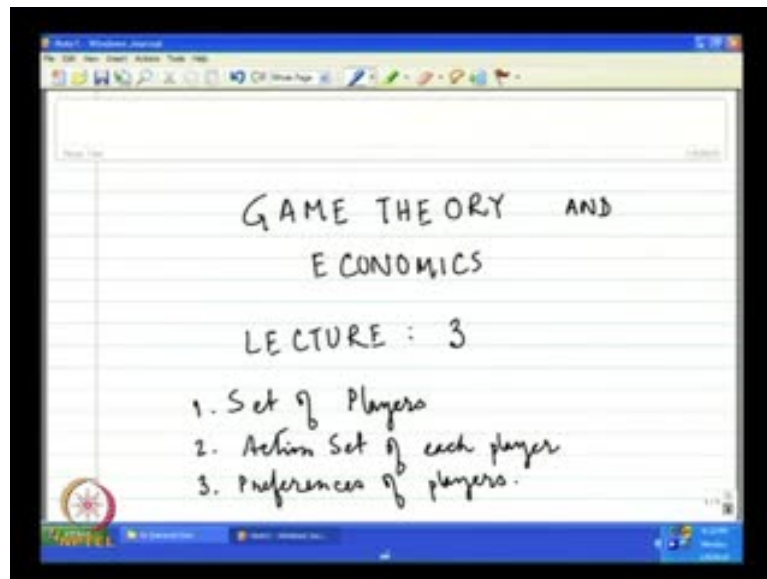


**Game Theory and Economics**  
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**Module No. # 02**  
**Strategic Games and Nash Equilibrium**  
**Lecture No. # 01**

Let me first recapitulate, what we have done in the previous lecture and then we shall continue with this lecture. What we have done in the lecture 2 is that we have introduced the basic idea of strategic games; we said that strategic game consists of 3 elements.

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Number 1 - the set of players, 2 - action set of each player and 3 - preferences of players. So, if we have an idea about these 3 elements, primary elements of a game, then that game can be represented as a strategic game.

**We have also said that** Another major important point about strategic game is that in a strategic game, the players move simultaneously; we can visualize that they move simultaneously.

We can as well visualize that they move sequentially, but then we have to assume that they have decided their action in the beginning of the game itself, once and for all. When the game moves, that is, if some other player makes a move, after seeing his action I cannot change my action; whatever I said in the beginning of the game is final.

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Strategic Games:  
PRISONER'S DILEMMA

		2	
		C	NC
1	→ C	1, 1	3, 0
	NC	0, 3	(2, 2)

Free Ride

Action Profiles:  $(c, c)$   $(nc, nc)$   
 $(c, nc)$   $(nc, c)$

So, that is another way of looking at strategic games and the need for having simultaneous move comes from this itself, that if people move simultaneously, if I take my action at the same time as some other player is taking his or her action, then I am not aware of what he is doing; it is as if I am blind to his action.

So, simultaneous move game basically tries to capture the fact that the people are not aware of each other's actions. This is the idea of a strategic game and while discussing strategic game, in an abstract sense, we started out by giving some particular example of strategic games.

The first example that we discussed in the previous class was the case of Prisoner's Dilemma. I have already described the game in the previous lecture. So, for the sake of continuity because we shall be referring to this game in this class also, let us just go over it very quickly and briefly. The idea is that there are 2 players, 2 prisoners basically, who are caught for a serious crime and they are kept in two separate prison cells. This is prisoner 1 and this is prisoner 2.

Now, what the prisoners can do is that either they can confess to the crime or they need not confess to the crime. If **they confess to the serious crime** both of them confess to the serious crime, let us call this action C, C, then both of them will go to jail for 3 years and this going to the jail for 3 years is represented by the number 1 for each.

In each of these cells here, each of the cells in this 2 by 2 matrix here, the first element 1, this element 1 represents the payoff of the first player and the second element is representing the pay-off of the second player. So, if both of them confess, each of them is going to the jail for 3 years; this 3 years is represented by this number 1.

Otherwise, they could not confess and if both of them do not confess, then what happens is that the police is not able to prove the case that these people are guilty and in that case they are going to the jail because they are petty thieves. So, they will go to the jail anyway for some petty crimes.

But in that case, the jail sentence is shorter. So, they go the jail for not for 3 years, but for 1 year. Since 1 year is better than 3 years, I represent this pay-off by 2; 2 is greater than 1 and 2 is preferable to 1, which means that going to jail for 1 year is better than going to jail for 3 years. So, these are the cases of C, C; this is called an action profile. So C, C is an action profile. NC, NC is another action profile. For both of them, NC, NC is better than C, C because 2 is greater than 1.

What happens if one of the players confesses, whereas, the other does not? Then we have a tricky situation. Suppose, player 1 confesses, player 1 is choosing C and player 2 does not, he is choosing NC. In that case, player 1 confesses and in a way, player 1 is cooperating with the police and by confessing he is not only saying that I have committed the crime, he is also saying that the other one, the player 2, who is a partner of player 1 was involved in the crime.

In this case, player 1 is freed because he is cooperating with the police. So, he does not get any jail sentence. So, he will get the pay-off 3; 3 means the best; the best that he can get. Whereas, the other player who has not confessed, but who is implicated by the first player will be sent to jail for longest possible number of years.

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Strategic Games:

PRISONER'S DILEMMA

		2	
		C	NC
1	C	1, 1	3, 0
	NC	0, 3	(2, 2)

Free Ride

Action Profile

(C, C)	(NC, NC)
(C, NC)	(NC, C)

So, in this case, he goes to jail for 5 years and this, we represent by the number 0. You can see that how the numbers are comparing to each other. There are 4 numbers involved here 0, 1, 2, 3; 0 is going to jail for 5 years, the worst possible; 1 is going to jail for 3 years, which is the second worst and 2 corresponds to the sentencing of 1 year to jail, which is the second best and the best you can do is you do not go to jail at all in which case, you get 3.

So this C, NC is another action profile; C, NC is the action profile which will you can see is good for player 1 but, it is bad for player 2 and the opposite thing will be NC, C NC, C is the case where player 1 is not confessing. So, he is getting 0, but player 2 is confessing and he is implicating player 1. So, he is rewarded in the sense that he is freed. This is the best thing that can happen to player 2.

So, this is the situation. This is a Prisoner's Dilemma game. This is the typical Prisoner's Dilemma game. We shall see that such kind of situations, qualitatively such kind of situations can prevail in others spheres of life as well.

Now, it is not the case that the players know what the other player is doing. For example, if I look very carefully into the game, I can see that if I compare between C, C and NC, NC then obviously, NC, NC is a better option for both the players.

