Game Theory and Economics

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Module No. # 06
Illustrations of Extensive Games and Nash Equilibrium
Lecture No. # 02
Stackelberg Duopoly Model

Welcome to the second lecture, of module 6, of this course - game theory and economics; this is going to be our last lecture of this entire course. What we shall do is to start off this last module. After we have ended with the lecture and discussion of this module, we shall do a brief talk of what we have discussed in this entire course. If you remember in the last lecture, we have been discussing a game which is a variation of the ultimatum game, which is known as the holdup game.

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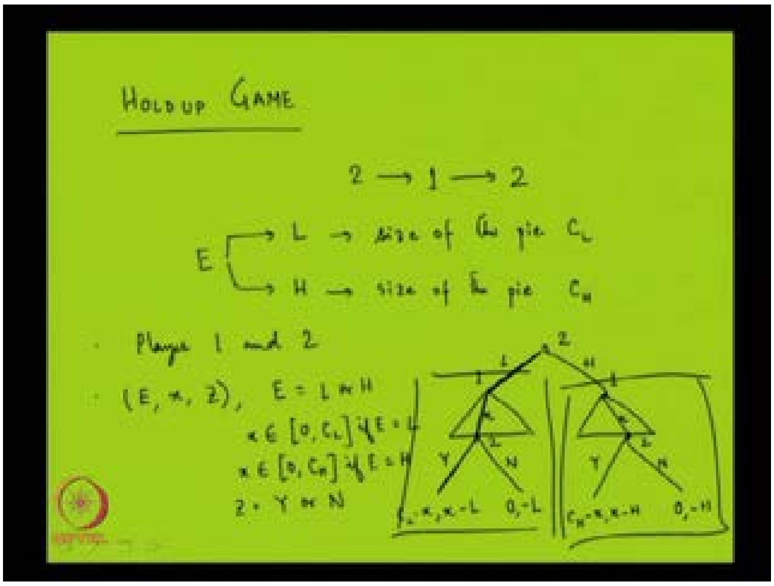

In this holdup game what happens is that, first player 2 moves, then player 1 moves and then again player 2 moves. What does player 2 do in the beginning of the game? Well, player 2 can do either of two things, player 2 can either exert little effort which we shall called effort E; E could be less, if he exerts very less effort then, the size of the pie is small, let us call it C L - C L is a total size of the pie. Otherwise, E could be high which means, C is exerting high effort in which case, the size of the pie is going to be large, let us call it C h.

Now, after player 2 has decided his effort level, that decides the size of the pie then in the second stage, the ultimatum game that we have discussed before starts, which means that after that player 1 makes an offer to player 2 which is an x and after that player 2 decides whether to accept that offer, if he accepts the offers, he gets x and if he rejects the offer both of the players gets 0, this is the game in short.

Question is, what is the sub game perfect Nash equilibrium in this game? Where, I have added a further step in the beginning of the game itself; to answer that let us look at the game in terms of the game tree. So, 2 is starting the game and there are two kinds of action that he can take either l or h, then player 1 moves. He can make an x offer in either of these 2 cases and then again player 2 moves, he can either say yes or he can say no and then the game terminates.

What are the payoffs in the terminal histories, if l is the effort chosen by player 2 in the beginning then the size of the pie is C L, in that case if player 2 accepts the offer he gets x and player 1 gets C L minus x. There is one thing further player 2 has already made an effort L which has cost him some effort and that is why I have to subtract that from that x that he is getting right now, so it is x minus L.

If he rejects the offer player 1 gets 0 whereas player 2 is getting minus L and likewise here also it will be C H minus x, x minus H, if he rejects the offer it is 0 minus H. This is how the game looks like in terms of the language of extensive game it will be like that two players, player 1 and 2. What are the terminal histories? It could be E, x and Z, where E is L or H, x belongs to 0, C L if E is equal to L, x belongs to the interval 0, C H if E is equal to H and Z could be either yes or no. So, it depends on the value of E what value x will take in the subsequent stage in which player 1 moves, so these are the possible terminal histories.

