Game Theory and Economics
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Module No. # 04
Mixed Strategy Nash Equilibrium
Lecture No. # 06
Rationalisability and Beliefs

Welcome to lecture 6 of module 4 of this course, game theory and economics. Before we start this lecture, let me take you through what we have been discussing in the last lecture.

So, what we have been discussing is various aspects of mixed strategy Nash equilibrium. We have first defined mixed strategy Nash equilibrium and looked at certain properties of it.

For example that in the mixed strategy Nash equilibrium, in a proper mixed strategy Nash equilibrium, the actions which have positive probability attached to them will all have same expected payoff and that expected payoff is the expected payoff to the player from that mixed strategy profile and we have also and the actions for which probability attached is 0, the expected payoff should be at most the expected payoff to the actions, where positive probability is attached.

This important property of mixed strategy Nash equilibrium helps us to find the mixed strategy Nash equilibrium in different games, even if there are more than one action, more than two actions of a player or even if there are more than 2 players in a game.

Subsequently, we have been discussing also the modification that is needed of the concept dominance. The fact that an action can be dominated by another action. It can be strict dominance or weak dominance. That we have discussed in case of pure strategy where no randomization is allowed.

But if randomization is allowed, if player can play mixed strategies then we have seen that that definition is to be modified. But qualitatively, the game does not the results do not change much in the sense that even if we have mixed strategies, then it is seen that the action which is strictly dominated by another mixed strategy, then this action which is dominated will never be played in the mixed strategy Nash equilibrium. Even a mixed strategy which assigns positive probability to an action, which is strictly dominated will not be played in a mixed strategy Nash equilibrium.

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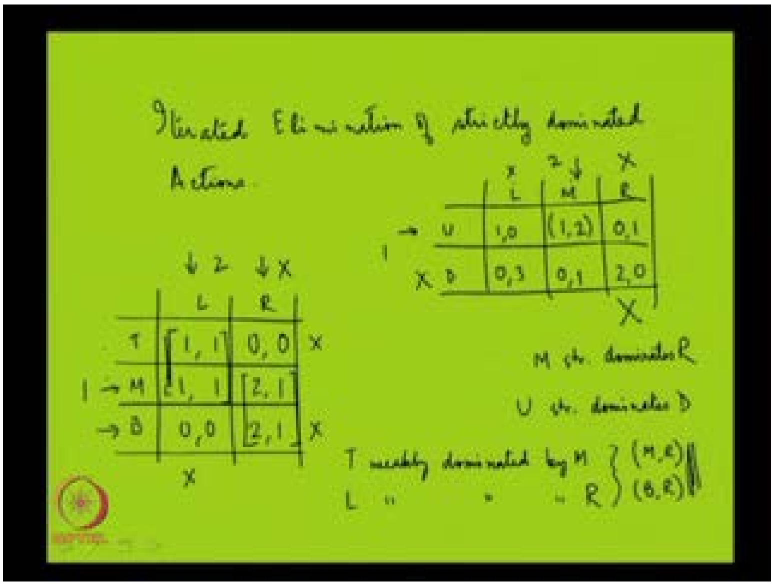

Talking about mixed strategy Nash equilibrium and strict dominance and weak dominance, we have seen that if we have strict dominance by not both the players, in a two player game, not both the players, but one player, then we can do what is known as iterated elimination of strictly dominated actions.

To take you through the example that we have given in the last lecture, this was the example.

Now, you can see that M strictly dominates R. So, if R is dominated, strictly dominated, R is not going to be played, in the sense that there is no circumstance under which 2 will play R because whether 1 chooses U or D, it is always better to play M than R. Now, if R goes out of consideration, then look player 1 now, has a strict dominance. U strictly dominates D; this is a sequential thing.

Initially, U was not strictly dominating D, but since R has gone out of consideration, then this column does not matter anymore and if this column does not matter anymore, then U is strictly dominating D and now D is going out of consideration because 1 is never going to play D.

If that is the case, then 1 is left with U and basically, 2 has to choose whether he will choose L or M and obviously, 2 is greater than 1. So M is chosen by 2 and U is chosen by 1 and 1, 2 is the payoff to the players. So, this is known as iterated elimination of strictly dominated actions.

Now, this may seem very simple way, but it is not that simple as it seems. The logic is a little bit complicated. When we are eliminating this actions what is needed is that both the players are rational they want a higher payoff than a lower payoff.

But it is not sufficient that I know I am rational. What is also required for this process of elimination is that I know that the other player is also rational and vice versa. The other player must be knowing that I am rational.

So, rationality is a common knowledge and only then, we can go on eliminating the actions in the sequential manner, which is strictly dominated.

So, it works in the following way. Firstly, player 1 knows that player 2 is rational. Since player 2 is rational, that is why R is not going to be played.

If R is not going to be played, then 1 thinks that U is better than D. Now, the fact that 1 thinks U is better than D, the fact that 1 is rational is known to 2 and that is why, since 2 knows that 1 is rational and that is why, he - that is 1, has figured out that 2 is not going to play R and that is why, 1 is not going to play D, that is why 1 is going to play U and that leads 2 with the choice between L and M and 2 chooses M.

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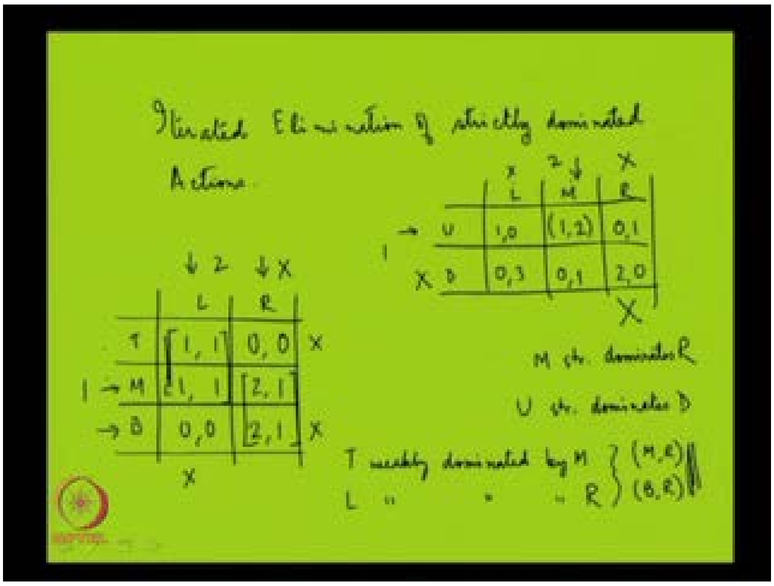

So, this rationality does not apply to a particular person on an individual basis only. It must be a common knowledge that I am as an individual rational, that has to be known to the other players also and then only we can go on eliminating these strictly dominated actions.

Now, this was the case of strict dominance. If we eliminate the actions, which are not strictly dominated, but suppose weakly dominated then can we get profiles, action profiles which are here in this case for example, 1, 2 is a Nash equilibrium, which we have got, while we eliminating strictly dominated actions.