So comparing between C, C and NC, NC, each of the players will like to have the outcome, where none of them is confessing.

This is a kind of outcome, this NC; NC is a kind of outcome in which we can say that the players are cooperating with each other. Instead of confessing to the police, as if they have made a pact and they are not confessing, they are cooperating with each other and cooperation is good compared to the case of both of them confessing to the police, but from NC, NC, each of them has an incentive to confess. For example, if this is the case where NC, NC is happening, but now player 1 will see that if he confesses then he gets 3; 3 is greater than 2.

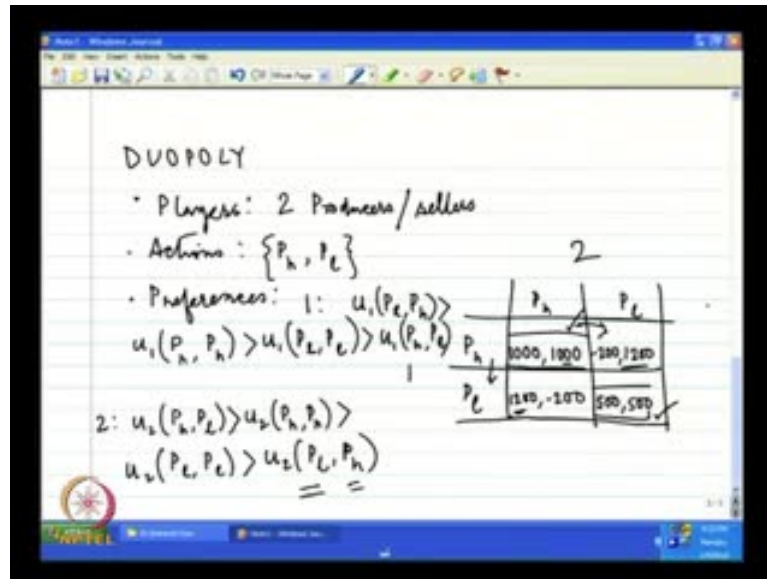
By the theory of rational choice, player 1 will be better off, player 1 in fact, will choose C provided that he knows that player 2 will choose NC, but this we do not know whether player 2 will choose NC or not.

But it is true that if player 1 knows that player 2 will choose NC, it is in player 1's interest that he chooses C and similar argument can be made for player 2 also. If player 2 knows that player 1 will choose NC, player 2 will be better off by choosing C because he is getting 3.

So, there is an incentive for both the players to Free Ride at the cost of his partner or the other player. Now, as I said before, it is not the case that we are very much interested to know how prisoners held in prison cells behave.

We are much more interested to know other real life situations, but which are qualitatively similar to this situation that we have describe right now.

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So, I can give you some other examples from real life, others spheres of life and the first example or other second example is the case of DUOPOLY. So, this is an economics example. What is a DUOPOLY? Let me clarify what does this mean - the word mean?

DUOPOLY is a market where there are 2 producers and sellers. DUO is related to the word two and so, there are 2 sellers. So, we call this is a DUOPOLY market

Similarly, monopoly will be a case where there is a market where there is only 1 seller - mono is 1 and we shall see that there are markets, where the number of sellers is more than 2, but not too many; that will be called oligopoly.

This is by way of introduction. This is an economics example where suppose there are 2 sellers in the market - producers cum sellers.

Here, the first element of the strategic game, the players is 2 producers/sellers. Secondly actions: what are the things that the players can do - these producers can do? Well, we are going to assume that the actions that they have access to, is that they can charge different prices for their products.

Now, in this connection, it is better to clarify that the products that they are producing, these 2 sellers are producing are basically similar in nature. It may be that one seller is producing one brand of cool drink and the other seller is producing another brand of cool drink.

It might happen that if I have been consuming cool drink a, now I find that cool drink b is very cheap. Then, it might happen that I do not consume cool drink a anymore and I shift towards cool drink b.

Goods are similar. So, actions are the prices that these two producers can charge and for simplicity, we shall assume that there are 2 prices, that each producer can charge and these prices are  $P_h$  and  $P_l$ .

So, this is the action set. It might be that  $p_h$  is 20 rupees,  $p_l$  is 10 rupees and this is same for both the producers. So, each of the producers can either charge 20 rupees or charge 10 rupees; again, this is just to make the story simple.

What are the preferences? Well, preferences are simple. What happens if both of the producers charge high price,  $P_h$ ?

In that case, what will happen is that the consumers that there are in the market, they will get divided between these 2 producers because the prices are the same. They cannot distinguish which producer they will go to because the goods are similar.

In that case, the producers are comparatively not better off or worse. They are in a similar position because both of them are charging high price.

However, if I compare this case of both of them charging high price with the case of both of them charging low price, then what is the difference? The difference is that in both the cases, the consumers are getting equally divided.

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DUOPOLY

- Players: 2 Producers/sellers
- Actions:  $\{P_h, P_l\}$
- Preferences: 1:  $u_1(P_h, P_h) > u_1(P_l, P_h) > u_1(P_h, P_l) > u_1(P_l, P_l)$
- 2:  $u_2(P_h, P_l) > u_2(P_h, P_h) > u_2(P_l, P_l) > u_2(P_l, P_h)$

		2	
		$P_h$	$P_l$
1	$P_h$	1000, 1000	-200, 1200
	$P_l$	1100, -1000	500, 500

But, in case of  $p_h$  that is when both of them are charging high price, profit margin for each of the producers is high because the cost in both cases is the same whereas, the price in  $p_h$  case is high. So, profit margin for each is high and we know what a producer wants to do.

In this case, it is to maximize the profit. So, any producer will like to have a situation where the profits are higher. So, I can write this in terms of a game; again in terms of pay-off matrix.

Let me just go over the action profile and the corresponding pay-offs. Any producer will like to have  $p_h, p_h$  than the profile  $p_l, p_l$  that is true. What are the other options? Other option could be  $p_h, p_l$  or  $p_l, p_h$

Now, where shall I place  $p_h, p_l$ .  $p_h, p_l$  is the case where first producer is charging a high price whereas, the second producer is charging a low price.

If I am charging a high price and my rival is charging a low price, obviously people will not come to me; they will go to the producer who is charging a low price, in which case my profit will be very low.

In fact, I can assume with some reasonable justification that in such a case, no consumer will come to me and since **there is a price of** there is a cost of production, in such a case



my profit will turn to be negative. So, this is the worst possible situation that can happen to me.

So  $p_h, p_l$  is the worst for player 1 and similarly, I can argue what happens if  $p_l, p_h$  is the case - is the action profile. I am charging a low price, whereas, the other producer is charging a high price.

