Game Theory and Economics

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Module No. # 05

Extensive Games and Nash Equilibrium

Lecture No. # 01

Extensive Games: Introduction

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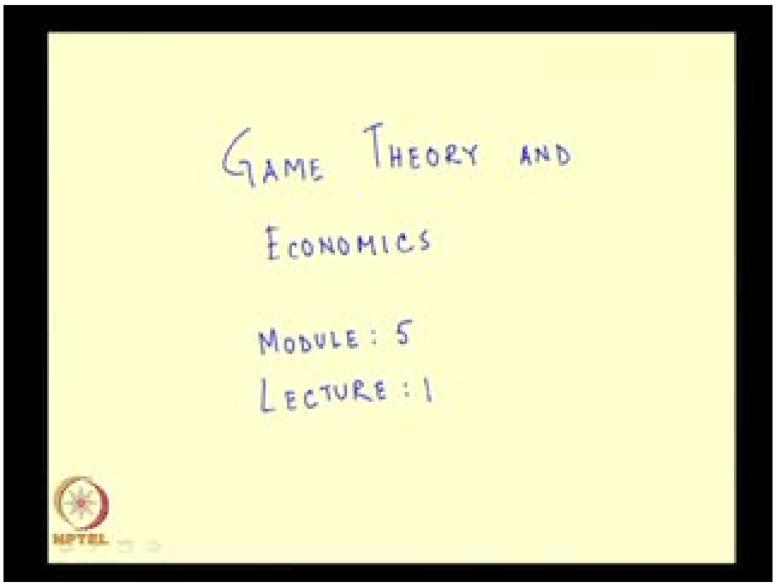

GAME THEORY AND

ECONOMICS

MODULE : 5

LECTURE : 1

Welcome to first lecture of module 5 of this course called game theory and economics. So, we are going to start a new module today. Before we start this new module, let me just briefly take you through out what we have discussed so far. So, that will give a prospective as to what we are going to do today. In this course, basically, we are going to cover two main areas of game theory; these are called and as a whole, these two areas of game theory can be summed up in what is known as non cooperative games. So, in this course, we are going to cover only non cooperative games. There is something called cooperative game and which we are not going to cover. Within non cooperative games, we can broadly divide this category of non cooperative games into two parts. One is this strategic games. They are also called normal form game.

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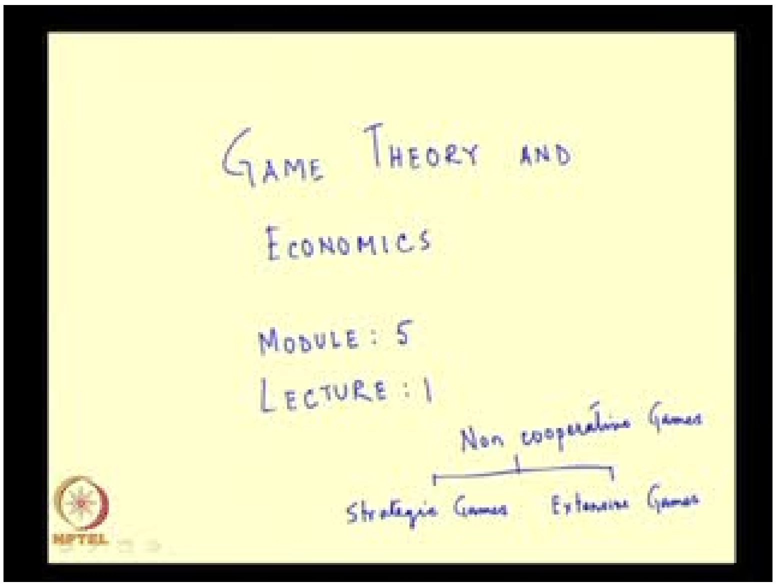
GAME THEORY AND

ECONOMICS

MODULE : 5

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$$\begin{tikzcd} & \text{Non cooperative Games} \arrow[dl, dash] \arrow[dr, dash] & \\ \text{Strategic Games} & & \text{Extensive Games} \end{tikzcd}$$

We have seen some examples of strategic games how they are dealt with by the notions of nash equilibrium or by other notions of solutions. For example, the case of the dominance, the strict dominance and weak dominance and how we can eliminate the actions or strategies which are dominated and that also gives some solutions. And there is another category within these non cooperative games, which is known as extensive games.

So, this is the second category and this is the, this is the group of games that we shall start discussing today. Extensive games are also called sequential games. So, what is the difference between strategic games and extensive games? The differences very simple and it is the following. If you remember in strategic games, we assume that the players when they take the decisions, they do not take into account the decisions made by the other players. Now, the fact that a player I for example, does not take into account the decision by other player suppose J can be justified in two ways - first is that, these two players I and J may be one and two. They are taking their decisions simultaneously.

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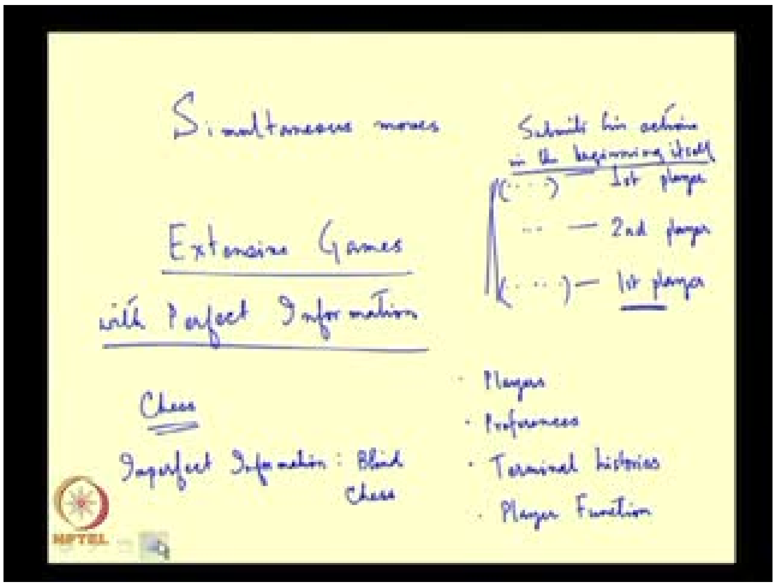

Simultaneous moves

Extensive Games

with Perfect Information

Chess

Imperfect Information : Blind Chess

Submits his actions in the beginning itself

(. . .) 1st player

. . — 2nd player

(. . .) — 1st player

· Players

· Preferences

· Terminal histories

· Player Function

So, if the decisions are being taking simultaneously, a player when he takes the decision, obviously will not know what is the decision that is being taken by the other player. And if he does not know, then there is no way in which he can base his decision; he can make his decision dependent upon the decision of the other player.

So, we also say that the strategic games are simultaneous moves game, move games. Another way of justifying the fact that the players do not taking to account the decisions of other players is to say that, well, it may happen that the decisions are taken simultaneously, not simultaneously but sequentially. So, one after another, but when a player says that I will take this decision, he submits his decision, the action that he will take right at the beginning of the game itself. So, right at the beginning of the game itself he says for example suppose player one makes a move in this stage, and in the second stage, suppose player two makes the move, and then suppose again player one makes the, makes a move in the third stage.

