Game Theory and Economics

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Module No. # 03

Illustrations of Nash Equilibrium

Lecture No. # 03

Further Aspects of Cournot Model

Welcome to the third lecture of module 3 of this course called game theory and economics. Before we start this lecture let me recapitulate what we have done in the previous lecture. Basically, we have been discussing the applications of Nash equilibrium, and the topic that we have been discussing is topic of oligopoly market called Cournot oligopoly model.

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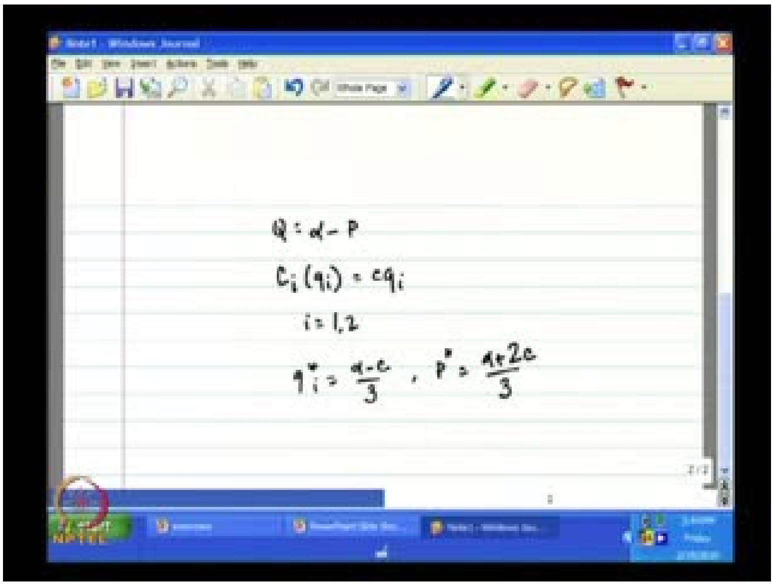

We have found that in the previous lectures we have found that in equilibrium in the Nash equilibrium in Cournot market. f there are two firms then both the firms will produce some output, that output level is given by if we have a linear demand curve given by Q is equal to alpha minus P, this is the linear demand curve, and cost function is given by this, where i can be 1 and 2. Then we have found at the equilibrium, output level is equal to alpha minus c divided by 3.

So, both the firms will produce the same level of output, and the price will be alpha plus 2c divided by 3, so this is the equilibrium quantity and equilibrium price by these two firms; and we have been discussing the various aspects of this Nash equilibrium. In this Nash equilibrium, the firms will produce some level of output, and the profits that they will earn will be positive which means, they are earning some profits, it is not that they are earning 0 profit. Of course, if they earn negative profits that is loss then they are not going to produce anything so that cannot be an equilibrium, if they produce negative profit. But it is possible that they produce some output and at that output level the profit that they earn is 0.

So, one may ask that if they are not earning any profit how come they are producing any goods at all, why do not they just quit. The answer is that even if the firms produce some output at which the profit is 0, it is assumed that in that cost the normal profit of the firms is included. So, by profit is equal to 0 what is meant is that they are super normal profit or extra profit that is 0, their normal profit or the minimum level of profit which is required to sustain themselves is earned if we find that the profit is 0.

So, these are some basic clarifications. Now, the question that we were discussing in the previous lecture is, suppose the firm instead of fighting with each other instead of competing with each other, decide that jointly what is the level of output that will maximize their joint profit; and if that is the output level which they can determine then each of the firms, there are two firms here, each of the firms will produce just half of that output and that output they will take to the market to get sold and they will earn some profit.

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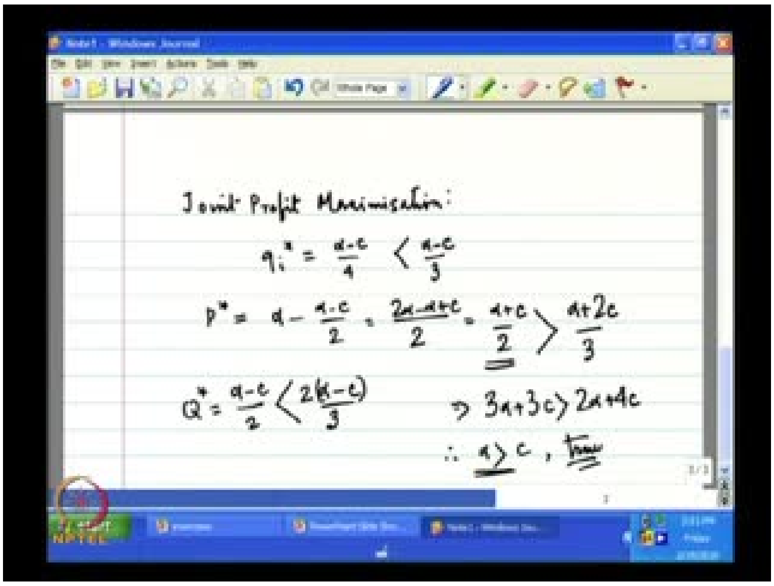

So, is that profit is going to be high or low, than the profit which they earned by producing individually, by just competing with each other, by not coordinating with each other, that is the question that we were discussing in the previous lecture; and we had found that…, so this is joint profit maximization, and we have seen that this is at alpha minus c divided by 4, which means that this is less than alpha minus c divided by 3.

So, if they decide that they are going to coordinate there actions, and try to produce an output level where the total profit is maximum, then the output that they are producing is less than what they do produce if they compete with each other, so that is the first thing. Secondly, what is the price here, we have found in the previous lecture that the price in this case will be alpha minus alpha minus c divided by 2, which is alpha plus c divided by 2, which means this price is less; how do I get this price, because price if you remember the demand curve is given by p is equal to alpha minus q, here the total output is alpha minus c divided by 2.

So, price here is alpha minus this, and so we have got this alpha plus c divided by 2, where as the price in which case they were competing was alpha plus 2c divided by 3, this is true; how do I know that the price here is higher, if the firms decide to produce jointly, the reason is that if they decide to produce jointly we see that the total quantity being produced is alpha minus c divided by 2, this is Q star, and this is higher if they decide to produce individually, this is the output that there are producing, this is less than this.

So, total output in the market that they are supplying is less if the total output they are supplying in the market is less price is going to be higher, that is why I know without doing the calculations I know that alpha plus c divided by 2 is going to be higher than alpha plus 2c divided by 3. So, that is that I can confirm that also for this to happen I must have 3 alpha plus 3c greater than 2 alpha plus 4c, which means alpha is greater than c and which is true.

