Game Theory and Economics
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Module No. # 05
Extensive Games and Nash Equilibrium
Lecture No. # 06
Backward Induction: Exercises

Welcome to the 6th lecture of module 5 of this course called Game Theory and Economics. Before we start this lecture, let me take you through what we have been discussing.

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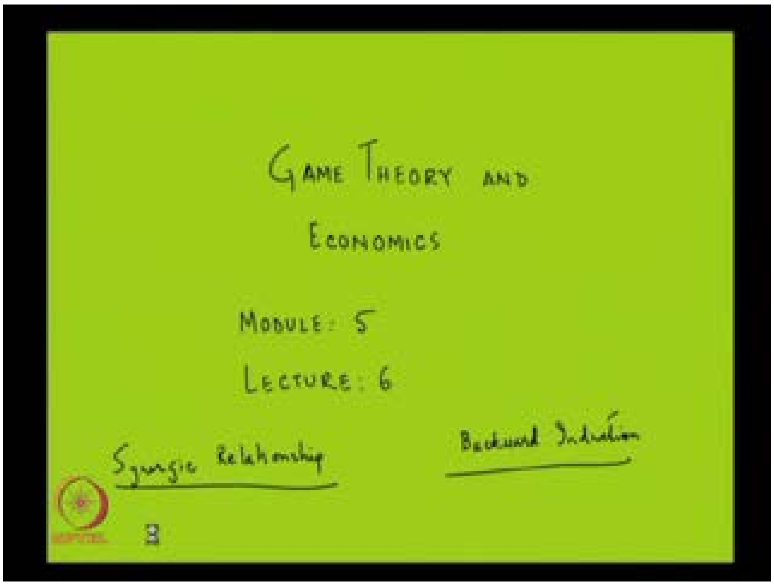

So, in this module and for last couple of lectures, we have been discussing sub game perfect equilibrium which is also known as sub game perfect Nash equilibrium, which we have seen is a refinement over Nash equilibrium; the idea of Nash equilibrium in the context of games which are extensive games which means, that the actions are taken sequentially and the players are aware of each other's actions, which have taken place before they are supposed to take any action.

So, there is no problem of information, people are aware of the other actions taken by other players. So, we have defined what is the sub game perfect Nash equilibrium, we have said that it is a strategy profile, where, in each and every sub game there is equilibrium in the sense that, if we define sub game, what is known as a sub game in an extensive game, there will be many sub games. Now, the strategy profile should be such that, the strategies of the players are optimal, given the strategies of other players and this should happen not only at the beginning of the game but, each and every sub game.

So, that is that and we have seen that, there is another easy way to find out this sub game perfect equilibrium through what is known as backward induction. And this method of backward induction can be used if the game is a finite horizon game, which means that, the length of the longest terminal history is finite; in that case, we can go back to the end of the game look at sub games of length one, find out what are the optimal actions of the players who are supposed to move at the beginning of those sub games of length one and taken taking those actions as given, we go backwards and try to find out in games of length two. In sub games of length two, what are the actions of optimal actions of the players, who are supposed to move in the beginning of those sub games, that is sub games of length two.

So likewise, you go backwards going to sub games of length three, length four and going backwards through this induction method, we go to the beginning of the game and the chain of optimal actions that we get from this process will be the sub game perfect Nash equilibrium or equilibrium as the case may be. So, this is the method of backward induction, what we propose to do today is to discuss some exercises of using this method of backward induction and how to find out the sub game perfect equilibrium through this method, one exercise that we have done in the last lecture which will be relevant today, first the exercise of Synergic Relationship.

So, in that exercise if you remember, there were two parties, two players basically and the actions that they were taking where such that, if you put more effort then, your output or your payoff will go up provided that, the other player also puts more effort and this going up of the output also is not a very linear, it does not to go up in a linear fashion, but in a non-linear fashion, and the actions are taken by not simultaneously as it was the case in a strategic game, but sequentially.

So, first player one moves and chooses A 1 and then player two moves and chooses A 2 and then the game ends and the players get their payoffs. So, in this case what we had is that, the actions where the set of actions, it was not a finite set, there could be infinite number of actions that a particular player can take is where there was a continuous variable, the action was a continuous variable.

And in that problem, what we applied, this method of backward induction, we try to go back to the end of the game, at the end of the game, it is player two who is supposed to move, we look at her optimal action that is A 2 given A 1 and which is nothing but, the best response function of player two given A 1, so A 2 is a function of A 1.

Now, this A 2 as a function of A 1 now will be used by player one to find out what is his optimal action in the first stage. So, in this case player one is basically, second guessing player two and taking a preemptive action such that, his payoff becomes more than what could have been the case had the taken their actions simultaneously.

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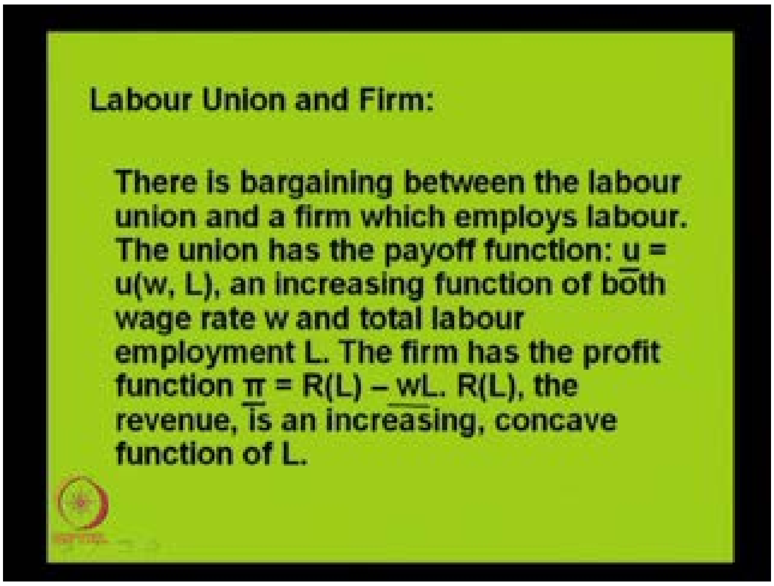

So, this is one important thing, conclusion that we derived in the last lecture. Today, we shall start with one exercise which is similar in spirit to that synergy relationship and this is called the problem of labour union and firm. So, let me first read out the exercise and then we shall try to solve this.

There is bargaining between the labour union and a firm which employs labour, the union has the payoff function u as a function of w and L. An increasing function of both wage rate w and the total labour employment L, the firm has the profit function i, which is equal to R L minus w L, R L is the revenue, it is an increasing but concave function of L.

