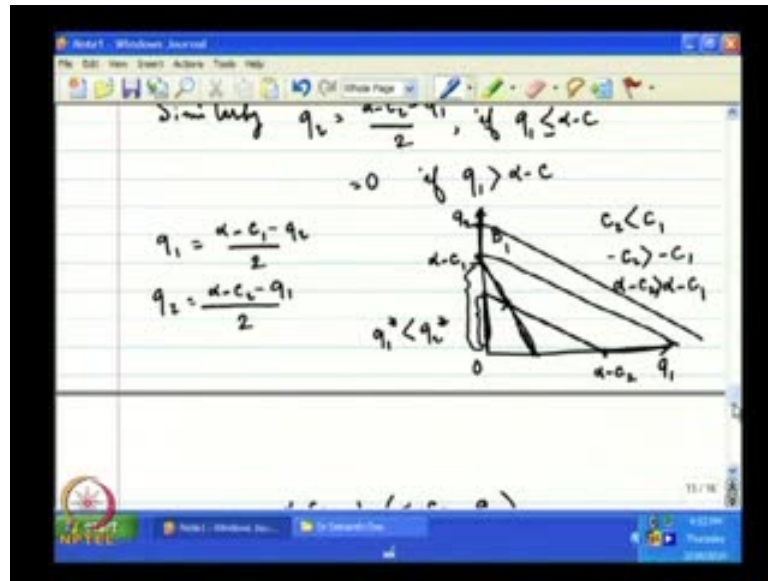


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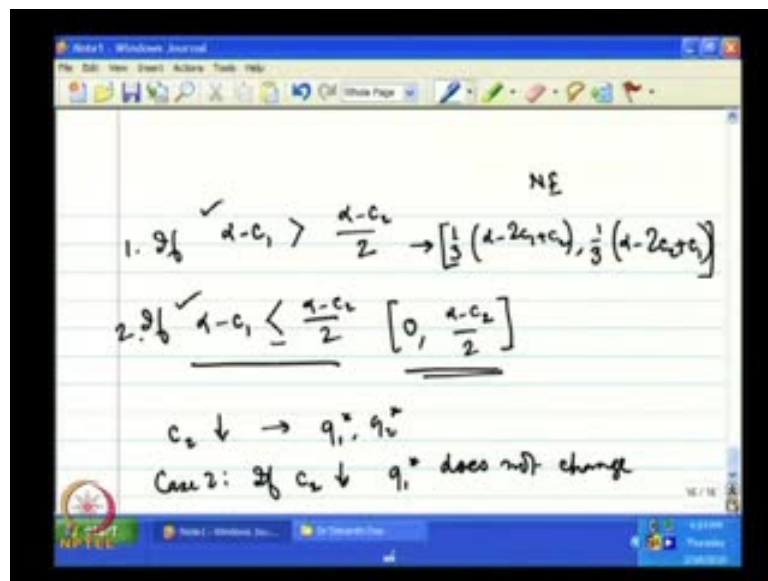


But remember this is not necessarily the case, this situation of intersection will occur in a particular condition, and that condition is that alpha minus c 1 is greater than alpha minus c 2 divided by 2, that is, this part is higher than this part, the bigger part is alpha minus c 1, and this smaller part is alpha minus c 2 divided by 2; so, only if this is true we are going to **have this Nash equilibrium of one-third of...** only then we are going to have this Nash equilibrium; if alpha minus c 1 is less than equal to alpha minus c 2 divided by 2 then what is the equilibrium?

