Game Theory and Economics
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Module No. # 03
Illustrations of Nash Equilibrium
Lecture No. # 06
Electoral Competition-1

Welcome to this third module sixth lecture of the course called game theory and economics. Before we start let me recapitulate what we have done in the previous lecture; we have been discussing the model of Bertrand oligopoly, and we have been discussing the application of Nash equilibrium in find out what will be the equilibrium in such a game, in such a market, and we have seen that the equilibrium in the Bertrand model will be such that all the firms will if there two firms all the firms will charge the same price in the equilibrium and they will earn zero profits.

If the number of firms go goes up if there are more than two firms then also all the firms which are there in the market will earn 0 profit; so, Bertrand model is a particularly model where if the cost of production of all the firms is equal, there is no difference in their technology and their cost of production then there is no reason why any firm will get any positive profit, all the profit all the firms will be on equal footing, and they will earn the same zero profit.

So, that is what we have seen in the previous lectures. We have already seen other aspects of the Bertrand model, for example, if suppose there are two firms but there cost of production differs, if there cost of production differs then there is we have an asymmetry, if there is asymmetry then we have seen that under certain conditions the firm which is having a lower cost of production will earn positive profit in the equilibrium, so that is there, so this is like the Cournot outcome.

If you remember in Cournot also the…, if the firms are having different cost of production and then the firm which is having a lower cost of production will have an upper hand, in the sense that it will produce more output it will earn higher profit; and eventually, it may happen that the other firm is not producing anything. Here also what will happen is that, if my cost of production is less than my rival, then I will cater to the market entirely and my rival will not produce any output, it will not be able to sell anything in the market; not only that in this case I will be earning some positive profit which was not there if the cost of production of the two firms was equal.

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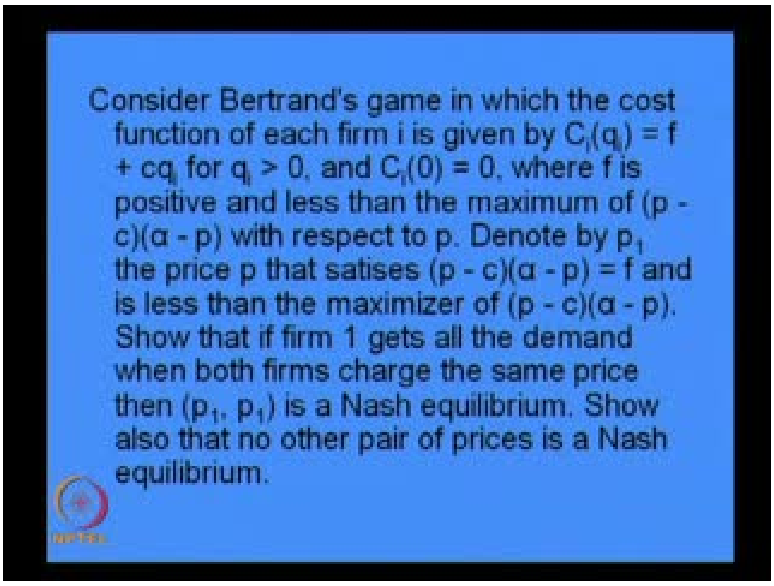

So, this is what we have done we have seen. Now, let us do another small exercise to look at another aspect of Bertrand model in particular what will happen if there is a fixed cost of production. So, this is the setting; consider Bertrand's model game in which the cost function of each firm i is given by C i q i is equal to f plus cq i for q i greater than 0, and c i 0 is equal to 0 where f is positive and less than the maximum of p minus c multiplied by alpha minus p with respect to p. Denote by p 1 the price p that should be satisfies that satisfies p i p minus c multiplied by alpha minus p is equal to f and is less than the maximizer of p minus c multiplied by alpha minus p.

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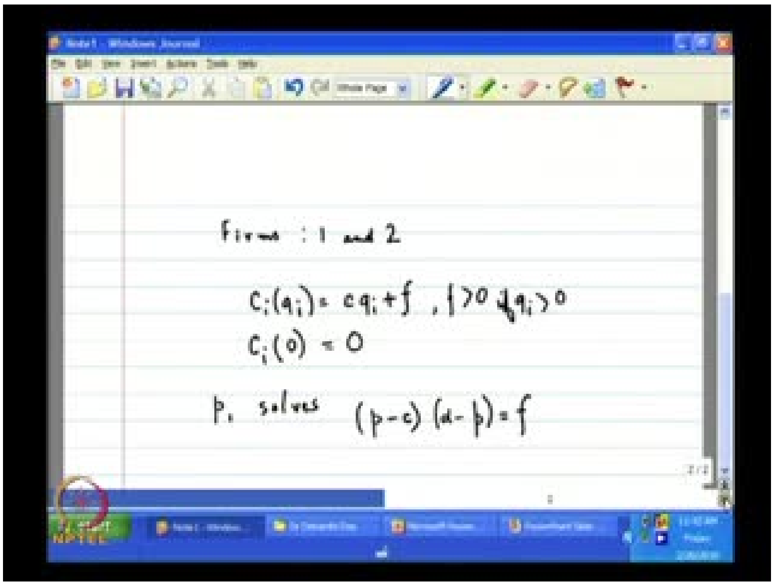

Show that if firm i gets all the demand when both firms charge the same price, then p 1 p 1 is the Nash equilibrium. Show also that no other pair of prices is Nash equilibrium. So, this is the question, I have to show, this in terms of diagram how will it look like, what is happening is that, there are two firms 1 and 2, and there cost of production is the following where, f is positive; one thing to notice is that, we are back to the old frame work where the unit cost of production of both the firms is equal it is given by small c.

So, the cost of production unit cost of production is not differing, what is important is that, here I have a fixed cost which is given by f; and we have already seen that if my level of output is 0, if I am not producing anything, then I do not have to bare that fixed cost, that is important; because generally in economic citizenship that if I am not producing anything then also I have to bare this fixed cost f, because f is not variant with respect to q i. So, even if q i 0 f is there, but here we are assuming that if q i is 0, then f also it just vanishes; and if q i is positive only then, we have this function.