So, does it apply in case of weakly dominated actions also? Do we get by eliminating weakly dominated actions, do we get profiles which are Nash equilibria and which are the only Nash equilibria, in the sense that in this game 1 2 is a Nash equilibrium that is U and M. This action profile is a Nash equilibrium and no other action profile here is a Nash equilibrium.

So, does this same feature apply to case, where weakly dominated actions are eliminated? We shall see that the same feature does not apply. There we shall see that the action profile that we shall be left with in case of a weakly dominated actions are not necessarily, the only Nash equilibrium; there could be other Nash equilibria also. Take the case here, this game. So, this is a game, where player 1 has three actions and player 2 has two actions.

Now, let us compare between T and M. Player 1 has three actions. If we take only these two actions T and M, then it is obvious that T is weakly dominated by M. So, T is not going to be played; T is weakly dominated.

If T is not going to be played then I am left with only M. If M is the action M and B are the actions that 1 is going to take, then basically, 2 has to consider between L and R and it is obvious that L is weakly dominated by R.

So, L goes out of the question because 1 is 1 and 1 here. If M is played by 1, if 2 plays L, he gets 1 and 2 plays R, he gets 1. If 1 plays B and 2 plays L, he gets 0; if 2 plays R, he gets 1. So, R is weakly dominating L. So, L goes out of the question.

So, this goes out of the question. So, we are left with these two action profiles and you can see that both of them are Nash equilibrium. We are left with M, R and B, R. So, these are the action profiles that we are left with and we can check that both of them are Nash equilibrium.

Now, here, we start with comparing between T and M and eliminating T, which was weakly dominated by M, but if we started by comparing M and B then what happens. Here, if we compare between M and B, we can immediately see that B is weakly dominated by M.

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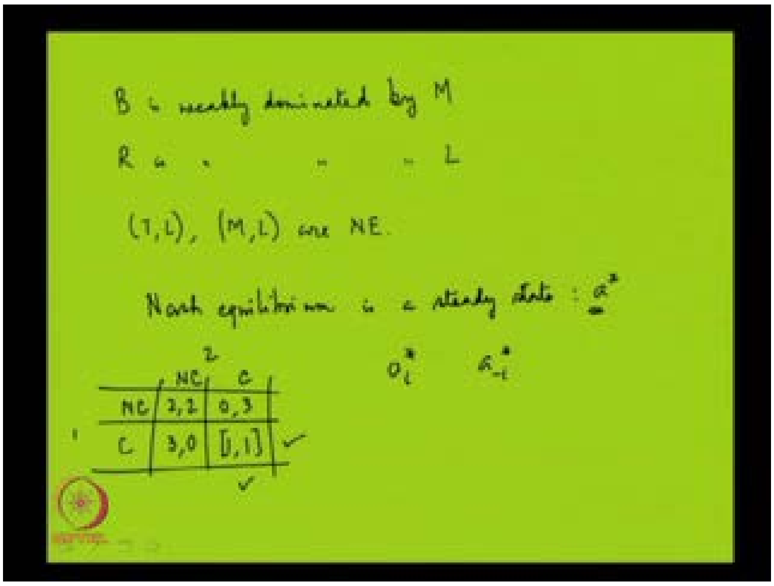

So, B is weakly dominated by M. (Refer Slide Time: 14:20) So, B goes out of question and if T and M are the actions to be played by player 1, then again player 2 will compare between L and R and he will observe that R is weakly dominated by L.

So, in a sequential manner, R is weakly dominated by L. This R now goes out of the question and we are left with these two profiles. It depends Both this T, L and M, L are Nash equilibria, but the point is that they are not the Nash equilibria which we got before which were M, R and B, R. So, the profiles that we are left with, that set varies depending on the order of elimination. If we had started with eliminating T, then we are getting these two profiles.

But if we had started with eliminating B, then we are getting these two profiles. So, in case of weakly dominated actions and elimination of iterative elimination of weakly dominated actions, the profiles that we shall be left with, the identity of those profiles depends crucially on the order of elimination, but this was not the case, in case of elimination of strictly dominated actions. There it does not matter which order you take; you shall be reaching a unique set of action profiles.

Now, one reason why we are discussing this weak domination and strict domination with such emphasis is also the fact that in case of Nash equilibrium, remember the idea of Nash equilibrium, it is a steady state. Suppose, we are talking about pure strategy Nash equilibrium, a star is an action profile such that given the other players are taking a naught i star, taking a i star while player i is optimal.

So, that was the idea of Nash equilibrium, but a crucial question that has not been answered is that how have we reached this a star. It is true that if a star is reached, if this particular action profile is reached and if this action profile is played for a number of times, then by looking at the experience, looking at the previous history of the play of this game, people who are taking the action at a particular play of the game will take a i star or a j star, whatever the actions at the equilibrium action profiles because he knows that other players will be taking their expected actions.

But the question that remains is how did we get into this a star. There must be some beginning of the game at point 0, when the game is starting and there is no guarantee that at the start of the game itself, a star will be played.

So, if there is no guarantee that at the beginning a star will be played, it may start with any action profile and from that arbitrary a, how do we get into a star; that question has not been resolved so far.

The advantage of this strict dominance or even weak dominance is that to get into this, strict the action profiles which we have got by iterated elimination of strictly dominated actions, we did not need the repeated play of this particular action profiles to say this is going to be played. At the beginning of the game at point 0 itself, players can figure out that they have to play that action profile.

So, to give you an simple example, take the case of prisoner’s dilemma. Here, we can see that player 1 has this as the strictly dominating action and this is for player 2, the action which is strictly dominating the other action. There are only 2 actions.

Therefore, this C and C, this action profile is going to be played in the beginning of the game itself. They do not need this C, C to be played over and over again to figure out that at a particular play of the game, the other player is going to play C and therefore, I should play C.

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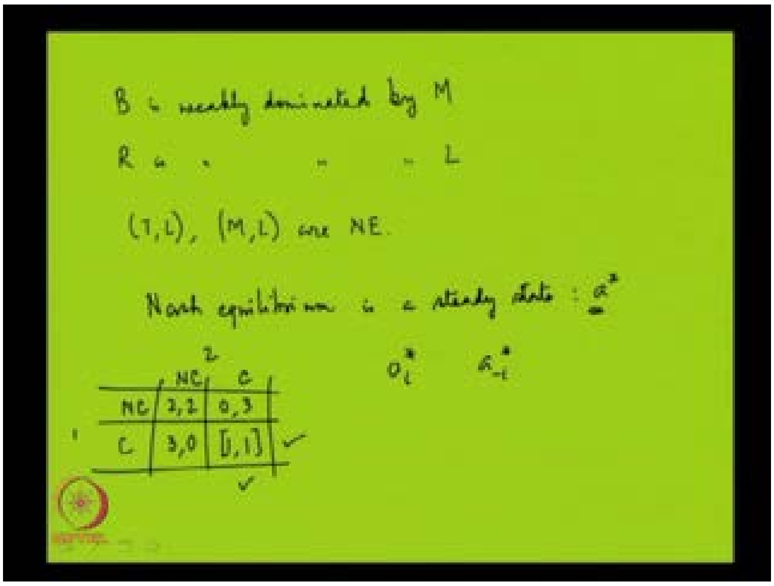

So, here at the beginning of the game itself, I can apply my rationality and I can play C and other player also can apply his rationality and right at the point 0 itself, they will be playing C and that will continue.