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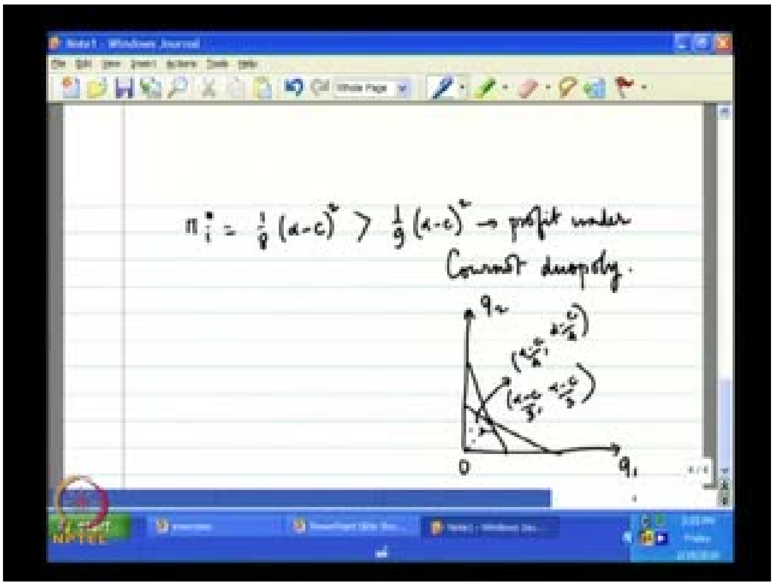

So, basically, price has gone up, total quantity has gone down, and whatever the individual profits, because that is what the firms are after all maximizing; in this case the individual profits we have calculate in the previous lecture, it was 1 divided by 8 alpha minus c divided whole square, and this I know is greater than alpha minus c whole square divided by 9, and this is the profit under Cournot duopoly.

So, the cracks of the story is that, if the firms jointly try to maximize their profit then the total output that they are going to produce is coming down, the price in the market is going up, and the individual profit is also going up. Now, one obvious question that may occur is that if the individual profits are going up, then why do not the firms produce that level of output, why do not the firms produce alpha minus c divided by 4, why do they produce alpha minus c divided by 3, which they do if they compete with each other, why do not they coordinate with each other, and the answer is that it is like the prisoner's dilemma came that we have seen in the beginning, given what the other firm is going to produce.

Suppose, I am firm 1, and there is another firm 2, and we have decided, well, let us maximize our profits ,and let us produce alpha minus c divided by 4; now, once c produces alpha minus c divided by 4, then it is not in my interest to produce alpha minus c divided by 4, this alpha minus c divided by 4 is not on the best response function of both the firms, because we have seen that there is only one point at which the best response functions intersect with each other which is apt alpha minus c divided by 3.

So, this is the 45 degree line, alpha minus c divided by 4 will be somewhere here, so this is the point of alpha minus c divided by 4; so. if the other firm is producing alpha minus c divided by 4, suppose the firm 2 is producing this, then what should the firm 1 do, firm 1 should be should try to be on its best response function.

So, what it should do is that, given this it will produce this much, so it is just steering away, it is just not staying on this alpha minus c divided by 4, alpha minus c divided by 4 line, it is deviate, and that is why this is not going to sustain, this alpha minus c divided by 4 is not going to sustain as the equilibrium; it is like that situation in prisoner's dilemma where the two prisoners both of them could have done better if they had not confessed, but the point is that if my partner does not confess, it is in my interest to confess.

So, I do not agree to that non confess impact, here also given that the other firm is producing alpha minus c divided by 4, I do not produce alpha minus c divided by 4, and so there is a deviation, and so that is why this is not an equilibrium; and finally, we shall reach this equilibrium which is the Nash equilibrium of alpha minus c divided by 3, where the profits have gone out gone down compare to the case where the outputs are alpha minus c divided by 4. The same thing can be shown in terms of a different diagram, which is the diagram having what are known as Isoprofit curves.

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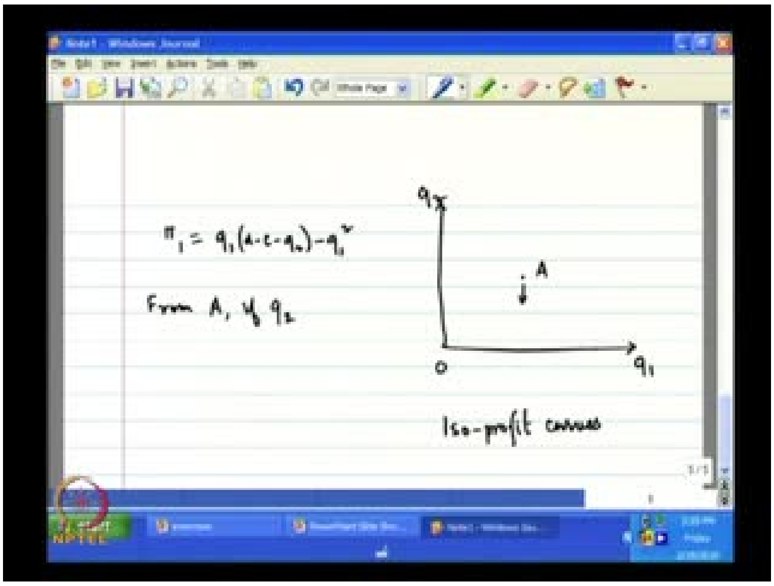

So, what are Isoprofit curves? Well, Isoprofit curves are those curves along which profit of a firm remains constant, it remains the same; so, suppose, I want to draw the Isoprofit curve of firm 1, I know that the firm one's profit function that is pi 1 is given by the following function, it is given by…, this is the profit function of firm 1, given that q 1 plus q 2 is less than equal to alpha.

Now, if this is the profit function of firm 1, how do I draw the Isoprofit curve of firm 1? Now, let us start from any point here; suppose, this is the point was starting from A, from A if q 2 if q 2 declines that is firm 2 is producing less output. If q 2 declines I can straight away c from this pi function pi 1 function that pi 1 is going to rise; so, if I go down this direction I am going to reach a higher level of profit for firm 1.

So, if I have to draw function Isoprofit function of firm 1, it has to be that Isoprofit function cannot be a vertical line, because if it is a vertical line that will show that the profit is remaining constant whether we go up or down, so it cannot be a vertical line.