So, total employment of labour if it raises, it increases the utility or the payoff of the union because, more of its members are getting employment. If more labour is employed what happens to firms profit? It rises, but not in a linear fashion, it rises in a declining fashion, because the reason is that, if you employee more and more labour, the productivity of the additional labour units goes on falling.

So, the first labour might be adding a lot to your output but, the 100 first unit of labour might not be contributing that much to your output and that is why your output will not raise that much, therefore your revenue will not raise that much. So, that is the effect on revenue and if I look at the profit function of the firm, revenue is the not the only thing that I should consider, I should also consider the cost, if more labour is employed, there is more labour cost.

So, unequivocally if labour raises, L raises, cost raises and we have assumed that, this relationship is a linear relationship, w multiplied by L. So, this is what the union wants to maximize u and this is what the firm wants to maximize and there are two variables here, one is w and other is L.

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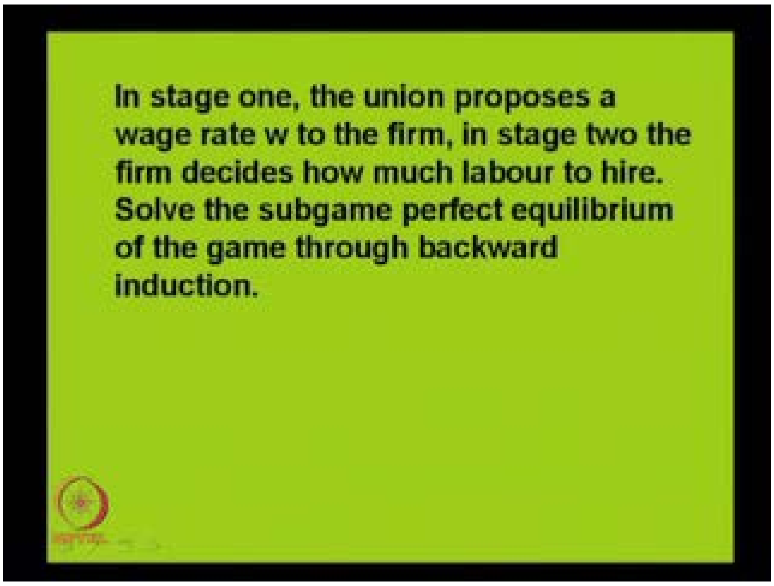

Now, how is the bargaining taking place? In stage one, the union proposes a wage rate w to the firm, in stage two the firm decides how much labour to hire. Solve the sub game perfect equilibrium of the game through backward induction. So, this is the exercise there are two stages to this game; in stage one, the union says we want this wage rate, this w for one unit of labour that you will employ and after the union tells its w, which it wants to have from the firm. The firm decides how much labour it will employed and so w and L are decided and then the two parties, two players get their payoffs, we have to decide which is the sub game perfect equilibrium through the method of backward induction.

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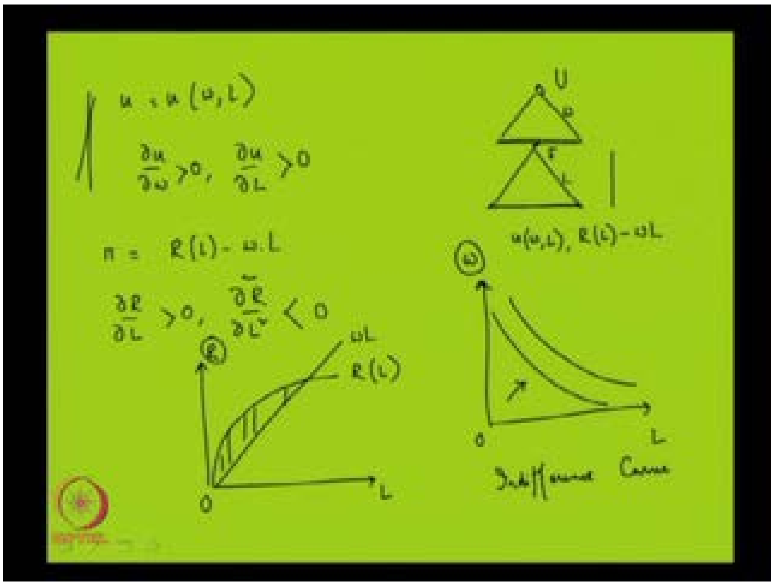

Now, why did i say that, this exercise is relevant to the synergic relationship exercise because, here there are many similarities; first there are two parties involved and there are two stages, the union, let us call it U is like the first player and here comes the second similarity, the variable that it is deciding, which is w is a continuous variable like it was A 1 in that synergic relationship exercise which was also continuous, w can presumably take any positive, it can take zero also hypothetically and in non-negative value. So, that is why I have this continuous range here for w and after it has decided w, then the firm moves in and decides how much labour it will employ which is L and again L is a continuous variable, it can take any value between zero and infinity. After w and L has have been decided, the two parties that is the union and the firm get their payoffs and these are the payoffs.

So, this is where the game ends and we have to figure out, how to find out the sub game perfect equilibrium. Now as before, what we shall do is that, we shall start with sub game of length 1 which is this sub game. Find out the optimal action of the firm and go backward and try to see what is the optimal action by the union, if the firm has taken its final optimal action. Now, before we go into that, let us look at the nature of this payoff functions. First let us look at u, this is the payoff function of the union. We know the following, that the payoff is increasing both in wage rate and the labour employment, if wage rate w raises, utility raises, if L raises, again payoff raises.

So, this can be represented by a downward sloping line like this, this line or this line these are these are called indifference curves. So, the idea is that, along any of these curves, there could be infinite number of such curves, which are parallel to each other, along any such curve the payoff of the union remains constant. So, that is why we are calling, then the indifference curves any one is indifferent between two points on the same curve. And we have to assume that, the curves are convex to the axis, but this does not necessarily follow from the utility function and the assumptions that are given.

So, the implicit assumption behind the convexity of the curves is that, the second derivatives that is del square u del w square and del square u del L square, they are negative. So, the utility margin what is called the marginal utility of wage rate and L are positive, but they are diminishing and therefore, we have this convex indifference curves and what does a union want to do, any union wants to maximize the utility. So, it wants to go as far as possible away from the origin, so it wants to reach as high and indifference curve as possible. The high indifference curves are to the northeast, more and more towards the northeast of this plane of L w.

So, this is what the union wants to do and what does the firm want to do, firm was to maximize this pi and pi is given by this and we have this relationship between L and R, that the first derivative is positive which means that, if more labour is employed revenue raises, but it raises at a declining rate because, if you remember the firm has a profit function pi, the revenue is increasing an concave function of L. Concave function of L means that, it is slope goes on declining, which means that the second derivative is negative.