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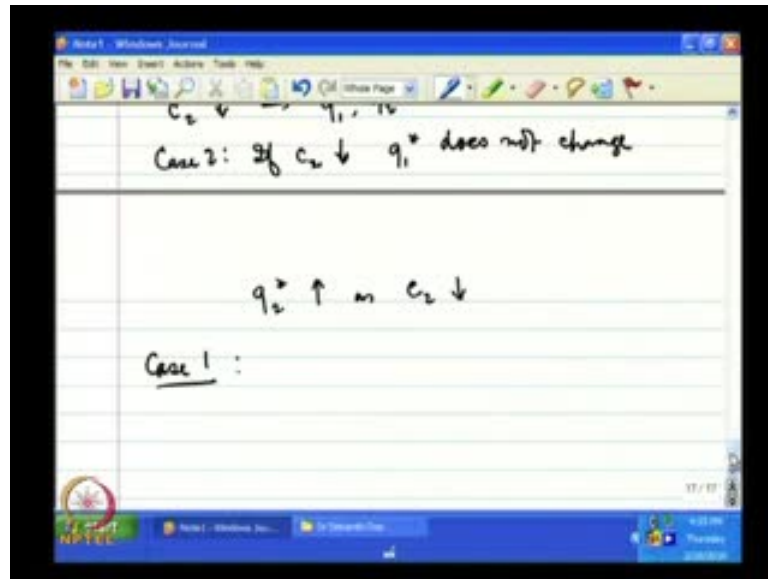
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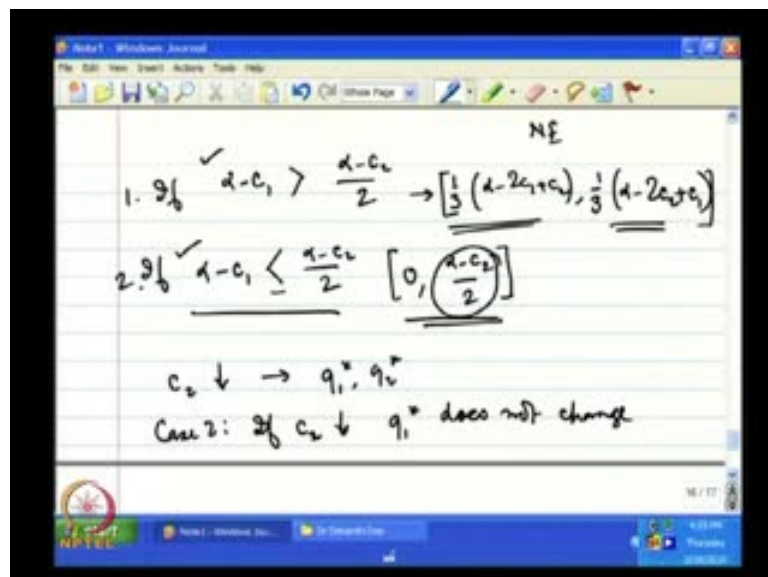
So, then we are going to have a corner solution here or so it is going to look like this, these are the points of equilibrium then on the vertical axis; then the Nash equilibrium we are going to have is 0, and the only firm 2 will produce, but how much will the firm 2 produce? Firm 2 will produce the amount which is given by its intercept, it is vertical intercept, this intercept, and what is that vertical intercept? It is just alpha minus c 2 divided by 2.

So, in all these cases, **the Nash equilibrium** in the Nash equilibrium the firm 1 is not producing the firm 2 becomes the monopolist, that is, it is the only producer in the market and the amount of output it produces is given by alpha minus c 2 divided by 2; so, what is the situation if this condition is satisfied, if this condition is satisfied both the firms are producing output.

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Now, what is what about the second part of the question; if c 2 declines what is the effect on q 1 star q 2 star, how do they behave? Now, we know that if we are having this case,

the second case, let us call it case 2, if c_2 declines further then c_1 remains same, does not change, because it is already 0, what about q_2 star? As c_2 declines further then this value is going to rise, so q_2 rises as c_2 declines; in case 1, as c_2 declines we can see that this is going to fall and this is going to rise.

