

Game Theory and Economics
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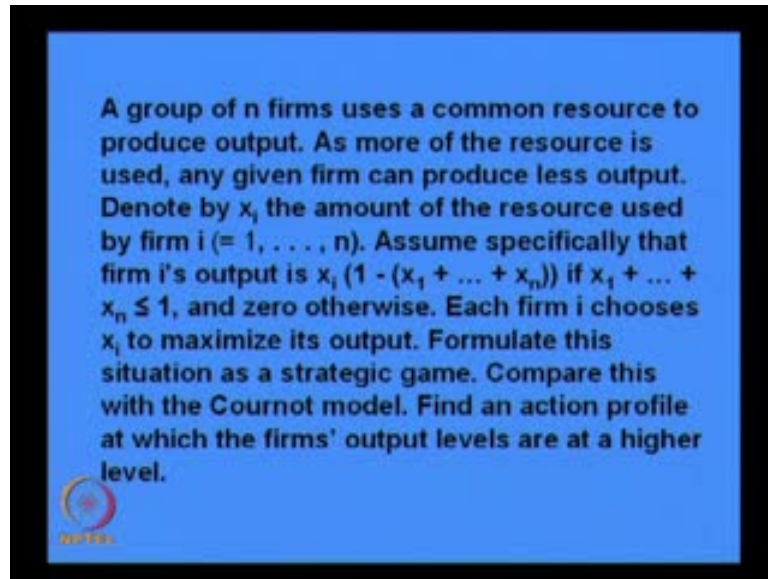
Module No. # 03
Illustrations of Nash Equilibrium
Lecture No. # 04
Cournot and Bertrand Models

Welcome to the lecture 4 of module 3 of the Course Game Theory and Economics. Before we start, let me recapitulate what we have done in the previous lecture. What we have done in the previous lecture is that we have discussed various aspects of cournot model. Cournot model is a model of oligopoly market, where there are few firms who are competing with each other.

We want to find out in this market, what is the output level that will be set by each firm? Consequently, what will be the total output produced by all the firms put together? Hence, what will be the equilibrium price in the market? Based on our observations regarding the various aspects of cournot model, we have found that the cournot model essentially depicts a situation of prisoner's dilemma, in the sense that the firms are competing with each other.

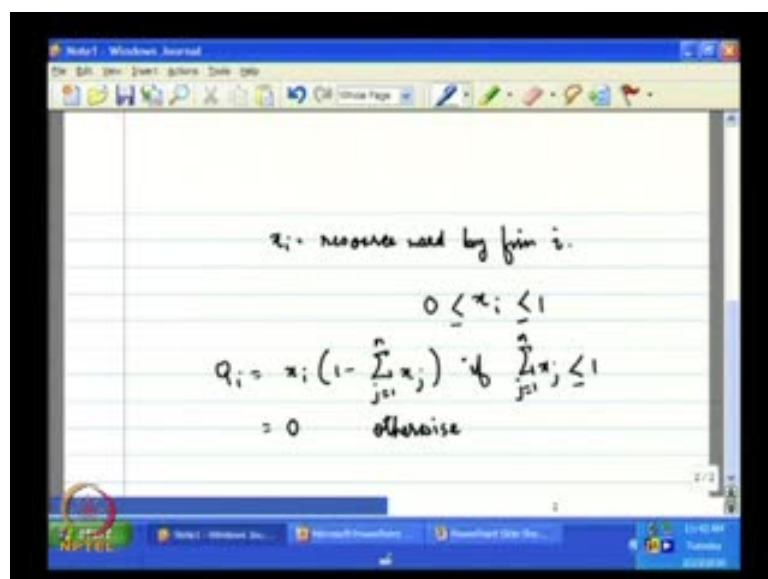
By competing with each other, they are earning a level of profit, which is less than what they could have earned, when they had come together and taken a joint decision as to what output they will produce and we have seen that. Today, we are going to conclude that this part of discussion of cournot model, by talking about a general model of this sort of behavior by different firms.

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Let me start with this discussion. This is the question: a group of n firms uses a common resource to produce output. As more of the resource is used, any given firm can produce less output. Denoted by x_i , the amount of resource used by firm i goes from 1 to n . Specifically, assume that firm i 's output is x_i multiplied by 1 minus x_1 plus x_2 dot dot dot x_n . If x_1 plus x_2 dot dot dot plus x_n is less than equal to 1 and 0. Each firm, i chooses x_i to maximize its output. Formulate this situation as a strategic game and compare this with the cournot model. Find an action profile at which, firm's output levels are at a higher level.

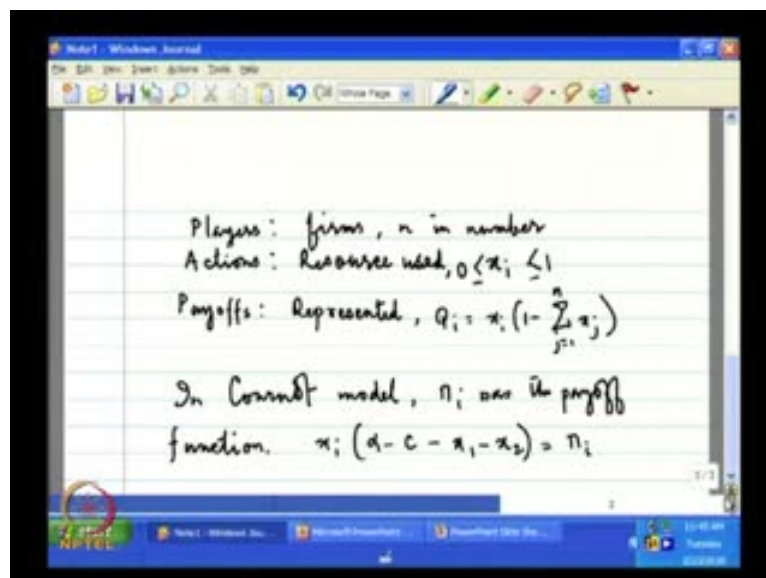
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Essentially, what we have is that x_i is the resource used by firm i and this resource can at most be 1. So, x_i can vary between 0 and 1. This is a natural resource, which means that the total amount of resource available in the entire economy is denoted by 1. What we further know is that If i th firm uses this resource x_i , it is able to produce some output. This output, let us call it as Q_i and it is given by x_i multiplied by summation x_j , so let us call x_j and j goes from 1 to n .

If this summation is less than 1 or equal to 1 or 0, each firm, for example, i th firm wants to maximize this Q_i , which is the output of that firm. What we need to do is that we have to model this situation as a strategic game. Find and compare this with the model of cournot model that we have seen before and find it is Nash equilibrium.