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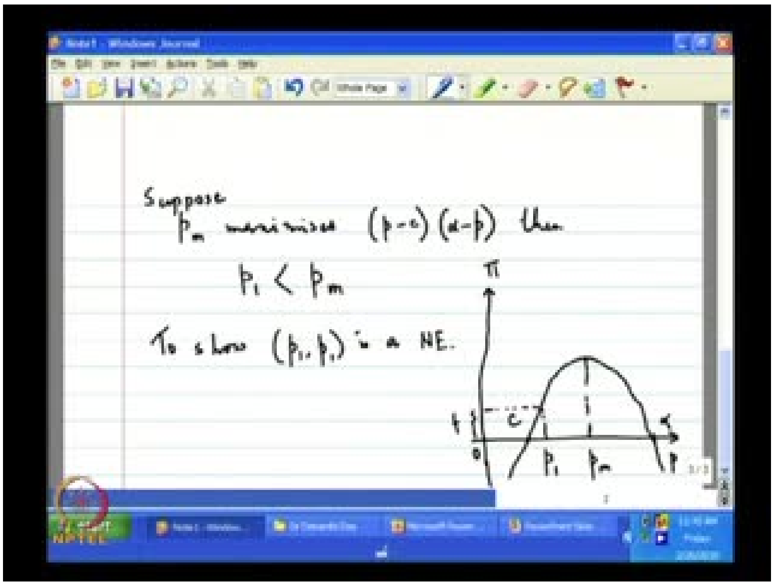

So, that is it, that is the setting; we are also inform that p i is that price which solves p minus c alpha minus p is equal to f, p 1 this is the price p 1 which solves this equation; and this price we are also told is less than the price which maximizes. So, suppose p m maximizes p minus c alpha minus p then p 1 is less than p m, that information we have; what we need to show is that, when both the firms charge the same price p 1, p 1, then that is Nash equilibrium.

So, to show and we are given the assumption that if both the firms charge the same price then firm 1 gets the demand all the demands; so, if both the firms are charging same price firm 2 is not getting any part of the demand, it is not getting c, it is not getting any profit therefore.

So, in terms of diagram this is the old Bertrand diagram, so this is c, this is suppose alpha, and this is the profit function at this level p m profit is getting maximized, so at this level pi is highest; suppose, alpha is f is given by this value, and what is the significance of f is that, it tells us what is that level of price at which the profit earn by a firm is just equal to 0.

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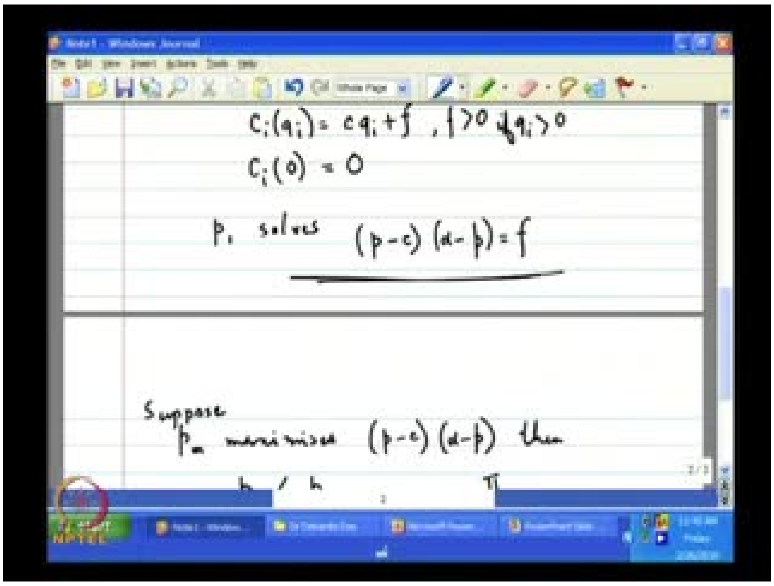

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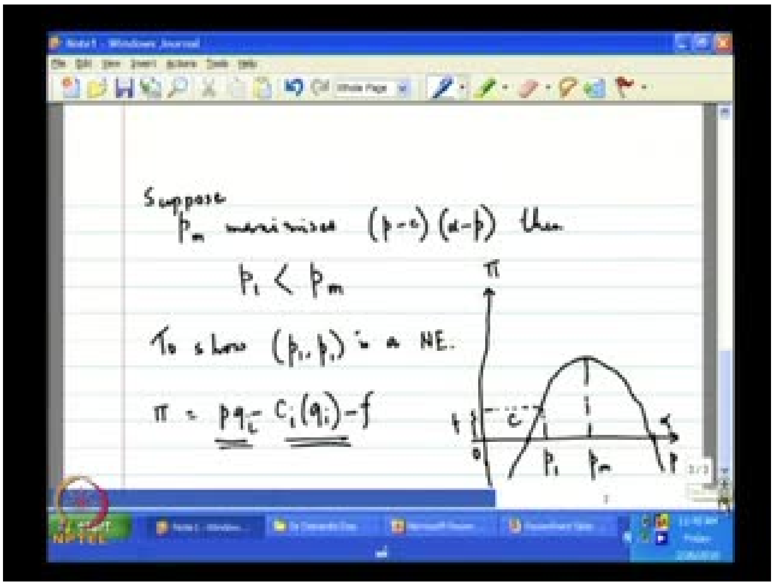

So if a firm is a monopoly, suppose there is no rival firm, and it charges price p 1 then its profit is going to be just equal to 0; why it is so? Because of this, because what is profit if I consider a monopoly’s, what is profit after all this pq i minus C i q i minus f, this is the total revenue, and this is the total cost.

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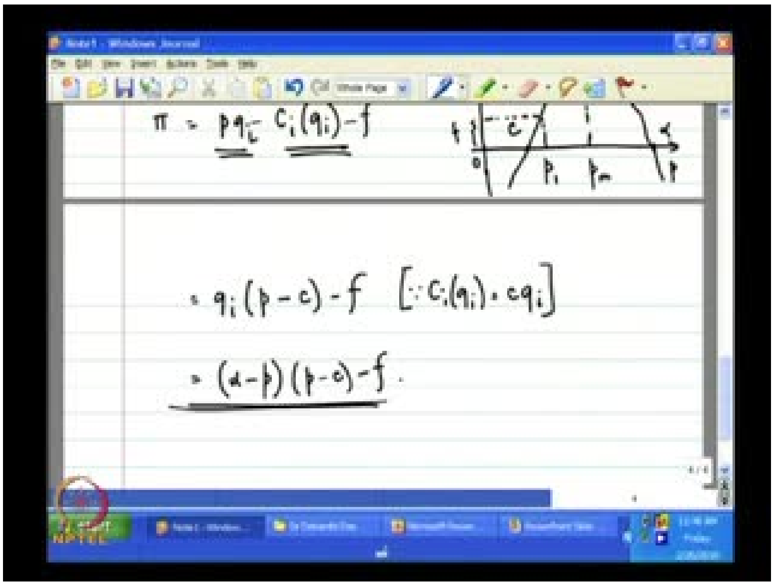

So, from here I can take q i common, so this is just p minus c minus f, because C i q i is equal to cq i, and what is q i? q i is nothing but the what is the demand? Demand in the market is alpha minus p p minus c minus f, so this is the profit under this sort of assumption where there is a fixed cost, and if there is no rival I am getting the entire market, this is the profit.