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$q_2^* \uparrow$ as $c_2 \downarrow$

Case 1: As $c_2 \downarrow$, $q_1^* \downarrow$, $q_2^* \uparrow$.

$$P = \alpha - (q_1^* + q_2^*) = \alpha - \frac{1}{3} [\alpha - 2c_2 + c_1 + \alpha - 2c_1 + c_2]$$

$$= \alpha - \frac{1}{3} (2\alpha - c_1 - c_2)$$

$c_2 \downarrow$, $P \uparrow$

So, as c_2 declines q_1 star falls q_2 star rises, so these are the effects on individual output; what about the price? So, let us look at what will be the price in case 1. In case 1, the price is given by α minus q_1 star plus q_2 star, and this is α minus I can take 1 by 3 common, I do not have to do any further; from there we can find out, what happens if c_2 declines, if c_2 declines now then the effect of c_2 , and P is going to be positive, that is P is going to raise, and the reason is the following that as cost of production of firm 2 goes down firm 2 produces more output, and as a result price starts to raise, but firm 1 produces less output, firm 2 produces more output, and the net effect is that in the market the price is going to rise.

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The image shows a digital notepad with the following handwritten text:

$$Q^* = \frac{1}{3}(\alpha - 2c_2 + c_2 + \alpha - 2c_2 + c_1)$$
$$= \frac{1}{3}(2\alpha - c_1 - c_2)$$

$c_2 \downarrow, Q^* \uparrow.$

$P = \alpha - Q \rightarrow P \downarrow \text{ as } c_2 \downarrow.$

Whatever the effect of Q total output, it is nothing but the summation of individual outputs, **which means it is one-third of...**, which means that if c_2 declines then Q^* is going to rise, so I was wrong about this effect on P that is price equilibrium price as Q^* rises what happens to price? Price is α minus Q , if this Q is rising, so it means that the price is declining as c_2 is declining.

(Refer Slide Time: 46:38)

The image shows a digital notepad with the following handwritten text:

Case 2 Only firm produces

$$q_2^* = \frac{\alpha - c_2}{2}$$
$$q_1^* = 0, Q^* = \frac{\alpha - c_2}{2}$$
$$P = \alpha - \frac{\alpha - c_2}{2}$$

Here as $c_2 \downarrow, q_2^* \uparrow, q_1^*$ is unchanged, $Q^* \uparrow, P \downarrow$

(Refer Slide Time: 47:50)

The image shows a digital notepad with handwritten mathematical equations. The top section shows the price P as a function of α , c_1 , and c_2 . The bottom section shows the total quantity Q^* as a function of the same variables, along with a note on its response to a change in c_2 .

$$P = \alpha - (q_1^* + q_2^*) = \alpha - \frac{1}{3}[\alpha - 2c_2 + c_2 + \alpha - 2c_1 + c_1]$$
$$= \alpha - \frac{1}{3}(2\alpha - c_1 - c_2)$$

$c_2 \downarrow, P \downarrow$

$$Q^* = \frac{1}{3}(\alpha - 2c_2 + c_2 + \alpha - 2c_1 + c_1)$$
$$= \frac{1}{3}(2\alpha - c_1 - c_2)$$

$c_2 \downarrow, Q^* \uparrow.$

So, that is how it turns out. In case 2 where only firm 2 produces, and the production is α minus c_2 divided by 2 and q_1^* is 0, the price sorry the quantity is obviously α minus c_2 divided by 2; what about the price? Price is this much, so here as c_2 falls, q_2^* is raising, q_1^* is unchanged, capital Q^* that is total output is going to raise, and P is going to decline as c_2 is going to fall.