So diagrammatically, it will look like the following; so, this could be different R curves for different parameters. In particular, in general R could be a function of not only L but other parameters also, so I can have many R curves, but since in this particular example no other parameter is given, let us suppose this is the only revenue curve that i am concerned with.

Now, what the firm was to maximize is not this R, but R minus w L,so one can draw w L curve also, this will be a straight line, a linear function of L w is constant in this in this diagram. So, it is an upward raising straight line going to the origin, what the firm was to maximize is the difference between these two. So, this this area the gap, the gap is what the firm was to maximize and we shall see how the firm was to maximize.

Now, talking about this, this R and w L, I can also find out what is the indifference curve of the firm, which means along what w and L, the firm is getting the same amount of profit, that will not be called an indifference curve per say but, it will be called an iso profit curve; iso means same, so along and iso profit curve of a firm, the profit of the firm remains the same, like the payoff of the union was remaining the same in an indifference curve.

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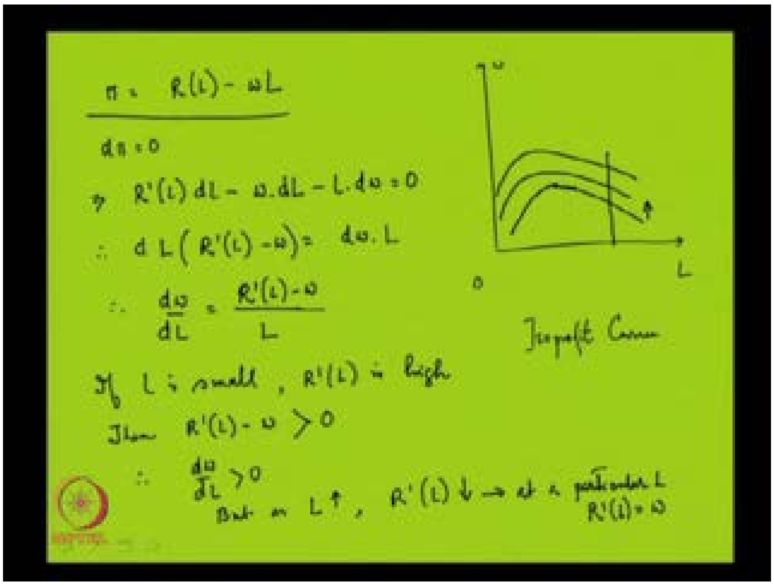

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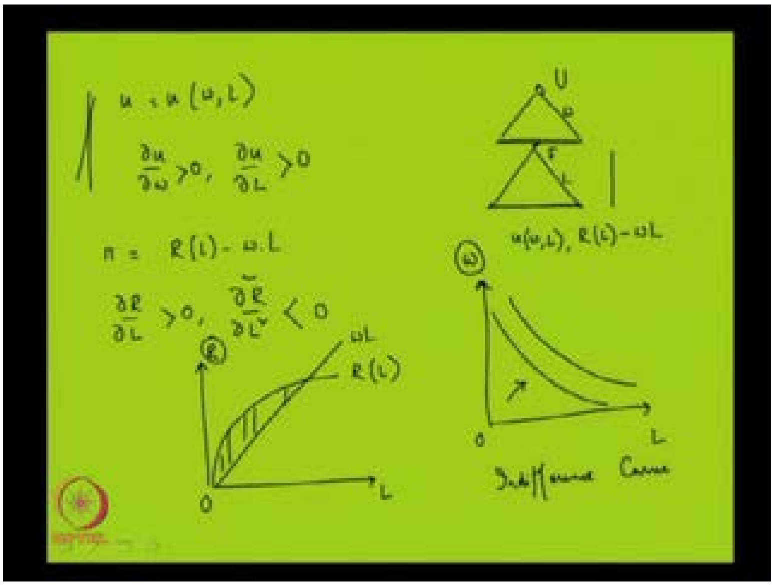

So, if I want to draw the iso profit curve of the firm, then along the horizontal axis I am having L, but notice along the vertical axis i am not having R, but I am having w. So, this is w like it is here, w is along the vertical axis in the indifference curve also. So, I want to find out along what combinations of w and L is the firm indifferent, so that those curves will be the iso profit curves and I have this function of profit. Now to figure this out, I have this for assumptions which are given that, R L is an increasing but concave function of L.

So, there are two ways to find out the shape of the iso profit curves, first is to look at technically, what will be the slope of the iso profit curve at any combination of w and L? For that I, What I do is to take the total derivative of this function and said that equal to 0, because along any iso profit curve, the profit remains the same, and form is the following, this what I get is by taking the total derivative equal to 0 (No audio from 21:30 to 22:06).

So, it is R prime n minus w divided by L, this is the slope of the iso profit curve at any combination of w and L, and how will this behave over if I take different combination of L and w? If I take very small value of L, then I know R prime is high, because R prime is as such is positive, but it goes on falling, so if the value of L is very low, the value of R prime will be high, if I take high value of L, this R prime is go on declining.

Now, if R prime L is high then, R prime L minus w it will be positive, because I can take L as small as possible, so that R prime L eventual becomes higher than w. So in this case, the slope of the iso profit curve will be positive, so it will be increasing function like this, but as L goes on rising, R prime L will go on declining. And eventually a situation will come where R prime L at a particular L it may it will happen definitely, that R prime L is equal to w and if R prime L is equal to w, the slope become 0, and you can figure out if L raises further, the slope becomes negative because, then w becomes higher than R prime L.

So, I have a declining part of this iso profit curve, so this is how the iso profit curve of the firm looks like and there will be plenty of such iso profit curves, each represents a different level of profit. How do I know which curve represents higher level of profit, if I go on taking more and more curves which are to the north, so I go upwards then, what is happening is that at a given level of L, for example, I can imagine a given level of L, w is rising, which means wage rate is raising and I know from this profit function that, if wage raises, the profit definitely falls, given L given a constant level of L.

So, the iso profit curves which lie to the north direction, they represents a lower level of profit, where as if I take curves to the to the south, or which are closer to the L axis, those iso profit curves represent a higher level of profit. So, this is how the situation looks like, the iso profit curves of the firm looks like. Now, what is optimal for the firm, let us come back to the original question.