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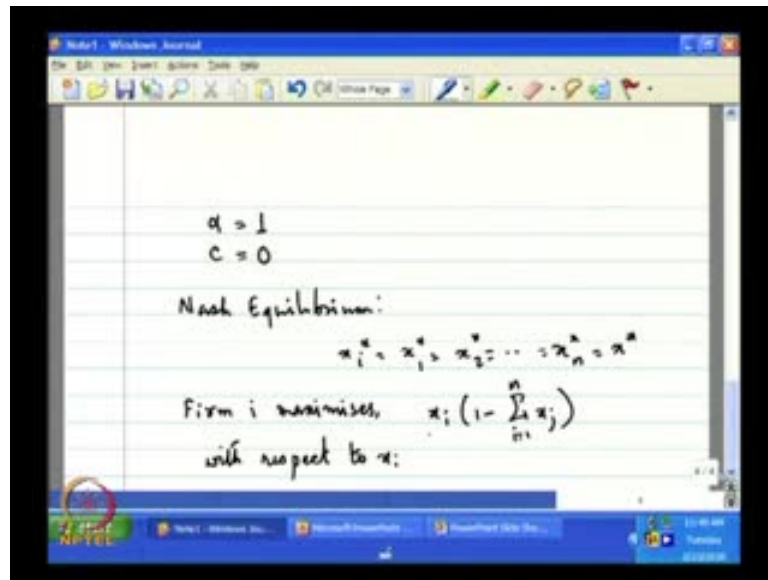


Here, if I have to see this as a strategic game, then I have to specify these three components. Here, the players are the firms that are n in number and actions are the resource used. It is given by x_i . It is always less than or equal to 1 and greater than equal to 0. Finally, payoff or preferences are represented by the output level. They want to maximize it, which is given by Q_i . So, this is the setting and this is the game theoretic representation of this situation.

If I have to compare this with the cournot model that we have seen before, what are the parallels? Remember, in the cournot model, the firms were maximizing their profit. So that π_i the profit of each firm is equivalent to Q_i here, the output that each firm is

producing. So, $\max \pi_i$ was the payoff function, which they were maximizing. It was given by x_i multiplied by $\alpha - c - x_1 - x_2$. This is the profit function of firm i . However, I used Q_i in and I did not use x_i , but nevertheless, I can assume that Q_i is equal to x_i without loss of generality.

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If I compare this with the setting that I have right now, then what are the parallels that we have? The parallel is that α is equal to 1 because the first constant we have in this setting is 1, whereas the first constant term in that setting was α . Secondly, the cost of each firm - cost of each firm in the Cournot model was c per unit cost. Here, I see that there is no such component, which means that c in the new setting is essentially equal to 0, while comparing this with the Cournot model. So, these are the two parallels that we have between the Cournot model and our present model.

Now, if I have to find out what is the Nash equilibrium, we can proceed as how we have done before in the previous lecture. Firstly, we show that in the Nash equilibrium, the output levels of each firm will be equal and that is not a very big deal. We can just follow the previous technique. The question is if they are equal that is x_i^* is equal to x_1^* and x_1^* is equal to x_2^* etc to x_n^* .

Suppose, this is the Nash equilibrium output level and very easily we can show that they are equal of each firm, what is the level and that is an important question. Now, we can find out again by looking at the first order conditions because like before, as in the

cournot model, the way to find out Nash equilibrium is that we are going to find out the best response function of each firm. From the best response functions of the firms, we can find out the Nash equilibrium by solving them simultaneously. For firm i, this is what maximizes with respect to x_i .

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The image shows a handwritten derivation of the first-order condition for firm i in a Cournot model. The text is written on a lined background and includes the following steps:

$$\text{First Order Condition:}$$

$$\frac{\partial Q_i}{\partial x_i} = 0 \Rightarrow 1 - (x_1 + x_2 + \dots + x_n) - x_i = 0$$

$$\therefore 1 - 2x_i - \sum_{j \neq i}^n x_j = 0$$

$$\therefore x_i = \frac{1}{2} \left(1 - \sum_{j \neq i}^n x_j \right)$$

First order condition is that this Q_i should be equal to 0, which means that 1 minus this entire thing, x_1 plus x_2 dot dot dot x_n minus x_i is equal to 0. Now, there is a x_i term in this bracket and also in this parenthesis. It means that 1 minus 2 into x_i minus summation x_j is not equal to j and it goes to n equal to 0. It means that x_i half of 1 minus this.

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In equilibrium $x_i = x^* \forall i$
 $\therefore x^* = \frac{1}{2} (1 - (n-1)x^*)$
 $\therefore x^* \left(1 + \frac{n-1}{2}\right) = \frac{1}{2}$
 $\therefore x^* \left(\frac{n+1}{2}\right) = \frac{1}{2} \Rightarrow x^* = \frac{1}{n+1}$

Now, what we know is that in equilibrium for all i , it means that x^* and all this n minus 1 x^* will have to be added up. It means that there are n minus 1 x^* . If I take this to the left hand side, it is 1 plus n minus 1 divided by 2. This is the Nash equilibrium output level resource used by each firm in the Nash equilibrium. It is given by 1 divided by n plus 1.

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In NE, output level of each firm,
 $Q^* = x^* (1 - n x^*) = \frac{1}{n+1} \left(1 - \frac{n}{n+1}\right)$
 $= \frac{1}{n+1} \cdot \frac{1}{n+1} = \frac{1}{(n+1)^2}$
 $x^* = \frac{1}{n+1}$

Now, output level is a little different concept. In Nash equilibrium, what is the equilibrium output level? It is given by Q^* , suppose forgetting about the subscript. It

is given by x^* minus all these terms here. It means n terms are there and multiplied by x^* , so this is 1 divided by $n + 1$. It is 1 divided by $n + 1$, so this is the output level in the Nash equilibrium, which the firms will be producing. Each firm will be producing Nash equilibrium. Now, there is a last part of the question, which says that find an action profile x_1, x_2, \dots, x_n at which, each firm's output is higher than the Nash equilibrium.

Basically, what is being asked is that is this the situation of prisoner's dilemma? It is in the sense that in the Nash equilibrium, we are having an output level, which is given by 1 divided by $n + 1$ whole square. Is it possible that the firms can produce some other output levels, at which that level is higher than this level? Can it be possible? So, it is in the terms of prisoner's dilemma. Is it possible that the prisoner's do not confess? Each of them will be having higher payoff, is that a probable situation? Can we find such an action profile? We can take the following, what we have here is x^* that is the resource used by each firm is given by 1 divided by $1 + n + 1$.