Now, obviously, since the game is unfolding in a sequential way, player one knows what player one, player two knows what player one has done in the first stage, but a way to justify the fact that player to cannot base his decision. Player one's decision is that he submits his actions in the beginning itself.

So, in the beginning itself, he says that I will play this action, and he is committed to that action; he cannot deviate once he says that am going to play this action. So, since he cannot deviate, he has committed. So, it may happen that first player takes some decision with respect to his, with, with respect to which player two's decision is not optimal. The decision that he said that he will take, but nevertheless he has to stick to what he has said before; he has committed.

So, this is another way of justifying the fact that players, it might happen that the player of the game is structured in a sequential manner. The players are not moving simultaneously, but nevertheless since I am committing in the beginning itself. There is no way other players changing decisions are going to affect my decision. Thus similarly, here also player one the action that he will take in stage three, he has committed this action in the first at the first, at the first, at the beginning of the game itself.

So, even if that action is not optimal, given what player two has chosen the second stage. Player one has to stick to his decision in the third stage. So, this is one way of justify simultaneous move games, which basically, if you, if you are careful with this demonstration have it basically takes away the fact that the game is in fact sequential. It basically makes the game a simultaneous move game, does not matter whether the game is structured in a sequential manner. It, the game boils down to a simultaneously move game because the actions are taken without taking into cognizance the actions of the other players.

So, now, what we are going to do from today and subsequent lectures is to discuss extensive games. So, here, in fact when a player takes a decision, he observes the decision taken by the other player and then he makes a move. So, he is not committing anything in the beginning itself. The game is unfolding in a sequential manner, and since it is unfolding in a sequential manner, everybody looks into the actions taken by other players before he takes an action. And therefore, the things are known to me and I can base my decision on the actions taken by the other players.

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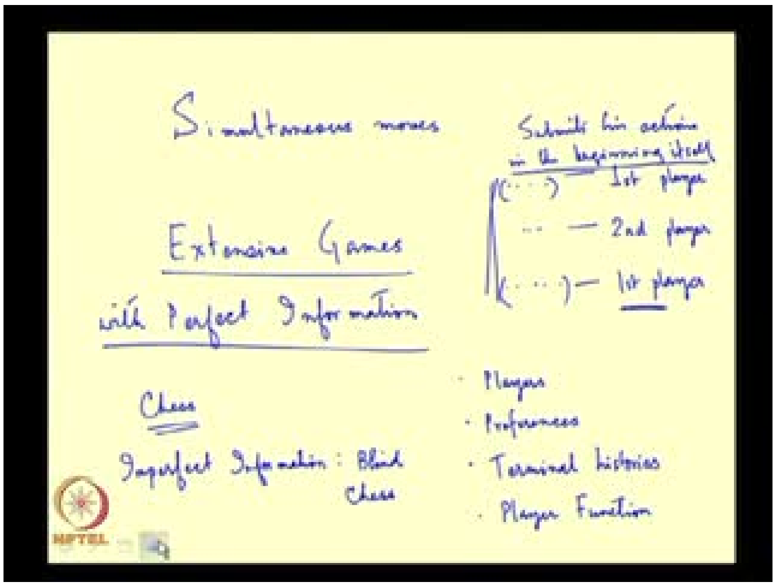
Simultaneous moves

Submits his action in the beginning itself

(· · ·) — 1st player

· · — 2nd player

(· · ·) — 1st player

Extensive Games with Perfect Information

Chess

Imperfect Information : Blind Chess

· Players
· Preferences
· Terminal histories
· Player Function

So, it is also to be made clear that for example, take this case again. It might happen it could have happened that player one is taking a decision in the first stage, but that action though it has happened before is not clearly known to player two, who is taking the action in the second stage.

So, there might be a problem of information, though the, the, action has happen before and I am taking that as action at a later point of time. The, the, information regarding the previous action is not clear to me. So, that would have been a game with imperfect information, but here, the game that we are going to discuss, the games that we are going to discuss are extensive games with perfect information. Meaning that whatever action that has been taken before I am taking my action is clearly known to me; there is no uncertainty.

So, this is the genre of games that we are going to discuss, now since the game is structured in a sequential manner, what are the key components that I need to know. First, obviously, I need to know the players, identity of the players were involved. Second, of course, I have to know the, the, the, preferences of the players. Different players are there. There, their likings and disliking will be different, regarding the outcome of the game. So, I have to know those likings and dislikings. Thirdly, what I need to know is how is the game sequenced, what are the sequences of actions that can be possible, because here, you see a player one in first stage could take a number of actions. So, what are the actions that are available to him? In player, in second stage also player two can take a number of actions, and in third stage, again you see player one is taking some actions and it is not necessary that these set of actions occurring in the first stage is going to be the same set of actions which is occurring in the third stage.

So, the set of action may be the same player is taking decisions at multiple stages, but the action set available to that particular player may go on change. So, instead of specifying the action set for each player, which we were doing in case of a strategic game, if you remember in strategic game, what were the, what were the, three components? One was the set of players; second was the set of actions available to each player.

Here, set of actions available to each player may go on changing depending on the stage at which he is in. So, what we do in this extensive games is that, we do not specify the actions set of each player because it goes on changing. So, it will be more complicated if we do, try to do that. Instead, what we do is specify all possible sequence of action that can happen in a particular game and these possible sequences of actions are known as histories in particular terminal histories.

So, terminal history is they, they, refer to the sequences of actions which are possible in a particular game, but merely, specifying the sequences of actions which are possible. Let us look at it then we are not basically specifying which player is taking which action.

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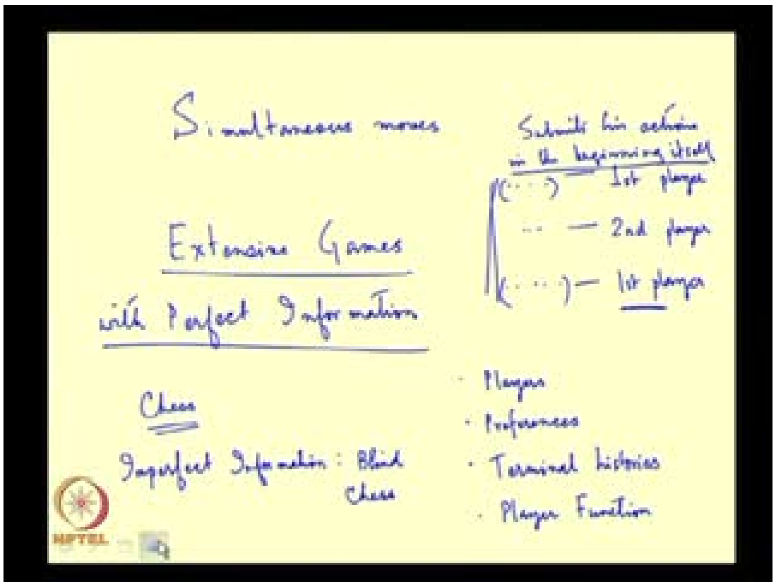

Simultaneous moves

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We are just saying that there is this set of players and there are these sequences of actions which are possible, but I have to know at, at a particular stage which is the player who is taking actions. So, that has to be made clear to me and that is made clear by this fourth component, which is known as the player function. What does it mean? It means that in a terminal history, there are many points because it is a sequence of actions.