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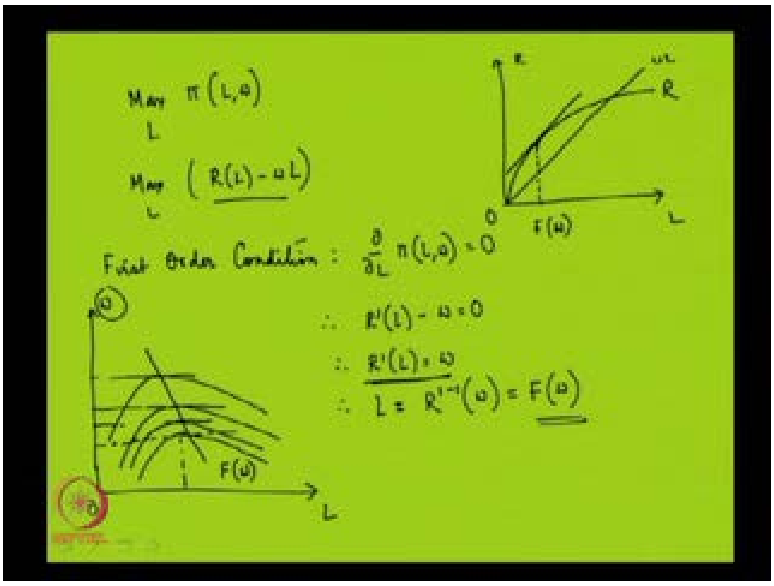

The original question was, in this sub game, what is the best the firm can do to maximize the profit. So, maximize pi which is a function of L and w, now what is the variable on which the firm has control, it is L, w is something which has been decided by the union in the first stage; so, on w the firm does not have any control. So, w is a parameter for the firm, it is given from outside and if you have a function, which is a function of just one variable which is L here and you want to maximize the function, can we depend on the first order condition? Here w is given, w is parameter, so set it equal to 0, so applying this FOC, your first order condition to this particular function, I get the following condition that L is a function of w, let us call this F of w.

So, this L as a function of w, this F function is the function which optimizes the profit of the firm in second stage given, the w which has been decided by the union in the first stage. So, this is the solution from the sub game of length 1, this can be also represented by this diagram, the diagram is something which you have seen before, this is R. So, you have this rising R function and you have this w L, now if w is given, what is the level of L which is maximizing R minus w L? It is given by this tangency point, where the slope of the R function which is R prime is equal to the slope of this line which is nothing but w.

So, this is the point, this is F w, given w this is the level of L, which maximizes the profit of the firm. What is the reflection of this analysis, this exercise, this F w in term of in terms of this iso profit curve? It will look like the following, remember there are plenty of iso profit curves; now, what this say is saying that, L is a function of w, at any given level of w. What is the L that maximizes the profit? Basically this is the L that maximizes the profit, any firm will like to have or like to reach that iso profit curve which is closest to the L axis.

Now, if it chooses a point like this, the corresponding iso profit curve is this, which is a lower level of profit. Here also the profit is even, this point represents even lower profit, so this is the point, where the profit is maximum. So, if I have different levels of w, I am essentially getting different levels of optimal L and this is the line that connects all this optimal L's and this is the line which is nothing but F w. Here F as a function of w, it is just the other way around, basically w is the depend independent variable here, whereas that independent variable is represented along the y axis.

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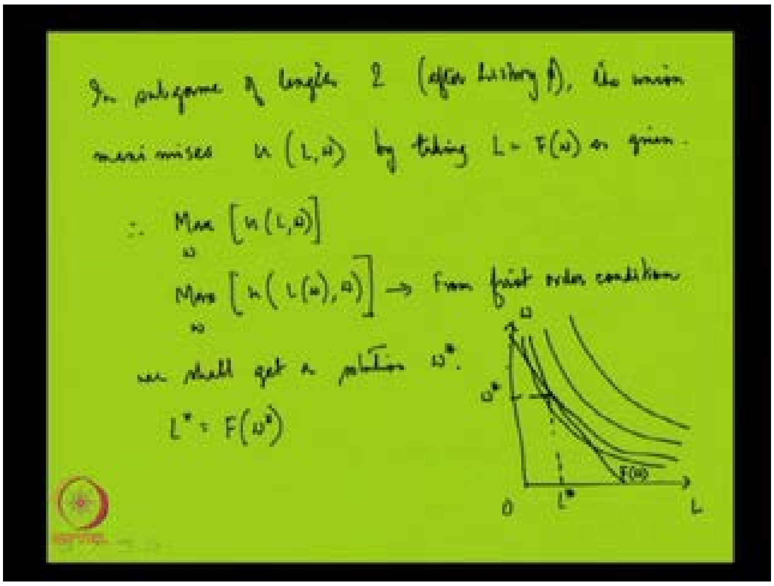

So, F w is the is the decision of the firm given a value of w, now we shall go backwards and try to find out what is the optimal action by the union. So, in sub games of length 2 and there is just one sub game of length 2, that is after history phi. The union maximizes u which is a function of L and w, but keeping in mind what the optimal action of the firm in stage two, by taking L is equal to F w as given. So, the idea is to maximize and what is the decision variable here, it is w is the decision variable so, w is the thing that is the union is deciding and by manipulating w it wants to maximize this.

So, and what is the additional information that I have is, that L here is a function of w itself, so this is just function of one variable and the union wants to maximize that function with respect to that variable. So therefore, we shall get from first order condition, just differentiating this function with respect to w and set that equal to 0 and that will solve for w star.

So, this is this is how it is done first, L is express as a function of w then, that relationship is taken by the union and used to maximize its own utility. So, this is the thing in terms of mathematics, but diagrammatically how does it look? So, we had different indifference curves for the firm downwards slopping convex indifference curves and we also had this relationship which is known as the F. F as a function of w so, what is the w in terms of this diagram, which is w star which is the optimal for the firm, well this relationship F w is given and it is taken as given by the union because, it is decided by the firm.

Now, taking this has given the firm was to maximize its utility and remember the indifference curvature to the northeast represents higher level of utility and therefore, it wants to touch the union wants to reach or touch as high an indifference curve as possible and again this is going to happen at the point of tangency. So therefore, this is might w star and given this F I have got this L star which is basically, the L which the firm will decide in the next period, so this is the solution.

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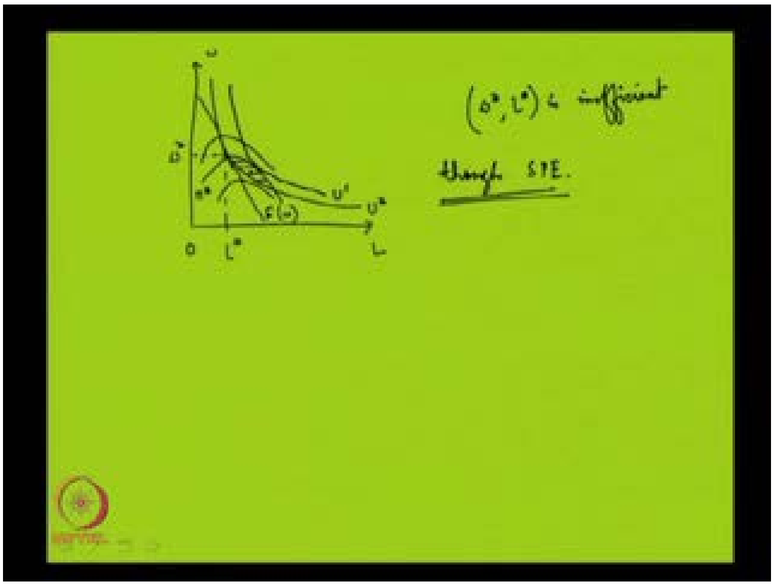

One interesting property of this solution is that, this solution is not an efficient solution, from this equilibrium, this w star and L star which is a sub game perfect Nash equilibrium. One can reach a better equilibrium more efficient equilibrium and this I shall show by diagrams that are, let me draw a different diagrams of the so that, the picture is not cluttered.