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$$\bar{x} = \frac{1}{2n}$$

$$\bar{x} < x^* \Rightarrow \frac{1}{2n} < \frac{1}{n+1} \Rightarrow 2n > n+1$$

$$\therefore n > 1$$

$$\text{At } \bar{x}, Q_i = \frac{1}{2n} \left(1 - \frac{n}{2n}\right) = \frac{1}{2n} \left(1 - \frac{1}{2}\right) = \frac{1}{4n}$$

Suppose, \bar{x} is equal to 1 divided by $n + 1$ but 1 divided by $2n$ is this greater than x^* or less than x^* . Why I claim that \bar{x} is less than x^* ? If this is true, then this must be true, which means that $2n$ is greater than $n + 1$. It means n is greater than, which is true. Number of firms in this model has to be greater than 1 because if it is equal to 1 , the model loses its core that there are firms, which are taking decision

interdependently. So, the number of firms must be greater than 1, otherwise it does not make any sense. Now, if the number of firms is greater than 1, then we know that if I have each firm's resource, it is given by 1 divided by 2 n. The resources are less than the Nash equilibrium resource used.

If each firm produces 1 divided by 2 n, then what is the output level? What is Q i? For example, it is given by x i that is 1 divided by 2 n multiplied by 1 minus... There are n such firms, each firm is producing 1 divided by 2 n. There are n such firms, so the total output is n divided by 2 n, which is equal to 1 divided by 4 n. The output level in this new set of resource use that is 1 divided by 2 n is 1 divided by 4 n.

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$$\frac{1}{4n} > \frac{1}{(n+1)^2} \Rightarrow 4n < (n+1)^2$$

$$\therefore 4n < n^2 + 2n$$

$$\therefore n - 2n + 1 > 0$$

$$\therefore (n-1)^2 > 0$$

Question is: 1 divided by 4 n greater than the output level that they were achieving before, which is 1 divided by n plus 1 whole square, is this true? If this is true, then it must happen that 4 n is less than n plus 1 whole square. It is true because n is greater than 1, the number of firms can at least has to be 2. It can be more than 2 and in those cases, n minus 2 whole square has to be greater than 0.

We have indeed found set of resource uses and a consequent set of output levels, which are those output levels that are higher than the Nash equilibrium output levels. It means the setting that firms are deciding on their output levels in a competitive environment. It gives their output levels lower value compared to an alternative set of output levels,

which means that it is indeed a case of prisoner's dilemma. It basically generalizes the Cournot equilibrium case.

People tend to produce more than the level, at which their output levels are. Their profit levels could have been higher and as a result, there is a question. There is an issue of over grazing and there is a common resource here. People tend to over use that common resource and by doing so, they produce less benefit for themselves and that is the general conclusion. So, this is more or less about the Cournot model.

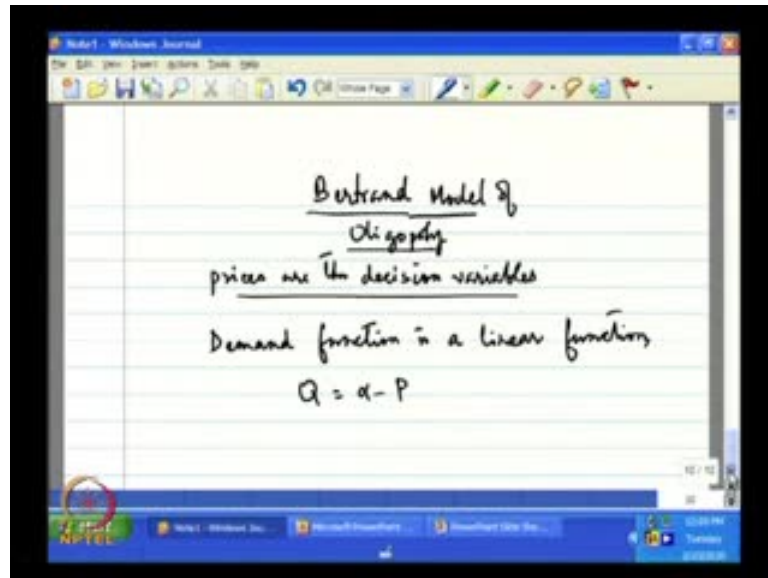
One basic fact about Cournot model was that we were trying to model a market, where firms are competing with each other by setting different output levels. This output levels are decided strategically. It is in the sense that I will set that output level, which maximizes my profit; whereas, I know that the output levels set by other firms might affect my profit. Now, this entire thing is happening through the setting of output levels, but this is not the only kind of way to formulate a market, when Cournot wrote his thesis on how to formulate a market in earlier 19th century. Later on, another French man called Bertrand, reviewed Cournot's work and he came up with an alternative way to model how markets behave and how we can understand market.

The next topic that we are going to cover is regarding Bertrand's model. So, this Bertrand model is essentially like the Cournot model; a model of market oligopoly market, but there are differences. Differences in the sense that here, it is not the quantity, which will be decided strategically by each firm, the output levels of each firm is not the decision variable, rather it is the price the firms decide. The individual prices are decided strategically and they try to maximize their profit, as they were doing before. So that is the thing and the basic questions that we are trying to answer in the Cournot model remains the same. For example, when should we look as how the market demand is going affect the equilibrium? What is the equilibrium to begin with, if the firms earn any positive profit in the equilibrium? What are the output levels? What are the price levels in equilibrium?

We are going to look if there are some other parametric changes in this setting like, if the demand rises or falls in the market, how is that going to affect the equilibrium? If there is some technological innovation, for example, if the cost of each firm changes, then how is it going to affect the equilibrium? If the competition changes, for example, if more firms

enter the market, is that going to change the equilibrium? So, these are the basic questions that we were trying to answer before and they remain the same, but the answers might be different here.