So, after some actions have taken place in that small sequence within the larger terminal history sequence. After the small sequence, I have to know now whose player, which players turn it is who can take the action. So, given a small set of actions, I have to know now who is the player who can take an action, and that function which tells with the identity of the player after a small sequence of actions that function will be called the player function.

So, obviously, now if I have a terminal history, I can apply this player function on each and every point on that terminal history and I will be able to know the identity of the player at, at, those nodes, at those points. So therefore, these are the four components in an extensive form game. If we in any extensive game, if I know these four components, then I basically know everything that is, know everything that is, know needed to be known about that game.

Now, what could be the illustration? What could be the examples of extensive game with perfect information? Check the game of chess. If I have to give a simple example, a game of chess is a game extensive game with perfect information, because the moves are made simultaneously, not simultaneously but sequentially and each player is aware of the moves made by the other player before he takes any action. And so, it is a game of not only, not only, it is a game which is extensive but it is a game with perfect information. What could be the example of game which is an extensive game but imperfect information?

So, example, blind chess what does it mean? Well, it is a, it is a, special kind of chess game where what happens is that I am unaware of the moves made by the other players. I cannot even see the pieces of the other player where those pieces are located. I can see my piece, my pieces on the board, but I cannot see other the, my rivals pieces.

When I want to make a move, if I want to make a particular move from this place to some other place, it may happen that the, that move is not valid move because the other player pieces might be intertwining. So, in that case, I will be told whether that move is a valid move, legitimate move or not. That information I will be given but nothing more than that. If it is a legitimate move, I will be able to make that move, but if it is a illegitimate move, then I cannot make that move. I have to make some alternative move.

So, in this game, see this game is a game of imperfect information because I am not aware of the moves made by the other player, though it is an extensive game because the moves are happening simultaneously, sequentially as it happens in a standard chess game.

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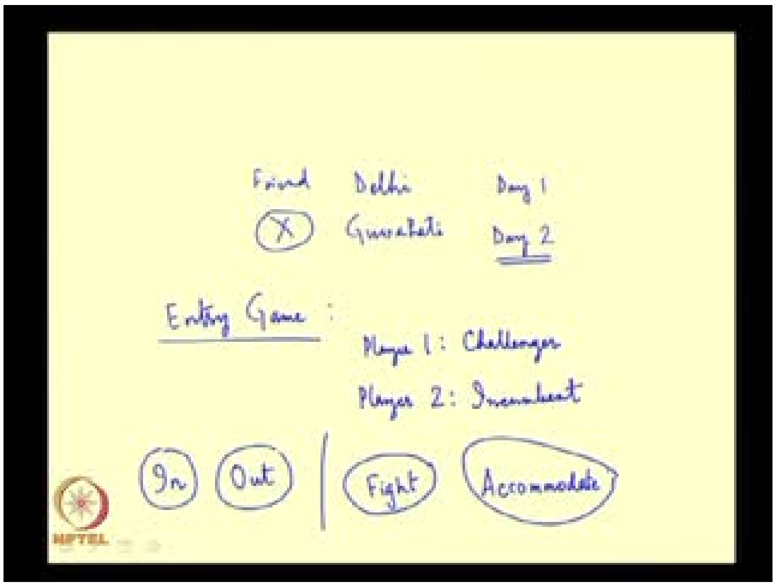
Friend Delhi Day 1

x Guwahati Day 2

Entry Game :

Player 1: Challenger

Player 2: Incumbent

In Out | Fight Accommodate

One can give more, more, routed real life example. For example, let us take the case that there is a person who lives in Guwahati for example, and he has a friend. So, this is x and this is x's friend who suppose lives in Delhi. And suppose in day one, this friend who is located in Delhi is suppose to send some money to this person x, who lives in Guwahati. So, this money you know if it is sent, it will definitely come to this person x in day two in his bank. So, money will be credited to his bank account.

Now, but there is uncertainty whether, whether, this friend who is in Delhi in day one has sent the money or not has sent the money. So, this person who is living Guwahati on day two suppose he has two actions - either he can go to the bank to check whether the money is there or he may not go to the bank. Now, this action of his going to the bank or not going to the bank, these two actions they will give him, they will be worthwhile going to the bank will be worthwhile if this friend has sent his him some money in, in, day one.

So, depending on the actions by the friend in day one, his action in day two may lead to different results. If he has indeed sent the money, then going to the bank and checking that account will be worthwhile. If the first person has not send the money, then going to the bank and checking the account will be a futile exercise. So, here, x also is not sure whether in day one the friend has send the money. So, he is not perfectly informed. So, this is the case a situation of sequential or extensive game where the information is not perfect. So, let me come back to the perfect information game and let me give another standard example of perfect information game, extensive game with perfect information, which is known as the entry game.

So, what is the story here? So, it is a two player game - player one is known as the challenger; player two is known as the incumbent. Incumbent is the one who is enjoying some privileges, some power, some payoff, and the challenger currently is not enjoying anything, but he is thinking whether he will challenge the incumbent and take a part of the power or privilege this incumbent is enjoyed. So, for the challenger, he has two actions to choose from either he can come in. In means he is entering the race and he is challenging this incumbent or he can take the action out, which means he is not coming in; he is not challenging the incumbent.

How, how, this model can be visualized in real life? Take the case of a market from economics if i have to give an example. Take the case of a market where there is a monopolist, which means he is the only producer in the market. So, all the profit in the market he is the, he is the, person who is enjoying all the profit. And there is one rival firm who is not presently in the market, but he, the, the, firm is thinking whether it will enter the market and try to compete with this incumbent firm who is the monopolist.

So, now, this is other firm who is thinking of entering the market will be called as a challenger and the monopolist is an incumbent. It can also be thought of as a political race for example, suppose there is a locality, where there is an undisputed leader who wins the election, no matter what; there is no one to challenge him; no one fights elections with him. So, he enjoys some privileges obviously. He is winning the elections. So, there are some privileges and money associated with that seat.

And suppose, there is a rival person leader who is thinking whether i will challenge this undisputed leader and fighten fight the election. So, that can also be, that example can also be given. So, here the challenger has two action either to enter the race, either to compete with the incumbent or not to compete, staying out. If he stays out, then there is nothing to be done by the incumbent; incumbent is happy; he is as before. The status queue is maintained.

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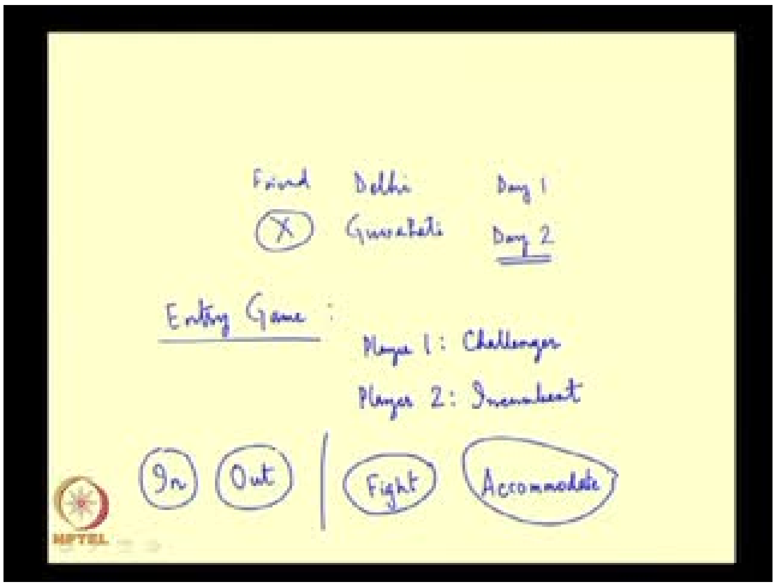

Friend Delhi Day 1

X Guwahati Day 2

Entry Game :

Player 1 : Challenger

Player 2 : Incumbent

In Out | Fight Accommodate

But if the challenger gets in, then the incumbent has to decide something. Now, what he decides on is basically a set of actions. We shall say that there are two actions that the incumbent can take. So, here, I am writing on the right side actions that can be taken by the incumbent only if the challenger comes in, only if the challenger you know he is interested in fighting.