This was the case where there was no iterated elimination, but in case of iterated elimination also same logic applies. In the beginning play of the game itself, right at the point 0, by applying rationality and by applying the fact that the other players are rational, players can figure out what are the action profiles that are going to be played.

So, therefore, this idea of strict dominance and iterated elimination of strict dominance or weakly dominated actions are helpful in the sense that we did not need a logic as to how come this particular action profile was played because this is going to be played, even if there is no history.

But whereas, in case of Nash equilibrium, we need this a star to be played for some period of time to justify that it is going to be played in future also.

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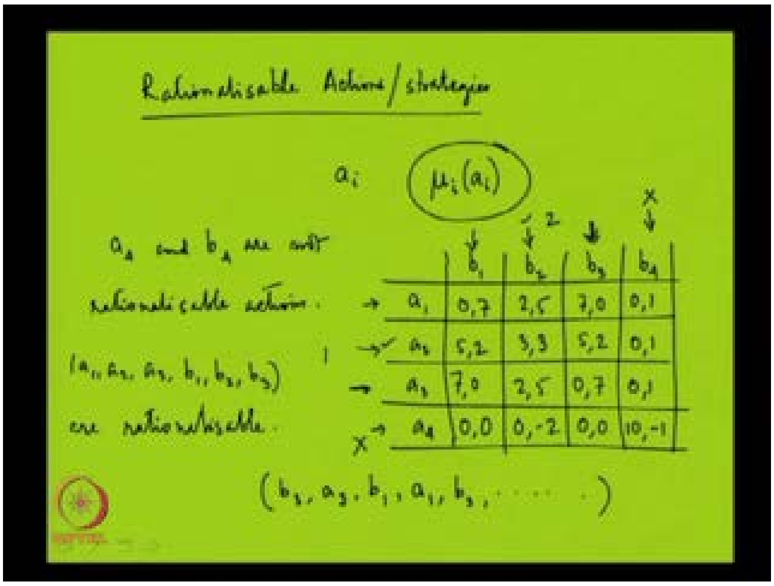

So, this is one advantage and related to this idea of strict dominance and weak dominance, there is another idea of rationalisability. We shall call it rationalisable actions. They are also called sometimes as rationalisable strategy. Let us call it strategies.

So, what is the concept here? The idea is that an action for player i, suppose a i is rationalisable for that player, if that action can be played by player i, given some beliefs of player i, regarding other players action.

So, if player 1 has some belief; let us call it mu i, which justifies the play of a i in the sense that if he has some belief regarding other player's action, a i is the best action that he can take, then a i will be called the rationalisable action for player i.

Now, this logic does not stop here. Remember, when i is saying that I have some belief regarding other player's action then that is why I am playing a i, this other player's actions have also to be justified. So, other players actions which are imputed in this mu i, they also have to be justified according to some belief of this other players.

So, we go on an in an infinite regress. a i is being played because of some belief regarding other player's actions and why these other players are taking these actions because they also have some belief regarding other player's actions of them and we go on in an infinite regress like that.

All these actions in this infinite sequence will be called rationalisable actions. This may seem a little vague. So, let me start with an example. How these rationalisable actions are found out in terms of real games?

So, every player, two players are there, each of the players has four actions. Now, what I claim is the following that in this game, a 4 and b 4 are not rationalisable actions whereas, a 1, a 2, a 3, b 1, b 2, b 3 are rationalisable.

How am I saying this? Let us look at b 4. Why I am saying that b 4 is not rationalisable? The point about b 4 is that no matter what is the action taken by player 1, b 4 is never going to be played.

If player takes a 1, then b 1 is the best action; b 4 is not being played. If player 1 is playing a 2, best action for player 2 is play b 2. If player 1 is playing a 3, the best action for player 2 is b 3. If player 1 is playing a 4 then the best action there are 2 best actions, either b 1 or b 3.

So, b 4 is never going to be played by player 2. So, player 2 can never justify under whatever beliefs that he might have, why he should play b 4 at all, at any circumstances. Since b 4 is not being played in any of this individual cases, b 4 is not going to be played even if player 1 mixes his strategies, mixes his actions.

So, therefore, b 4 cannot be justified; it cannot be rationalized under any circumstances by player 2; therefore, b 4 is not rationalisable. Why a 4 is not rationalisable is that this is dependent on the fact that b 4 is not rationalisable. a 4 by player 1 can be played; it is not that a 4 cannot be played.

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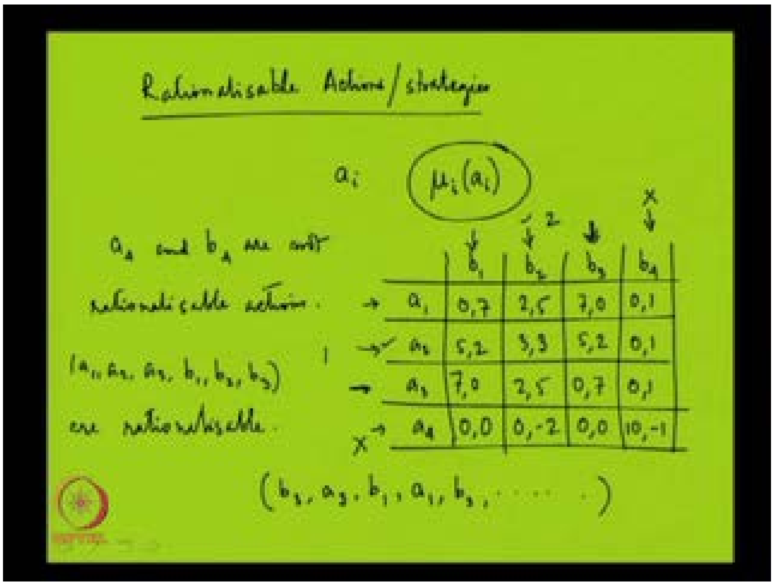

a 4 can be played, if player 2 plays b 4. You see b 4 is played, a 4 is the best response for player 1. However, there is no rationality for b 4 to be played. We have just said that b 4 cannot be played under any circumstances.

Therefore, the justification of a 4 also goes because justification of a 4 is dependent on the justification of b 4 and there is no justification of b 4. Therefore, a 4 is also not rationalisable.