(Refer Slide Time: 45:15)

The image shows a digital notepad with handwritten mathematical equations. The top section shows the total quantity Q^* as a function of α , c_1 , and c_2 . The bottom section shows the price P as a function of α and Q^* , with a note on its response to a change in c_2 .

$$Q^* = \frac{1}{3}(\alpha - 2c_2 + c_2 + \alpha - 2c_1 + c_1)$$
$$= \frac{1}{3}(2\alpha - c_1 - c_2)$$

$c_2 \downarrow, Q^* \uparrow.$

$$P = \alpha - Q \rightarrow P \downarrow \text{ as } c_2 \downarrow.$$

So, the effect is like before that as cost of production of the firm 2 goes down, that is the firm which is the more efficient firm, firm 2 was the more efficient firm, c_2 was less

than c_1 , that is the cost of production of firm 1; as cost of production of firm 2 declines firstly in equilibrium firm 2 produces more output, that is irrespective of the case where firm 1 was in fact producing some output in equilibrium, that is the irrespective of that firm 2 always produces more output if its cost of production declines as a result total output tends to raise, and as total output tends to rise total the market price that tends to fall.

(Refer Slide Time: 46:38)

Case 2 Only firm produces

$$q_2^* = \frac{d - c_2}{2}$$

$$q_1^* = 0, \quad Q^* = \frac{d - c_2}{2}$$

$$P = d - \frac{d - c_2}{2}$$

Here as $c_2 \downarrow$, $q_2^* \uparrow$, q_1^* is unchanged, $Q^* \uparrow$, $P \downarrow$

So, this is the overall conclusion of this model where cost of production differs; and what is basically happening here is, one may ask that if firm 2 is producing more output because its cost of production is declining, why is the case that firm 1 is producing less output, well, the answer is that as firm 1 as firm 2 is producing more output in the market the supply is going up; as the supply is going up in the market the price is going down, and as the price is going down if the firm 1 is getting more and more discourage to produce any output, so that is why q_1^* is going down when c_2 is declining.

So, that is basically the logic which is in operation here, so this is the cracks of the Cournot model, how one firm behaves, how much output one firm produces, that effects the other firms production level; the logic is this, if one firm produces more output in the market the supply rises, if the supply rises then the market clearing price will have to go down, and if the market clearing price goes down then the other firm finds it difficult to sustain the same level of output, it then cuts down its output, and this is what exactly

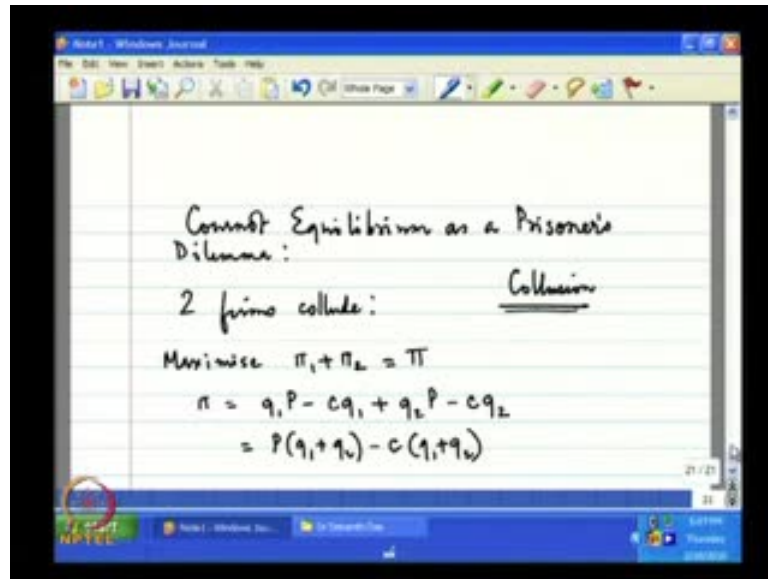
what is happening if the more efficient firm that is firm 2 is producing more output, because cost of production has come down; the first firm is getting shut out from the market, and we have seen that in case 2, that is, this case firm 1 is producing nothing, and remember this condition what this condition is basically saying is the following.

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$$\alpha - c_1 < \frac{\alpha - c_2}{2}$$
$$\alpha - c_2 \geq 2\alpha - 2c_1$$
$$\therefore 2c_1 \geq \alpha + c_2$$

This is alpha minus c 1 is, this can be alternatively written as, so this condition translates to this that 2c 1 is greater than equal to alpha plus c 2, which means that related to c 1 c 2 is very little it is a small value, which means that the cost of production of firm 2 is has become so less alternatively the cost of production of firm 1 relatively has become so high that firm 1 is not finding it profitable to produce in the market only firm 2 is producing in the market, so that is the logic of a topic.