The incumbent can do two things - one is to fight. So, the rival is coming into the market. The rival firm is entering the market. The incumbent firm is fighting with him in the sense that suppose the rival firm comes in and the rival firm charges a price of little less then can then the incumbent firms price. Then, he will be able to get a large share of the market because customers will slope to the challenger instead of the incumbent.

But if he does so by fighting, I mean that the incumbent also follow suite, he also reduces the price. So, it might hurt his small run profit because he is charging less price for the time being. So, his profit margin might be adversely affected but he is now giving a fight to the challenger; he is not taking it lying down. However, the incumbent can also choose not to fight and this not to fight we shall denote this by accommodate.

So, for example, in the case of market, may be the challenger comes in and charges a price with which is a little less than the incumbent price. Then he is getting a large share of the market; he is getting some profits, but the incumbent does not fight with him. He sticks to his price may be. He still get some loyal customers because he is a old firm for. So, all the some of the customers will still come to him.

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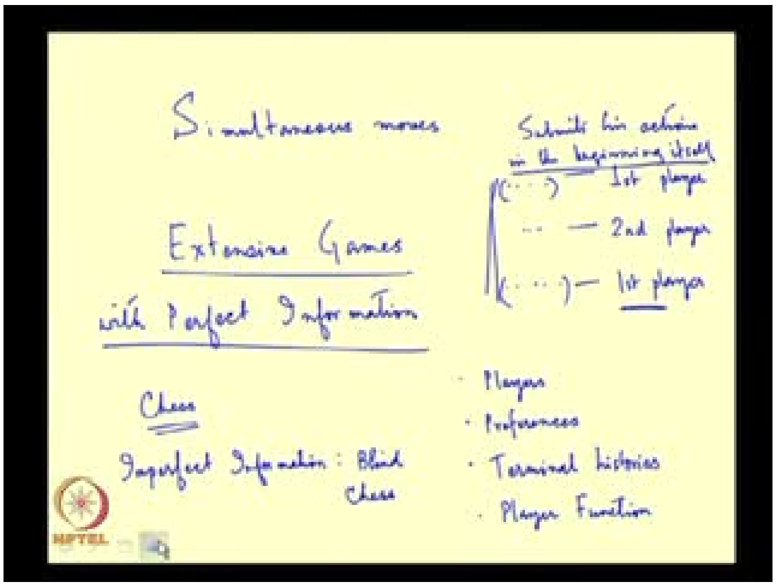

Simultaneous moves

Submits his actions in the beginning itself

(. . .) — 1st player

. . . — 2nd player

(. . .) — 1st player

Extensive Games with Perfect Information

Chess

Imperfect Information : Blind Chess

- Players
- Preferences
- Terminal histories
- Player Function

So, this is a case of accommodation. The incumbent is not keen to fight with the challenger. So, in this case, if I have to translate this story in terms of the elements that I have seen here - the players, the preferences, the terminal histories, the player function, how shall I write them? The Players are two - one is the challenger and the incumbent. The second component let us write the terminal histories.

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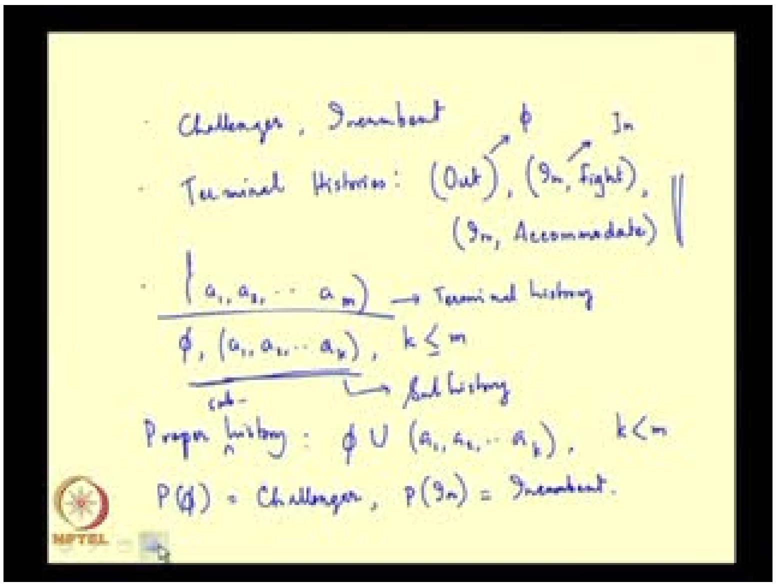

- Challenger, Incumbent
- Terminal Histories: (Out), $(\text{In}, \text{fight})$, $(\text{In}, \text{Accommodate})$ ϕ In
- $(a_1, a_2, \cdots a_m) \rightarrow$ Terminal history

ϕ, $(a_1, a_2, \cdots a_k)$, $k \leq m$ $\rightarrow$ sub history

Proper history: $\phi \cup (a_1, a_2, \cdots a_k)$, $k < m$

$P(\phi) =$ Challenger, $P(\text{In}) =$ Incumbent.

If the challenger does not get in, if he is stays out, then the game ends there, because there is nothing to be done by the incumbent then; he is happy. So, here, one terminal history could be out. The incumbent does not, the, the, challenger does not get in and we are maintaining, maintaining, the status queue.

Other sorts of sequence of actions which are possible is that the challenger gets in and the incumbent fights with him. So, this is one sequence. After the fight has happen, then the game, game, gets over. We are reaching the terminal of the game. They, these two firms divide their profits in some way. And what could be the other possibility is that the challenger gets in and the incumbent does not fight with him. He accommodates the challenger. So, let us write it down in an accommodate. So, these are the three possible sequences of actions there, there, are in this game.

What about the player function? If I have to talk about the player function, one thing was be made clear is that as I told before, player functions are dependent on some sequence of action, something has happened and then I have to know who is the player now who can take an action. So, that players identity is given by the player function.

So, suppose there is a terminal history of this form, suppose m, m, actions are there. Now, this is the terminal history which is ending at m, but I can construct many sub histories which are known as sub histories, a sub set of this big terminal history. Now, what are the subsets of this history, this terminal history? One could be phi itself which means a null history, nothing has happen. We are beginning, at the beginning, and how are the other sub histories defined? They will be of the form a k - where k is less than equal to m. So, this is a general form of a sub history of this terminal history. So, this is a terminal history and this is a sub history including this null set; all these are sub histories.