What is the proof that the other actions are rationalisable? We can start with b 3, for example. Is b 3 rationalisable? Well, player 2 is going to play b 3, if player 1 plays a 3 because if player 1 plays a 3, the best response of player 2 is to play b 3; he is getting 7 here which is the highest of 0, 5, 7 and 1.

What is the justification that player 1 is going to play a 3? So, let me write the sequences here. b 3 is being played because player 2 believes that player 1 is going to play a 3. Why is a 3 being played or what is the justification for player 2 to believe that player 1 is going to play a 3?

Well, player 1 can play a 3, if player 1 beliefs player 2 is going to play b 1. If b 1 is played then best response for player 1 is to play a 3. What is the justification for player 2 to play b 1? b 1 is going to be played because player 1 is going to play a 1.

If player 1 plays a 1, b 1 is the best response and why is a 1 being played? a 1 is being played because player 2 is suppose believed to be playing b 3, then a 1 is played. So, you see we are back to b 3 here. So, this way it will go on like this. We have an infinite regress that I talked to you about and each action is being justified on a belief which is dependent on the previous action in the sequence.

So, that is why these actions b 3, not only b 3, from here we can see that b 3, a 3, b 1, a 1 - all these four actions are justified; they are rationalisable. Similarly, we can show that a 2 and b 2 are also rationalisable. So, this is the definition of rationalisable and you can see that rationalisability - the actions which are rationalisable, it is related to the idea of best responses.

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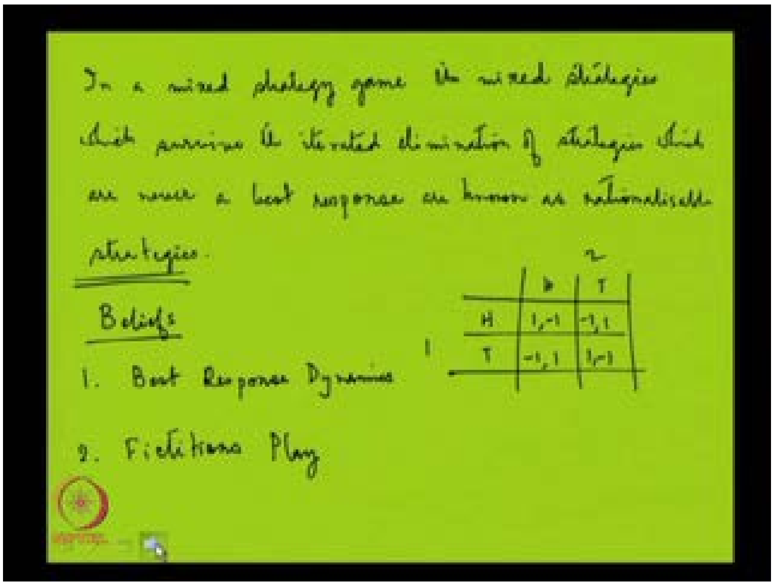

Therefore, we have a result which is the following. In a mixed strategy game, the mixed strategies which survive the iterated elimination of strategies which are never a best response are known as rationalisable strategies.

(Refer Slide Time: 33:22) An example is this game itself. Here b 4 was never best response. So, we eliminated this and consequent to the elimination of b 4, we can eliminate a 4 also because now a 4 is never a best response to any of the actions by player 2.

So, a 4 and b 4 are eliminated and we can see that none of these other actions a 1, a 2, a 3, b 1, b 2, b 3 can be eliminated. So, they are never which are not best responses. Therefore these a 1, a 2, a 3 and b 1, b 2, b 3 are called rationalisable actions or strategies.

Remember any strategy can also be an action because in that strategy, any particular action is being played with probability 1.

So, this is some discussion about rationalisable actions or rationalisable strategies, but what is the upshot of this that by applying these ideas that iterated elimination of dominated actions or rationalisable actions, the idea of rationalisable actions of strategies, we can pin points some action profiles which can be played without any history.

Because in Nash equilibrium, we required a history to justify that a particular action profile is being played and that is why it is called a steady state, but in case of rationalisability or strictly dominated elimination of strictly dominated actions, we did not need any previous play of the game. We just apply the idea of rationality and the fact that rationality is a common knowledge and we try to find out to what are the action profiles which are left.

But at the same time we also must keep in mind that this is not giving us tremendous amount of predictive power because we are left with so many action profiles - nine of them and all of them, obviously are not Nash equilibria.

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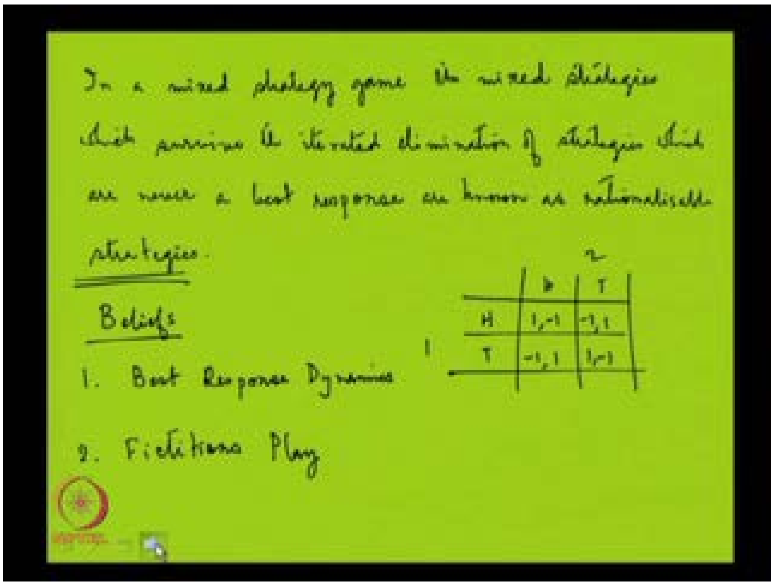

Nash equilibrium will pin point us to a very few of the action profiles. For example, to take a simple example, take the case of matching pennies. This was the game and in this game by rationalisability, by dominance, we cannot eliminate any of the actions.

So H, T by player 1 and H, T by player 2, both these actions for each of these players remain there and so, it means that in terms pure strategy, all these four action profiles remain; we cannot eliminate any of them.

Whereas, if I apply Nash equilibrium, the idea of Nash equilibrium, I have a unique solution which is half of a mixed strategy equilibrium.

So, Nash equilibrium has basically pointed, it has pin pointed to certain action profile or action profiles whereas, this idea of rationalisability and strict dominance, they do not pin point at a very small set, but they take into account a very large set and in that large set, maybe the set of Nash equilibrium is a subset, is a small subset.

So, as far as predictive power goes, as far as pin pointing a particular set of actions goes a Nash equilibrium is better. It is basically condensing the solutions to a smaller set.

Therefore, the idea of Nash equilibrium is powerful and it cannot be thrown away. So, one has to figure out how any game for example, converges to a Nash equilibrium profile.