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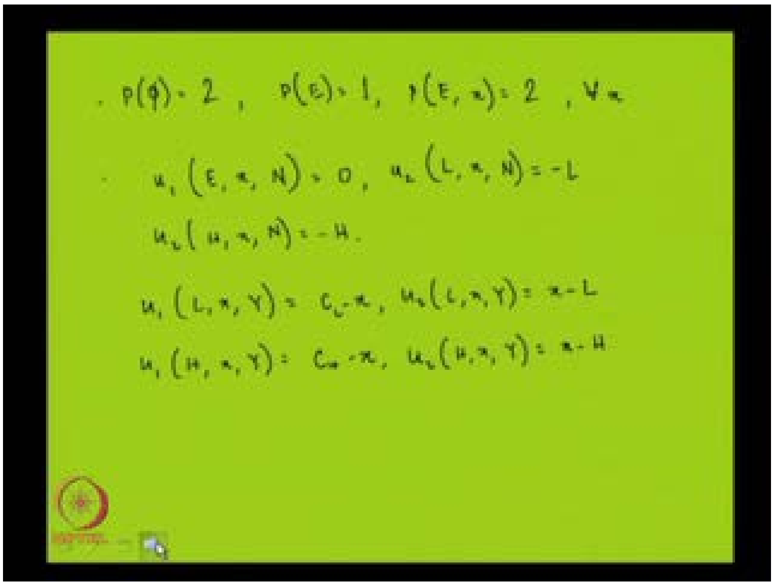

Now, let us look at the player function, it is player 2 who moves first, then player 1 moves and then player 2 moves for all x. Finally preferences, what is the kind of preference that we are having here? These are given by these payoff functions. So, in general, I can write the following that u 1 E x N is equal to 0 - if N is the decision of player 2 in the last stage then player 1 will get 0 everywhere.

If his effort is L in the beginning and he is saying no in the last stage, his payoff is minus L. In this case, it will be minus H, these are the cases where N is reached. What happens there? is yes suppose, player 1 is saying yes in the last, player 2 is saying yes in the last stage whereas, he has put a low level of effort in the first stage then player 1 is getting C L minus x.

In this case, what is player 2's payoff, he is getting x minus L and likewise, if high effort has been put then it is C H minus x, this is going to be x minus H. This is how the payoffs are represented, everyone likes to maximize the payoff of his and these are the payoffs and that is how the preferences are defined.

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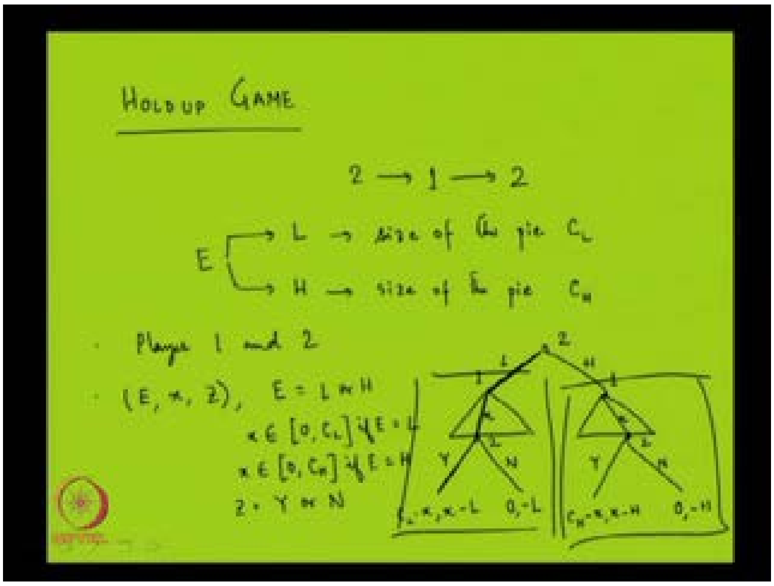

Now, what is the sub game perfect Nash equilibrium here? It is not very difficult to see, after player 2 has decided in the first stage, whether the effort is going to be low or high the game becomes the familiar, this game or this game, both these games are the familiar ultimatum games. We know that if we have an ultimatum game then, the sub game perfect equilibrium is that player 1 is going to offer 0 to player 2, that is, x equal to 0 and player 2 is strategy is that accept any offer if x is greater than or equal to 0.

This is going to be the sub game perfect equilibrium in each of these 2 sub games whether we are starting with the history L or history H, the equilibrium is going to be the same that 1 is going to offer 0 to player 2 and player 2 will accept that. However, player 2's payoff from both these equilibria are different in - the first sub game - this sub game, which is beginning. After the history L, player 2's payoff in the equilibrium is minus L, whereas if the sub game is of beginning after H then player 2's payoff is minus H.