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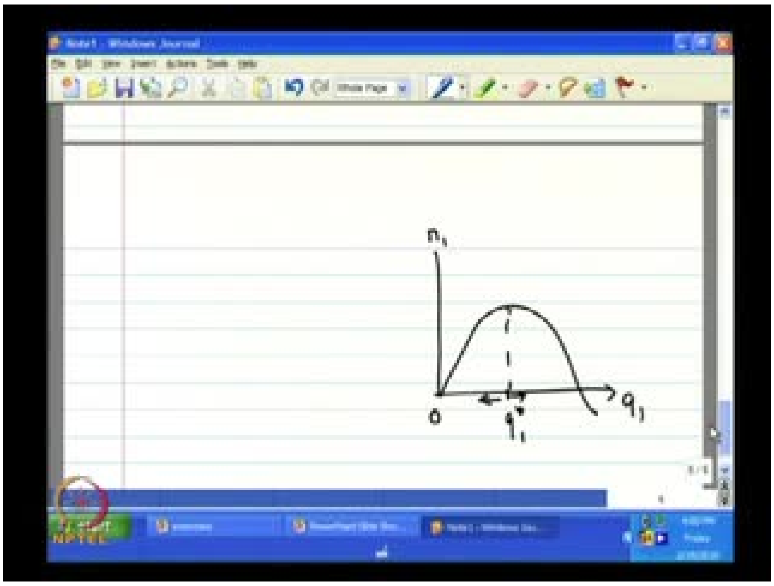

Then what kind of line is it, if for example q 1 rises or falls how does pi 1 respond to that, we know that the answer to that is it is not very clear sometimes it can rise it can fall; remember, that other curve we have drawn before the profit function of firm 1, it looks like this; so, at any q 1, if this is the q 1, q 1 star, suppose if I go on go to this direction, if I go to this direction pi falls.

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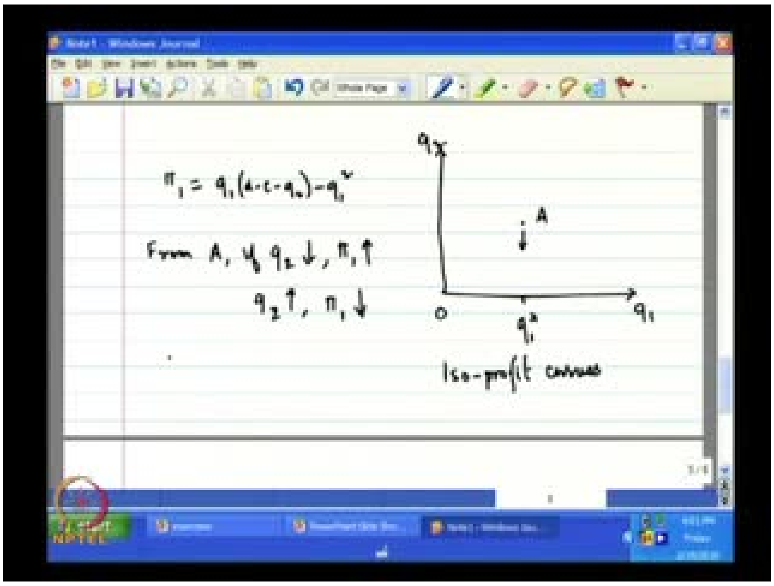

So, sometimes it rises, at some times it falls, but at certain level of pi which is suppose pi star that profit becomes maximum; so, let us call this as pi 1 star; so if I go to this direction to the right of A profit falls, if I go the left of A again profit falls. So, these to the left or to the right if I draw the Isoprofit functions they will signify lower level of profit; and if I go to towards down to the southern direction from A the profit should raise, those curves should represent higher level of profit for found.

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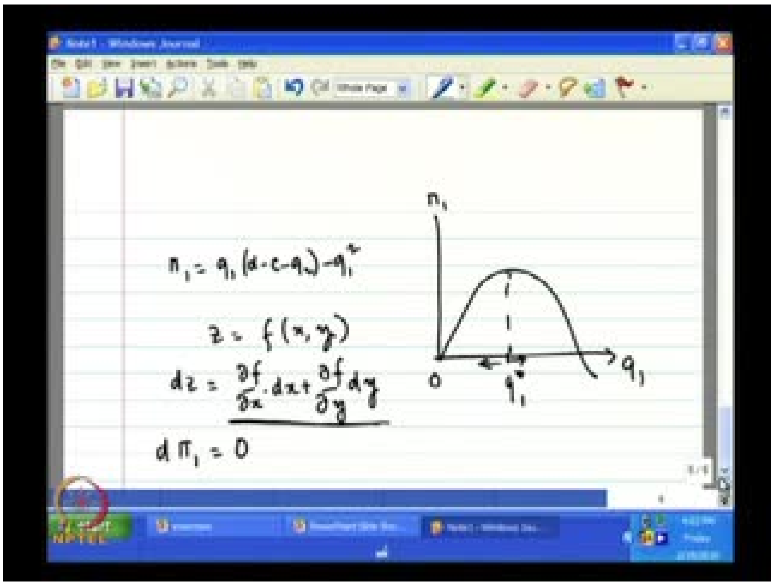

So, these are some of the general conclusions that we can draw from the things that we already know; so, if it falls now, can we have some specific idea about this function, how does this function look like, for that what we do is to find out what is a total derivative of this function, we have this pi 1 is equal to q 1. Now, by total derivative what we mean is that, suppose I have a function z as a function of x y, then if I want to find out what is d z, it is the partial derivative of f with respect to x multiplied by d x plus partial derivative of the function with respect to y multiplied by d y, so this is total derivative. I take d pi 1 to be 0, because along the Isoprofit curve I know profit does not change, so that I apply, then I apply this formula, here instead of x I have a q 1, and instead of y I have q 2.

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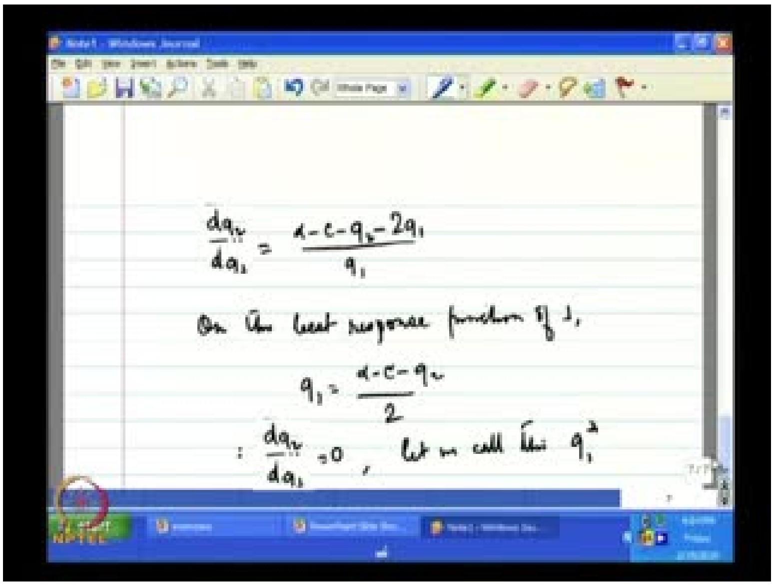

So, if I apply that formula and do some derivation what I get will be the following, d q 2 divided by d q 1 will be equal to alpha minus c divided by q 1; and from this formula for slope of the Isoprofit function of firm 1 we can immediately see that if we are on the best response function of player 1, that is on the best response, what do I know in on the best response function of player 1 I know that q 1 is equal to alpha minus c dividing minus q 2 divided by 2. Now, if this is true then on the best response function when the firm was is maximizing the profit whatever the slope d q 2 by d q 1 then will be equal to 0 right; the profit when the profit is maximum for firm 1 the slope of the profit function the Isoprofit function is 0.