One way to do that how a game converges to a Nash equilibrium profile which is then getting repeated because of steady state properties is that people tend to form beliefs because they have seen the previous play of the game. So, beliefs are formed and this formation of beliefs in a sequential manner leads us to the Nash equilibrium.

So there are two hypothesis, how the beliefs are formed and we go to a Nash equilibrium. One can be discussed, which is called the best response dynamics and the second is known as fictitious play.

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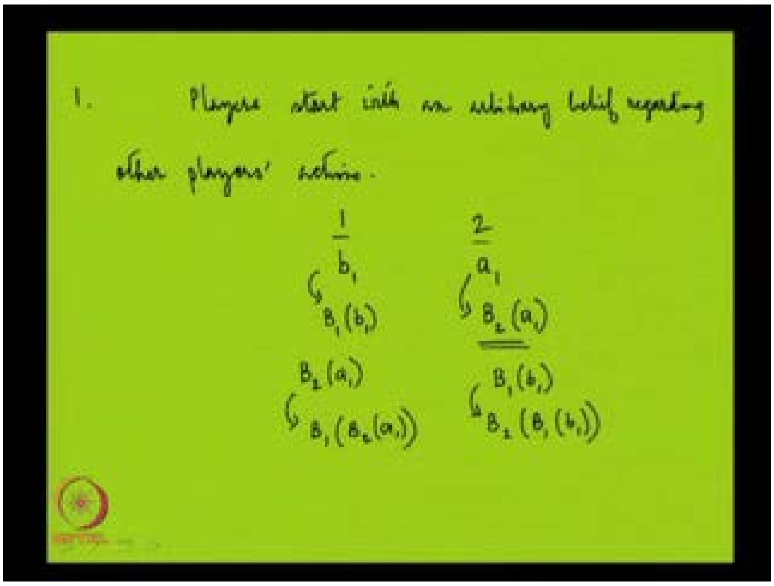

So, let us start with this first one best response dynamics. In each of these two, that is fictitious play and best response dynamics, players start with an arbitrary belief regarding other players’ actions and after they have started with an arbitrary belief regarding other players’ action, after the game has been played, then they observe the other players’ action and observing the other players’ action, they believe that this action is going to be repeated by this player in the next stage also, in the next play of the game also.

This happens for each of the players and this game goes on. So, suppose player 1 and there is player 2. Suppose, there are two players only and player 1 suppose starts with the belief that player 2 is going to take action b 1 and player 2 beliefs that player 1 is going to take any action a 1.

These are arbitrary and believing this, they take the action. Suppose, this is the best response function of player 1, this and believing this he takes the action, this. So, this is what happens in period one.

After the game has been played, player 1 now observes that B 1, the belief that he thought action of player 2 might be different from B 2 a 1.

And barring the case of coincidence, small b 1 will be different from B 2 a 1 and therefore, in period two, player 1 now starts with the belief that that player 2 is going to play this thing.

So, this is his now changed belief and accordingly, he takes the action, this. Similarly, player 2 will now have the belief that player 1 will take this action, right and therefore, he should take this action, alright and this way the game progresses.

Now, it may same a little naive that the players are just taking the other player's action in the previous play of the game as given and thinking that that action is going to be repeated.

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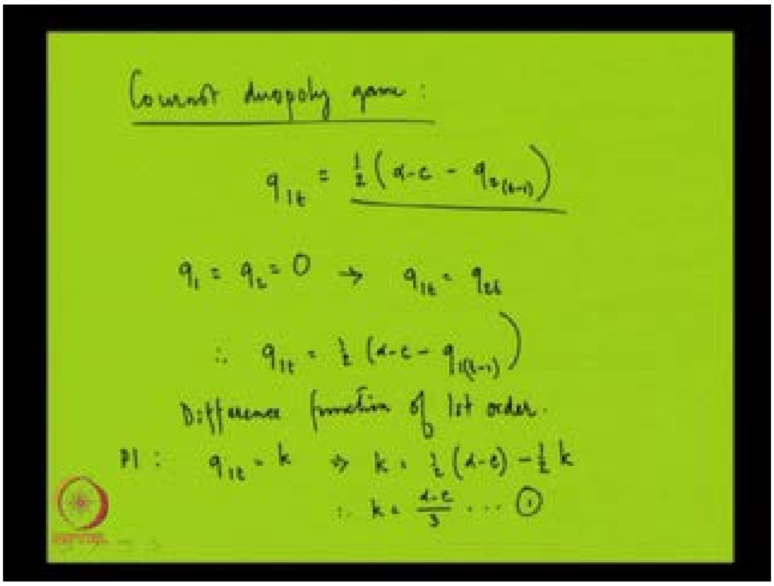

That is a simple very naive kind of belief formation, but never the less in many games, this in fact converges us to the Nash equilibrium. One game in which it leads to a convergence to the Nash equilibrium is the Cournot duopoly case - duopoly game.

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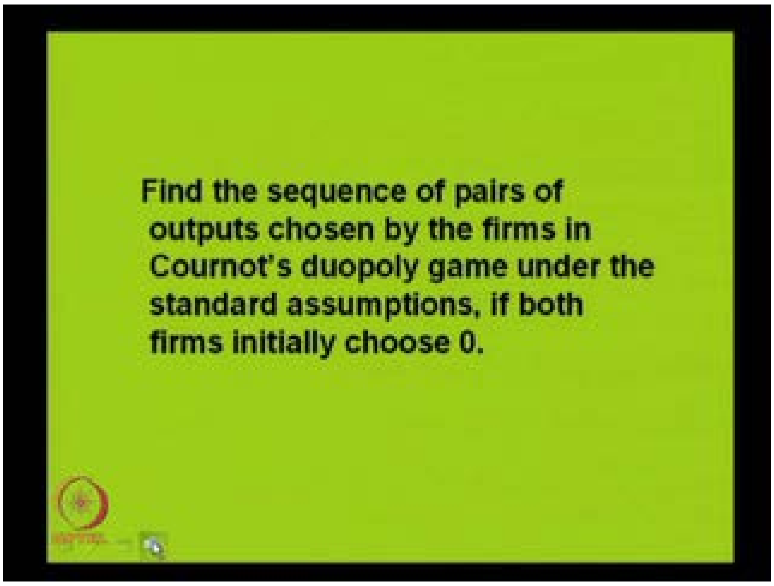

I have one question regarding this, in fact. Find the sequence of pairs of outputs chosen by the firms in Cournot's duopoly game under standard assumptions, if both firms initially choose 0.

So, here both the firms are choosing the actions 0, 0 and the game is starting and we have to find out what is the sequence of actions taken by both these players and does it converge to the Nash equilibrium. That is what we want to see.