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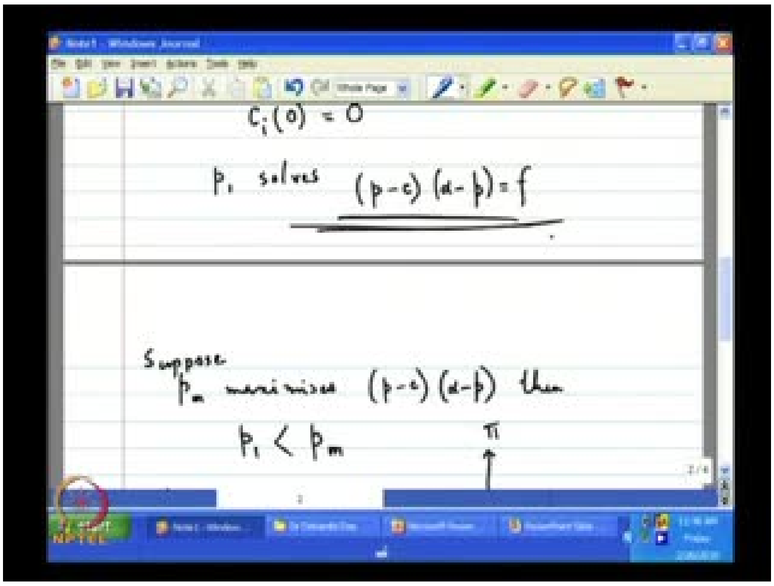

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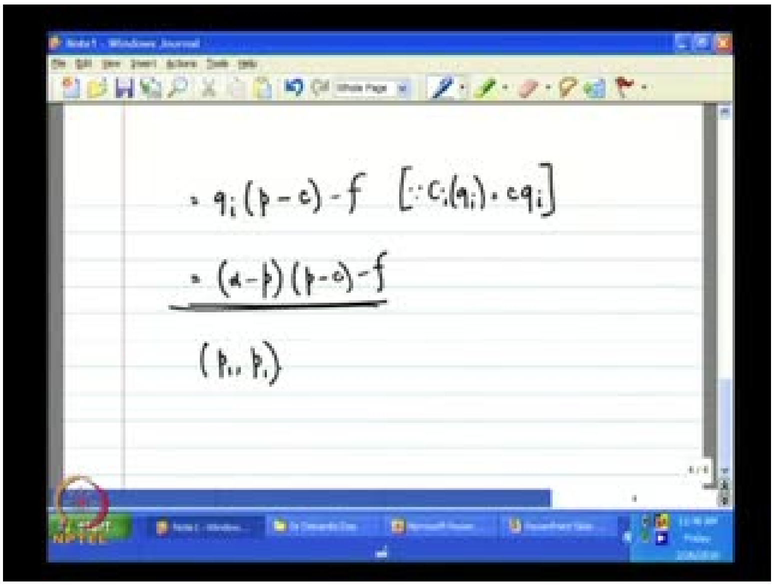

So, in this case if I charge a price p 1 then this entire thing become 0, because of this fact; therefore, p 1 is given by this point on the horizontal axis; what we need to proof is that p 1 p 1 is Nash equilibrium, what is the proof? One can take this as a homework and try to see their suppose the rule of dividing the market was same as it was before, for example, if the prices are equal the market is split equally between the two firms.

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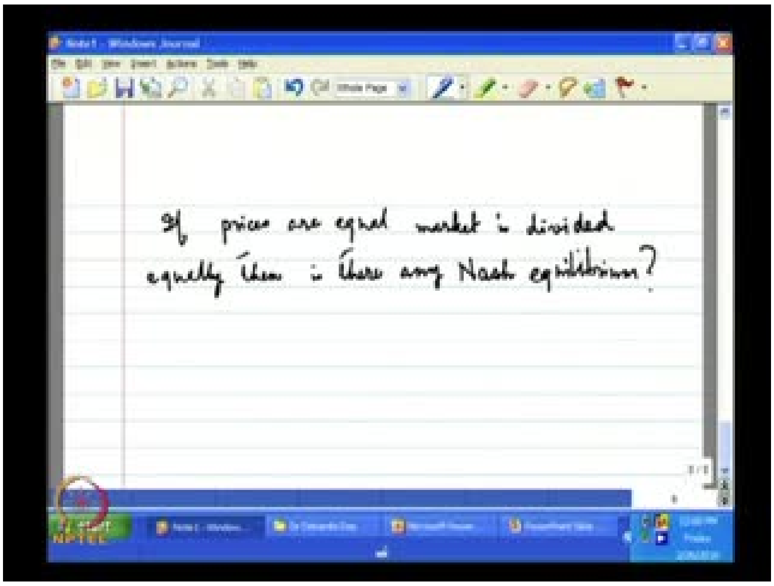

So, if prices are equal market is divided equally then is there any Nash equilibrium? And one can show it that if the rule is that if the two firms are charging the same price, then the market is divided equally, then there can be no Nash equilibrium. In this case, in the exercise we are having Nash equilibrium, because when the prices are equal firm 1 is getting the entire market.