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Handwritten notes on a lined paper background, titled "Cournot Equilibrium as a Prisoner's Dilemma". The text describes a collusion scenario between two firms and provides the profit maximization equation.

Cournot Equilibrium as a Prisoner's Dilemma:

2 firms collude: Collusion

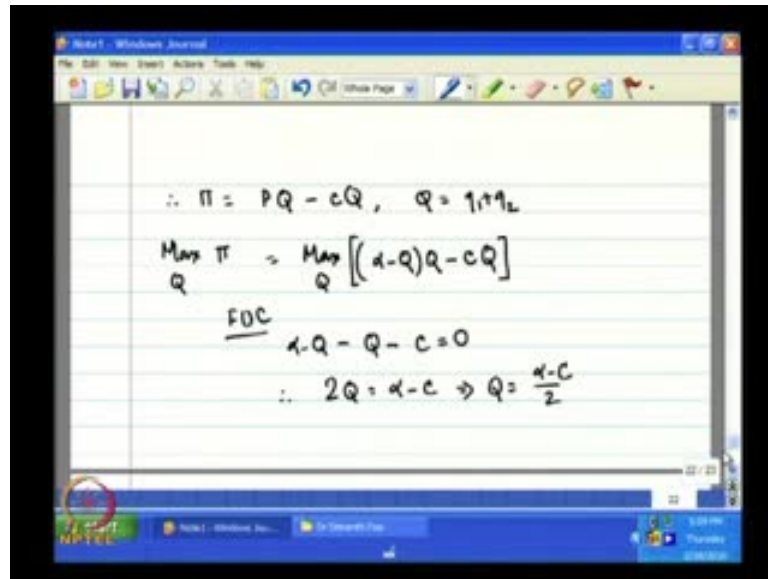
Maximize $\pi_1 + \pi_2 = \Pi$

$$\pi = q_1 P - c q_1 + q_2 P - c q_2$$
$$= P(q_1 + q_2) - c(q_1 + q_2)$$

Now, one more interesting property of Cournot equilibrium, it is a case of Prisoner's dilemma, we are going to show that in terms of an illustration; suppose, that instead of competing with each other these two firms collude, what it means is that, these two firms instead of fixing their output separately and trying to maximize their individual profit, taking the quantity of the other firm to be given, suppose they decide that let's us meet together, and let us decide what the total output in the market is going to be, what is the total supply in the market going to be, because we have the control over the total supply in the market.

So, let us decide what is the total supply in the market going to be which is going to maximize the total profit, and once the total profit is maximized we can divide that total profit equally that can be an alternative way instead of fighting with each other, so that is called a collusion, which means the firms are taking their decision in an united fashion. So, what happens then? So, what the firms are deciding now is to maximize, how to maximize, that is, π_1 plus π_2 the total profit, let us call it π , that is what they maximize.

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The image shows a digital whiteboard with handwritten mathematical equations. The equations are as follows:

$$\therefore \pi = PQ - cQ, \quad Q = q_1 + q_2$$
$$\text{Max}_Q \pi = \text{Max}_Q [(a - Q)Q - cQ]$$
$$\text{FOC} \quad a - Q - Q - c = 0$$
$$\therefore 2Q = a - c \Rightarrow Q = \frac{a - c}{2}$$

Now, what is π_1 and π_2 it is $q_1 P$ minus cq_1 , we are retaining the old assumptions, that is, the unit cost of production of both the firms is equal and we have this simple linear demand function; so, the total profit the united profit is let us call q_1 plus q_2 as capital Q .