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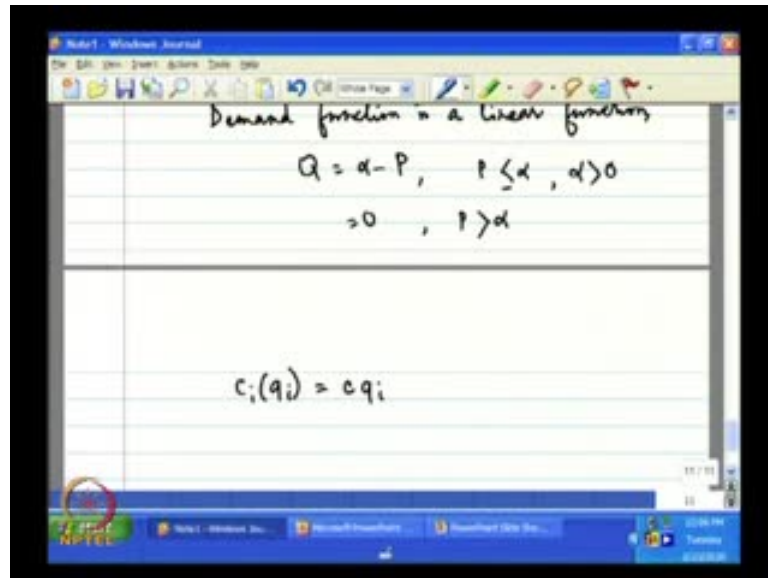


This is the Bertrand model and we will begin with firms setting their prices. So, different firms set their prices differently. They might set their prices differently and the output they producing are of similar quality. It means that if I set a price as 5 rupees for my product and my rival sets a price of 4 rupees for his product. If we take our goods to the market same market and the customers are not able to distinguish between my product and his product that is there is no qualities difference between the products. If that is the case, then there is no reason why any customer will come to me because my price is higher. It means that the producer who is charging a low price is going to capture the market. This is one basic inference that we can draw, if the prices are the decision variables, so that is the first thing.

Secondly, we can also assume that if the prices set by me and my rival is the same price, then we can safely assume that the market is going to be divided between us. So that is an assumption. We can play around with this assumption or we shall do it later on in some more extension version of this model. Like before, the demand function is a linear function; a simple linear function given by Q is equal to α minus p . Remember, this

was the demand function. While talking about the cournot model, we just inverted it and used p as a function of Q .

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The image shows a screenshot of a Windows Journal window. The title bar reads "Notes - Windows Journal". The main content area contains handwritten text in black ink on a white background with horizontal lines. The text is as follows:

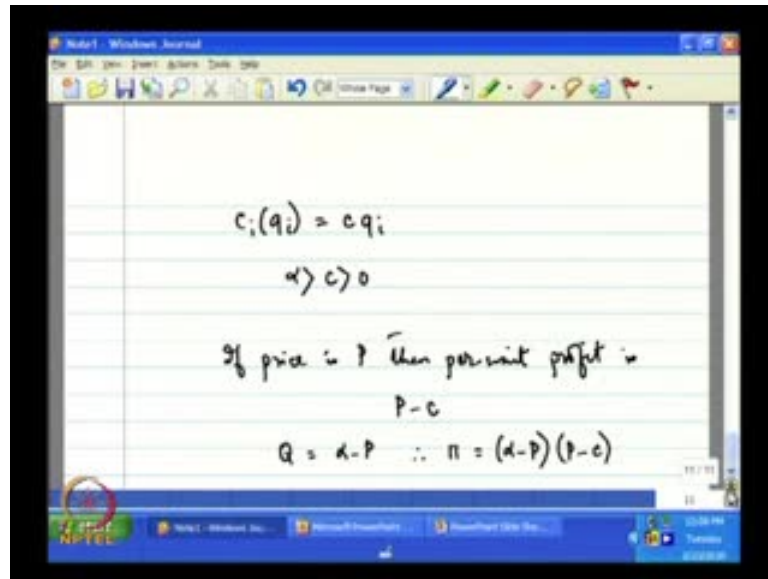
Demand function is a linear function
 $Q = \alpha - P, \quad P \leq \alpha, \alpha > 0$
 $= 0, \quad P > \alpha$

$c_i(q_i) = cq_i$

The screenshot also shows the Windows taskbar at the bottom with several open applications and the system tray on the right.

This Q is now taken as a function of p , it is the original demand function and not the inverted function. This is true, if p is less than or equal to α and α is greater than 0. It is equal to 0, if p is greater than α and like before, we are going to assume a very simple cost function that is c_i of q_i . It is the output level of firm i and c_i of q_i is the cost of production of firm i , which produces the level q_i of output. It is given by small c multiplied by q_i , which means that for one unit of output, the cost that firm i has to bear is small c .

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$c_i(q_i) = cq_i$
 $\alpha > c > 0$
If price is P then per unit profit is
 $P - c$
 $Q = \alpha - P \therefore \pi = (\alpha - P)(P - c)$

We are going to assume alpha is greater than c, greater than 0. From this, it is clear that if the price that I said is p, then per unit profit is p minus c because by selling one unit of output, I am earning p for producing that one unit of output. I am bearing a cost of c, which means that p minus c is my profit per unit. How much profit will be mine? This is the profit per unit and I have to multiply this with the total output that I am producing totally that is the aggregate level of output I am producing. How much units I am producing? If I set a price of p, what is the quantity that I am going to sell in the market? This is just given by the demand curve. So, Q is equal to alpha minus p and this is the quantity that I am going to sell in the market.

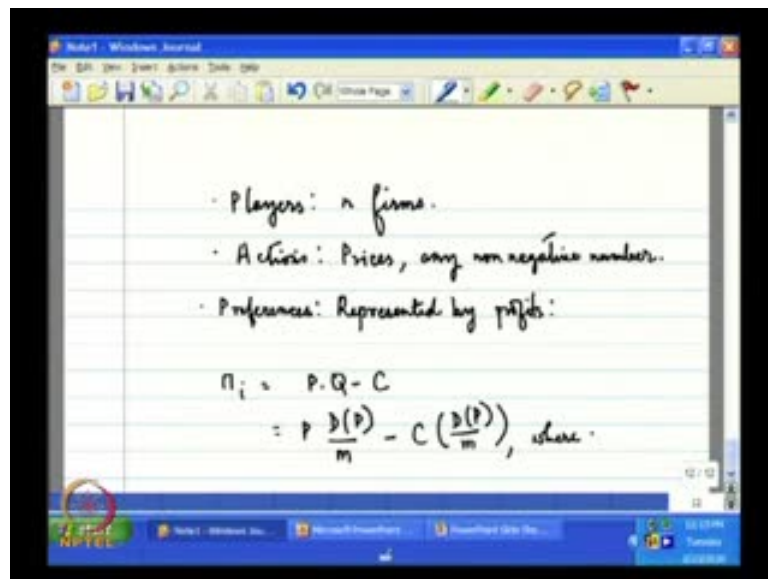
If the price that I said is p, assuming that there are no other firms in the market. Suppose, I am the only firm in the market and in that case, profit for me will be alpha minus p, which is the output multiplied by profit per unit of output. So, this is the total profit that I am going to earn, if I am the sole producer and the price that I am setting is the price in the market. This is the general formula for profit, if there is no rival.