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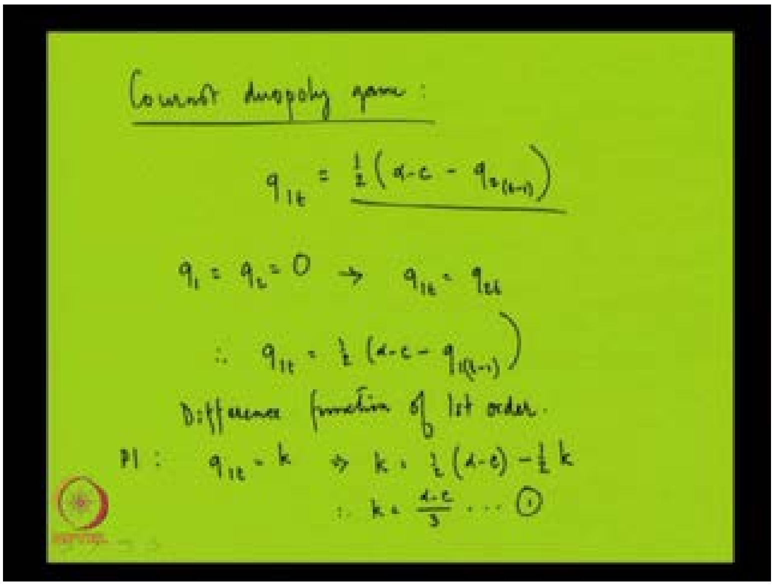

Now, in Cournot duopoly game, if you remember, the best response, if I want to write it for player 1 it was given by and the rest of the formula as shown in the slide. This was the best response function.

Now, in this case what I need to do is that player 1 when he is deciding his output, he is looking at the output of player 2 in the previous period and believing that to be the output in the next period also. There is a time involved here. So, I can write it as t minus 1 and this as t.

So, he looks at the output produced by player 2 in the previous period, which is q 2 t minus 1 believing that to be repeated in this period and therefore, q 2 t minus 1 is equal to q 2 t and therefore, he is taking the best response according to that and this I know is the best response function ( )

One more thing that is needed to be remembered is that they are starting with this initial output level of 0 and remember, the best response of these 2 players are symmetric, in the sense that player 2 also has the same kind of best response function. I need not write that you just have to replace 1 by 2 and 2 by 1.

Now if the best response functions are symmetric, if they are starting from the same output levels, then it means that in each period this will happen.

In each period, their output levels will be same. Now, therefore, I can write this as the following. So, this becomes my function, which is a difference function of first order.

If I solve this function then I can chart out what is the trajectory that q 1 t takes and that will be the same trajectory of q 2 t. So, how to solve this? There are two parts in a difference equation in the solution of the difference equation.

One is the particular integral and the other is complimentary function. In particular integral, the value remains constant. The solution - this is the inter-temporary equilibrium value.

So, let us suppose, this value is k. So, if I substitute this here, which means k is equal to alpha minus c divided by 3; this is number 1.

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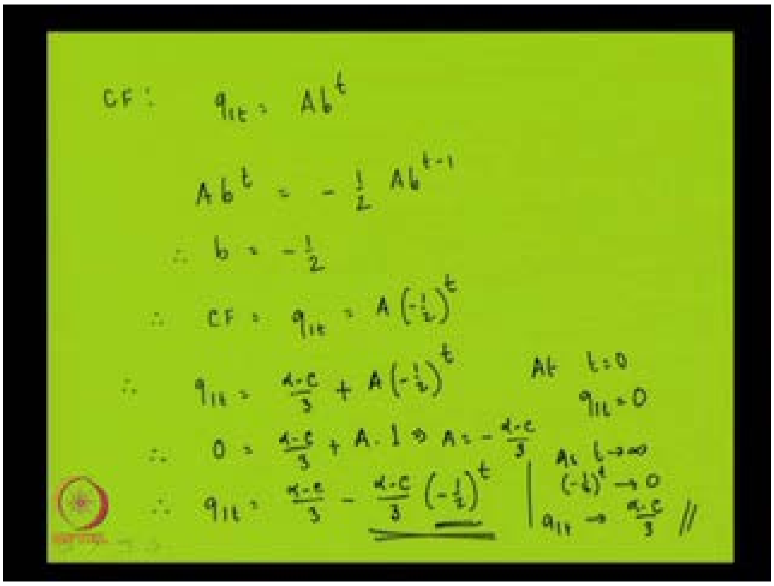

The second is complimentary function and let us write it as any arbitrary function capital A multiplied by small b to the power t, where A and b are parameters and if we substitute this in this function, what we get here. I do not have to include the constant part, what I get is the following.

A b t is equal to minus half A b t minus 1, right because here the substitute t minus 1; here, it is t.

So, from here, what do I get is b is equal to minus half because I can divide both sides by A b to the power t minus 1.

Therefore, CF is equal to q i t is equal to minus half to the power t. So, the complete solution is the following. It is equal to particular integral, which is alpha minus c divided by three plus the complimentary function t.

We have the information that at t is equal to 0, q 1 t is equal to 0. They are starting with 0, 0 output level. So, 0 is equal to alpha minus c divided by 3 plus 1 because if I put t is equal to 0 this becomes 1 so which means A is equal to minus of alpha minus c divided by 3.

So, q 1 t will be equal to this. Now, what is important is that as t goes to infinity, this part goes to 0. As this part goes to 0, which means this part also goes to 0. So, as t goes to infinity, minus half to the power of t goes to 0 and which means that q 1 t tends to alpha minus c divided by 3. So, we basically converge to the Nash equilibrium because if we remember alpha minus c divided by 3 was the Nash equilibrium.

The same logic applies for q 2 t also because as we have just figured out that since they are starting from the same output level 0, 0 and the best response functions are symmetric. Therefore, the dynamics of these 2 outputs, q 1 t and q 2 t will be same.

So, q 2 t also will converge to alpha minus c divided by 3. So, in this case, this naive kind of belief that my rivals action in the previous period is going to be repeated by him in the next period, this kind of belief is helpful in this case of Cournot equilibrium.

One thing we have not said, which was there in the question is that what are the sequence of actions, what are the actions to be produced, what are the outputs to be produced by these two players, by these two firms.

So, at t is equal to 0, I know the output level is 0. So, that is there. If t is equal to 1, I can put t is equal to 1 here and I can find out what is the value of output for player 1 in the first period. I can put t is equal to 2 and I can find out the value of output for period two etcetera. So, the sequence can be derived from this equation itself.

But in many games, this is not true. In many games, this kind of naive beliefs will not lead us to equilibrium, for example, to the Nash equilibrium.