We know minus L is greater than minus H because H is greater than l. Therefore, player 2 will have to choose between L and H and by choosing L, he is getting minus L and by choosing H is getting minus H in equilibrium, and since minus L is greater than minus H, so player 2 will choose L.

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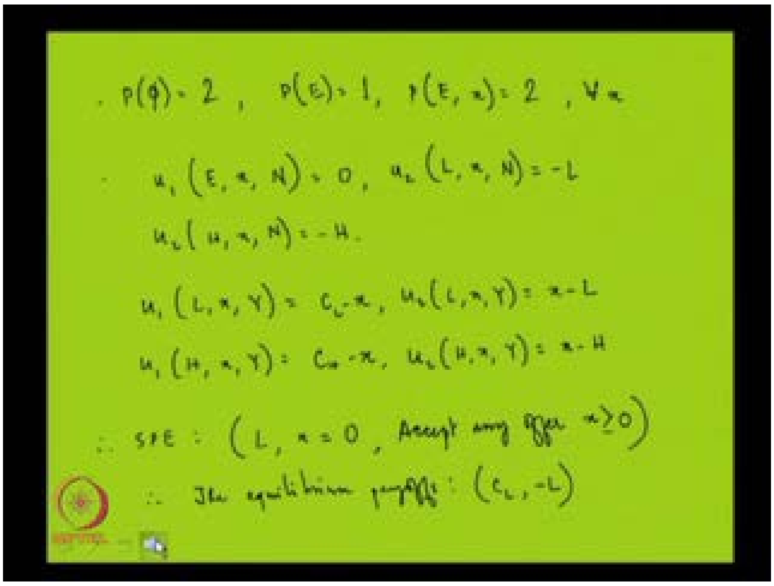

So, the sub game perfect equilibrium in this case is the following L - L is the action chosen by player 2 in the first stage. In the second stage, that is, when player 1 is making an offer, he is going to make an offer x is equal to 0 to player 2 and what is the strategy of player 2 in the third stage. Accept any offer x which is greater than or equal to 0 and the equilibrium payoff of these 2 players are C L and 0, sorry, not 0 minus L.

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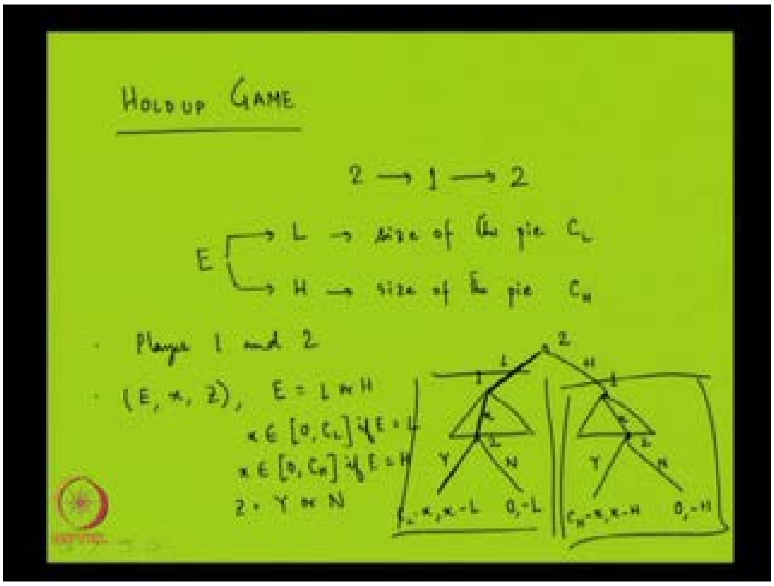

Here, we are reaching basically this, this and this, where x is equal to 0, so if I put x is equal to 0 it becomes C L comma minus L, so this is the equilibrium payoff.

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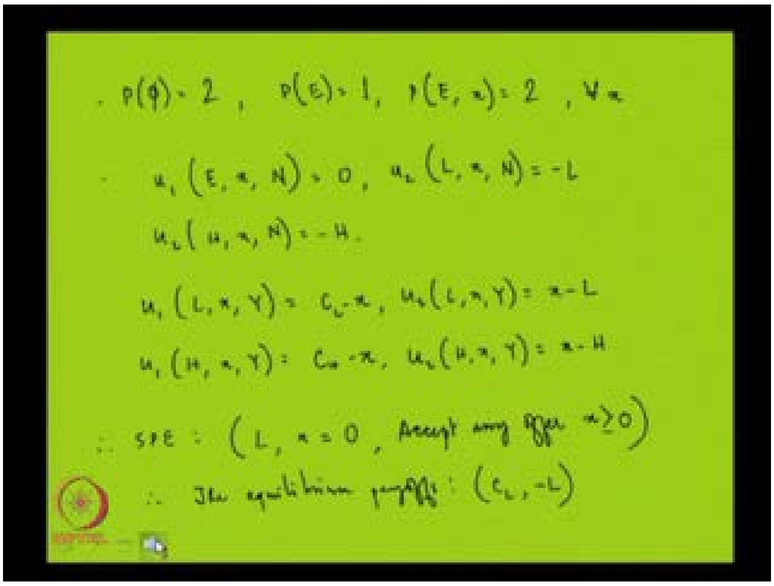