If k is equal to m, then we are including the main terminal history itself, but as we know any set is the subset of its own. So, that sub history is considered; the, the, set itself the, the, terminal history itself is a sub history of its own, but we can consider some what is known as a proper sub history. So, proper sub history will be what? It will include phi plus it will include all the sub histories of this firm, where k is strictly less than m.

So, the, the, terminal history itself is not counted but all the sub histories of the firm a 1, a 2, a k - where k is strictly less than m. They will be proper sub histories of the terminal history, sorry, this is sub history, proper sub history. Now, why did I mention all this? The reason I mention all this is that a player function is defined only over a proper sub history, because if I take into account the terminal history itself, I cannot define a player function on that because the game is ending there. There is no one, no player left who can take an action when the game ends. So, I have to remove this terminal history and look into the sub histories, proper sub histories, and if there is a proper sub histories, which means that if there is a sequence of actions which has happened, then I have to know who is now the player who is going to move.

So, in this case, I have these three terminal histories, and what are the proper sub histories here? In case of out, the proper sub history is just one phi. In this case, if I forget about phi because which has been included before, the, there is only one proper sub history which is in, and from here also I will get two proper sub histories - phi and m.

So basically, in this game, the, which is called the entry game, there are two proper sub histories if I take into a consideration all sorts of terminal histories. So, one proper sub history is phi, and what does the player function on phi give us? How do I know? Well, I know that if phi is the history, then who are the players, who is the player that is making a move? In the beginning of the game itself I know it is the challenger who makes a move. So, this is the challenger. So, this is the one player function, and if in which is the others proper sub history is there, then the player function defined over this gives me the incumbent.

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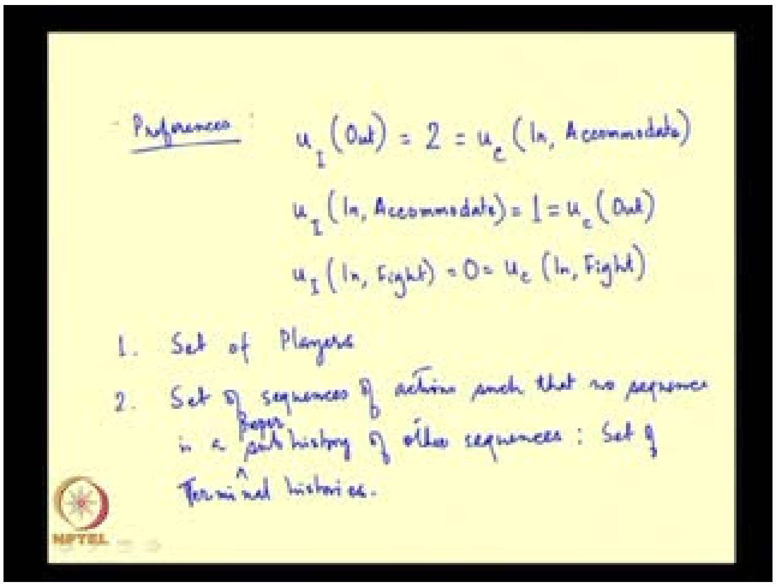

Preferences :

$$u_I(\text{Out}) = 2 = u_c(\text{In, Accommodate})$$
$$u_I(\text{In, Accommodate}) = 1 = u_c(\text{Out})$$
$$u_I(\text{In, Fight}) = 0 = u_c(\text{In, Fight})$$

1. Set of Players
2. Set of sequences of actions such that no sequence is a proper sub history of other sequences : Set of Terminal histories.

The reason being that, if I, if you remember the description of the game, if in has happened, that is, if the challenger has taken the action in, now the incumbent has to decide whether to fight with him or not to fight with him. So, that is why the player who makes a move after him is obviously the incumbent.

And finally, the preferences in this game, entry game. Now, there is nothing sacrosanct about the preferences, but typically, in this entry game, it is assumed the standard assumption is that, and which is quite reasonable that suppose the challenger does not get in, he is stays out. Then, what is the, who is the player, who is getting the maximum sort of payoff in this case? It is the incumbent because he is just this maintaining the status queue. So, that is the best for him.

So, if I write this as i and out is the, is the terminal history. Now, one thing I should make clear here, here, is that how is the preference defined, on which it is defined? A preferences are defined only if some terminal history has already happened. So, something has happened; the game is not stuck in the midway. Only in that case, I can say whether player, how much a player likes that that particular outcome.

So, u i out is the best for player i. So, I shall denote it by 2. And which is the case which is best for the, for the, challenger. In this game, it is assume that the challenger is best his best off, that is, he is getting the maximum payoff. Let us write it as c. If he moves in and the incumbent does not fight. So, this is the another terminal history in an accommodate. And that is the outcome which is best for the challenger, because the fighting is bad. Fighting is bad for the both, both, the parties, and in typically, we shall assume that fighting is the worst possible case for the both the parties.

So, this is one. What is the second best for the incumbent? We shall assume that the second best for the incumbent is that the challenger comes in and the incumbent accommodates. If there is a fight, then that is the worst thing that can happen to the incumbent. So, in an accommodate, is the second best we shall, which we shall denote by 1, and what is the second best for the first player? That is the challenger, is that he stays out, because you see there are three possible cases - either in fight, in accommodate and staying out. For the challenger, the best is in accommodate, and for the challenger, the worse this the fight fighting case.

So, thus middle one, the second best is the out terminal history. And so, what is left to be done is just specifying that fighting is the worst for both the players where they are getting zero. So, this is how the preference of the players are define in this entry game. So, just to recapitulate and get back to the precise definition of these three components of extensive firm game. First, I have to specify the set of players who are the players, who are involved. Second, what is known as terminal histories and how is it defined sequence set of sequences of actions such that no sequence is a sub history of other sequences. I should write proper sub history. So, these set of sequences of actions such that no sequence is a proper sub history of other sequences. The set of sequences is called the set of sub history, the set of terminal histories.

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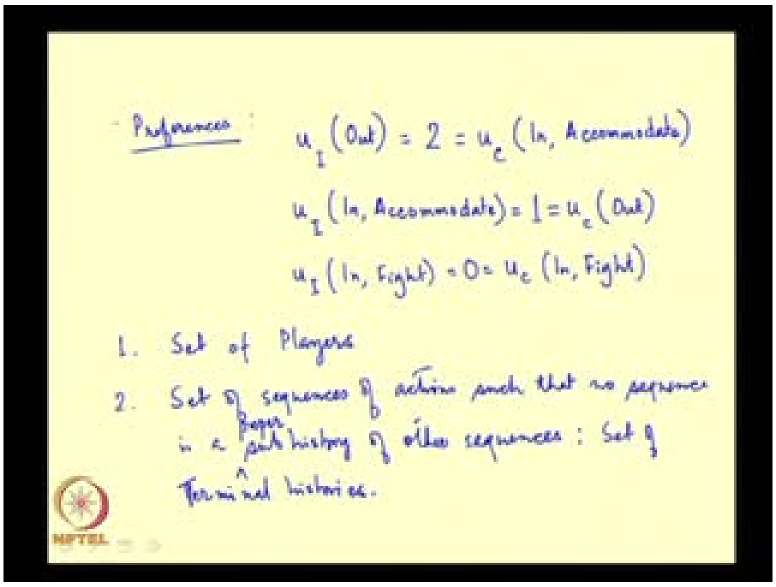

Preferences : $u_I(\text{Out}) = 2 = u_C(\text{In, Accommodate})$

$u_I(\text{In, Accommodate}) = 1 = u_C(\text{Out})$

$u_I(\text{In, Fight}) = 0 = u_C(\text{In, Fight})$

1. Set of Players
2. Set of sequences of actions such that no sequence is a proper sub history of other sequences : Set of Terminal histories.