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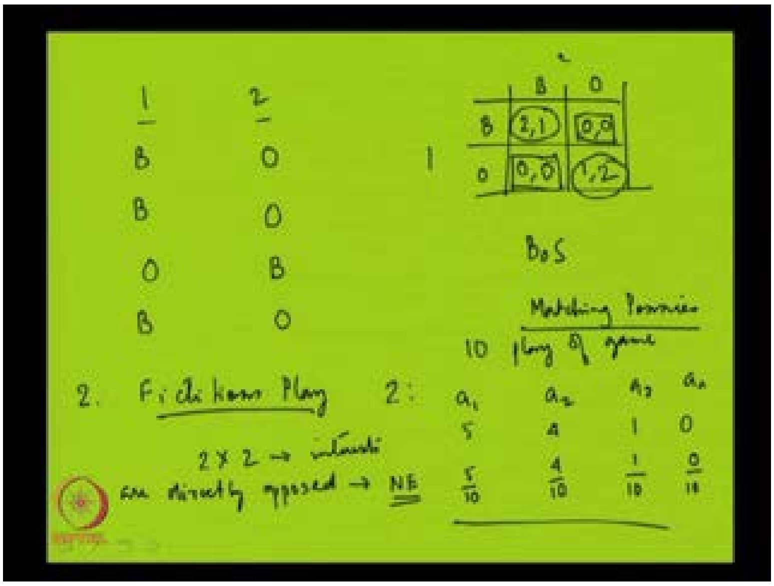

For example, take the case of battle of sexes. In this game, there are two players and I know these are the pure strategy Nash equilibrium. Now, if I start with an arbitrary belief that player 1 believes that player 2 will play B and player 2 believes that player 1 will play O; then, do we get to a Nash equilibrium?

If B is the belief of player 1, he will play B and if O is the player of player 2, he will play O and we basically reach here. In the next period, what player 1 will do, he will play O and player 2 will play B and we shall reach here and again this is going to be repeated.

So, we are basically fluctuating between 0, 0 and 0, 0; that is, O B and B O; none of them are Nash equilibrium. So, this naive belief may not be helpful in some cases and therefore, we need the second concept called fictitious play. Here, the beliefs are the following.

Suppose, I have seen my rival player 2 in the 10 play of the game, he has taken a 1, 5 times; he has played a 2, 4 times; he has played a 1, 1 time and a 4, 0 time. So, in the 11th play of the game, I shall believe that he will play a 1 with probability this, a 2 with probability this, a 3 with this probability and a 4 with this probability.

So, I believe that the frequency in which he has played the actions will be proportional to the probabilities he will attach to each of the actions when he plays the next play of the game. I take into account that he can play mixed strategy. Is it helpful? Is this kind of little more sophisticated belief structure leads us to the Nash equilibrium? Well, the result is the following.

That if I have a 2 by 2 structure, that is, there are 2 actions 2 players and if the interest are directly opposed, which means that if I gain, my rival looses then this kind of belief structure leads us to the Nash equilibrium. So, take the case of matching pennies.

In matching pennies, in fact, infinite play of the game with this kind of belief structure leads us to the Nash equilibrium no matter where we start from. That is, no matter what arbitrary belief we start from.

So, these are some of the attempts that have been made to justify how from point time 0, we converge to Nash equilibrium and there are lots of work to be done, still to be done in this field.

So, this is more or less the module four 4, which module was about mixed strategy Nash equilibrium. What we shall do from the next lecture, we shall take up another important topic of game theory, which is sequential games and we shall see what the various facets of that game are.

Thank you so much.

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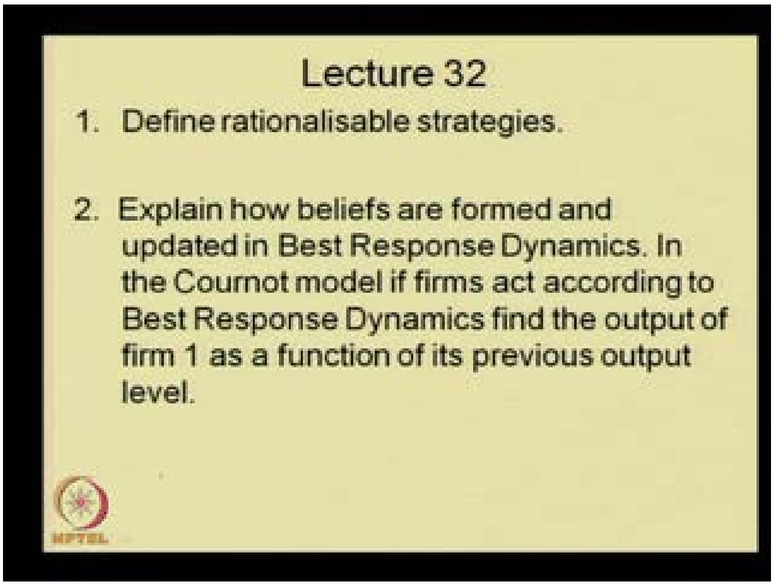

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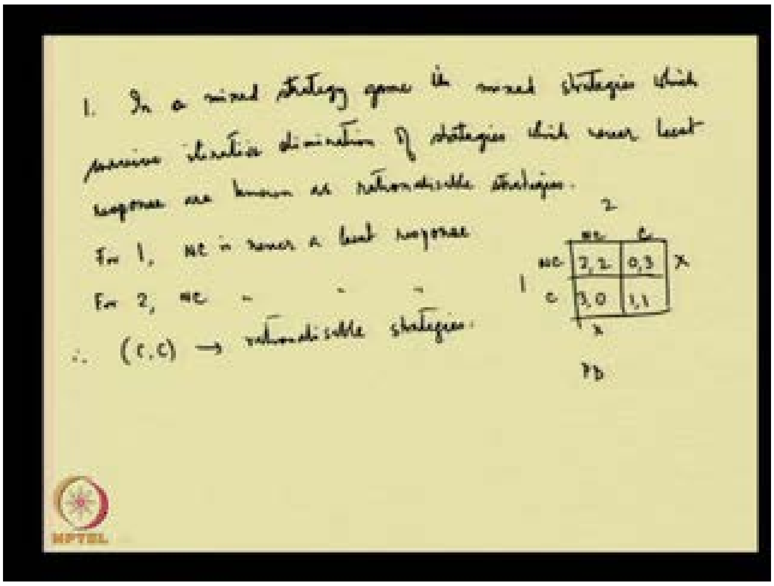

First question, define rationalisable strategies. So, let us look at the definition carefully.In a mixed strategy game, the mixed strategies which survive iterated elimination of strategies, which are never best response are known as rationalisable strategies.

So, we can give some examples. The game, the strategies which are not best responses, they will be eliminated and the rest will be considered as the rationalisable strategies. So, very simply, if you remember the prisoner’s dilemma game. So, this was the prisoner’s dilemma game. Here, we see that for 1, the action N C is never a best response. So, this is eliminated.

Similarly, for 2 again, N C is never a best response. So, 1 is left with C, C. It is the rationalisable strategies. Here, since it was a 2 action by 2 action game. So, the game was simple, but one can think of more complicated games, where mixed strategies are eliminated because those mixed strategies are never a best response.