Now, this might seem a little awkward because player 2, it could be argued that if he puts some effort then, the size of the pie could increase so much by large amount that the player 1 could have also benefited by getting high portion of the pie. That large portion of pie he could have shared with player 2 and which could have made player 2 go for high effort, but what is happening is that in equilibrium player 2 is getting 0 here.

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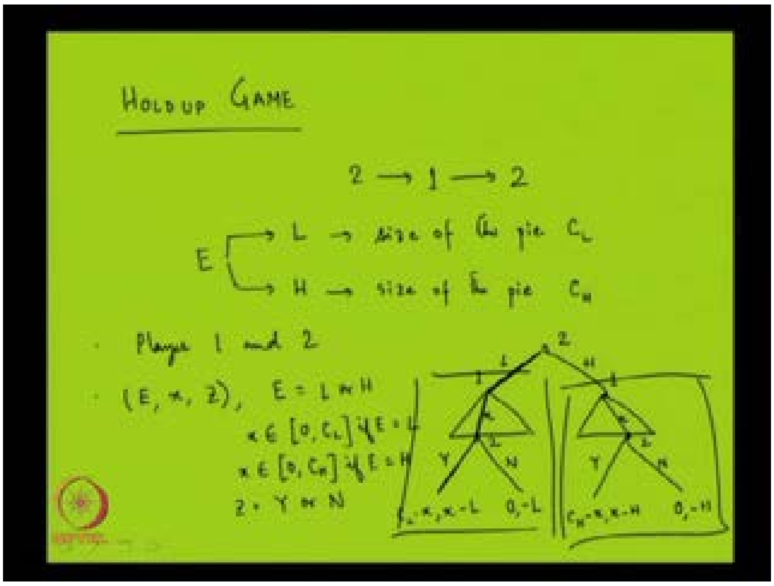

So, there is no incentive for him to put high effort and neither can player 1 convince him that look if you choose high effort if the size of the pie is so much high that I am going to share a part of that pie with you, because that is not going to be credible because once H is chosen the equilibrium is that going player 1 is going to offer player 2 only 0 that is, the equilibrium.

So, this may seem a little inefficient kind of equilibrium that we are having here, that player 2 is always choosing a low effort level and that is why the total size of the pie remains small, so this is again 1 of the many examples where the equilibrium is not very efficient.

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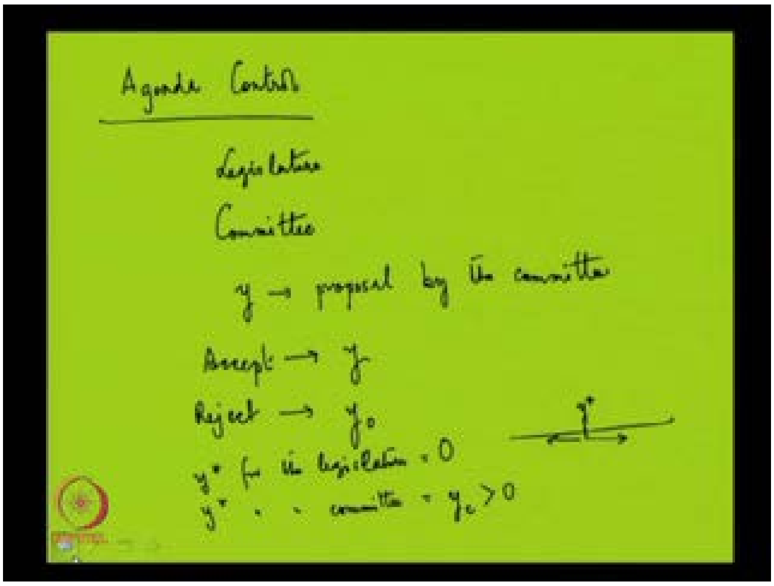

Now, related to this ultimatum game which is we have another game which is called the Agenda control game. I am simply going to describe the game here and without doing any exercise, so what is the story? The story could be told in the following way, suppose there is a legislature, which decides what policy the government will take and the legislature, when a implementing a policy may appoint a Committee.