This is suppose the F function and this is the tangency point, does not look like tangency point but, suppose this is the tangency point, so, therefore, since this is a tangency point, this is w star, this is L star, this is the equilibrium. If on the same diagram, I want to impose, I want to superimpose the iso profit curves of the firm, then how will it look like? I know that, along this F function, the firms are having their maximum points of the iso profit curves, look at this, the highest points on the iso profit curves they on the lie on this line, the F w line (Refer Slide time: 37:34).

So, it will be the following this and here is the problem, this is the equilibrium, so the in the equilibrium, what is the level of utility obtain by the firm, let us call this u star which is represented by this indifference curve. And what is the profit, that is obtain by the firm? It is represented by this iso profit curve, so let us call this pi star. The point is that, both the union and the firm could reach better levels or more improve levels, higher levels of u and pi than this equilibrium of u star and pi star, how do I know this? Look at this lengths, this shaded region, if I take any point in this shaded region, let us call, let us take this point here, this is a point; now, in this point the characteristic of this point is that it is on pi star.

So, the profit of the firm is same as in w star L star, but at this point, the firm, the union is having a higher level of utility, because this is u dash which is a higher level of utility than u star and this is this kind of improvement of one persons utility, one players utility, keeping the utility of the other player constant is possible for many such points in this shaded region. In fact, it is possible to take some point inside, where both the firms profit goes up and the union’s utility also goes up. So, that is why I am saying this w star and L star is inefficient, though it is sub game perfect equilibrium, so this is just a common that sub game perfect equilibrium may be inefficient.

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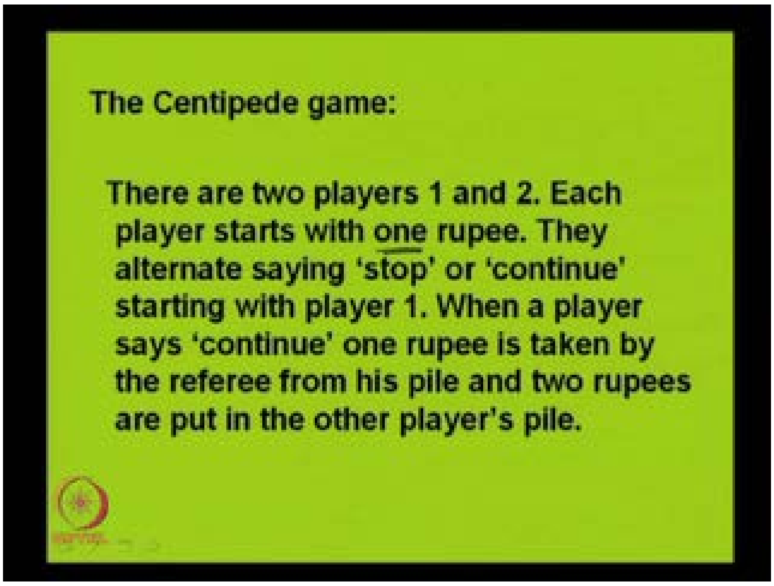

So, this was one exercise, let me take another interesting exercise, which again uses in the idea of backward induction, this is the centipede game. So, here you have two players, there are two players; player one and two, each player starts with 1 rupee, they alternate saying stop or continue, starting with player one. So, player 1 is the first mover, it either says stop or continue, when a player says continue, 1 rupee is taken away by the referee from his pile and 2 rupees are put in the other player's pile. So, in first stage, if player one says continue, then his 1 rupee which he started with because, he starting with 1 rupee that will be taken away from him, so he will be left with 0 rupee and the other player, the second player will be given 2 rupees.

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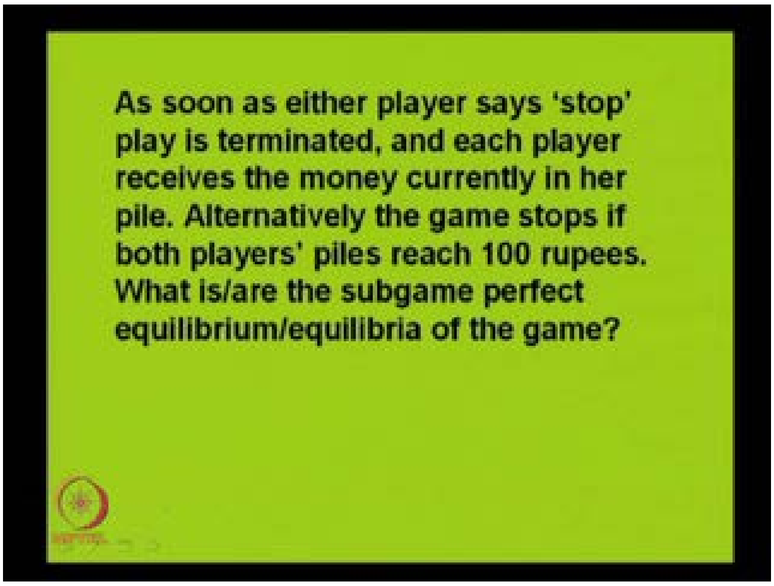

So, the second player will be getting in all 3 rupees in the second stage, as soon as either player say stop, the game is terminated and each player receives the money currently in her pile. So, if in the second stage, player two says stop and does not say continue, then player two will get all those 3 rupees and player one will be left with 0 rupees. Alternatively, the game stops, if both players piles reach 100 rupees, so this this way the game continues in stage two, player two does not says stop but says continue, then 1 rupee will be taken from him and 2 rupees will be given to player one.

So, in the third stage player two will be left with 2 rupees and player one will be left with 2 rupees and then it is up to player one to continue or not to continue; if he says continue again, 1 rupee will be did deducted from his pile and 2 rupee will be given to player two like that. Why does the game end? If everyone continuous to say continue, then the games stops, if both players piles reached 100 rupees, so once a both the players are getting 100 rupees each, the game is stopping there.