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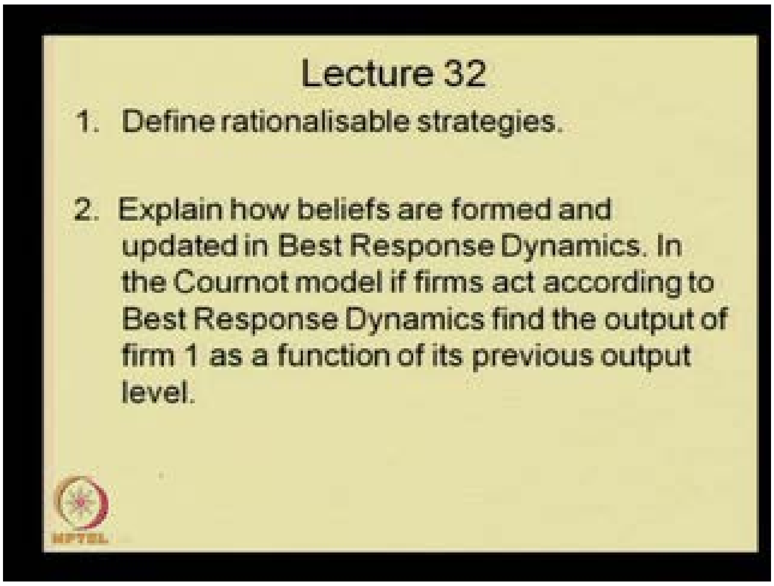

Explain how the beliefs are formed and updated in best response dynamics in the Cournot model, if firms act according to best response dynamics. Find the output of firm 1 as a function of its previous output level.

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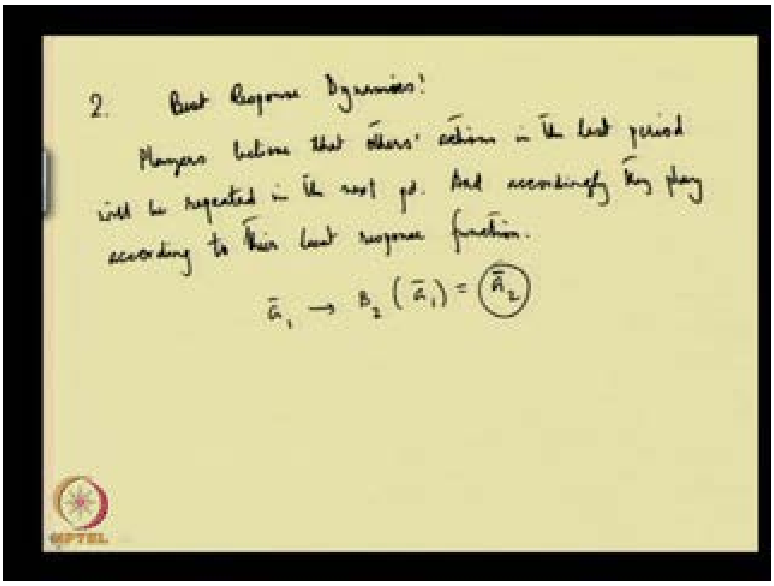

So, in best response dynamics, what happens is that players believe that others' actions in the last period will be repeated in the next period and accordingly, they play according to their best response function.

So, whenever they see someone, some other player is playing some actions, suppose, say a 1 bar then player 2 thinks that in the next period also, player 1 will play a 1 bar and then she uses her best response function and suppose, this is equal to a 2 bar and so she plays A 2 bar. So, this is how best response dynamics works.

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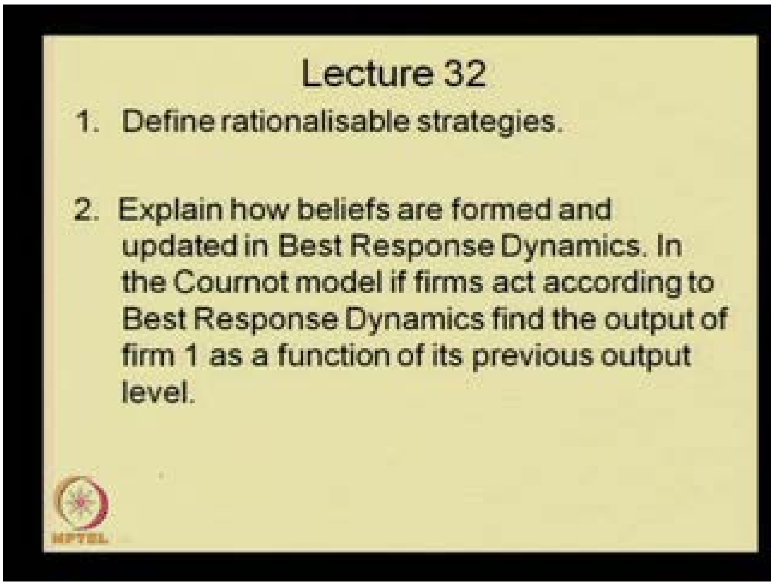

In Cournot model, if the firms act according to the best response dynamics, find output of the firm 1 as a function of its previous output level.

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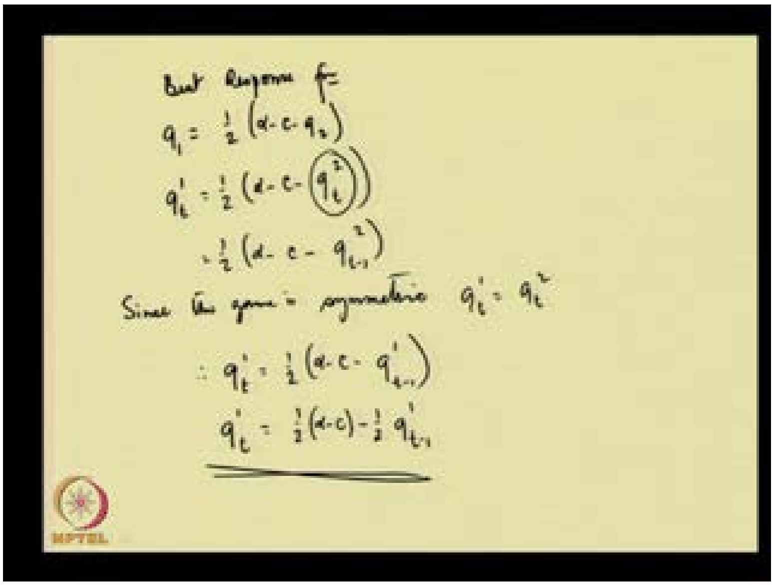

So, in Cournot model what we know? We know the following. This is the best response function.

Now, remember here, we are talking about periods, different periods. So, one has to incorporate the time dimension here. So, let us call this q t 1; that is, the output of firm 1 in period t.

Now, she will decide her output according to the output of firm 2, but the output that firm 2 will decide is not known by firm 1 from beforehand and this is simply therefore is going to be what was found to be the output in the previous period.

So, that is what the best response dynamics tells us and since the game is symmetric, q t 1 is equal to q t 2. So, by simplifying this a little bit more. So, that is how the output of firm 1 depends on its own output in the previous period.

Thank you.