Now, notice if I charge a low price, obviously my profit margin is low. The difference between cost and profit is low. So, that is the bad part of it, but the good part which is now overwhelming the bad part is that, all the consumers are coming to me. They are not going to the producer who is charging a high price.

So, for player 1, the best possible case that can happen is the case where he charges a low price and the other player charges a high price.

It will look like this, he charges a low price whereas, the other producer is charging high price. So, this is the case for player 1; for player 2, it will be similar, but a little different. So, for player 2, the best case is that he charges a low price and the other producer charges a high price.

So, the worst possible case is that other player is charging a low price; the other player is getting all the consumers. No consumer, no customer is coming to the second player because he is charging a high price.

Now, this  $p_h$  can be represented here and  $p_l$  can be represented here and instead of using 0, 1, 2, 3 we can just point out what are the profits or loss that are being made because I know that what is important for ordinal preference is the relative grading of the profits or losses.

So, you can as well use the profit figures here, higher the profit the better. If both the producers are charging high price, let us assume that the profits that they are getting is 1000. If both of them are charging low price, the profits are less, let us say 500, 500 each.

If player 1 charges a high price, he will make losses. Suppose that, this is 200 rupees which is a loss and I have a negative sign here, whereas, the other player is making

higher profits; so, it is 1200 rupees and similarly, 1200, minus 200. So, this is how it looks, in case of DUOPOLY.

You can see that this situation is qualitatively similar to the Prisoner's Dilemma situation. How we can say that? Well, compare again between p h, p h and p l, p l.

In p h, p h situation, it is as if they are cooperating with each other that they are going to charge a high price and if they indeed charge, both of them charge high prices then they are getting 1000 rupees each.

Whereas, if this pact does not take place and if both of them charge low price, then profit to each is less – 500. (Refer Slide Time: 21:52) So, this is like the situation where there is no cooperation between the prisoners; they are confessing; so, this is the case like C, C.

This is the case like NC, NC where it is as if they are cooperating with each other and they are deciding that they will not confess in which case their pay-off is higher 1000.

But you can see like the Prisoner's Dilemma situation, if both of them decide not to confess, then from the point of view of each individual player, it is better to charge a low price.

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The slide contains the following text and a payoff matrix:

**DUOPOLY**

- Players: 2 Producers/sellers
- Actions:  $\{P_h, P_l\}$
- Preferences:
  - 1:  $u_1(P_h, P_h) > u_1(P_l, P_h) > u_1(P_h, P_l) > u_1(P_l, P_l)$
  - 2:  $u_2(P_h, P_l) > u_2(P_h, P_h) > u_2(P_l, P_l) > u_2(P_l, P_h)$

		2	
		$P_h$	$P_l$
1	$P_h$	1000, 1000	-200, 1200
	$P_l$	1200, -200	500, 500

For example, for player 2 instead of charging high price, if he charges low price, he is getting 1200 which is greater than 1000 whereas, from the point of view of player 1 also, if he charges a low price then here he is going to get 1200 as profit which is higher than 1000.

Like in the Prisoner's Dilemma the situation where there is cooperation between the players can be in fact subverted by the logic of individual rationality. Each of them might like to deviate from that situation and do something else. Again this is a situation which is we say, Prisoner's Dilemma like situation.

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The image shows a handwritten payoff matrix on a piece of lined paper. The title is "Joint Project". Below it, it says "Players: 2 Students" and "Action: {HW, S}". To the right, it defines "S = Shirking" and "HW = Hard Work". The matrix is a 2x2 grid with "HW" and "S" for both players. The payoffs are (2,2), (0,3), (3,0), and (1,1). There are checkmarks in the (2,2) and (1,1) cells, and a circled 'G' in the top-right cell.

		Player 2	
		HW	S
Player 1	HW	(2, 2) ✓	(0, 3) ✓
	S	(3, 0) ✓	(1, 1) ✓

Let me give you another example which is a case of suppose, joint project. What is the story here? The story here is the following that the players, again 2 in number. Let us say there are 2 students and what these 2 students are doing is that they are trying to write a project.

What are the actions that they are having access to? Well, either, they can work hard, let us say, HW - hard work or they can shirk; let us call this S, S which is for shirking; they are not working hard; HW is Hard Work.

These are the actions that they can take and what are the preferences? Well, it so happens that if both of them work hard then the project quality is very good.

So, that is a situation which both of them will like compared to the case where both of them are shirking. If both of them do not work, obviously the project quality will be very poor. So, I can write 2 here and 1 here; 2 is greater than 1. Each of them will prefer a situation where both of them are working hard to a situation where both of them are shirking.

What happens in other cases? Other two cases: If player 2 works hard and player 1 does not then the project quality is not so good, not so good compared to the case when both of them are working hard.

So, that is a bad thing for player 1 also, but on the other hand if he does not work then it is saving his effort and that is something which he likes and this addition to his benefit because he is not working hard is higher than the loss that he suffers because the project quality is not so good.

So, this shirking is then basically, a better outcome for him provided the other person is working hard. So, here it will be 3, similar for player 2 last element will be 3. If he shirks, he is better off compared to the case when he is working hard, provided the other player is working hard.

What about the other player? Who is working hard and whereas, his partner is shirking then it is the worst possible thing for him because firstly the project quality is bad; secondly, which is compounding this situation even further is that he is now feeling cheated; he is now feeling exploited.

So here I shall put 0. Notice between this S, HW and S, S, the project quality is better in S, HW. So, here the project quality is better because at least one of them is working, but nonetheless player 2 is feeling cheated, that is why his pay-off from this, which is 0, is less than his pay-off from S, S, that is where both of them are shirking and the project quality is poor.

So, you can see again this situation is like the Prisoner's Dilemma situation, where HW, HW is compared to the case where Prisoner's are not confessing and S, S is the case where they are confessing.