What are the sub game perfect equilibrium or equilibria of the game? So, this is a game of perfect information, and let us see, what could be said about the sub game perfect equilibria of this game, if we apply the idea of backward induction. First try to draw the game tree of this centipede game.

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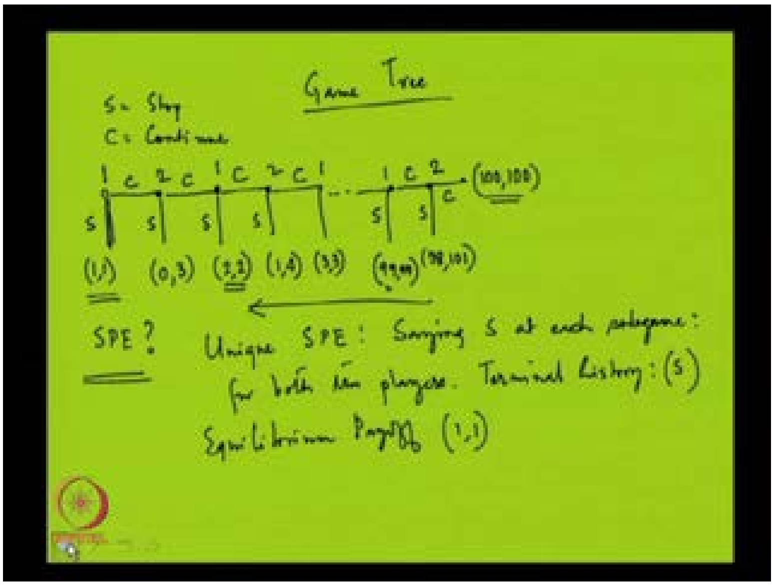

So, I know the starting point is player one and player one can says stop, s is stop and c is continue and it can say continue, if it continues the game reaches player two, if stop is the decision by player one, the game ends there, it is terminated there. Player two again can say stop, again the game ends here, it can say continue the game progresses, then again its player ones turn to move, stop and continue.

So, that is the way it moves on and you can see, why it is called the centipede game. Why does it end? It ends somewhere here, where 100 rupees is by each of the players is reached. Now, who is the last player that moves and therefore, the piles reach 100 rupees, something we can know up if we look at this pattern of the payoffs. So, let us look at the pattern of the payoffs; in stage one, if player one says stop, then there is no game, there is no progress of the game and the initial endowment of 1 1 is the payoff that they are getting.

If player one says continue, then obviously 1 rupee will be deducted from his pile and player two if he says stops stop, now then he will get this 2 rupees and player one will be left with 0, instead if player two says continue and in the next stage player one says stop. Then basically, 1 rupee had been taken away from player two and this is transformed into 2 rupees and given to player one, so this is what the payoff after stage three.

If player 1 says continue again 1 rupee will be taken away from him and if player two says continue in the next stop in the next stage, 1 4 will be the payoff. So, likewise it goes up, the question is how does the game look like in the termination point. One thing is important is that, when the payoffs are equal, that is 100 100, that is a decision taken by player two. See here 2 2 is happening, and this was the payoff which was taken by player two he said continue and that is why it became 2 2; likewise here also player two, if he says continue, then only and if player one says stop, the game stops here, then only it becomes 3 3.

So, the last decision must be by player two. So, this is he said continue that is why it became 100 100 if he had said stop then, what would have happened? That can also be figured out, if he had stopped the game before that, it would have been 98 101, because 1 rupee will be taken from him.

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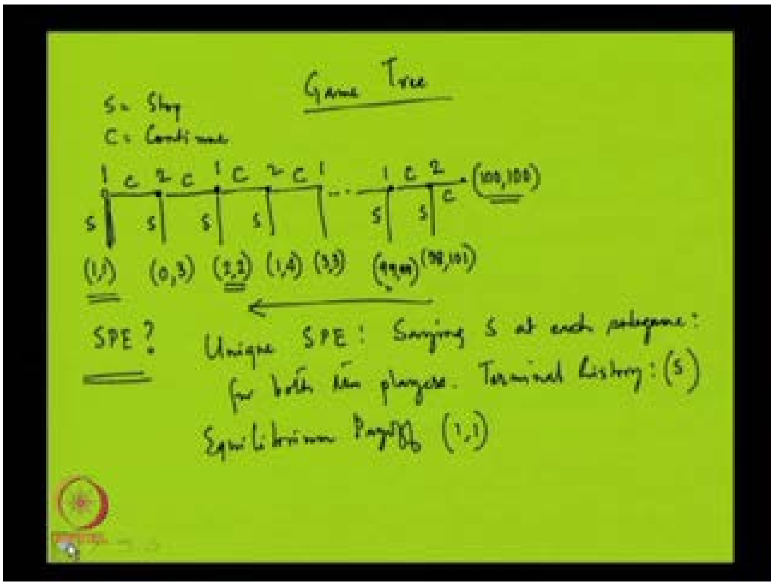

So 100, from 101 it becomes 100 and 2 rupees will be added to player one’s pile, so that is why it becomes 100 from 98. So, if I look a little backwards, then this is C, this is 1, this is s and it will be 99 99, because 1 is saying continue, if he had not said continue and if he had said stop, it would have been 99 99, because from 99, 1 rupee is taken from player one’s pile, so it becomes 98 and 2 rupees are added to player two’s pile, that is why from 99 it becomes 101.

So, this way there is a gap between these two ends, this way the game looks and you can see it looks like a centipede. Now, what is sub game perfect equilibrium here? Interestingly, here there is a unique sub game perfect equilibrium and that is saying stop at each sub game and for both the players, why is this so? Because again I can go back at the end of the game, at the end of the game player two has to choose between 101 and 100 and definitely he will stop at 101, which is more than 100 and given that player two is going to stop at 101, which gives player one 98. There is no reason for player one to continue at this stage because, he is getting 99 here, so why should he you know pass on the game to player two, where he will get 98.

So, the same reasoning, basically take us back and basically it means that the game ends here itself, which means the terminal history is this S. So, this is the terminal history and payoff, equilibrium payoff is obviously 1, so this is again one important and interesting result that we are getting, that the sub game perfect equilibrium could be very inefficient. This game if they had trusted on each other, they could have got 100 each and which is much better than 1 1. So, because the players are thinking that the other players are rational, and depending on the rationality of the other person, therefore gives them payoff which is much sub optimal than, what they could get at 100 100.