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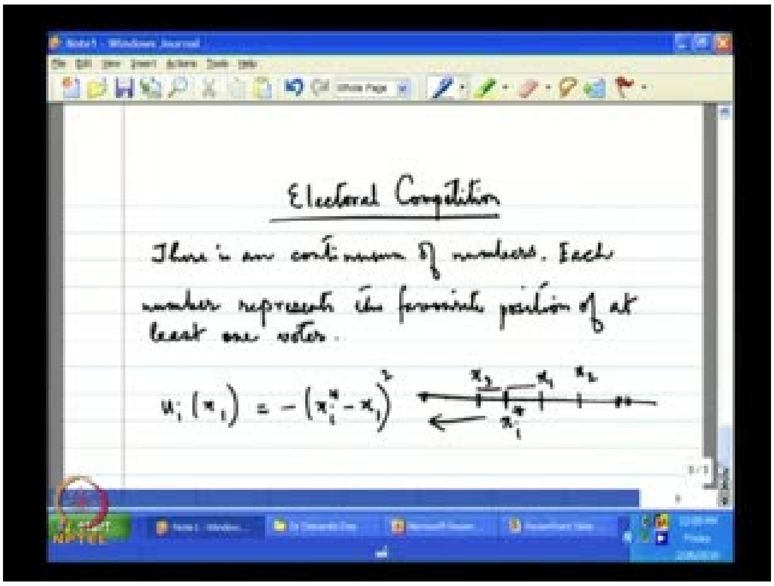

So, that is more or less what we had to discuss about Bertrand model. Let me take up the next topic which is the case of electoral competition; so, electoral competition by this what we mean is that we are trying to look at how people vote, and how number of candidates when the people are voting, number of candidates are decided, more importantly what are the agenda said by the candidates if they want to win the election or if they have some ideological persuasions.

So, what will be the agenda that will be said by the candidates who will win in a particular equilibrium situation, so these are some of the points and if suppose there are some costs involved in standing in the election; if you want to be a candidate you have to bare some cost, then does it hamper the election process thus the number of people who are running for the election does that go down.

So, there are these important issues which we want to address in a very elementary manner in this section of electoral competition; so, the frame one that we have is the following, there is a continuum of numbers each number represents the favorite position of at least one voter.

So, I have this real line, for example, in this real line if I pick up any point then this point represents a number, this number will be the favorite position of at least one voter, so we can imagine that each point in this real line is basically corresponding to at least one voter that can be more than one voter who has the same favorite position.

Now, when we say favorite position what exactly do we mean, what we mean is that, the preference the political preferences of the voters can be represented in a unit dimensional scale; so, this is a very simplifying assumption mind you because my political preferences might be multidimensional it can be having 2 or 3 or more than that set of points set of numbers; and this set of numbers might be a representative of what my political preferences are but since this is a very elementary exercise what we are proposing is that my entire political preferences what I like what I dislike can be represented by a single number, this number can be higher, this can number can be lower etcetera; however, this may seem a little outlandish to begin with, it is to be remember that when we discuss political issues we talk about leftist and rightist.

So, at the back of our mind we have this unit dimensional scale, some persons is preferring some policy which are to the left which means may be in this line you are going to this direction, we are preferring some point here, and someone is rightist which means c is preferring some point here, so that can be visualized in that sense, so it is not so uncommon to visualizes straight line and each point on the straight line is representing the political proclivities how a particular voter.

So, this is that every point is representing the favorite position of a voter, we can think of this number to be suppose defense buzzed, so the amount of money that the country which spent on defense is represent it by a single number; now, I am if I am rightist it is possible that I like that number to be very high, where as if I am a leftist tie like that number to be not as high, and if I am centrist my preferences will be between these two numbers, so this is just an illustration and instance of how political preferences can be represented by a single number.

So, suppose x i star is this point, this is the favorite position of individual i porter i. Now, the point is that he likes the choice of the entire country to be x i star, but if it is not x i star then how does he ranks how does he rank those positions; suppose, there is another point x 1, and there is other point x 2, in this model we are going to assume that he ranks the x 1 and x 2, these two numbers in the following sense that further those numbers are from his favorite position which is x i star the less he likes those numbers; so, in this case x 1 will be prefer to him than x 2.

So, in a sense that the distance between his favorite position and any arbitrary position represents his dislike for that arbitrary position, so I can write it like this, that suppose u i x i represents how do I like the position x i, how not let us not write x i suppose x 1 otherwise will be confusing with I suppose x 1; so, u i x 1 is representing what is the payoff of player i from the position x 1. So, if the country is taking of this policy x 1 then how does individual i like that, this can be represented by x i star, x i star is his favorite position, and let us take this distance, and what we are going to do is to take a square of that.

Now, if I take just the distance on the square it will mean that more the distance is better I prefer that policy which is not in fact the case, so it is just the other way the more distant x 1 is from x i star my dislike for that policy goes up, so that is why I have put this negative sign; and why did a square that, why did I square that is because suppose there are two policies x 3 and x 1, and there equidistant from x i star so this part is same as this part, then our model is going to assume this setting is going to assume that my dislike for x 1 and x 2 or my liking for x 1 and x 2 is the same, so I do not differentiate, I do not distinguish whether that position is towards my left or towards my right, as long as they as equidistant from my favorite position my liking slash disliking is the same.

However, we can relax this relax this assumption will cribbed, because one can imagine that I do not like my the policies which are to the right that much as I like the policies which are towards my left, so those existential, those added complications can be a included, and we can see how they can be included in a later exercise.

So, this is the setting that every voter has a favorite position, and he likes the policy of the country to be closes to his favorite position, the more distant the policy of the country from his favorite position the greater is his dislike for that policy, and he does not differentiate whether that policy is towards his right or left, this is just a simplifying assumption.

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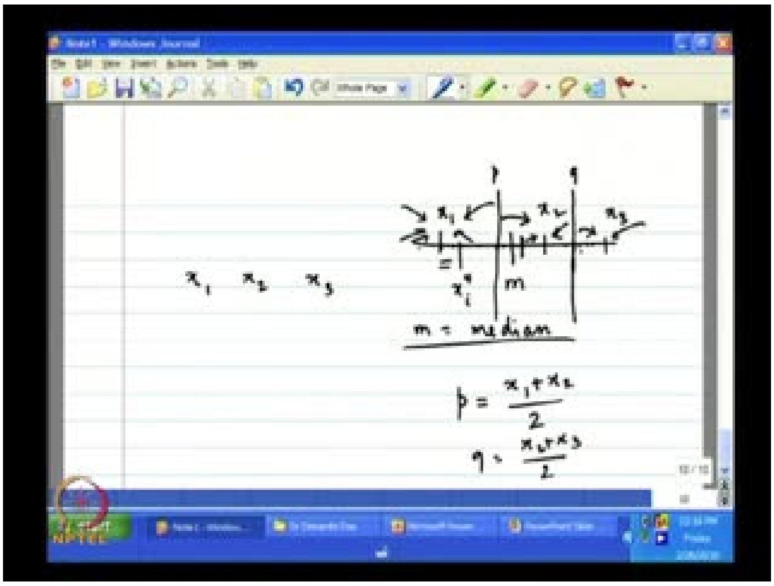

So, given this case we have seen that all the voters are having some favorite positions over this line. Now, we are not going to assume whether this distribution of favorite positions is of a particular kind, it is just a continuous distribution, there is no gap between any two points in this line that is all we are trying to say the distribution can be of any sort that is an open ended thing.