Before we look into the various integrities of this model, one more important assumption that I need to clarify is that in this model, if I announce a price p, it may happen that the price is less than the cost of production. The price is less than c, but nevertheless, if I have announced a price, many customers will come to me knowing that price. I have to

cater the demand irrespective of whether I am making loss or a profit. So, it is now my owners to cater to that demand.

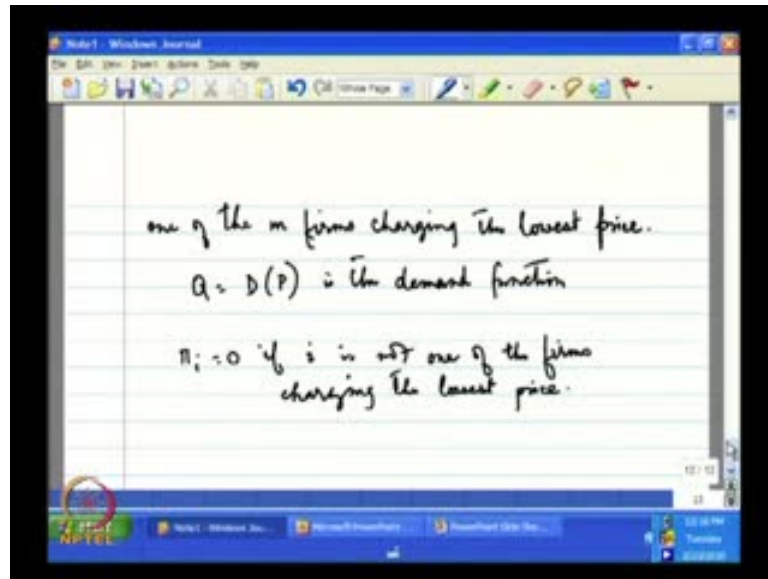
We are going to assume that there is no constraint on the firms to produce that level of output. It means that if the market demand is 1000, I can produce that 1000 unit of output. There is no restriction on that. It may happen that by producing that 1000 unit of output, I am making losses, but that is immaterial to this setting. If I am getting demand, I have to cater to that. So, this is the setting. Now, if I have to model this setting in terms of game theory, again as in any strategic game, I have to specify three components of this game.

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One is player and here, the firms are the players. In general, there are n firms and two prices. So, what are the prices in this model? Prices are any non-negative number. Sorry, I should not write as prices as prices are the actions and it can be 0 also, hypothetically. Finally, preferences are represented by profits. How do I represent that profit? Suppose, I am talking about firm i , the profit of firm i is given by price. It is given by p multiplied by Q minus cost and that is the first principle. If I charge a price of p , then what is the quantity that I get? This is given by demand function, but there is something else.

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Here, m is the number of firms, which are charging the same price and this price is the lowest price in the market. Minus q of this, where i is one of the So let me explain, firm i is charging price p . If firm i is one of the m firms, which are charging the lowest price, then the demand in the market is going to be divided between these m firms. So, each firm will get the total demand in the market divided by m because the market is being shared equally. Each firm is going to get $D(p)/m$. $D(p)$ is the demand function minus the cost and your cost of production depends on the quantity that you produce.

How much quantity are you producing? It is given by $D(p)/m$ and we divide it by m , the portion of the market that you are catering to. That is why the total profit is p multiplied by q minus $c(q)$, which is same as p multiplied by $D(p)/m$ minus $c(D(p)/m)$, which is a function of $D(p)/m$. This is true, if i is one of the firms charging the lowest price. If not, then what happens? π_i is equal to 0. If i is not one of the firms charging the lowest price, i is not getting any demand in the market. In that case, it does not produce any output. If it does not produce any output, its cost is 0, the revenue is also 0 because it is not catering to any demand. So, this is the story notice. If i is the only firm, which is charging the lowest price, m is just equal to 1. It means that the profit function is just p multiplied by $D(p)$ minus $c(D(p))$ if m is equal to 1. This is the general setting, these are the preferences and the preferences are represented by the profit function.

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$n = 2$ $Q = \alpha - p$
 $\pi_i = (\alpha - p_i)(p_i - c) \quad \forall p_i < p_j$
 $\left(\frac{\alpha - p_i}{2}\right)(p_i - c) \quad \forall p_i = p_j$
 $0 \quad \forall p_i > p_j$
 $i = 1 \text{ or } 2$

Now, what we are going to do now is that we are going to find out Nash equilibrium of this model. While doing so, we are going to simplify this model a little bit. We are going to assume that there are only two firms. So, n is equal to 2 and like before, the demand function is given by D equal to α minus p and the cost function is given by c q . Cost of production is just unit cost small c multiplied by the level of output. If that is the case, then what is the profit that each firm is earning? I can succinctly write this as follows: If firm i is the π_i , which is charging the lowest price. We have just seen that the price and the profit that it is going to be earned is given by this, where p_i is the price charged by firm i , if p_i is strictly less than p_j .

Remember, there are only two firms 1 and 2. If i is equal to 1, j is equal to 2 and vice versa. What is it equal to? Here, p_i is equal to p_j and if p_i is just equal to p_j , then the market is getting divided. Each firm is getting half of the demand, which means that q was equal to α minus p . Each firm will get only half of it, which means that this first component - α minus p_i the first term within the bracket. Each firm will get only half of that and it means that this is going to be α minus p_i divided by 2 multiplied by p_i minus c if p_i is equal to p_j .

Finally, if i th firm's price is greater than the j th firm's price, nobody is going to come to in and it is demand that it caters. Now, 2 is going to be 0 that is cost of production is going to be 0 and so, profit is going to be 0 . This is the last case, so here i can be 1 or 2.

Now, we are going to find out the Nash equilibrium for the given setting. Before we systematically find what is the Nash equilibrium, let us try to look at the response of each firm, given the price charged by the other firm.

We know that the unit cost of production of each firm is given by small c . If my rival charges a price, which is greater than c , then what will be my best response or what price shall I charge to maximize my profit? Remember, price that is p_i or p_j is a continuous variable, it can take any fractional value, no matter how close it is to a particular value. It can take any closest possible fractional value. If the price of the other firm is any value greater than c , then if I charge the price that he is charging, we share the market equally. So, I get some profit because the price in that case is greater than c .