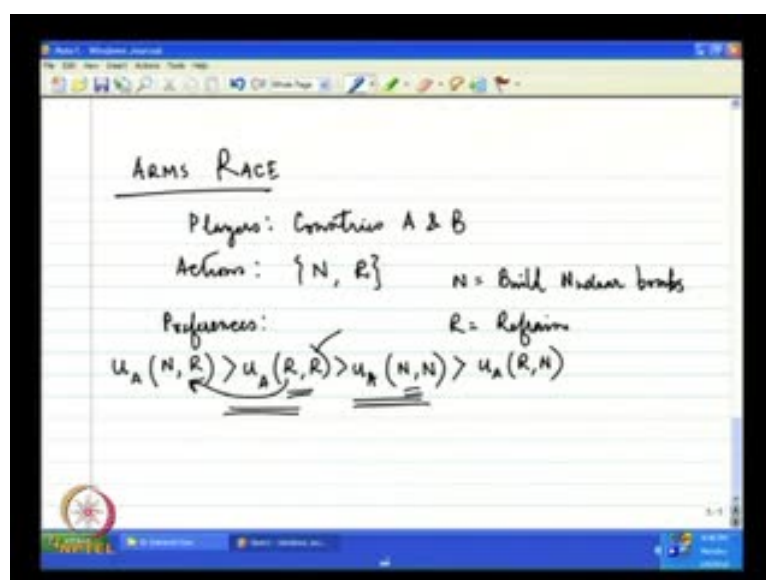
Not confessing is better than confessing. Though this is better than the other thing, both of them confessing, but from the situation of this HW, HW, where both of them are getting 2 and 2; each of them has a tendency to free ride.

They will like to now shirk so that the other guy works the entire thing and this fellow just enjoys his time.

So, this is the other case of Prisoner's Dilemma. Now, let me clarify a simple thing here. I am not claiming that in joint projects, where 2 people are involved, preferences are always like this. It might happen that if the other fellow is working hard, I get more benefit and if I also work hard compared to the case where I do not work hard.

So, that is not a Prisoner's Dilemma kind of situation. Prisoner's Dilemma kind of situation will prevail when exactly this kind of pay-off occurs, where if the other fellow is working hard, I am better off, if I do not work hard.

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Another example: let us take the case of Arms Race. Again, this is another application or this is similar to the Prisoner's Dilemma kind of situation.

Here, what we have is neither students nor producers, but suppose, 2 countries. So, 2 countries - countries A and B. This model was studied to look into the cold war tactics between USSR and USA. So, A could be USA and B could be USSR, but A could as well be India and B could as well be Pakistan.

There are 2 countries. What is the story? The story is that they can spend money and build nuclear bombs, nuclear arsenals; that is one action and the second action is that they do not spend money and they refrain from building any nuclear arsenal.

So, we can say that actions are two in number. One is, let us suppose N and the other is R, where N is build nuclear bombs and R is refrain. and preferences. Preferences are the following.

Firstly, let us compare like we have done before. Two situations where both of them are building nuclear bombs which is N, N and both of them are refraining which is R, R. Now, which is better N, N is better or R, R is better?

Such is the case that if both of them are building nuclear bombs, that situation is similar to the case of both of them refraining because in either of the cases, none of the countries is having a tactical advantage over the other country.

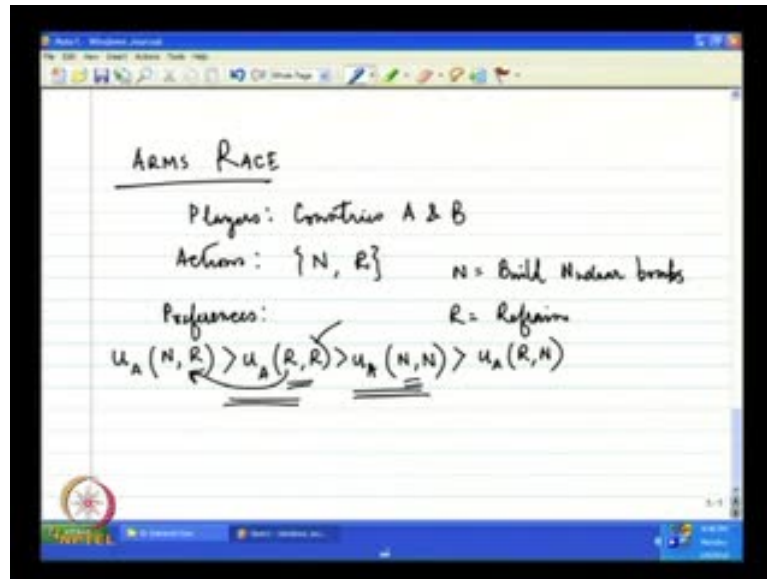
Because, ideally what any country will like to do is that it builds its nuclear bombs and the other party does not, so that this country can arm twist other country; may be launch a war on the other, if needed or even if there is no war, it can pressurize the other country to follow its own dictates.

The best possible thing that can happen to any country is that it has built its nuclear bombs whereas, the other country has not. It is like this; u A that is the pay-off, player 1 gets from any action profile, and the best is that N, R.

What is the second best? Second best is the case where both of them are refraining from building nuclear bombs. Why this is better than the case, where both of them have built nuclear bombs? Because building nuclear bombs takes money.

If you have built nuclear bombs, you have spent a lot of resources of your country into building them. That is bad compared to the case, where none of them have built nuclear bombs, where none of them have spent any resources; so R, R is the second best.

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Third best is N, N that is, both of them have built nuclear bombs. Here, you see there is no tactical advantage in either of these cases, but N, N is worse because you have spent your resources.

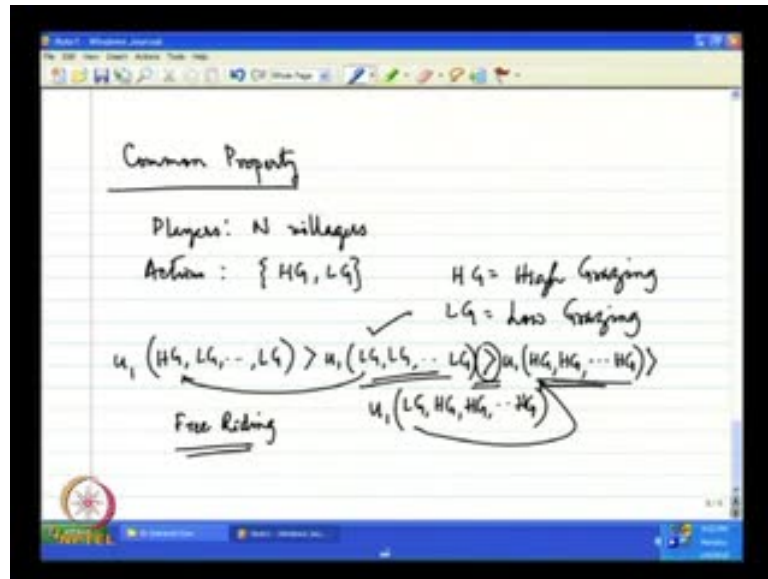
The last is R, N where I have refrained from building nuclear bombs. Now, if I have, then obviously, I am saving on the resources, but what about the disadvantage.

Disadvantage is that now I am in a vulnerable position. The other country has built nuclear bombs and it may pose threat on me and that might be more costly than the benefits; that is in fact, more costly than the benefits that I get by not building the nuclear bomb.