What will be important is that in this distribution there is going to be a median of this distribution. Let us call that median to be small m, so the definition as we have known before is that half of the voters in this country will have their favorite positions either equal to m or less than m, and half of the voters will have their favorite positions either equal to m or greater than m, that is how the median is define; we are going to see that this m position is going to be of major importance.

Now, let us talk about the candidates; the candidates what they are trying to do is that they are trying to win the election very obviously, and suppose there is a number of candidates, may be there are n number of candidates, and they are going to announce the positions, suppose x 1, x 2, x 3, this three positions that they are announcing and their promises that if they related to the office they are going to implement this positions; so, if candidate 1 wins is going to implement x 1, and if 2 wins he is going to implement x 2 like that.

Now, after they have announce their policies the voters vote, so suppose x 1 is here, x 2 is here, x 3 is here, how will the voters vote that is the point; well, given the assumptions that we have so far is not very difficult to see that the voters will vote for that candidate which is closes to his favorite position, his their favorite positions.

So, if I am here suppose this is x i star, then the closest position closest candidate for me is x 1, so I am going to vote for candidate 1 who as announce the x 1, because x 2 is very far away from x i star, x 3 is even further from x i star, and you can see x 1 the difference between x 1 and x i star is not much, so these votes from here also all will go to candidate 1, but how much when this voter vote for candidate 1 answer is obviously no, he is going to vote for candidate 2.

So, if I think about it little more carefully, I can figure out that they will be one point before that point that is to the left of that point all the voters will vote for x 1, and after that point the voters will vote for x 2, a similarly they will be some point here between x 2 and x 3 such that suppose this point is p, and this point is q.

So, before q and after p the voters will vote for x 2, and after q every voter will vote for candidate 3, that we can figure out because all this points are closer to x 3 than to x 2, and it is not very difficult to figure out that p is nothing but x 1 plus x 2 divided by 2. So, p is dividing this distance between x 1 and x 2; if you are to the right of p you are closer to x 2 than to x 1, if you are to the left up p the closer to x 1 than to x 2, that is why p is dividing this line of x 1 and x 2, and similarly q is dividing x 2 and x 3.

So, all these voters here are going to vote for 1 voters here, vote for 2 and 3, so this is how the voting takes place, and the candidates know this, and since the candidates know this what they want to do is that they want to win the election, and so they want to garner they want to get as many votes as possible; in this context, it is important to note that there are no ideological persuasions we are going to assume that the candidates are not concerned about the positions.

So, positions that they take, only thing that there interested in is to win the election; we can again relax this assumption a little bit, but this is a simplifying assumption to begin with, we are going to assume that candidates do not bother about the positions that they take as long as they win that is the best thing that can happen to them.

Now, this is the setting, then in this case if we have this setting then the questions that we want to ask are how the candidates will choose their positions that they are going they are announcing, for example, how x 1, and x 2, x 3 are decided; and if they have decided their announce their positions then how the voters are going to vote, and who will win and if at all people will win or will there be a tie.

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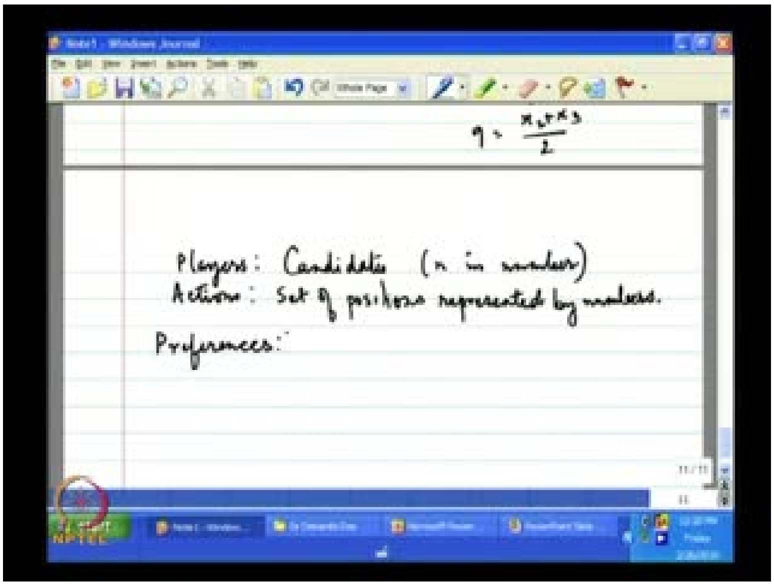

So, this is the question that we are interested in; so, remember we are talking in terms of game theory where in a game theoretic frame work, so it is better to specify this setting in terms of the language of game theory; so, players, the candidates n in number, this is important, because we are going to assume that the voters are not playing the game, the voters are going to vote in a non-strategic fashion.

So, they are not going to calculate this, other voter is voting for him so I should vote for this candidate so that my favorite candidates wins nothing like that; the voters are just looking at the announcements made by the candidates who are running the election, and they are choosing that candidate who is closest to their favorite position, that is all. It is the candidates who are trying to act strategically and deciding what position and what announcements to make, so that they can win. So, what are the actions that they are taking the actions are basically numbers, so my actions is set of positions, and this positions are represented by numbers.