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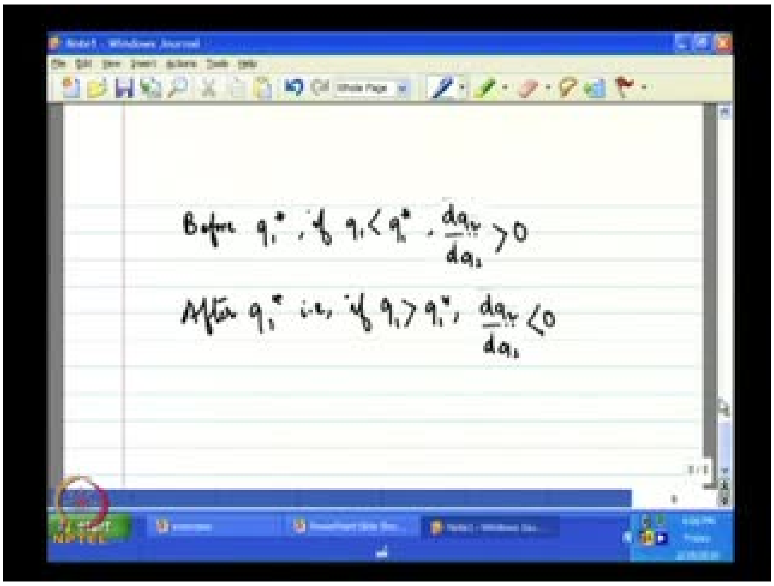

Before let us say let us call this q 1 star, this is the q 1 star that is coming here, and this is the same q 1 star which is coming here also, so at q 1 star the slope of function of firm 1 is 0. Before q 1 star, that is if q 1 is less than q 1 star, what do I have is this is less this q 1 is less than what could have made this slope 0, which means that this is going to positive, the slope is going to be positive right; and after q 1 star that is if q 1 is greater than q 1 star, so q 1 has exceeded the value for which this is equal to 0, it has is exceeded that value which means that this term is high now, which means this is going to be negative.

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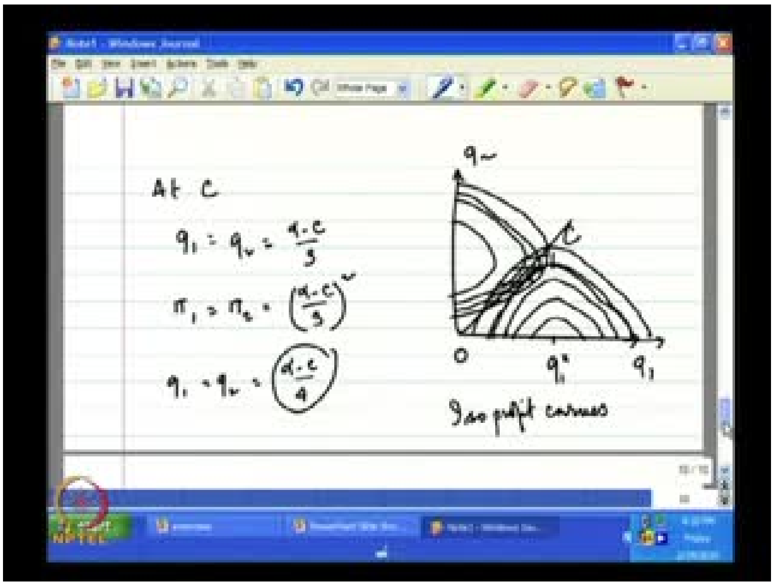

So, these are the information that I have, and from this I can now draw the Isoprofit function; suppose q 1 star is here, and suppose this is the q s q 2 we are starting from then before that I have a upwards rising part and after that I have a downward sloping part, this is how the Isoprofit function of firm 1 will look like; and if I go down then I know that the profit is going up.

So, there will be other Isoprofit functions, but they will signify higher and higher level of profit, where as if I go up then they will signify lower and lower level of profit, so these are the Isoprofit curves. By the same logic just as we have use the logic of firm 1 same logic could be used for firm 2, and firm 2's Isoprofit functions will be someone like somewhat like this.

Now, the question is where is the Nash equilibrium? Well, I know the Nash equilibrium will be at a point where both the firms are at there best response functions, and which means that the curves this two sets of Isoprofit curves I have to find out that point where the Isoprofit curve of firm 1 will have a 0 slope, and the Isoprofit curve of firm 2 will have a slope of infinity, and that I can see it is happening here.

So, this point let us call it c, this corresponds to this point c here, this c the Cournot equilibrium point; and from this immediately it is possible it is obvious that at c the firms are earning some profit at the equilibrium at c they are earning q 1 is equal to q 2 is equal to alpha minus c divided by 3, and profit is equal to I know alpha minus c divided by 3 whole square this is the profit, but it is possible that the both the firms can earn high profits; and how do I know this, because this is the region, this shaded region is the region where they can earn both of them can earn high profits, because if you remember if I go down from c then firm 1 is having greater profit.

So, if I consider this Isoprofit curve is passing through the shaded region, and which means that this Isoprofit curve is showing high profits and which includes points in the shaded region, but this point on the shaded region also signify high profits for firm 2; so, both the firms can earn high profits in this shaded region inside the shaded region, and our solution of joint profit maximizing output which was alpha minus c divided by 4 is somewhere here, in particular I am not going to show that in particular that point can we found by using two things first I join 0 and c, that will o and c, origin and c, this is the 45 degree line.

I know this point alpha minus c divided by 4 has to be on the 45 degree line, because both the firms are producing the same level of output here, and what additional property it must satisfy is that at this point that is alpha minus c divided by 4, the Isoprofit curves of both the firm should be tangent to each other; if that is satisfied then there is going to be only one unique point, and that unique point is the point where this point profit is been maximized.

So, two summit up what is happening is that, in Cournot equilibrium the firms are competing with each other and producing some output level and earning some positive profit, that is well and good; the great point is that, if they get together and try to produce joint output and try to maximize there total profit not individual profits, then their output levels are different; and that at that output levels the total profit that they are going to earn will be such that their individual profit that is share of there that the total profit is also going to rise compare to the case where they were competing which with each other; so, there is a possibility that both of them can earn better profits, but that outcome is not sustainable, because that point at that point nobody is maximizing his or her called individual profit; and if nobody is maximizing his or her individual profit, it is profitable for both of them to deviate if the other firm is sticking to that joint profit maximizing output, so this is therefore is the situation like the prisoner's dilemma situation.