If the price is greater than c that is positive profit margin and if there is positive profit margin, the total profit is obviously positive. How and what is the price that a charge, which maximizes my profit? If I charge the price just equal to his price, I share the market on positive profit. Can I do better? The answer is yes. If I charge a price, which is just a little bit less than c , I capture the entire market because if my price is less, the customers are going to come to me and not to him. In that case, I cater to the entire market. Instead of sharing the market with him, it is better for me to charge a price, which is a little less and get the entire demand in the market, so that is my best response.

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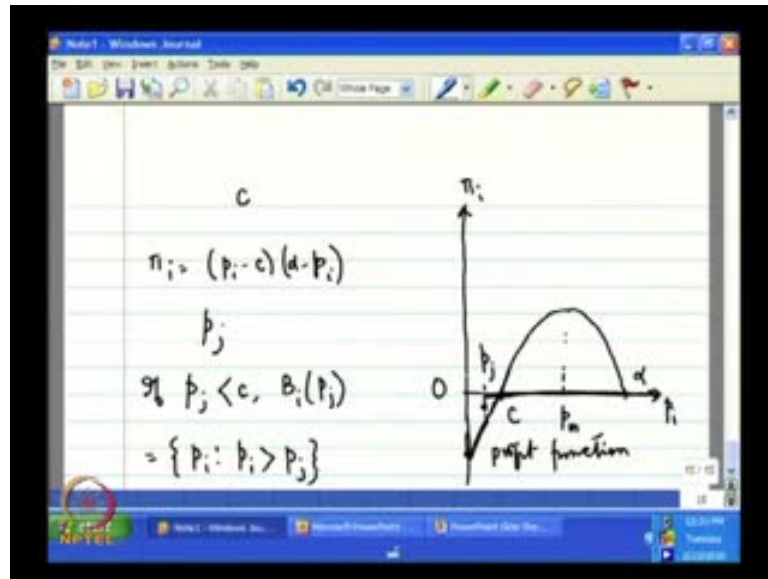
$n = 2$ $Q = a - P$
 $\pi_i = \frac{(a - p_i)(p_i - c)}{2}$ if $p_i < p_j$
 $\frac{(a - p_j)(p_i - c)}{2}$ if $p_i = p_j$
 0 if $p_i > p_j$
 $i = 1 \text{ or } 2$

However, if I look at this profit function, this π_i is equal to $\alpha - p$ multiplied by $p_i - c$. This profit function tells me that at a particular price, this profit function must be attaining its maximum and it is going to be a concave profit function. It means that if my rival is not there in the market, there is an optimum price at which my profit is maximized. It is the monopoly price that is the price I should have charged, if there was no competition, which means that if my rival is charging a price greater than that monopoly price.

My optimal response should be not to charge a little less than his price, but charge that monopoly price. Monopoly price might be quite a less; it might be much lower than the price that he is charging. In those quick cases, my strategy is not to undercut him, but forget about his price and charge the price that maximizes this π_i . So, these are the two cases, where his price is more than c . If his price is less than c , then what should I do? If his price is less than c and if I get a price less than his price, I get the market. What is the profit? It is negative because the profit margin is negative. Here, it is a and the price is less than c . So, it is not optimal for me to charge a price, which is less than his price. What is the optimal thing to do? I shall not charge a price less than his price or shall I charge a price, which is just equal to his price? If I charge a price just equal to his price, the market is divided.

If the market is getting divided, I get the half of the negative profit that he is earning, which is not a very good thing to do. In that case, what I should do is to charge a price, which is just a little higher than his price and that is what I should do. Finally, if his price is just equal to c , there is no profit margin and he is getting 0 profit. It is not optimal for me to charge a price, which is less because if I do, I get negative profit. In that case, either I can charge his price. In which case, my profit is 0, but it is equally well. If I charge a price higher than his price, I get 0 profit. This is the general discussion. From this general discussion, can we find out what is the best response function, function of each firm and what is a Nash equilibrium? So, let us look at that.

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To begin with, let us try to draw the profit function. So, this is p_i and this is π_i . I know the profit function is given by this. Now, if p_i is equal to 0, this profit is a negative quantity, which is $-c$. It has a negative intercept, given that it has a negative intercept, at what points does this profit function intersect this horizontal axis that is p_i is 0? If p_i is either c , in which case the first term is 0 or if p_i is equal to α , in which case a second term is 0 and I know that α is greater than c . If c is here somewhere, α should be somewhere at a higher level. So, these are the points through which, the curve will go through and it will have a negative intercept. Let us call this to be $-c$, whatever the shape of the curve is.

If this curve is going to be concave curve, concave to the p_i axis. I know that because if I take the second derivative of π_i with respect to p_i , I get a negative term, which is -2 . If the second derivative is a negative quantity, then the curve is a concave curve. So, the curve will be most likely having a shape like this. This is the shape of the profit function and there is a point in between, which I shall call p_m , at which the profit is maximized. It means that if there is no other firm, there is no rival to this firm i . Firm i should charge a price p_m because at that point, the profit is maximized. So that is the ideal situation for the firm.

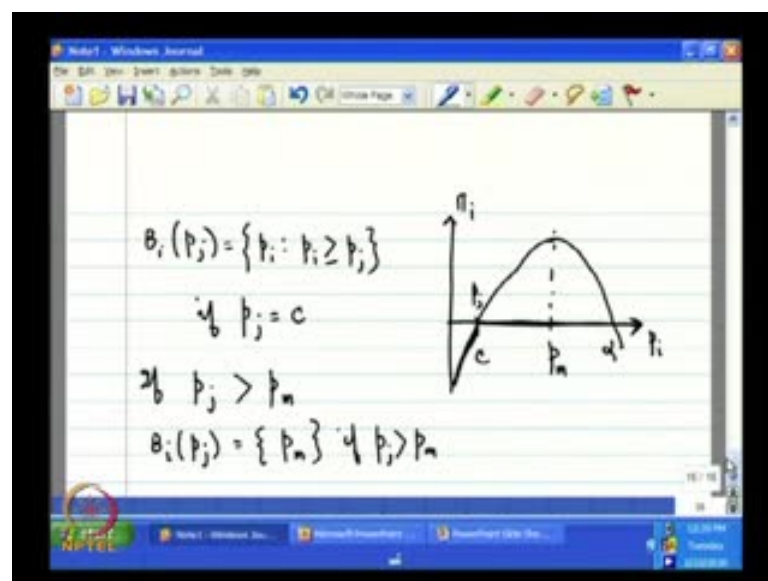
What happens if there is a rival? If there is a rival, then the rival charges a price p_j . depending on the location of p_j . How p_j compressed p_i ? The response of i will be

different and that is what we have seen from our discussion. The crucial thing to decide or the crucial thing to consider is the relationship between p_j and c . For example, p_j is here, the rival firm is charging a price less than the unit cost of production. In that case, what is the price that I should charge? If the rival firm is charging a price less than equal to p_j , which is less than c , then this part of the curve after p_j is getting is not relevant anymore. If I charge a price greater than p_j , my profit is not given by this curve, but my profit is given by this horizontal axis because I am getting 0 profit.