The Committee may recommend a particular policy, the Legislature can do the following, either it can accept that policy in which case the recommendation of the Committee is accepted. On the other hand, the Legislature can choose to reject the policy recommended by the Committee in which case, in case of rejection, we go back to the what is known as the status quo policy.

Suppose, y is the proposal by the committee, this is the first stage where as if the player 1 is making a movement and he is making an offer to player 2, why is that offer? Player 2 in this case legislature, can either accept in which case y becomes the policy or reject. In which case, we are back to the status quo policy with the policy which was prevailing before - let us call it as y 0.

So, these are the things that these 2 players can do, what about the preferences? Well, we are going to assume that these 2 players Legislature and Committee they have what is known as a single peak symmetric preference. What does it mean? It means that you have a one dimension variable where you like, suppose, this y star and if you are at y star that is best for you.

The further you go from y star does not matter it is to the left or to the right, you are worse off in an increasing fashion, so that is why it is symmetric - it does not matter whether we go to the left to the right - this is called single peak to a symmetric preference. Remember, this is the kind of preference we have seen in case of voting games, also before what we discussed in case of discussing the application of Nash equilibrium in strategic games, there we talked about electoral competition and there also we talked about this kind of preference pattern.

Now, we are going to assume that favorite or this y star for the Legislature could be suppose, 0 and y star, that is, the favorite for the Committee is different it is suppose equals to y c - y c is greater than 0 suppose. So, this is the setting of the game in the first stage committee proposes something, some y in the second stage Legislature accept or rejects and that is what the game ends, so that is how it looks like.

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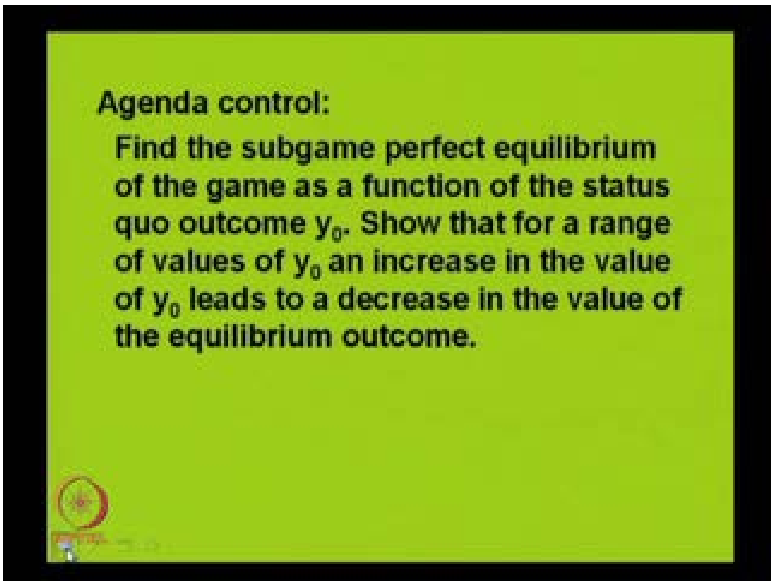

We can have the following exercise, from this sort of game this is the exercise. Find the sub game perfect equilibrium of the game as a function of the status quo outcome y 0. So, whatever sub game perfect equilibrium outcome and finds out which has to be expressed as a function of y 0, which is the status game status quo outcome.

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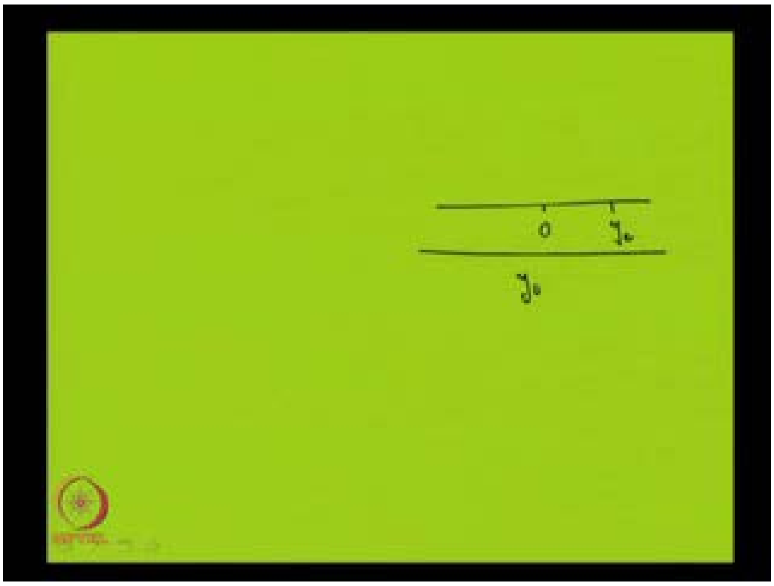