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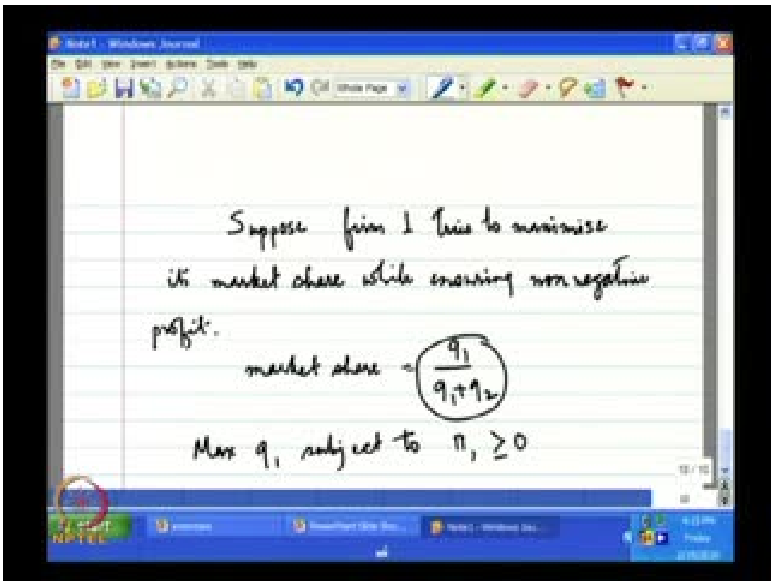

We shall do some other properties of Cournot equilibrium; one exercise that we are interested in, suppose, firm 1 tries to maximize its market share while ensuring non negative profits. Suppose, so for the setting that we have is that both the firms are maximizing their individual profits. Now, suppose firm 2 is maximizing it its profit, but firm 1 tries to maximize its market share, which means it wants to maximize q 1 divided by q 1 plus q 2, q 1 is the amount that it is able to sell to the market, and q 1 plus q 2 is a total goods being sold, so this is the market share.

Now, one can show that if q 1 is maximized then this market share is also going to be maximize, which means if q 1 goes on rising this thing is also going to rise; now, if that is the case then what the firm 1 is doing is that it is maximizing q 1 but there is a condition subject to this the profit has to be either 0 or positive profit cannot be negative, that is the firm 1 is ready to get its normal profit which is the minimum profit to sustain itself provided that it can capture the market, it can capture as high portion of the market as possible.

Now, in that case what will the firm 1 produce, what amount of goods will the firm 1 produce; one can extend the module by a little bit by saying that, suppose both the firms instead of only firm 1 both the firms want to maximize their market share, which means that firm 1 is maximizing q 1 provided pi 1 remains non negative, and firm 2 is also maximizing q 2 provided pi 2 remains non negative, then what could be the equilibrium, and I leave that to the viewers to find out.

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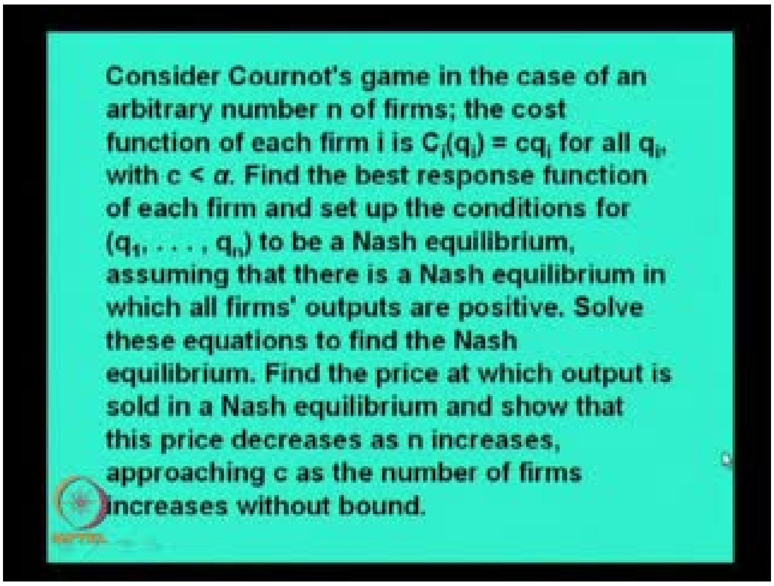

The last application of Cournot equilibrium that we shall do is the following: this is the question, consider Cournot's game in the case of an arbitrary number n of firms; the cost function of each firm i is C i q i is equal to cq i for all q i, where c is less than alpha. Find the best response function of each firm and set up the conditions for q 1, q 2, dot dot dot q n to be a Nash equilibrium, assuming that there is a Nash equilibrium in which all firms outputs are positive. Solve these equations to find the Nash equilibrium. Find the price at which output is sold in a Nash equilibrium and show that this price decreases as n increases, approaching c as the number of firms increases without bound.

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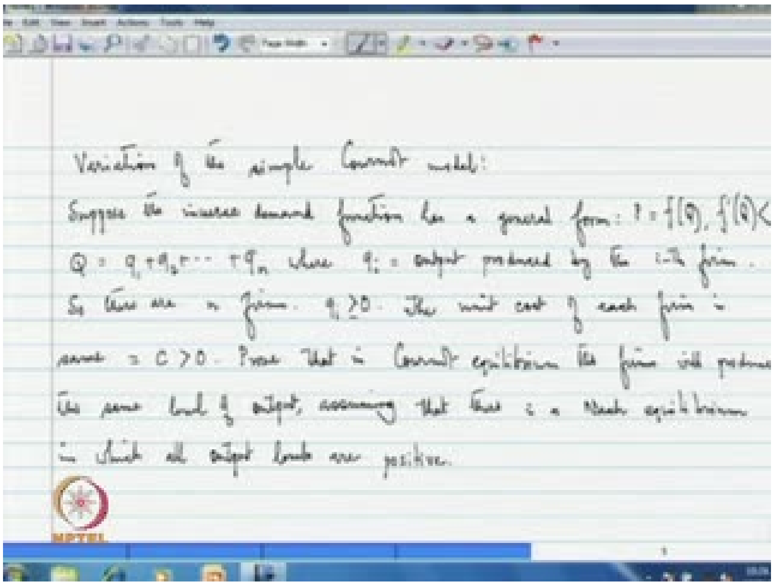

So, this is an exercise; let me first write down the exercise, then we shall solve this exercise, this variation of the Cournot model of the simple Cournot model let us say; so, what we have here is the following, so what we have here is that the inverse demand function instead of being a simple linear function has a general firm p is function of capital Q f of capital Q, where f prime Q is less than 0, which means that it follows the law of demand.