What was the reason for writing this the such that no sequences are proper sub history of other sequences. By this, basically we are ruling out those sequences which are not complete, which are not reaching the end of the game, because if you are including some incomplete sequence which is basically a proper sub history, then the other sequence will also be included which is the terminal histories, and the first one will be a subset of the second one which we are basically ruling out by this criteria that no sequence can be a proper sub history of other sequence. And these sequences will be called the terminal histories.

Now, before we mention the third point, one point, one small point that can be made here is that we are not ruling out infinite sequences. Basically, the definition of the terminal histories that we have that it is a possible sequence of actions. That may include the case where that action. This is, this sequence is a, is an infinite sequence. Take the case of chess. Suppose there, there, are two players and they are just repeating their moves. Same moves of these two players are being repeated. Now, this game can go on and go on to infinite periods of time. So, that can also be terminal history where it is not in fact terminating. It is going on to a, to an infinite extent. So, those things are there and in that infinite sequence also we can talk about proper sub history.

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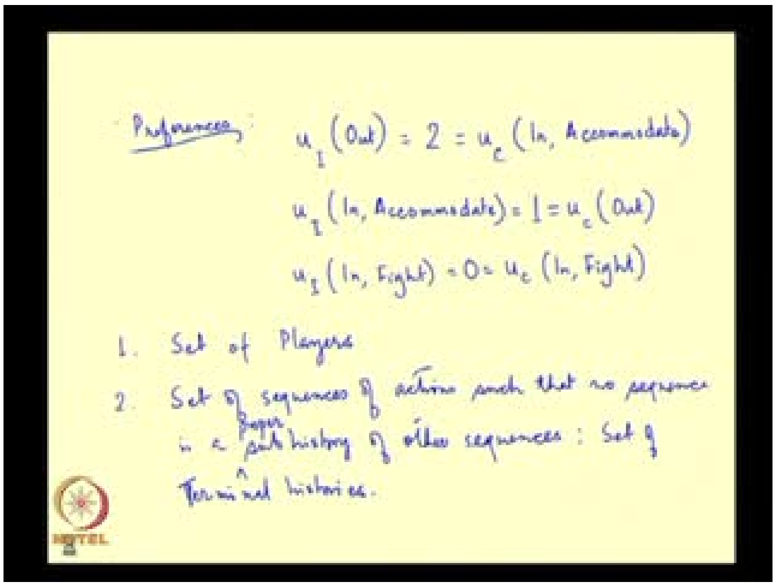

Preferences: $u_I(\text{Out}) = 2 = u_C(\text{In, Accommodate})$

$u_I(\text{In, Accommodate}) = 1 = u_C(\text{Out})$

$u_I(\text{In, Fight}) = 0 = u_C(\text{In, Fight})$

1. Set of Players
2. Set of sequences of actions such that no sequence is a proper sub history of other sequences : Set of Terminal histories.

So, the third point which is to be mentioned is the player function. So, it is a function that assigns a player to each proper sub history of a terminal history. So, if I have a terminal history, then there are many proper sub histories of the terminal history. For each proper sub history, I have to know who is the player now who can make a move. That is player function.

And finally, the preferences - preferences are defined over terminal histories. If the game is somewhere stuck in between and some players are still there who will take an, take actions, then we cannot talk about preferences because the game has not completed. So, preferences are defined over the set of terminal histories. So, these are the three important, four important components that are to be mentioned in case of any sequential game, any extensive game. One useful way to represent extensive game is to draw what is known as a game tree.

A game tree basically it summarizes all these four components in a very neat way. If I have to draw the game tree of the entry game which we have just discussed, it will be like the following. So, we start with a circle here and this is an empty circle. This is where the game starts. If you remember the game starts with the movement made by the challenger, he has to take a decision.

So, in this node, this is also known as a node. In this node, I have to write the identity of the person who is making a move here. In this case, it is the challenger. Now, how many actions does the challenger have in the first node? Now, one important thing is that in case of extensive game, we do not specify the action set of the players, but from the terminal histories which, for which the information we have and the player function, we can figure out what are the action sets, what are the action sets available to the players at different nodes.

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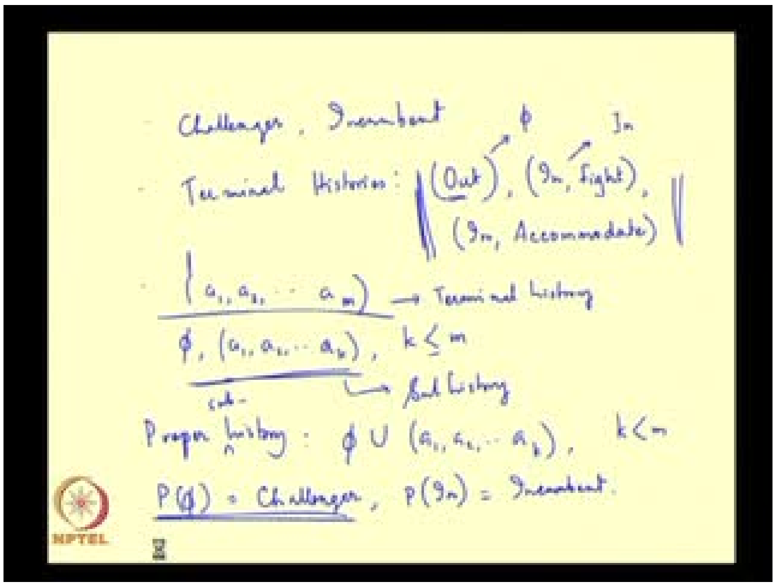

- Challenger, Incumbent
- Terminal Histories: $\left\{ \underline{(\text{Out})}, (\text{In}, \text{Fight}), (\text{In}, \text{Accommodate}) \right\}$ ϕ In
- $\underline{(a_1, a_2, \cdots a_m)} \rightarrow$ Terminal history

$\underline{\phi, (a_1, a_2, \cdots a_k)}$, $k \leq m$ ↳ Sub history

Proper $\overset{\text{sub-}}{\wedge}$history : $\phi \cup (a_1, a_2, \cdots a_k)$, $k < m$

$\underline{P(\phi) = \text{Challenger}}$, $P(\text{In}) = \text{Incumbent}$.

For example, I know, if you look at the game that these are terminal histories and the terminal histories are written in a particular way. First action that can be taken is the first component; it can be out; it can be in; it can be in. So, basically we get two sorts of action and who is the player who is taking the first action. It is the challenger. P phi is the challenger, which means if nothing has happened the game is in the beginning itself. It is the challenger who is making a move. How many moves can you make? That we can, that information we can get from the set of terminal histories. He can make two sorts of moves.