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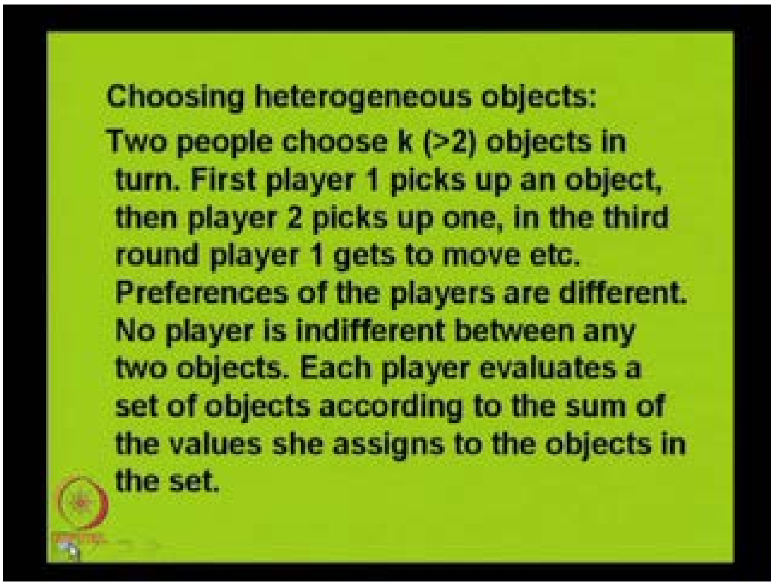

Let me take another exercise and then we shall close this lecture, this is the exercise of heterogeneous objects, let me just describe the game, may be will not have time to finish this exercise. Two people choose k which is greater than two objects in turn, first player one picks up an object, then player two picks up one in the third round, player one gets to move etcetera. Preferences of the players are different, two player no player is indifferent between any two objects, each player evaluates a set of objects according to the sum of the values she assigns to the objects in the set.

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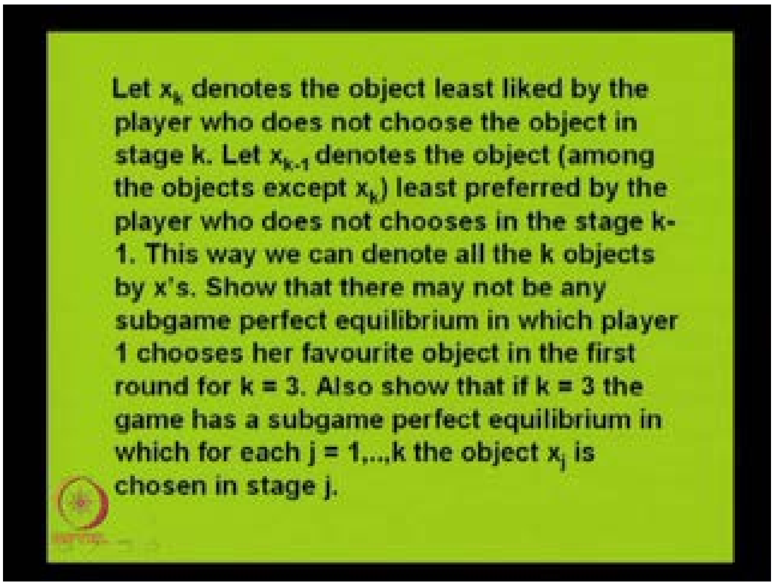

Let x k denotes the object least liked by the player, who does not choose the object in stage k. Let x k minus 1 denotes the object among the objects except x k least preferred by the player, who does not choose in the stage k minus 1. This way, we can denote all the k objects by x's, show that there may not be any sub game perfect equilibrium in which player one chooses her favorite object in the first round for k is equal to 3.

Also show that, if k is equal to 3, the game has a sub game perfect equilibrium in which for each j going from 1 to k, the object x j is chosen in stage j. So, it is quite a long question, let me try to do the first part of the exercise, which is the following, that there are two players.

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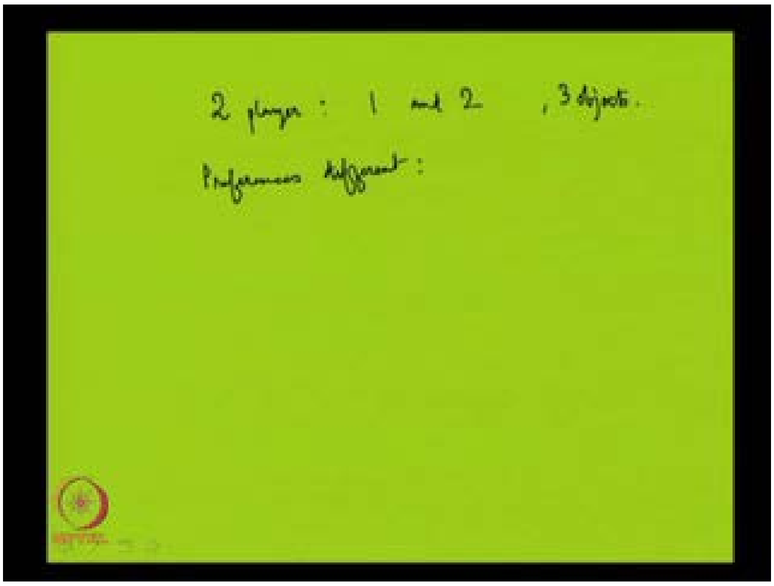

Suppose, there are 1 and 2, their preferences are different, now what we have to show is that, there could be preference patterns for this two players, such that player one will never pick up his favorite object in the first round. These two players are picking up their choices, their objects from a set of 3 objects, so 3 objects are there. In first round, player 1 picks her object, now this object may not be her favorite and that is what we have to show and in second round, player 2 picks up her object and in the third round player one again picks up her object and then the game terminates, because there only 3 objects.

What we need to show is that, in sub game perfect equilibrium, it can happen that player one does not pick up her most favorite object in the first round and that is what we want to show in the next lecture and that there is also the exercise that, if I define this three objects in terms of x 1 and x 2 and x 3 in a particular way, which we shall discuss in the next lecture, then x 1 will be chosen in the first round, x 2 will be chosen in the second round and x 3 will be chosen in the third round. So, that is where we shall begin in the next lecture, thank you.