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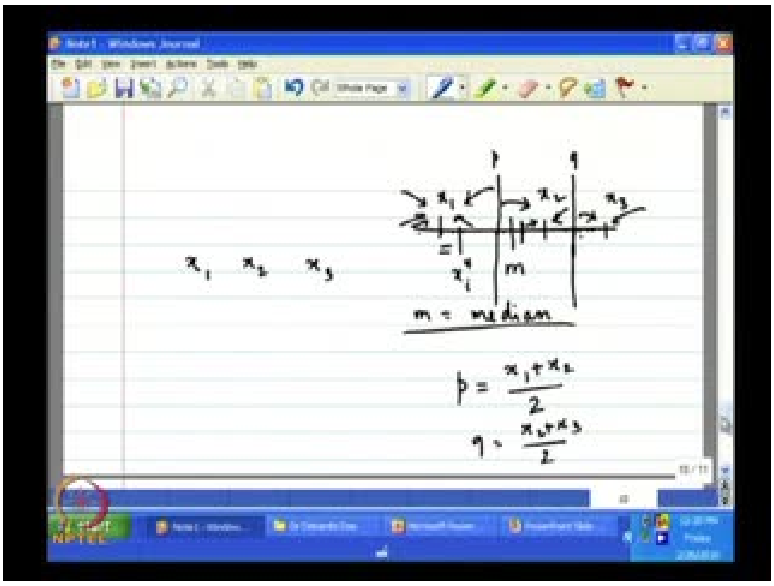

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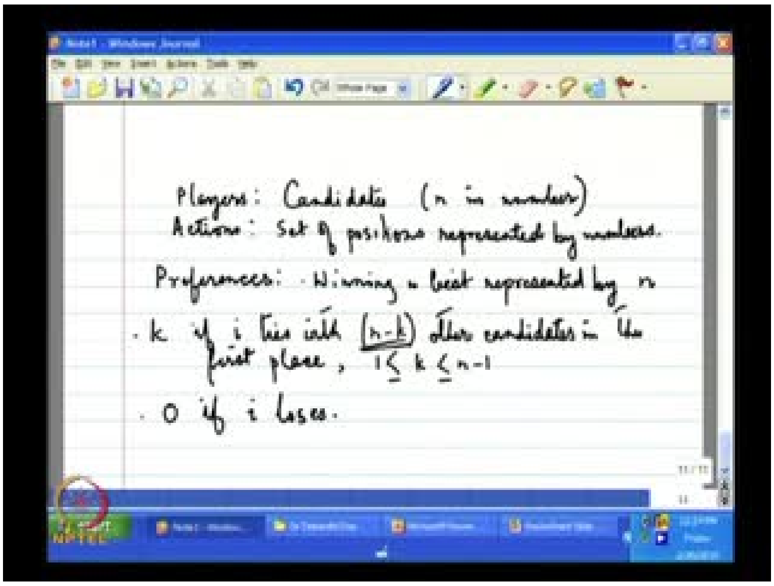

So, what I the actions that I take, I can take are basically some numbers, I can choose any of those numbers and that will be my action; another thing I did not mentioned is if there are some voters on this line p, p is the middle point between x 1 and x 2, then this voters are going to be equally divided between candidate 1 and candidate 2, because they are on the border line. Preferences: preferences are very intuitive and obvious, the candidates we want to win, so winning is the best possible thing that they can do, so winning is best by n suppose.

So, if I win, I get n what is the second best? Second best is that, I do not win out rightly but I tie with some other candidate in the first place, so I am in the first place that is true, but this is not and outright wins for me. So, this will be represented by k, if i ties with n minus k other candidates, so this is the second case of this is the winning, and this is the tying if i tie then i get k, k is any an integer, and k can vary between n minus 1 and 1; if k takes the highest value that is n minus 1, then basically by putting k is equal to n minus 1 from here i get 1.

So this is the case when i is tying with just one other candidate in the first place, so there are two winners here; and if there are winners the payoff that each of them gets is given by n minus 1, and this goes on rising, this payoff goes on declining as the people who are tying in the first place goes on rising; for example, let us take the lowest possible value of k; if k is equal to 1 then i is tying with n minus 1 other candidates; basically he is tying with everyone, nobody is winner is the winner and nobody is a loser either everybody is tying in that case he is getting payoff of just 1.

And lastly, 0, if i loses, so this is the worst possible situation that you contest the election and there is at least one candidate who has got more votes than you, and therefore you lose, and therefore you get 0, so this is the set of preferences. Now, what will be the Nash equilibrium, what we are going to do is that like before in the while trying to find out the Nash equilibrium; in other problems we are going to find out what are the best response functions of candidates and try to see at what point they intersect with each other, but to make it tractable we are going to assume that n is equal to 2.

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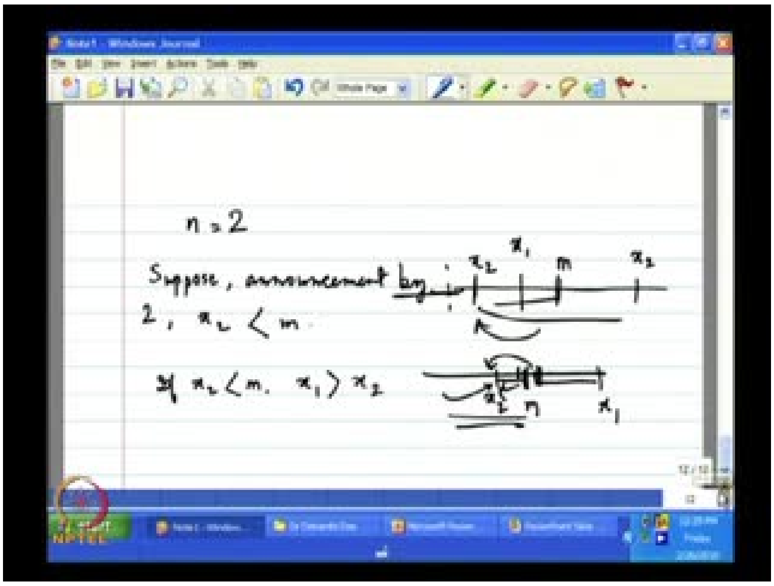

So, there are just two candidates who are competing with each other and trying to win the election. Now, let us look at this problem from the point of view of player 1 that is candidate 1. Now, depending on the position announced by candidate 2 his best response will be different, and this announcement by candidate 2 can take different kinds of values, what is important is that, that value is it greater than or less than m or is it just equal to m, that is an important question, sorry, that is an important question to ask.

So, x 2 is the announcement made by candidate 2, x 2 can be of different values, it can be less than m it can be more than m; if it is less than m, suppose announcement by 2 x 2 is less than m, then what is best for candidate 1 to do, candidate 1 will obviously in this case announce something more than x 2, because if it announces something less than x 2 then it is getting these votes, but all these votes are going to x 2 then which is not a good thing to do, because I know that m is here so half of the voters are to the right of m, these votes person 2 will get these votes person 2 will get, so he is going to win, so person 1 that is candidate 1 he is never going to announce something less than x 2; will he announce equal to x 2, if he announces equal to x 2 there is going to be tie, because in that case all the voters are going to be divided between the two candidates, so that is going to be a tie.