I need not specify the pay-off matrix here. It is just like the Prisoner's Dilemma situation here C, C is compared to the case that is confess, confess is compared to the case of N, N where there is no cooperation, both of them are distrusting each other and they have built nuclear bombs.

R, R is the case where both of them are in a sense, cooperating with each other and they have not built any nuclear bombs. Best is that when I know the other fellow is not building nuclear bombs, I shift from R to N that is instead of refraining, I now build a nuclear bomb and I pose at threat to the other player and that is a tactical advantage to me. So, this is arms race.

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Before I go to the next game, I will just give you another important application of Prisoner's Dilemma situation. It is the problem of common property and the story is the following.

This story is a little different from the other stories that we have encountered so far, in the sense that in the other stories, if you have noticed, there were only 2 players.

Now, it is not necessary that in a game theory, there are only 2 players and in this game, in this common property game, let me have a case where **there are more than there is more than 1 player**, there is more than 2 players.

Here, players are the following; suppose, there are n villagers. Actions: well, what these villagers do? How they make their living is that they rear sheep and each villager has suppose, same number of sheep with him and there is a common grazing ground in the village.

The problem is that, if all the villagers graze their sheep in that ground by a high amount then there is no regeneration of grass in that grazing ground. There are two actions basically one is more grazing, let me write it as HG for high grazing and LG which is low grazing.



So, these are the actions. Each player has the option to let his sheep graze the grazing ground by high amount, in which case, he is choosing HG or he could choose to make his sheep graze the grazing ground by a low amount, in which case, he is choosing LG.

The preferences are: I need to basically compare between 4 action profiles. Here, there might be plenty of action profiles, as you can see that there will be n number of elements in each action profile because there are n villagers.

So to say that this game is akin to the game of Prisoner's Dilemma, I need to compare between four action profiles. Which are they?

First, in that suppose, I am talking about the first villager – so, u 1. First is the case, where he is grazing his sheep by a high amount whereas, all the other villagers are choosing LG. So, this is the best for him because since all the other villagers are grazing a little, then there is no problem of regeneration of grass.

So, grass is a plenty. He is getting the benefit of the fact that there is more grass, whereas, his sheep are going for HG. So, this is the best that he can hope for.

What is the second best? Well, second best is the case that all of the villagers are grazing their sheep by low amount. So, LG by all is the second best. What is the third best?

From the point of view of the first villager, the third best or among the four that we are comparing, the third best is that all of them are making their sheep graze by a high amount.

LG, LG is better than HG, HG because if HG, HG is the case, then this common property that is there will exhaust very shortly whereas, if LG, LG is there, well, **part** period might be the sheep are grazing less, but it will last - the property will last.

The last is that other people are going for HG, but he is going for LG. So, other people have gone for high grazing, but he is going for low grazing.

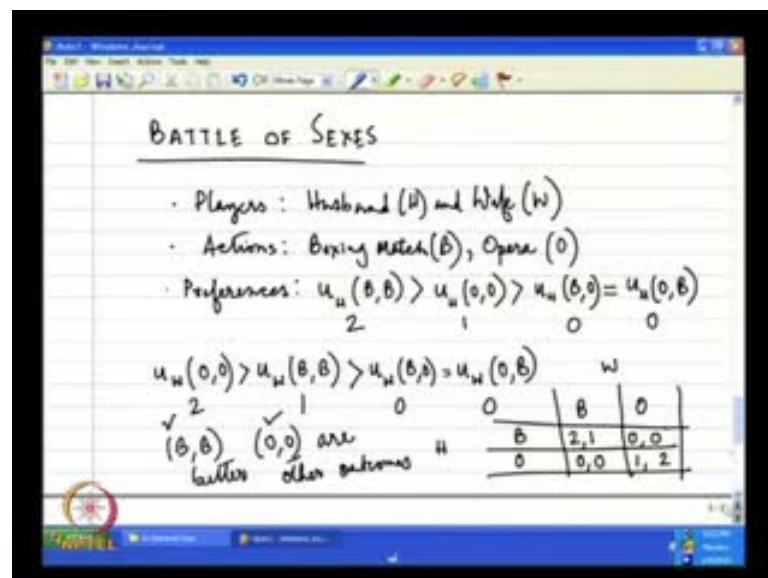
Now, this is similar to the case of Prisoner's Dilemma because you can see that this is the cooperation outcome; all of them co-operated and they have decided that let us preserve this common property, but if I know all of the other players are going for low grazing, it will be better for me to go for high grazing; this is free riding.

I am exploiting the other players because they have gone for LG whereas, I am taking advantage of that situation and whereas, LG, LG, LG everything cooperating is better than HG, HG, HG - that is. if people are not cooperating that is worse for each of them that they understand this; this is given by this greater than sign.

It is also the case that if all of them are going for HG - that is, none of them is interested in preserving the property; it is individually rational for me to go for HG as well.

So I will not be a fool and let my sheep starve, if the other people are going for HG. So, you can see that this is a very similar game to the Prisoner's Dilemma game.

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I think we have more or less discussed the different aspects of Prisoner's Dilemma and now, let me go to the second kind of game, second genre of game which we shall call, Battle of Sexes game.

Let me just first introduce the game. Then I shall talk about what is the similarity and differences of this game with the game of Prisoner's Dilemma.

Here, like the P D, the players are two in number and who are they? Let us call them husband or H and wife or to be more gender sensitive, I should have written wife and husband and what are the actions that they can take?

Well, the story is that there they are planning to spend their evening in their own way and they have the following options: that they can go to a boxing match; let us call this B and a musical performance; let us call this opera O.

What about the preferences? Since we have 2 actions and 2 players here, so, there might be 4 action profiles now, from the point of view of each player, which is better and which is worse. Well, it is the case that the husband prefers the outcome where both of them go to the boxing match; he likes to watch boxing.

Whereas, the wife likes the outcome, where both of them go to the opera and so, B, B where **both of them are going for** both of them are taking the action B; that is, both of them are going to the boxing match is preferred by the husband to the outcome where both of them are going to the opera house.

So, I can write this  $u_H$  that is, the preference of the husband is that B, B is strictly greater than O, O; that is very clear. What about the other outcomes, where husband is choosing B and the wife is choosing O or vice versa husband is choosing O and the wife is choosing B.

Now, we are assuming that in these cases, what is happening is that they are not going to the show together - either of the show together and this is the worst possible for both of them. (Refer Slide Time: 46:40) So, it is like this.

Whenever I write this in the blackboard in the class, when I try to teach this, the students protest well, we know that the husband prefers the boxing match.