Now, so, what the firms will now do is to maximize π with respect to capital Q instead of bothering about individual profits and maximizing the individual profit with respect to the small q 's; and if it does so what we get is the following, this is $a - Q$ multiplied by Q minus this, and what is the solution? $a - Q$ this is the first order condition minus Q minus c is equal to 0, which means that $2Q$ is equal to $a - c$, capital Q is equal to $a - c$ divided by 2, so this is the first order condition. We can check that the second order conditions will be satisfied, because if I differentiate this with respect to Q once more I get minus 2, which is negative, so the second order condition is satisfied.

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A screenshot of a Windows Journal window showing handwritten mathematical derivations. The equations are as follows:

$$q_1 = q_2 = \frac{1}{4} (\alpha - c)$$
$$\pi_1 = \pi_2 = \frac{1}{2} \pi$$
$$\pi = PQ - cQ$$
$$= \left(\alpha - \frac{\alpha - c}{2} \right) \frac{\alpha - c}{2} - c \frac{\alpha - c}{2}$$
$$= \frac{\alpha - c}{2} \left(\alpha - c - \frac{\alpha - c}{2} \right) = \frac{\alpha - c}{2} \cdot \frac{1}{2} [2\alpha - 2c - \alpha + c]$$
$$= \frac{\alpha - c}{4} (\alpha - c) = \frac{(\alpha - c)^2}{4}$$

So, this is going to be the total output if the firms decide to maximize their joint profit - their united profit - so individual output level will be half of this; what about individual profit? Individual profit will be half of the total profit, what is the total profit here, total profit is PQ minus cQ which is...

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A screenshot of a Windows Journal window showing handwritten mathematical derivations. The equations are as follows:

$$\pi_1 = \pi_2 = \frac{1}{2} \pi$$
$$\pi = PQ - cQ$$
$$= \left(\alpha - \frac{\alpha - c}{2} \right) \frac{\alpha - c}{2} - c \frac{\alpha - c}{2}$$
$$= \frac{\alpha - c}{2} \left(\alpha - c - \frac{\alpha - c}{2} \right) = \frac{\alpha - c}{2} \cdot \frac{1}{2} [2\alpha - 2c - \alpha + c]$$
$$= \frac{\alpha - c}{4} (\alpha - c) = \frac{(\alpha - c)^2}{4}$$

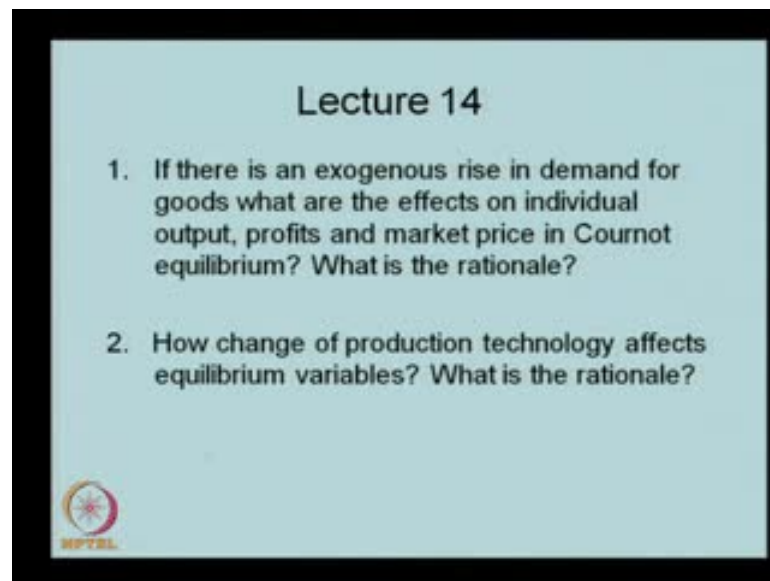
$$\pi_1 = \pi_2 = \frac{(\alpha - c)^2}{8} \quad ||$$

So, this is the total profit, which means that the individual profit, that is its going to be half of that, which means α minus c whole square divided by 8; so, this is, let us stop here, so this is the case where the firms unitedly decide how much they produce they will

produce, and we have seen that the output they are going to produce is $\alpha - c$ divided by 2 and the profits that they earn is $(\alpha - c)^2$ divided by 8; before we finish just recapitulate what we have done.