What is capital Q? Capital Q is the sum of all small q's where q i is the output produced by the ith firm; so, that basically means that, there are n firms, so we have a general case here, instead of two firms we have n firms in the market, and of course q i is greater than or equal to 0, it is a non-negative quantity.

The cost the unit cost of each firm is same is equal to small c, c is greater than equal to 0; what we have to show is the following, prove that, so this is what we have to show that assuming that there exists a Nash equilibrium in which all output levels are positive, we have to show that in that Cournot equilibrium the firms will produce the same level of output. So, what we have here is a general case as we have said before, general in the sense that the inverse demand function as been taken as a general function instead of simple linear function, and we have n firms instead of two firm.

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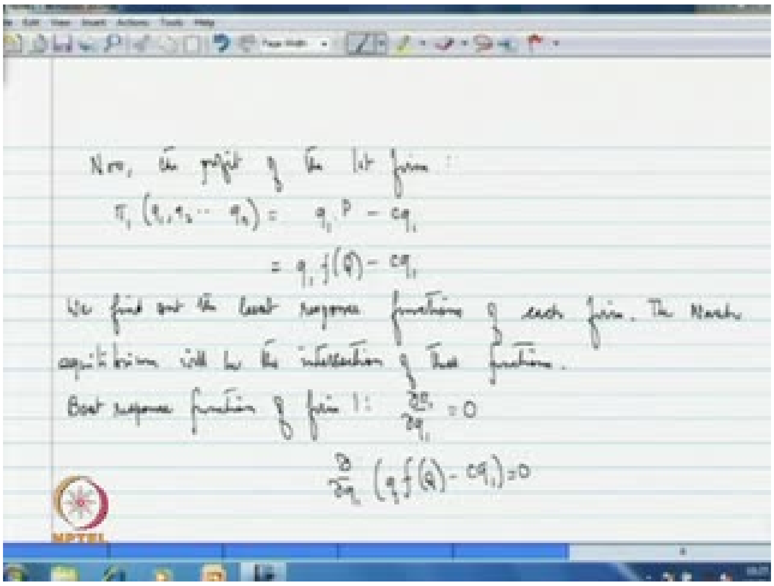

Now, how to proceed the profit of the first firm? What is the profit of first firm? It is let us call it pi 1, and we know this will be a function of n variables q 1, q 2, etcetera,

etcetera, q n, and what will be the particular firm? It will be q 1 multiplied by p, p is the price, this is the total remaining and minus the cost which is small c multiplied by q 1 all right, and we know the form of p it is f of capital q minus cq 1. So, this is the profit function of first firm, how do we proceed we find out the best response functions of each firm, and we know the Nash equilibrium will be in the intersection of all this best response functions.

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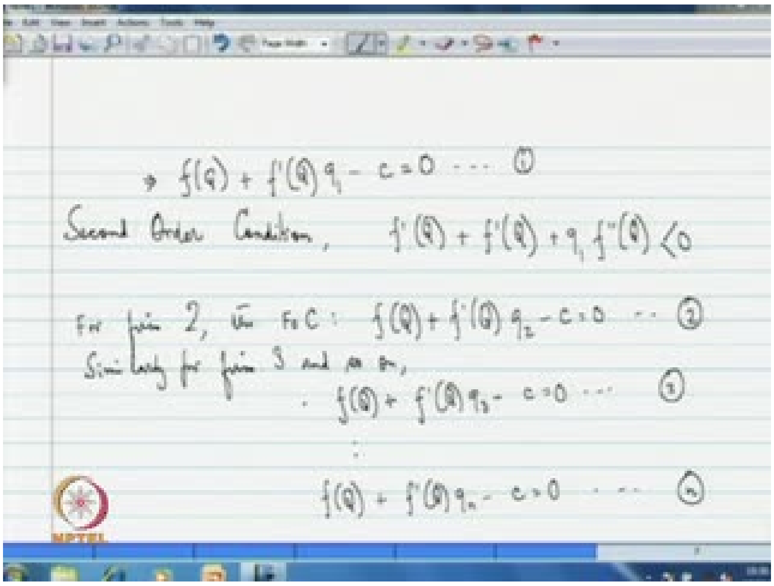

Now, to find out the best response function of let us say firm 1, what we need to do is to partially differentiate pi 1 with respect to q 1 and said that equal to 0 where maximizing profit of firm 1 with respect to the output of firm 1; and from this what we shall get is the following right, this is what we shall get; and from this we get call this 1; and what about the second order condition? That will be satisfied because what we are going getting here is f prime Q plus f prime and this will be less than 0 with a reasonable…., if the function is not a very awkward function, this will be less than 0 because f prime q is less than 0.

So, this is what we have, and this is for firm 1, for firm 2 what will be the second order first order condition, it will be f Q just call this 2; and similarly, for firm 3 and so on we shall get f capital Q plus f prime last for the last firm it will be a f Q plus f prime Q q n minus c is equal to 0.

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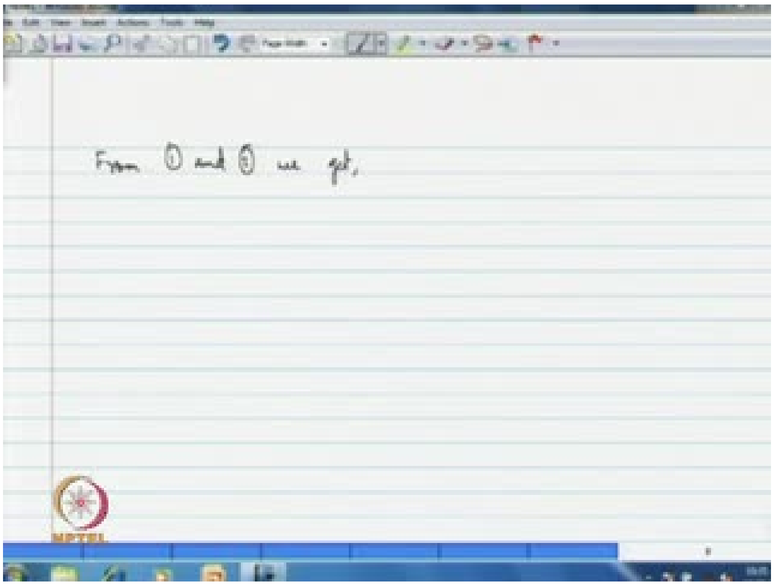