So, why is he indifferent between these two? Why is he indifferent between B, O and O, B? In B, O, though the wife is not there, he can enjoy the boxing match. Now, this might be rational from the student's point of view, but this game is such that we are going to assume that if they are not together, they do not enjoy anything.

So, that is the worst possible. It does not matter, if you are going to the boxing match. If the wife is not there, you are going to get the worst possible thing.

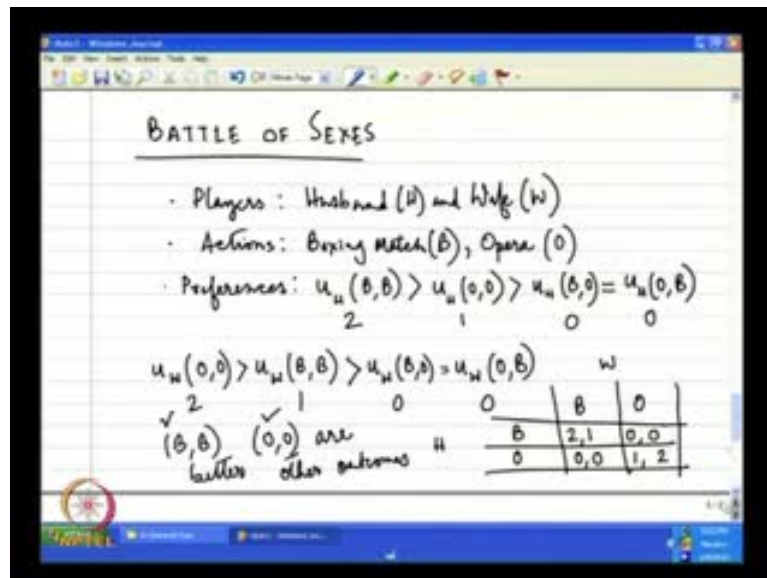
So, these are the 3 numbers that I need to assume. I need to assign to these four action profiles they can be 2, 1, 0; both are 0 here, in the last element.

What about the wife? It is just opposite. So, she will like this outcome to be the best; both of them going to the opera house. Second will be both of them are going to boxing match. This is how it looks. So, this will be 2, this will be 1 and this will be 0.

Now if I write this term in terms of a payoff matrix. So, the first profile is B, B. B, B is the best one for the husband. So, this is 2 and from the wife point of view B, B is good, but not as good as O, O. So, O, O is the best for her; B, B is 1 here. If O, O is the outcome, the husband is getting 1.

If there is no meeting of them; that is, they are going to different places; they are going to get 0 each.

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This is how this game looks like. Now, if I compare this game with the game of Prisoner's Dilemma then I can see that there is one difference with the Prisoner's Dilemma game.

In the Prisoner's Dilemma game, there was confusion between the players whether they will cooperate or not cooperate. Here, both of them are in agreement that cooperation is better. What is cooperation here? Cooperation is that both of them are going to the boxing match or both of them are going to opera house.

So, they agree that this B, B or O, O is better than other outcomes; that is, O B and B O. There is no disagreement regarding that, but the point is that they are in disagreement as

to whether B, B is better or O, O is better. From the point of view of the husband, well, B, B is better than O, O and from the point of view of the wife, O, O is better than B, B.

Now, this is different from the Prisoner's Dilemma game because in Prisoner's Dilemma what was happening is that if they were not confessing, that is the cooperative outcome; cooperative in the sense that there is cooperation between the prisoners.

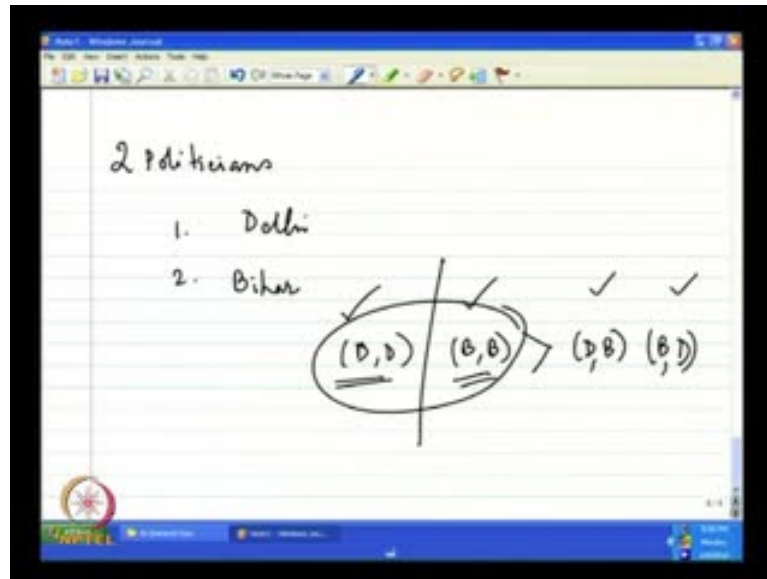
If that cooperation is the outcome and I know that the other people are cooperating then I am not going to cooperate. So, there was a dilemma regarding whether cooperation will at all take place because it is individually rational for me not to cooperate.

Here, there is no such problem. Here, cooperation is better for both of them compared to the other outcomes, but there is a disagreement as to between these two cooperation, which is the better. Now, this is the generic case. This is the case with the proper Battle of Sexes story.

But as I have remarked earlier, it is not the case that in game theory we are too much interested in knowing how husbands and wives decide whether they will go to boxing matches or opera houses.

The point is that this kind of situation can prevail in their life with a little, you know, variation. So, what can be the examples of applications of B O S or Battle of Sexes game?

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Let me give you two examples. One could be the case of Suppose, there are 2 politicians. They belong to the same party - 2 politicians and they are going to run for elections and they come from two regions of the same country. For example, suppose, one of them is from Delhi and the other suppose is from Bihar and both of them suppose belong to the congress party.

Now, what they are going to announce is that suppose, politician one is to announce that after getting elected to the office, his party spending a lot of money in Delhi.

What is the other option? The other option is that the party if elected to the office, is going to spend a lot of money in Bihar. So, these are two announcements that the party can make through these 2 politicians.

Now, you can see that the politician it is better for both of them if they take a stand which is same. For example, the first politician says that the party is going to spend money in Delhi and the second politician says that the party is going to spend money in Bihar.

Then there is a disagreement and the people who are the common voters, they will get a mixed message. In that case, the party might suffer because they will think that the party itself is not very clear as to what it wants to do. So, it is better for both of them that they take a common stand - one stand.