If I charge a price just equal to p_j , then my market is divided. The market is divided between him and me. So, I get half of the profit that he is earning. If he is earning a negative profit, I get half of that. So, I get profit, which is half of that and my profit function is this part. If I charge a price less than his price, my profit function is coinciding with this curve. If my price is just equal to p_j , I get half of the market given by this point. If my price is higher than p_j , I get profit represented by the horizontal axis.

In that case, what should I do? The ideal thing for me to do will be charging a price greater than p_j , strictly greater than p_j . That price can be any price and it does not matter, what price it is. So, this is what I can infer from this diagram and our argument that if he is charging a price less than c , I should price my product strictly greater than his price.

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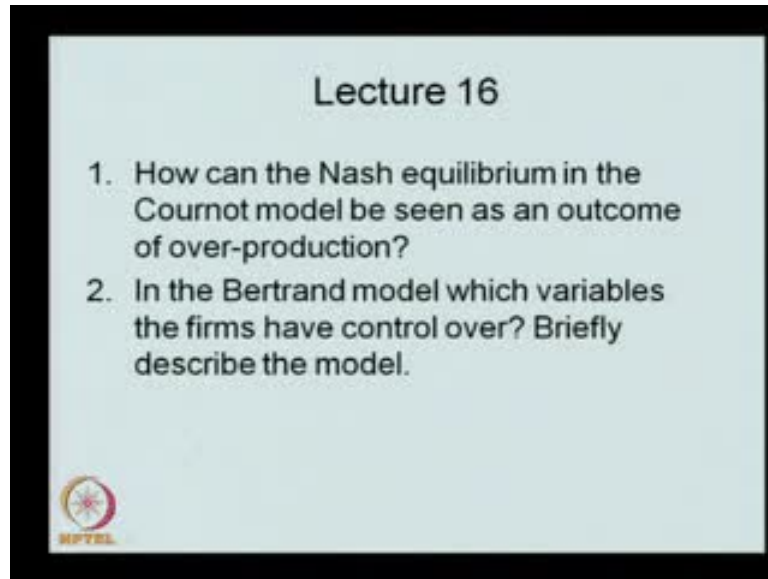
What, if his price is just equal to c ? This is c and this is α , suppose this p_j is here, if the price is just equal to c , then if I charge a price less than c that is p_i is less than c , then I get the market. This part of the curve becomes relevant and if this part is relevant, I can immediately see that my profit is negative and it is below the horizontal axis. So, I should not charge a price, which is less than p_j . If his price that is p_j is just equal to c , what should I charge? If I charge price equal to c or greater than c , then my profit is not given by the curve that I have drawn, but by this horizontal axis. I am getting 0 profit in that case. It means that I should do that because I am not earning positive profit in any case. So, what I can do is to earn 0 profit. But p_i is equal to p_j , such that p_i is greater than or equal to p_j if p_j is equal to c .

We have two cases, one is p_j less than c and the other is p_j equal to c . We have seen that the best response functions are different. Thirdly, suppose this price is p_m . If p_j is greater than p_m , suppose the price that he is charging is more than p_m , what should I do? My maximum profit is being attained at p_m , so I should charge a price, which is p_m because by charging p_m , I am getting the market at the same time and I am maximizing my profit. So, in that case, my optimum price is p_m . It is just a single value, if p_j is greater than p_m .

What happens if p_j less than p_m or equal to p_m , but greater than c ? What should the firm do? Firm I will do something that we shall take up in the next class. So, before wrapping up this particular lecture, let me take you through what we have done in this lecture. We have concluded the section on Cournot equilibrium and Cournot model. We have seen that the Cournot model essentially depicts the situation of overgrazing of common resources in the sense that people tend to be over producing the Cournot equilibrium. What they should have produced, if they were interested in maximizing their joint profit? There is a general model, in which we can fit in the Cournot model.

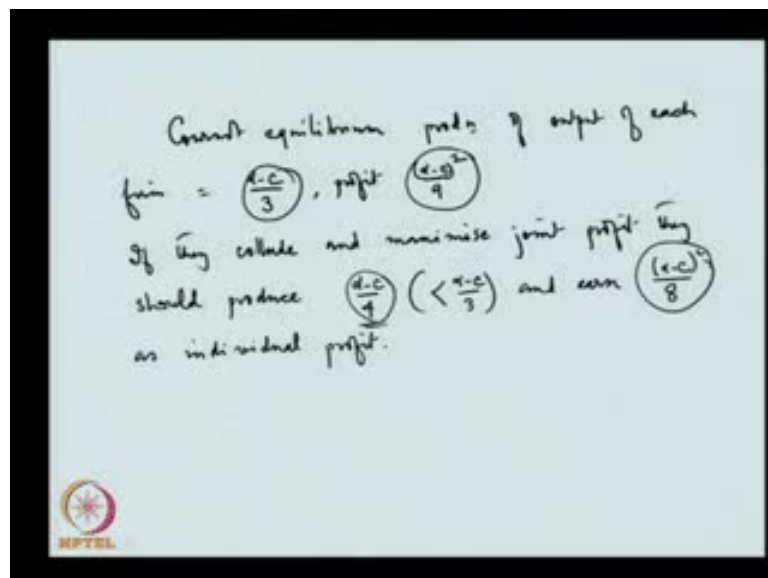
Secondly, we have started to our discussion of Bertrand equilibrium, which is different from the Cournot model, in the sense that here the producers are deciding their prices and trying to maximize their profit. We have started the discussion and we shall continue with this in the next lecture. Thank you.

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Question: how can the Nash equilibrium in the cournot model be seen as an as an outcome of over production?