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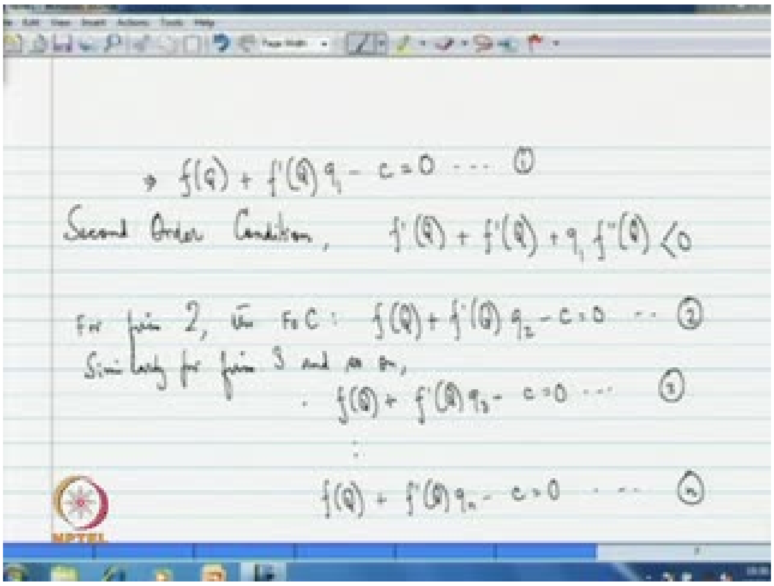

Now, from this n equations what we can find is the following, from 1 and 2 we get, just look at 1 and 2 from this, and this we can subtract 1 from 2 and we shall be easily able to find out that q 1 is equal to q 2.

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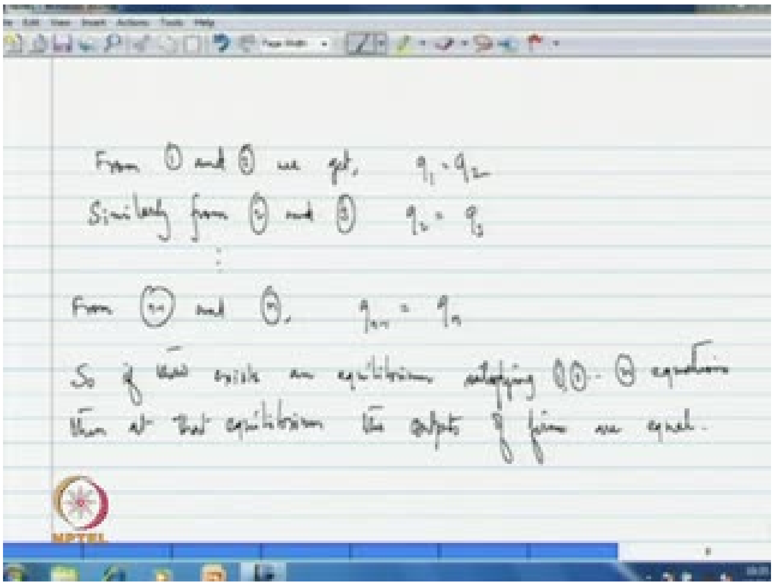

Similarly, from 2 and 3 q 2 is equal to q 3 and so on, from n minus 1 and n q n minus 1 is equal to q n; so, if there exists an equilibrium satisfying 1, 2, dot dot dot n equations then at that equilibrium the outputs of firms are equal; so, this is what we were suppose to prove that if we have a Nash Cournot equlibrium Cournot model setting, where we have a general inversed demand function, and there are n firms with constant cost then if the Nash equilibrium exists then at that Nash equilibrium the output levels of all the firms will be equal.

Just to recapitulate what we have done in this lecture we have looked at different aspects of Cournot model, we have found out some very interesting properties of this model; firstly, we have seen for example that, in the Cournot competitive model the firms are less profit than that than what they could have earned if they had maximize that joint profit, so we are having a situation like a prisoner's dilemma situation in the competitive case; secondly, we have seen that if any firm tries to maximize its market share instead of profit then it becomes the monopolist if the other firm continuous to maximize its profit and not maximize the market share.

And thirdly, we have seen that as if we have more than two firms if I have the general case where there are n number of firms then the quantity produced by each firm, and the price produce that will prevail in the market and the profits can be if calculated; and as the number firm rises the price declines in the market and it approaches the cost of production, the average cost of production of each firm as price, as the number firm goes to infinity, that is all for today. Thank you.

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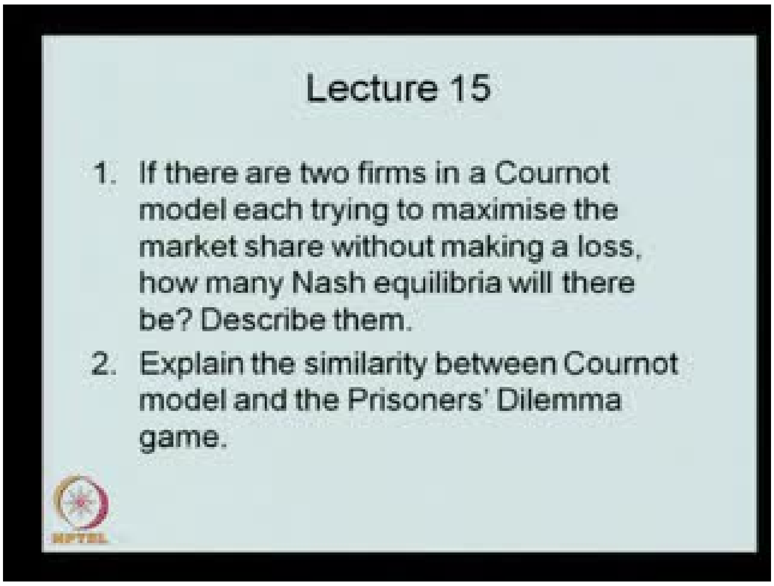

The first question, if there are two firms in a Cournot model each trying to maximize the market share without making a loss, how many Nash equilibria will there be? Describe them.

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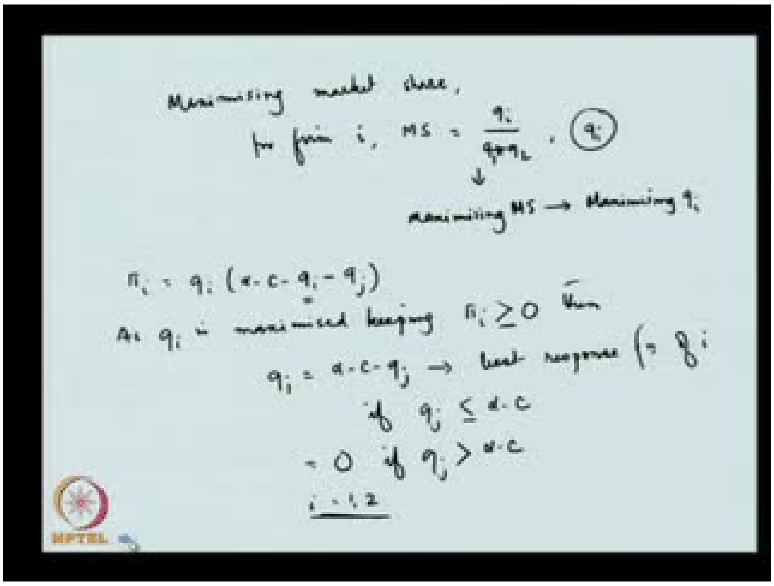

So, firms here are maximizing market share, the market share for each firm, let us say for firm i market, share it just called MS is given by q i divided by q 1 plus q 2, where there are all two firms for simplicity. Now, if this is going to be maximize, then one has to maximize q i, so maximizing MS is same as maximizing q i, however the profit cannot turn out to be negative that is the constraint that we have. Now, what is profit if you remember in the Cournot model is given by this for firm i.