For example, both of them are saying that we are going to spend the money on Delhi or both of them are saying we are going to spend the money on Bihar. (Refer Slide Time: 54:44) If D, D is there and B, B is there, both of them is preferred to the case like this.

Here, both of them are saying we are going to spend the money in Delhi, both of them are saying we are going to spend the money in Bihar and here they are disagreeing. So, this is clear.

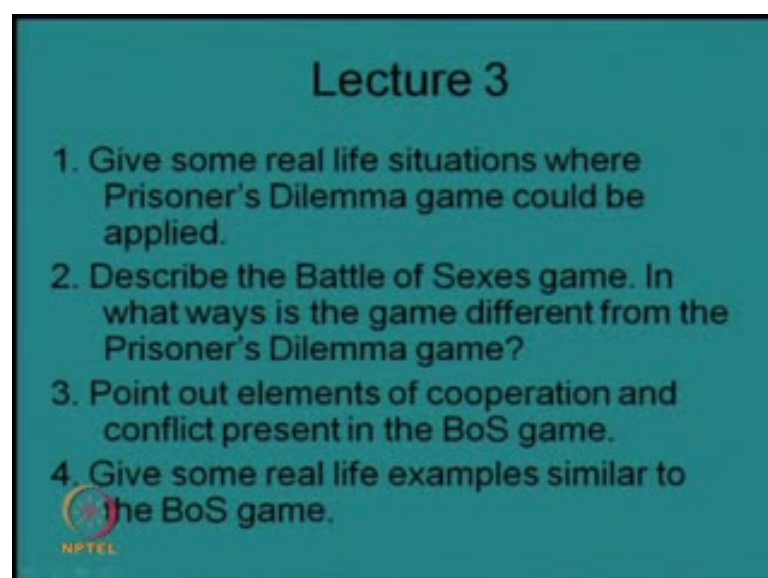
What is not clear is that from the first player's point of view obviously, this is better because he is a politician from Delhi. So, he will like the money to be distributed in his constituency .So, **providers** for the second player, it is better B, B is the action profile.

So, here you have an example of battle of sexes. They agree that these outcomes are better, but which one, that is not clear. One of them is preferring this one and the other one is preferring the other one.

**That is more or less and** I shall talk about other applications of B O S in the next class.


So, what we have done in this class or in this lecture is the following. We have discussed the Prisoner's Dilemma situation. We have talked about the various applications of Prisoner's Dilemma. In real life situations, how Prisoner's Dilemma situation can prevail and we have also introduced the Battle of Sexes game and we have seen one application of the Battle of Sexes game.

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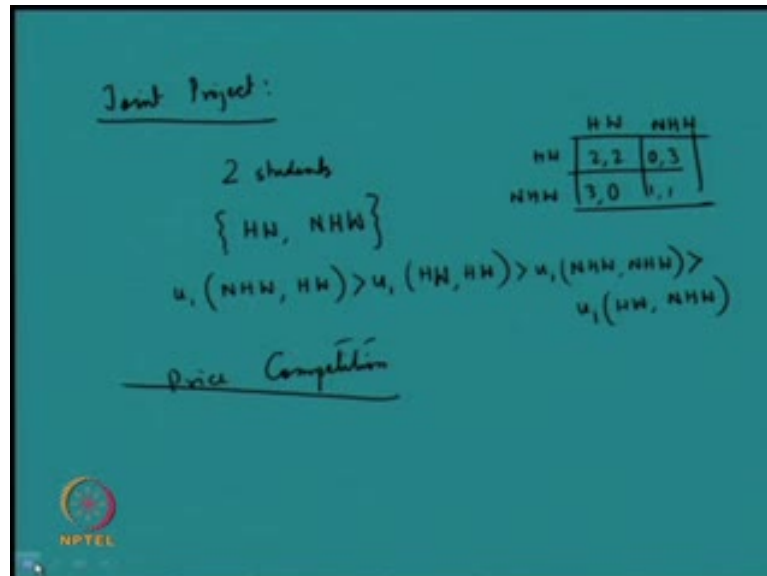
**Lecture 3**

1. Give some real life situations where Prisoner's Dilemma game could be applied.
2. Describe the Battle of Sexes game. In what ways is the game different from the Prisoner's Dilemma game?
3. Point out elements of cooperation and conflict present in the BoS game.
4. Give some real life examples similar to the BoS game.

 NPTEL

This is the exercise for the third lecture. We have 4 questions. First is, give some real life situations where Prisoner's Dilemma game could be applied.

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We can think of this case of joint project. What is the story here? There are 2 students who can take 2 actions either put hard work, just call it HW and no hard work, let us call it NHW- may be working not as much as hard work demands.

The preferences are like the preferences in the Prisoner's Dilemma. That is, suppose, I am student 1 then I like the situation best when I am not putting any hard work, but my friend is putting hard work and the quality of the project is good. So, that is the best thing that the quality is not bad and I am not putting any effort for it.

What is the second best? Second best is both of us are putting hard work. Here, the project's quality is good, but remember NHW, HW is better than HW, HW in the sense that though in the first case, the project quality is not very good, but the fact that I am not working as much as my friend is doing, gives me some pleasure.

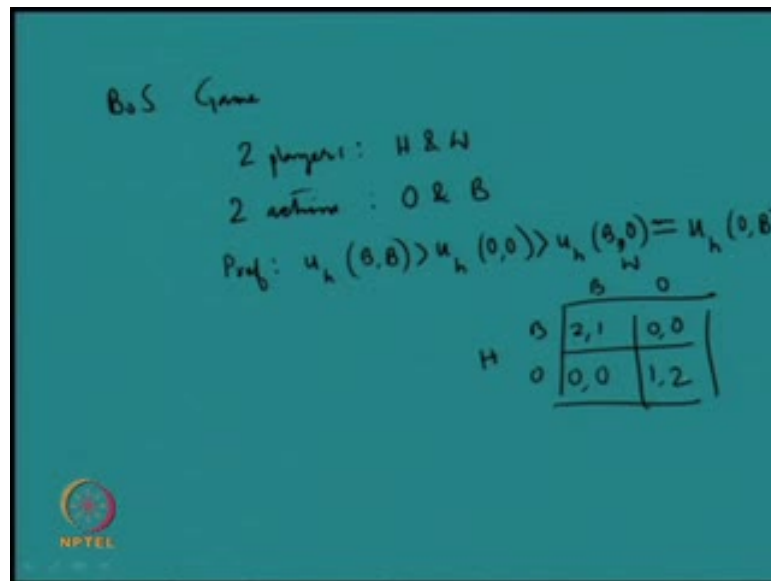
What is the third best? Third best is neither of us is putting any hard work. The quality is bad, but you know we are saving on efforts. So, I am not that much unsatisfied. What is the worst possible case? That I am putting hard work, but my friend is not putting any hard work.