So, the thing to do is that, for candidate 1 that he will announce something more than x 2; so, if that is the first conclusion that we can draw for here, but is it an arbitrary value is it an arbitrary value as long at as it is greater than x 2, the answer is no, because suppose here is m, and here is x 2, if 1 announces something here greater than m and too much greater than m, x 1 is too much for away from m, then all this votes will come to candidate 2, and there will be some votes from here also which will come to candidate 2, because for this voter this distance is less than this distance.

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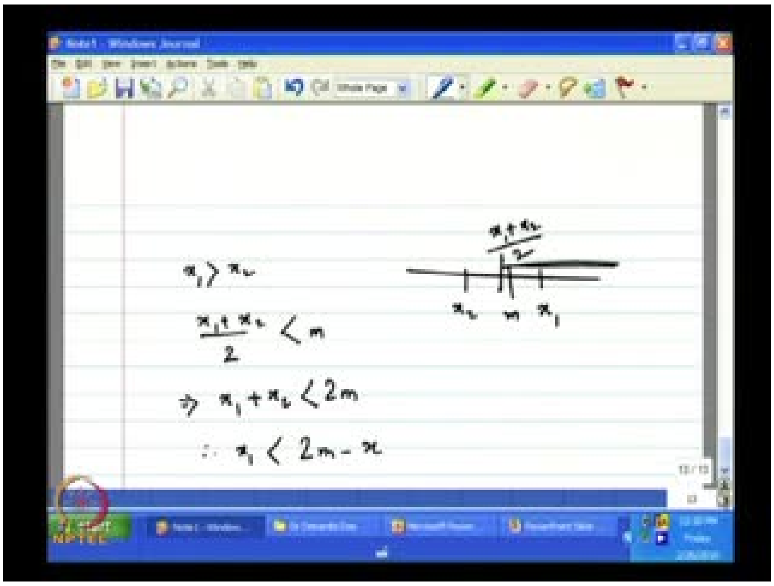

So, he will vote for candidate 2, and these voters are in addition to the half of the voters that candidate 2 is already getting, so in that case candidate 2 will win; so, for candidate 1 to win, he will announce something more than x 2 that is true, but at the same time that more should not be too much, he will keep himself close to x 2 and not go too much further away from x 2; so, if I have to draw this in a more clearer diagram, if m is here, x 2 is here, then x 1 should be here such that x 1 plus x 2 divided by 2, if it is less than m then candidate 1 is getting all these votes which is greater than half; if it is equal to m then again the voters are going to be equally divided, so one thing is that x 1 is greater than x 2, and second thing is that x 1 plus x 2 divided by 2 should be less than m, and if I simplify this x 1 plus x 2 less than 2 m or x 1 less than minus x 2.

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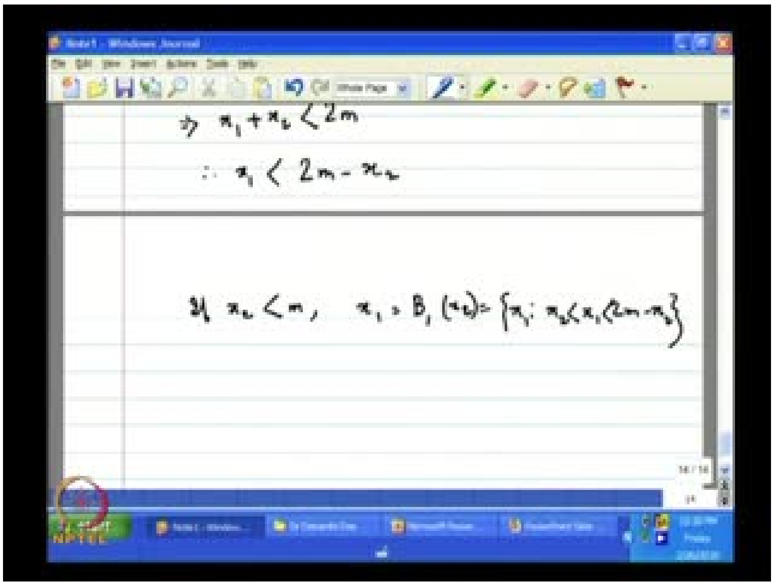

So, in short if x 2 is less than m x 1 which is equal to B 1 x 2 should be is given by x 1 such that x 1 is greater than x 2 but less than twice a minus x 2. So, this is one best response function we are going to construct other best response functions in the next lecture and try to find out what will be the equilibrium in this game. To recapitulate what we have done in this lecture is that we have finished our discussion of Bertrand oligopoly and we have started discussing the electoral competition games and we have trying to find out what are the best response functions. Thank you.

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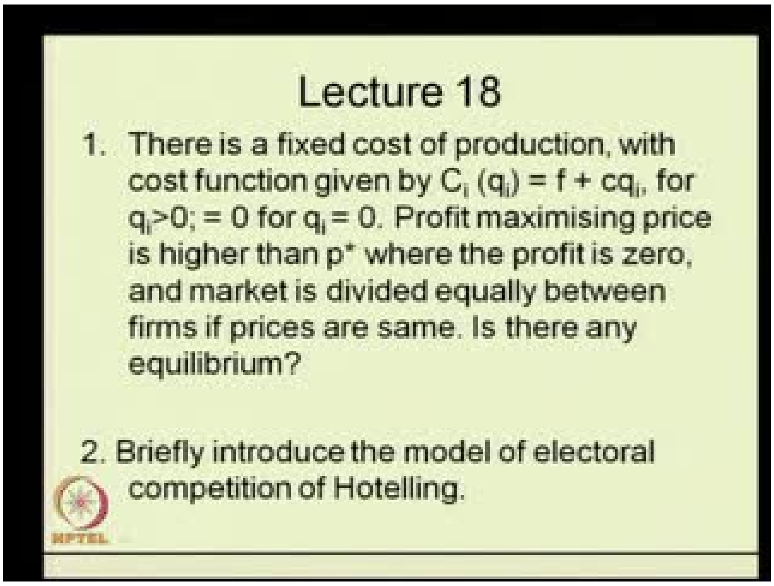