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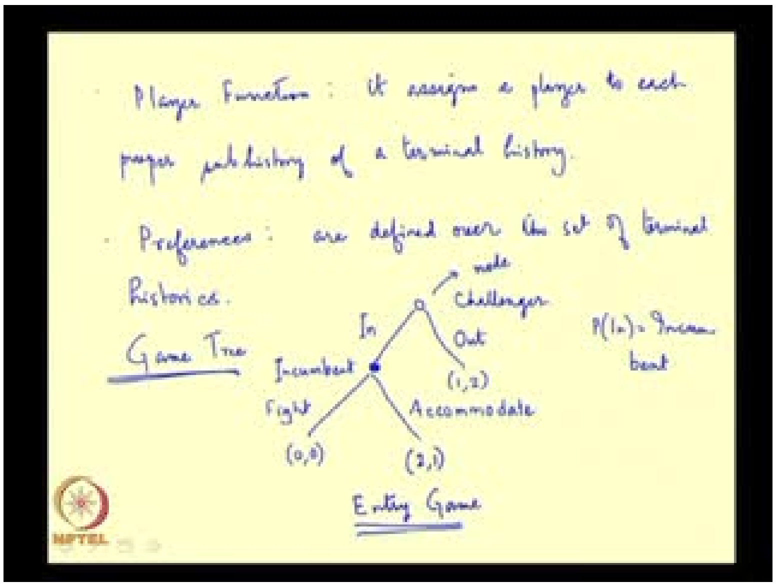

- Player Function : it assigns a player to each proper subhistory of a terminal history.
- Preferences : are defined over the set of terminal histories.

Game Tree

So, I write, I draw two lines, two slanting lines in for the first action and out for the second action. Now, if he chooses out, the game ends there. So, I do not have to write; I do not have to draw anything else. The tree ends there. However, if he plays in, then it is the turn of the second player to make a move, because how do I know this? Because I know that in is followed by two sorts of action, two possible actions, phi can accommodate. And after in the player function is incumbent.

So, from these two informations, I know what to draw. I have to write the, the, the, identity of the player here who can make an action, who can take an action and this is incumbent, because I know P in is the incumbent. And from these two sorts of histories can follow. So, basically there are two actions available to the incumbent here - one is fight; the other is accommodate, and there the game ends because this is the longest sorts of terminal history that can happen; it can possess two components. There is no terminal history which has more than two components. So, this is where the game ends.

And then, I write down the payoffs of the players at the term, at the termination points. So, I have to write the payoffs of the players here, here and here. For that also I have information. If the challenger is choosing out, then he is getting 1. So, I write 1, and if out happens, then the incumbent is getting 2. So, there is a order that I maintaining here.

The first element in the payoff functions in this, in this bracket, within the bracket. The first element denotes the payoff to the first player. Who is the first player? The player who makes the first move, the challenger. So, one is the payoff of the challenger, and a second component 2 is the payoff to the incumbent. If the terminal history is in fight, then what are the payoffs? The payoffs are 0 0, both of them.

And if the terminal history is in an accommodate, then the challenger is getting 2. This is the best for the challenger and the incumbent is getting 1. So, it is 21. So, this is the game tree for the entry game. So, this is how, the, the, all the information, the four information that are needed to be known for any extensive game can be represented in terms of a diagram.

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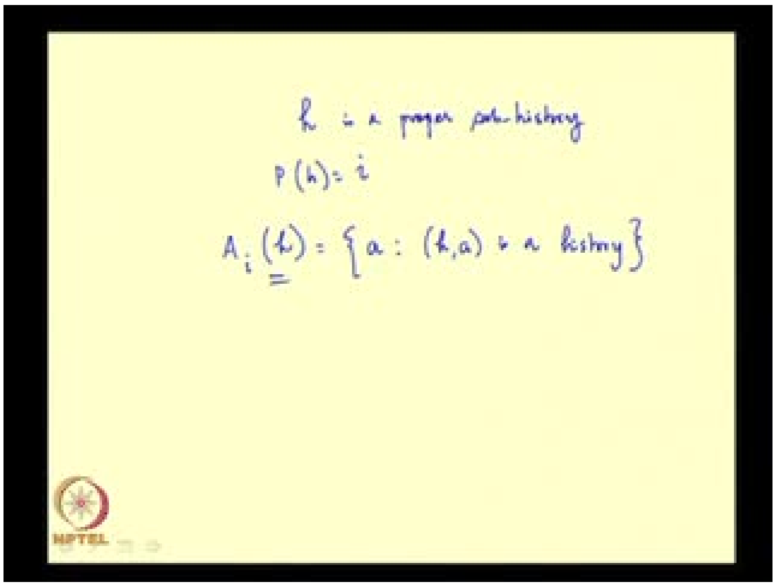

h is a proper sub-history

$P(h) = i$

$A_i(\underline{h}) = \{a : (h, a) \text{ is a history}\}$

One more thing regarding this action set of a player and how it is defined. This is very simple. Suppose h is a sub history, is a proper sub history and suppose p h is i. Then the action set available to i after the sub history has happened is the, is defined in the following way. It comprises of the actions of the firm a such that h a is a history. This history, it is not necessarily proper sub history; it might be a terminal history also, but the, the, point is that, if there is a proper sub history which is denoted by h and if it so happens, that h a is a possible sequence of actions, is a feasible sequence of actions which means it is a history. Then a will be in the action set of player i after h has happened. So, action set of any player is defined over a proper sub history.

So, that is more, that is more or less what we shall be ending within this, in this lecture. Before we finish, what we have been discussing so far? We have basically started with discussion of extensive game where the actions are taken step by step in a sequential of sequential manner. And if that is the case, then I have said that there are four components that have to be mentioned - one is the set of players who are the players were involved. Second is related to the terminal histories the set of terminal histories, which means what all possible sequence of actions which are, which can be possible in this game and such that no sequence of action is a proper sub history of sequence of actions. Third is the player function which are defined, which is defined over a proper sub histories of terminal histories, and fourthly, the preferences of the players which are defined over terminal histories. So, we shall pick up the thread in the next class. Thank you.

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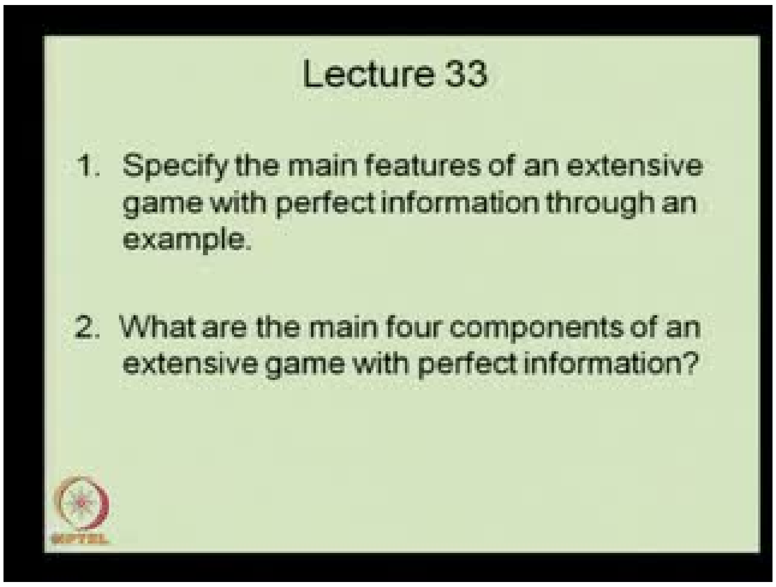