Now, if i is maximizing q i keeping pi i greater than or equal to 0, then what should it do, it can go on raising q i until this term becomes 0; if it crosses that particular value of q i where this is equal to 0, then this term becomes negative; and if this term becomes negative the profit becomes negative.

So, what the firm 1 should do firm i should do is that it should q equate q i is equal to alpha minus c minus q j, so this is nothing but the best response function of firm i; now, this is true if q j is a less than or equal to alpha minus c, and it is equal to 0 if q j is too high; suppose, it is greater than alpha minus c then obviously this term will turn out to be negative, and this cannot be negative because we are talking about output here.

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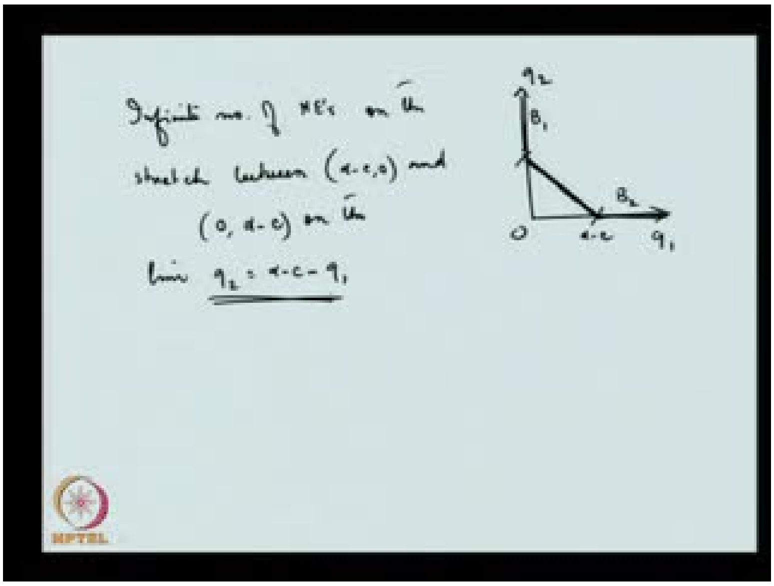

Now, this is for each of the two firms, i can be either 1 or 2; so, for firm 1 the best response function will look like this I am not going I not writing the best response function of firm 1 explicitly, but the diagram will look like this, it will be like this B 1 best response function of firm 1, and for firm 2 it will be this and this all the way this is B 2.

So, there are infinite now number of Nash equilibria, because this is the part where this best response functions are coinciding on the stretch between this point is alpha minus c, alpha minus c 0 and 0 alpha minus c on the line q 2 is equal to alpha minus c minus q 1.

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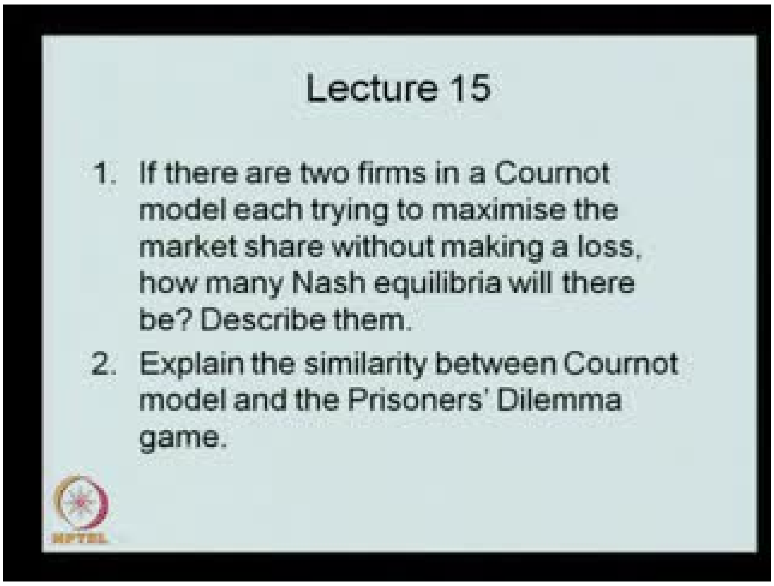

What was the second question; explain the similarity between Cournot model and the prisoners' dilemma game.

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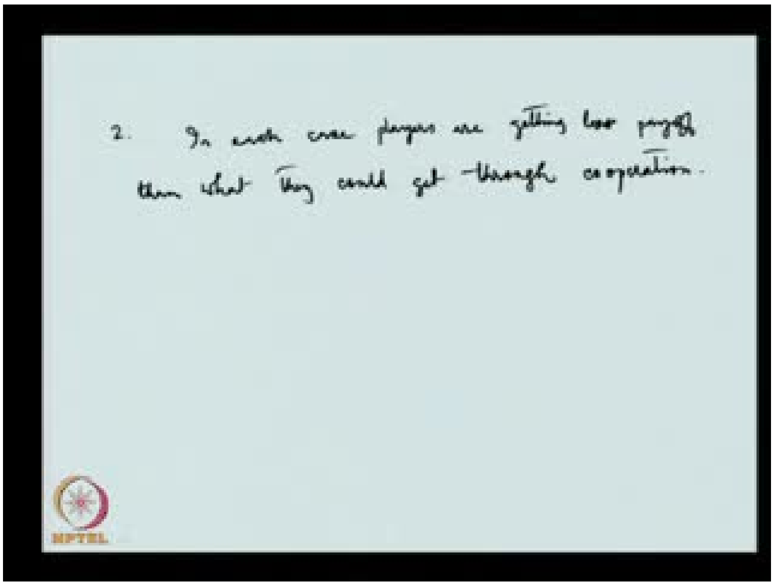

Well, the similarity is very simple, in each case the firms are getting less profit than what they could get through cooperation; let us say in each case not firms but players, because in prisoner’s dilemma the players are the prisoner’s in Cournot equilibrium the players are the firms, and in equilibrium they are getting a less profit or less payoff than what they could get if they cooperate it. Thank you.