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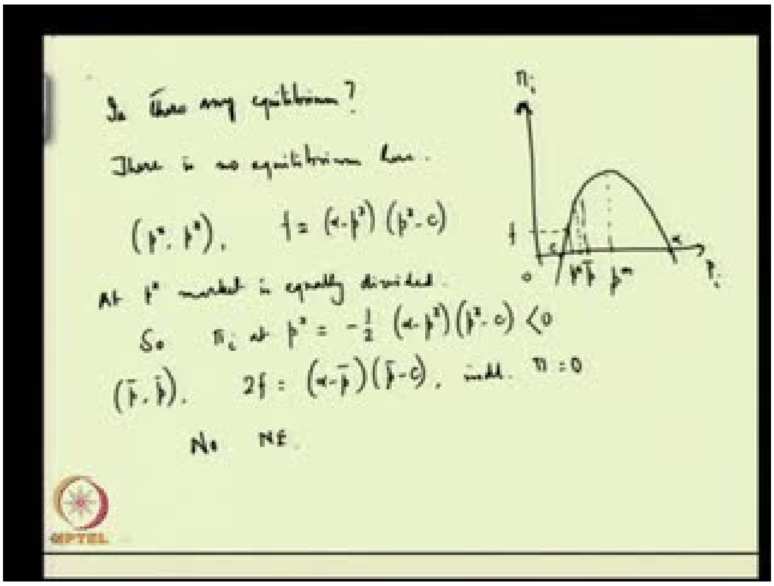

There is a fixed cost of production, with cost function given by C i q i is equal to f plus cq i for q i greater than 0, and cost is equal to 0 for q i is equal to 0. Profit maximizing price is higher than p star what is p star p star is the price where the profit is 0, and market is divided equally between firms if the prices are same. Is there any equilibrium? Let us try to visualize this, what is happening here?

So, what I am drawing is the profit function of any firm, and this is the price that is charged by that firm, this is price at which profit is maximized; so, let us call it p m, and suppose this is the p star price, and this is the value of f the fixed cost, and this point of intersection is the point of c, and this is the point alpha. Now, is there any equilibrium? And our claim is that there is no equilibrium here; what could have been the probable candidate for equilibrium, for example, p star p star could have been a probable candidate for equilibrium, because apparent reason is we know that f is equal to alpha minus p star p star min c, cap p star the total profit is 0.

So, this much sold the fixed cost is equal to the profit from the variable cost component. But here at p star market is equally divided, so profit at p star is we can calculate this to be alpha, this which is negative. So, rather than earning a negative profit a firm will deviate and charge something more and earn 0 profit, so p star p star not equilibrium; if we take any other pair of prices higher than p star then some firm earning positive profit if the prices are different the lower price charging firm is earning some positive profit; in that case the other firm we under cut that firm, so that cannot be an equilibrium another consideration we could take is this price pair where this holds.

So, what is happening is that, at p bar if both the firms charge p bar, let us suppose this is p bar then their individual profit is 0, but this is not an equilibrium for the reason that each firm will then will have a tendency to undercut the other firm and earn a positive profit, you know at this any price less than p bar you are getting the entire market rather than sharing the market and your profit will be positive.

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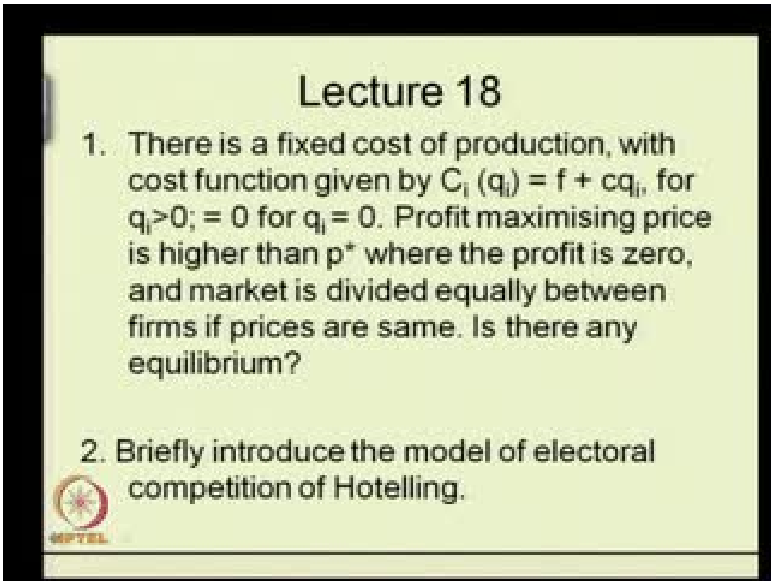

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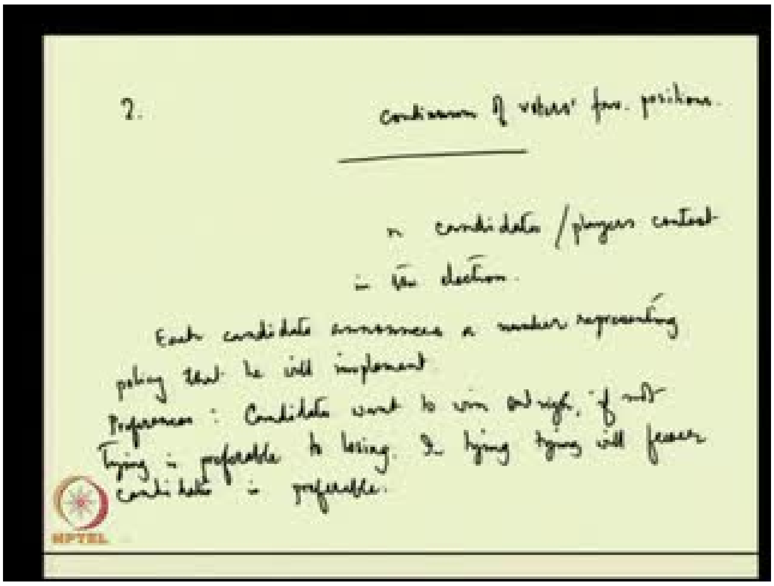

So by this logic there is no equilibrium. Briefly introduce a model of electoral competition of hoteling, so what we have a continuum of voters favorite positions and n candidates players they are the players of the game contest the election; in the election and, each candidate announces a number representing the policy that he will implement; and depending on the announcement of the candidates the voters vote are the closer the position is to your favorite position the better off you are; if the candidates announcement is something far away from your favorite position, you are less likely to vote for that candidate; and what are the preferences of the candidates - candidates want to win outright, this is the first, if not tying is preferable to losing tying is the first place is preferable to losing. In tying, tying with fewer candidates is preferable, so this the game, and this is the basic setting of the game. Thank you.