Lecture 33

1. Specify the main features of an extensive game with perfect information through an example.

2. What are the main four components of an extensive game with perfect information?

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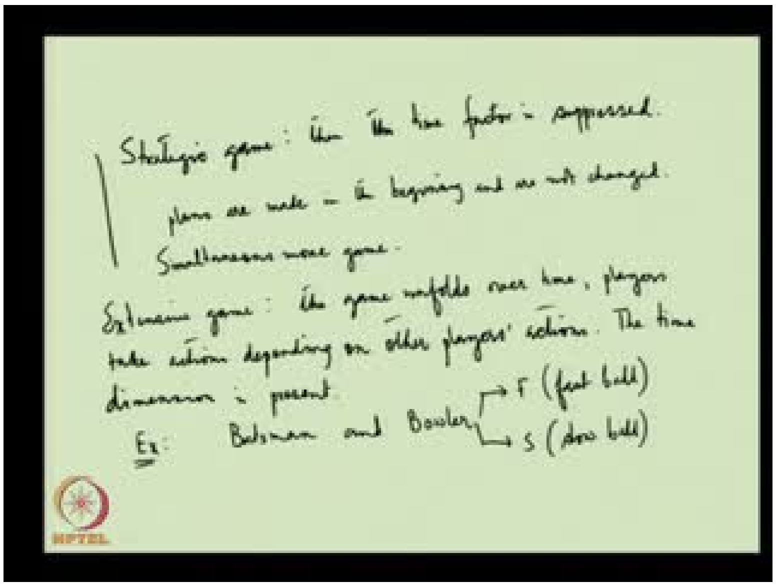

Strategic game : then the time factor is suppressed.

plans are made in the beginning and are not changed.

Simultaneous move game.

Extensive game : the game unfolds over time, players take actions depending on other players' actions. The time dimension is present.

Ex: Batsman and Bowler $\rightarrow$ F (fast ball)
$\rightarrow$ S (slow ball)

First is specify the main features of an extensive game with perfect information through an example. Now, we know that in a strategic game, if we have a strategic game, then the time factor is suppressed. So, it is as if the players decide what they will play in the very beginning of the game, and then if the game is unfolding over time, they do not deviate from the initial announcement.

So, plans are made in the beginning and are not changed. So, whatever be the action of other players, I do not waiver from my initial announcement of actions. So, this is one interpretation of strategic game. The other interpretation of strategic game is that it is a simultaneous move game that the players are making their moves at the same point of time.

So, even they, if they wanted to change their move according to other players actions, they cannot do so, because the actions are taken simultaneously before hand you cannot know what the other player is going to play. So, you cannot make your action dependent on the action of the other players because you do not know what is the action of the other player. So, this was the case of strategic game.

In extensive game, it is different here, but we have an extensive game is that the game unfolds over time. So, you do not have a simultaneous move game but a game which progresses stage by stage and players take actions depending on other players actions. So, the time dimension is present. Let me give you an example. So, suppose a batsman and a bowler, these are the two players. The bowlers can bowl a fast bowl. We shall call it f or a slow ball. And looking at the ball, the batsman can play two shots. Let us call s 1. This is suitable for f, that is for a first ball or he can play s 2 which is suitable for a slow ball s.

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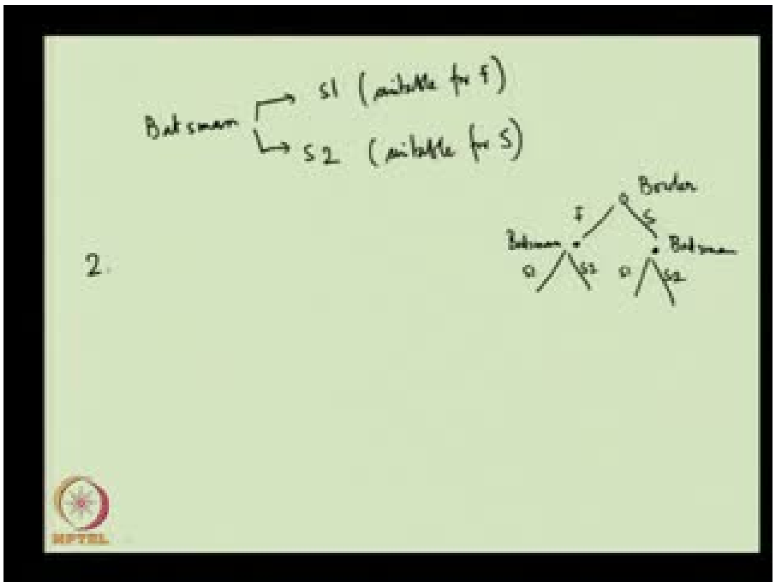

Now, here, what is happening is that first the batsman is making a move and then the bowler is making a move depending on what, sorry, first the bowler is making a move, then the batsman is making a move, and the batsman is making the move depending on how the bowler is bowling. So, I can show this in terms our game tree, so f or s. If f is played, then the batsman moves. If s is played, again the batsman moves and he has neither s 2 sorts of action s 1 or s 2. So, here, the actions of the second player, it is depended on the action of the first player. Second question - what are the main four components of an extensive game with perfect information?

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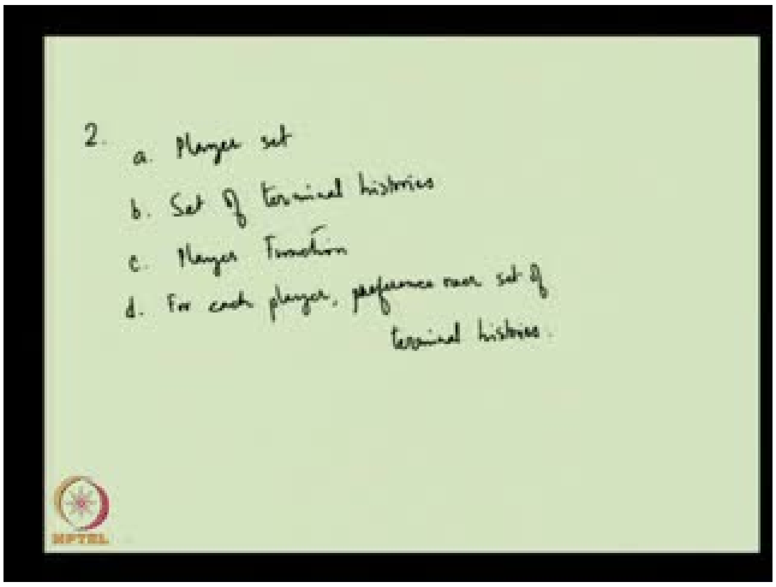

2. a. Player set
b. Set of terminal histories
c. Player Function
d. For each player, preference over set of terminal histories.

Main four components is the player set. Second main component is the set of terminal histories. Each terminal histories sequence of actions such that no sequence of action is a subset of some other sequence of actions. Thirdly, player function given a non terminal history. We should know who is the player now who is required to make a move, and fourthly, for each player preference over set of terminal histories. So, each player must specify which terminal history it prefers the most which second, second, most etcetera, etcetera. So, these are the four important components of an extensive game with perfect information. Thank you.