So, that is how, the preference of first player looks like and you can figure out that this is going to be the same as the Prisoner's Dilemma game. Just the names of the actions are different. (Refer Slide Time: 58:53) This is hard work; this is not hard work.

We can think about price competition also, where hard work can be represented by high price and not hard work can be represented by low price and the same story could be told.

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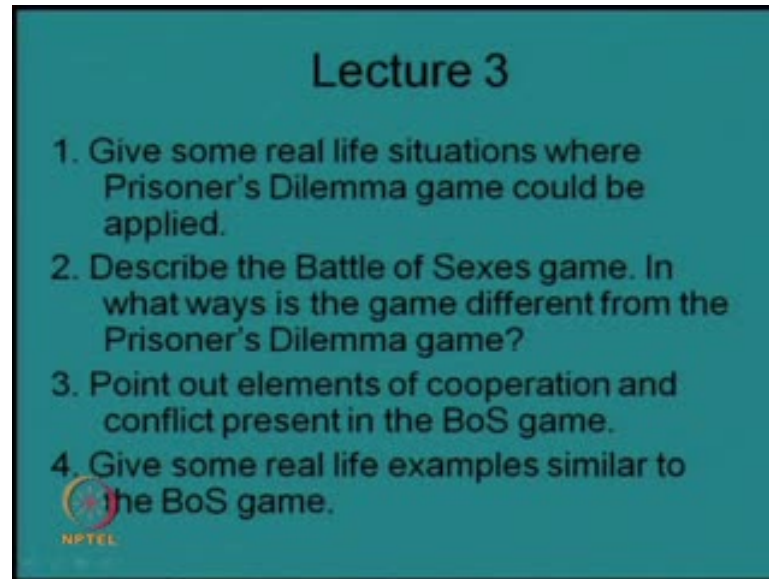


Second question: describe the Battle of Sexes game. In what ways is the game different from the Prisoner's Dilemma game?

Battle of Sexes game, B O S game: Here, we have 2 players, husband and wife; 2 actions for each. They can go either to opera or to a boxing match. Preferences: well, for the husband, let us write it as  $u_H$ , the best is both of them go to the boxing match, the second best is both of them go to the opera, third best is they do not meet anywhere that is B, O and this is the same as O, B.


So, the payoff matrix looks like the following. Let us call this the wife, the husband and B, O.

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**Lecture 3**

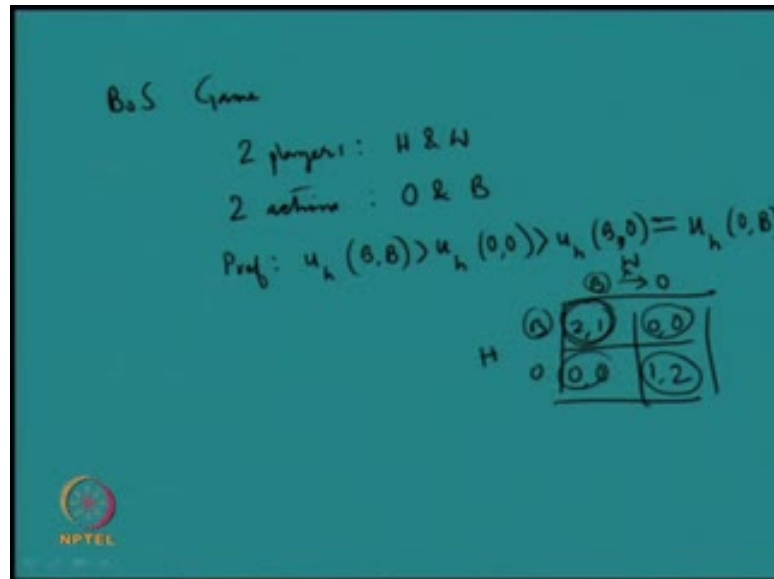
1. Give some real life situations where Prisoner's Dilemma game could be applied.
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 NPTEL

I am going to answer the last part of this question. How is it different from the Prisoner's Dilemma game? Well, the difference lies in the fact that in this game compared to the Prisoner's Dilemma game, the players are unanimous that taking the same action is better.

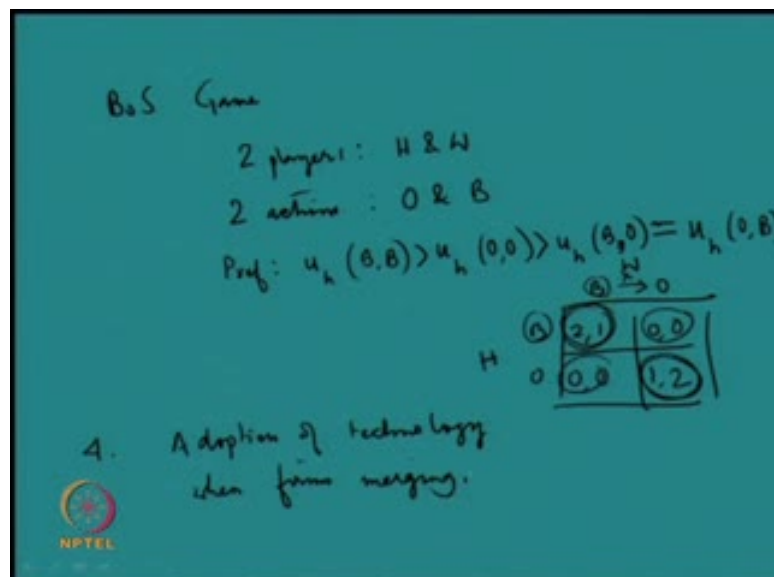
Remember in the Prisoner's Dilemma game, they were not sure whether taking the same action was better. Here for example, if they take the same action then each has an inclination to deviate, but here if they take the same action nobody will deviate. So, there is this difference.

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What about the third question? What are the elements of competition and conflict? Well, the elements of competition are there in the sense that the husband likes this outcome to prevail; the wife likes this outcome to prevail and what is the cooperation? The element of cooperation is both of them are unanimous that compared to this and this, this and this are better.

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Give some real life examples of Battle of Sexes game? One can think of the case of adoption of technology, when the firms are merging. When firms are merging, if you

have a technology, which the other firm does not have, to begin with you will like that other firm to adopt your own technology so that you are in the same technology which gives you more advantage whereas, the other firm likes you to adopt its technology.

So, there is this competition and there is the case of cooperation also that they do not want, these two firms do not want to adopt different technologies. That is the end of it.  
Thank you.