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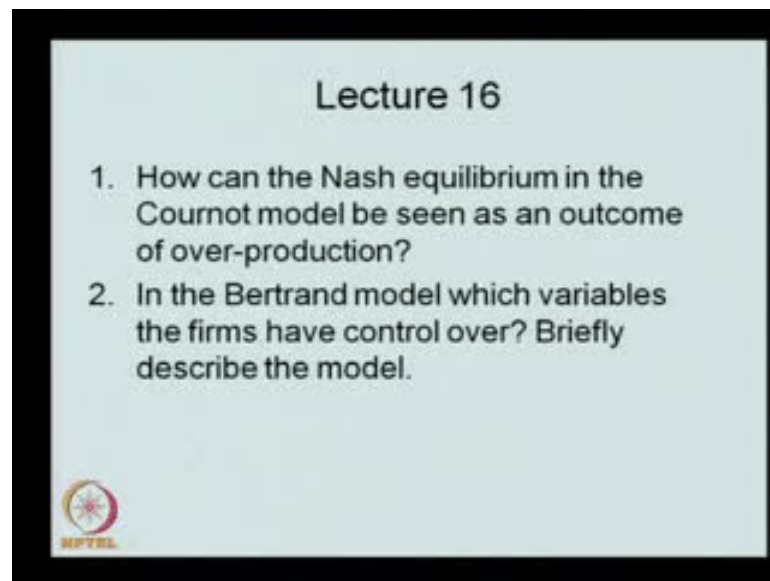


In cournot equilibrium, production of output of each firm is equal to alpha minus c divided by 3. The profit is alpha minus c whole square divided by 9. Now, we have also seen that if they cooperate and if they collude and maximize joint profit, they should produce alpha minus c divided by 4. It is in fact less than alpha minus c divided by 3 and earn alpha minus c whole square divided by 8 as individual profit. It turns out that when

they are not colluding and acting as individual players, they are producing more $\frac{\alpha - c}{3}$, compared to the case, when they collude. When they collude and maximize the total profit, they produce $\frac{\alpha - c}{4}$, which is less.

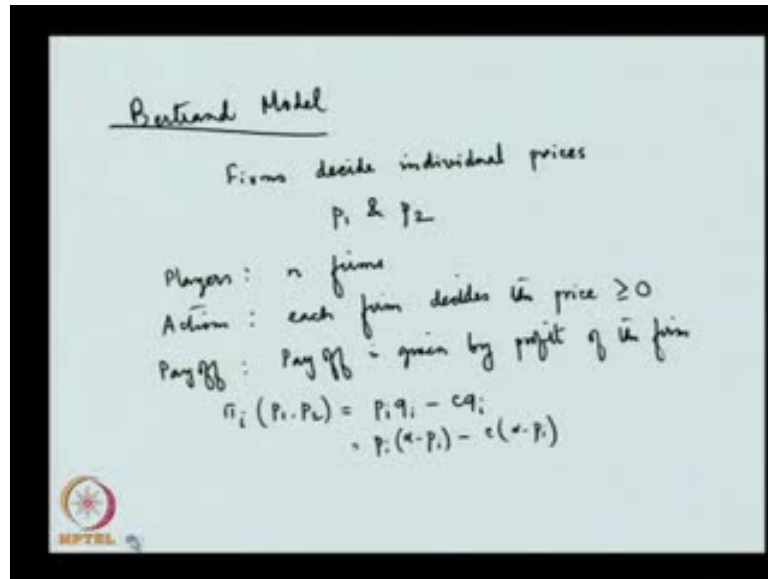
In fact, in collusion, their profit is more than what they get operating individually, so that is why we say that in the Cournot equilibrium, the firms are ending up by producing more. When they produce more, the prices go down and profit also is less than 1.

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The second question in the Bertrand model, which variables the firms have control over? Briefly describe the model.

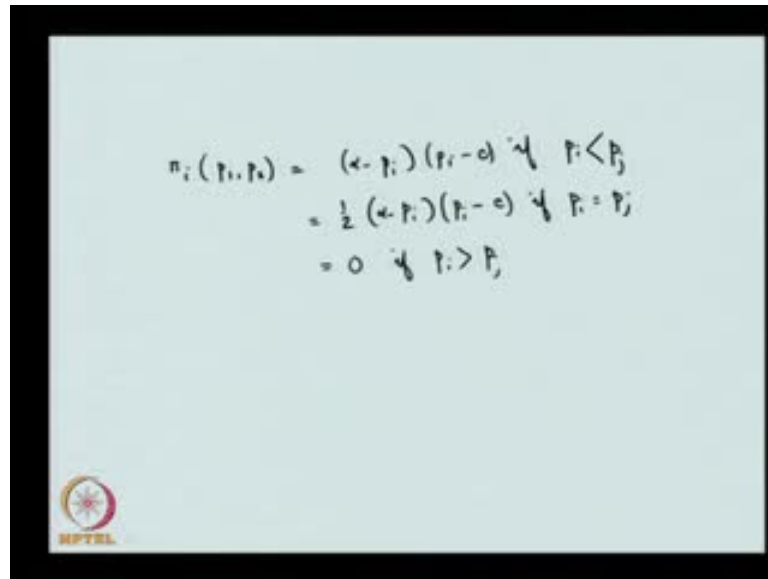
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Firms here decide individual prices, so if there are 2 firms - p_1 and p_2 , they do not decide by their own wish, what the output that they are going to sell in the market. Once the prices are announced, the consumers decide which price is the lower price. If they go to a particular firm deciding on the fact that that firm is charging the lower price, then that firm will have to make for the demand in the market. So, the quantity produced by the firm is not decided by the firm. It is decided entirely by the demand in the market; this was the part one of the question.

Briefly describe the model. In terms of game theoretic language, there are n firms. Suppose, which are the players actions? Each firm decides the price, which is non-negative number and payoff. Payoff of each firm is given by profit of the firm. So, let us say π_i is a function of p_1 and p_2 . It is given by the following $p_i q_i$ minus $c q_i$ that is $p q$ is the market, nothing but market demand, which is given by α minus p_i minus c . So, this is equal to α minus p_i minus c .

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$$\begin{aligned}\pi_i(p_i, p_j) &= (a - p_i)(p_i - c) \quad \forall p_i < p_j \\ &= \frac{1}{2}(a - p_i)(p_i - c) \quad \forall p_i = p_j \\ &= 0 \quad \forall p_i > p_j\end{aligned}$$

The image shows a slide with a light blue background and a black border. It contains three lines of handwritten mathematical equations. The first line is $\pi_i(p_i, p_j) = (a - p_i)(p_i - c) \quad \forall p_i < p_j$. The second line is $= \frac{1}{2}(a - p_i)(p_i - c) \quad \forall p_i = p_j$. The third line is $= 0 \quad \forall p_i > p_j$. In the bottom left corner, there is a small circular logo with a red and yellow design and the text 'NPTEL' below it.

If p_i is less than p_j and if my price is less than the competitor's price, then I get the entire market. So, this demand function is valid for me. If the prices are equal, then we share the market. So, it is half of the same price and same profit. If my price is more than the other firm's price, then I get 0 share of the market. I do not sell anything and my profit is also 0. Thank you.