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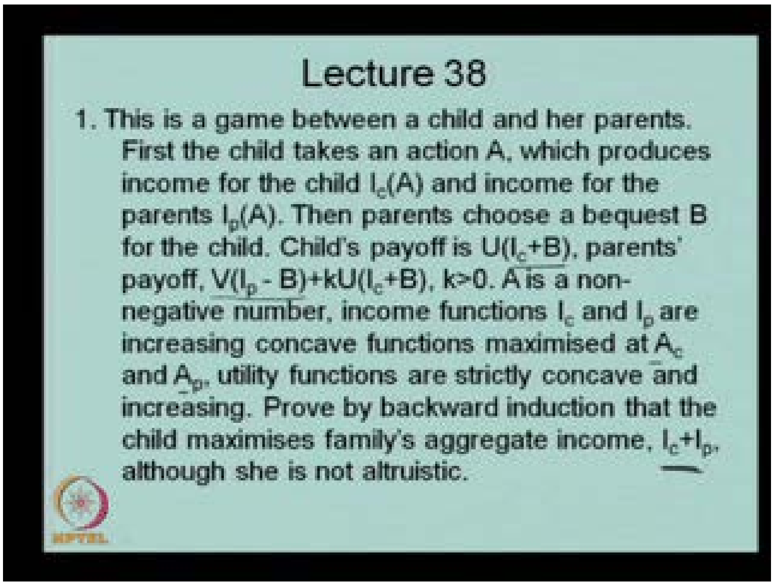

This is called the rotten kid problem, this is a game between a child and her parents. Fine, first the child takes an action A which produces income for the child is denoted by I c A and income for the parents I p A, then parents choose a bequest B for the child. Child's payoff is U I c plus B, because this is the total income and parents payoff is something more complicated, it is V of I p minus B plus k multiplied by u of I c plus B, where k is greater than 0.

A is a non-negative number, income functions I c and I p are increasing concave functions maximized that A c and A p. Utility functions are strictly concave and increasing, prove by backward induction that the child maximizes family's aggregate income I c plus I p although she is not altruistic.

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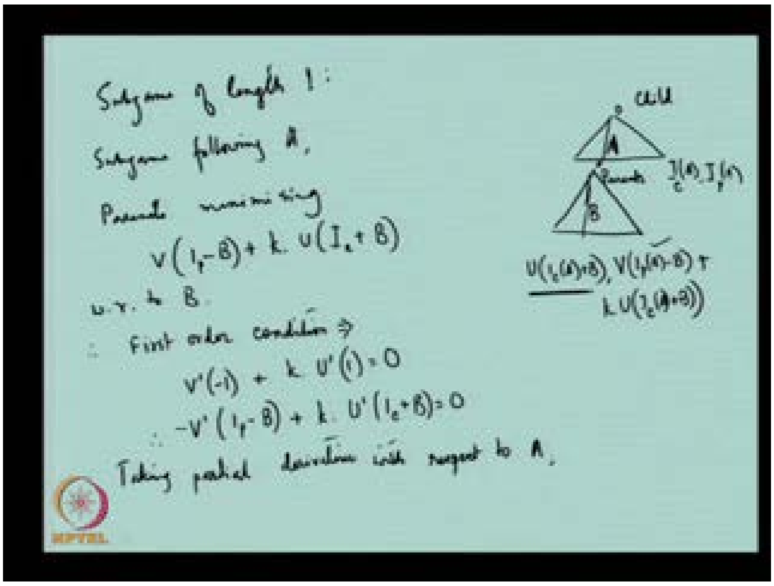

So, let us first briefly in terms of nutrition describe the game, first move is made by the child and the variable that she chooses, it is a continuous variable it is called A. Once, A is chosen, that A is determining two things: one is called I c, the income of the child, which is a function of A and I p the income of the parents, which is also function.

So, after incomes have been decided, then the parents, here are the parents, they make a move, so parents are here and the variable that they choose this called B. B is the bequest and once the bequest, what is the bequest? Bequest is the money that the parents are leaving for the child.

There, that is capital B, once the bequest has been decided the what is the income of the child, then income child becomes I c plus B. So, then we talk about the utility, that is the payoff of the child, which is given by capital U I c, which is a function of A plus B and the payoff of the parents, which is in fact composition of two things; one is v which is I p, which is a function of A minus B plus small k multiplied by U, which is the utility of the child.

So, this is the game, just before trying to solve the game, what is happening is that the child is determining the income of both the child and the parents and the parents then are giving away some money from their income to the child. Now, the child is not altruistic, we can see that, his payoff is a function of his own income, it does not depend on the utility, that is the satisfaction of the parents, whereas the parents payoff is their own payoff, the individual payoff plus k, k is positive multiplied by the payoff of the child.

So, if the child is getting more payoff, that satisfies the parents, because k is positive, so the parents are altruistic but the child is not. What we have to show that, even in this case the child is going to maximize not his individual income, not I c, but I c plus I p that is the income of the combined income of the child and the parents. So, we have to use backward induction here, so let us look at sub game of length 1. So, we are talking about the sub game, following A in this sub game, the parents are maximizing their payoff, so they are maximizing this ip function with respect to B, B is the variable that they have control over.

So, from first order condition, we shall have V prime sorry, this will be minus, this is minus because, V prime minus multiplied minus 1 plus k of U prime multiplied by plus 1 is equal to 0, which means that, V prime which is a function of i p minus B plus k multiplied by U prime, which is a function of I c plus B is equal to 0.

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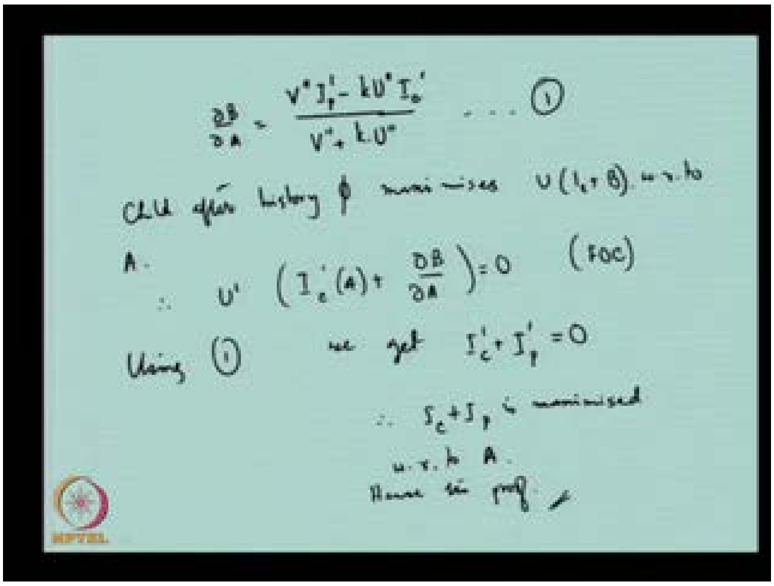

Now, we are going to manipulate this relation little bit, we are going to take partial derivative with respect to A and if you do that, we get the following; after some manipulation, we get this relation, it is V prime I p dashed minus k U role prime I c dashed divided by V prime plus k U role prime.

Now, this is what is known by the child, that the parents are maximizing their payoff here. And now, taking that into account, the child is going to maximize his payoff, so child after history phi maximizes U I c plus B with respect to A. So, what we get is the following; this is the first order condition and then we use 1 and using 1 we get the following, after some manipulation and jumping the steps, I c dash plus I p dash is equal to 0, which means that I c plus I p is maximized with respect to A, hence the proof. Thank you.