We have basically discussed the various aspects of Cournot equilibrium, Cournot oligopoly model, we have seen that in equilibrium the firms produce the same level of output; and if one of the firms is more efficient then that firm produces more output than the other firm which is not very surprising, and if that firm becomes too much efficient than the other firm may be out of the market this, more efficient firm becomes the monopolist, and we are in the process of discussing what happens if the firms decide their output levels united in an united fashion, so we shall pick up the thread from here in next lecture. Thank you.

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First question, if there is an exogenous rise in demand for goods what are the effects on individual output, profit and market price in Cournot equilibrium? And what is the rationale?

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Equilibrium in Cournot Model

$$q_i^* = \frac{\alpha - c}{3}, \quad P^* = \frac{1}{3}(\alpha + 2c)$$
$$\pi_i^* = \frac{(\alpha - c)^2}{9}$$

$Q^* = \alpha - P$

Exogenous rise in market demand $\rightarrow \alpha \uparrow$

As α rises $q_i^* \uparrow$, $P^* \uparrow$, $\pi_i^* \uparrow$.

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To understand this let us recall what is the equilibrium in the Cournot model; so, if we have the standard assumption that linear demand curve and constant unit cost, then we know that individual quantities going to be alpha minus c divided by 3, the equilibrium price is going to be one-third of alpha plus 2c and profit is going to be individual profit alpha minus c whole square divided by 9. Now, if there is an exogenous rise in demand, this basically means alpha is going up because demand function if you remember was $Q = \alpha - P$, so alpha rising means the demand curve is shifting upwards.

Now, we can see that as alpha rises q_i^* is going to rise, P^* is going to rise, π_i^* is going to rise; and what is the reason why is it happening, what is the rationale, as alpha rises there is an exogenous shock upward shock on the demand, people are ready to buy more; if they are ready to buy more obviously in the market price is going to rise, and as price is rising the firms find that their profits are rising, and when the profits are rising their going to respond by rising quantity, the amount of output that they produce, that is why q_i^* rises as a result.

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Lecture 14

1. If there is an exogenous rise in demand for goods what are the effects on individual output, profits and market price in Cournot equilibrium? What is the rationale?
2. How change of production technology affects equilibrium variables? What is the rationale?



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Production Technology:
If prodⁿ technology improves, cost of production, c , is going to decline.
 $q_i^* = \frac{a-c}{3}$, $p^* = \frac{1}{3}(4+2c)$
 $\pi_i^* = \frac{(a-c)^2}{9}$
As $c \downarrow$, $p^* \downarrow$, $q_i^* \uparrow$, $\pi_i^* \uparrow$.



How change in production technology affects equilibrium variables? What is the rationale? So, this is similar question, production technology - how is it reflected in this particular model; now, if production technology becomes more improved, if it improves then **simplistically** very simplistically we can say that the cost of production is going to come down, cost of production which is small c is going to decline, so this change in technology is basically manifesting itself in terms of change in c ; now, how does it affect our variables? Let us again recall, what is the quantum of each of the variables.

So, these how the variables look like; now, unlike before here if c declines then the effect on each of these variables is not in the same direction as c declines, we can see that P^* is going to decline, as cost of production declines each of the firms find it easy to reduce the price a little bit, and it is not the case that there controlling the prices; but what is actually happening is that as the cost of production declining, then they want to produce more, because it is possible for them to profitably produce more when the cost is declining to maximize the profit.

So, basically what is happening is that, q_i^* is going to rise, and that rise in q_i^* is basically raising the total supply in the market and reducing the price in the market and p_i^* is rising because cost is declining, that is the intuition. Thank you.