Show that for a range of values of y 0, an increase in the value of y 0 leads to a decrease in the value of the equilibrium outcome, so it is like the followings. I have 0 here and y c here, so these are the favorite position of the legislature and the committee, y 0 could be anywhere here, so y 0 I do not know it can be anywhere in this single dimension line. I have to find out what are the sub game perfect equilibrium outcomes, if y 0 keeps on changing, so that is the exercise.

I am not going to solve this exercise just to make you aware of what kind of questions can be asked from this set up, that is more or less from the part of ultimatum games or to be more precise the setting of ultimatum games, because we have seen that we can go beyond ultimatum games and talk about other sorts of games, which are related to the ultimatum games, but not exactly the ultimatum game.

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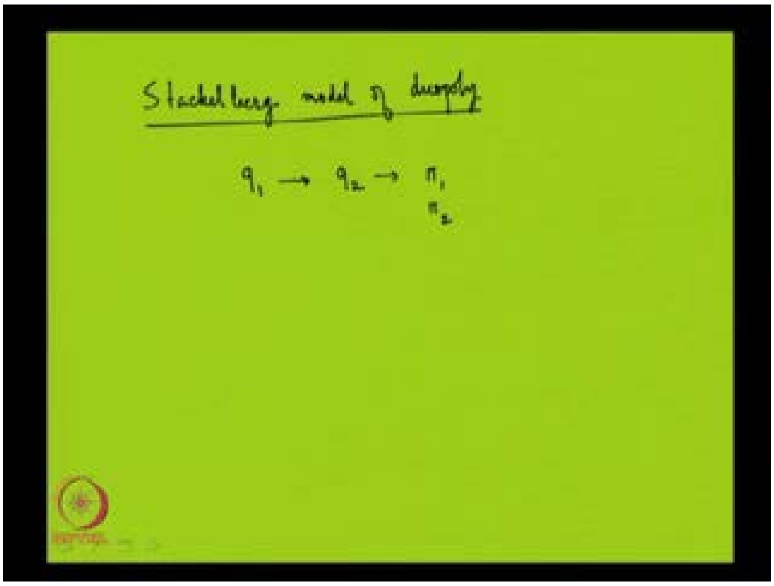

The last topic we shall cover is what is known as Stackelberg model of duopoly, we are back to markets, if you remember duopoly is a market where there are only 2 firms which are selling goods. What is special about Stackelberg model is that, it is a variation over the Cournot model. What was Cournot model? In case of a duopoly there were 2 firms which were selling goods in the market and simultaneously they were taking their decisions.

They were deciding on the quantity that they are going to produce and in the market given the demand function in precisely the inverse demand function, the equilibrium price was getting determined and therefore, they were getting their payoffs - profits.

In the Stackelberg model, instead of simultaneous actions by these 2 firms: firm 1 and firm 2, we are going to assume that the decisions are being taken sequentially. Player 1 may be the firm 1 is moving first and deciding on the quantity of output that this firm is going to produce. So, q 1 happens first - the amount of output firm 1 is producing. Given the value of q 1 in the second stage player 2 that is firm 2 decides how much he will produce, so then q 2 is being decided. Therefore, we have q 1 and q 2 and therefore we have the profits pi 1 and pi 2, so that is where the game it is.

So, it is a 2 stage game, it is a sequential game, that is, itself extensive game where the first mover is firm 1 and second mover is firm 2 and there the game ends. What we want to look at is, that in this changed environment where the Cournot model has been changed a little bit - paltered a little bit - what are the outcomes do the profits of the firms remains same or they change.

If they change in what direction does the total output change, so these are the questions that we want to ask and what about the price does the equilibrium price also change, so these are the questions that we want to ask in this Stackelberg model of duopoly.

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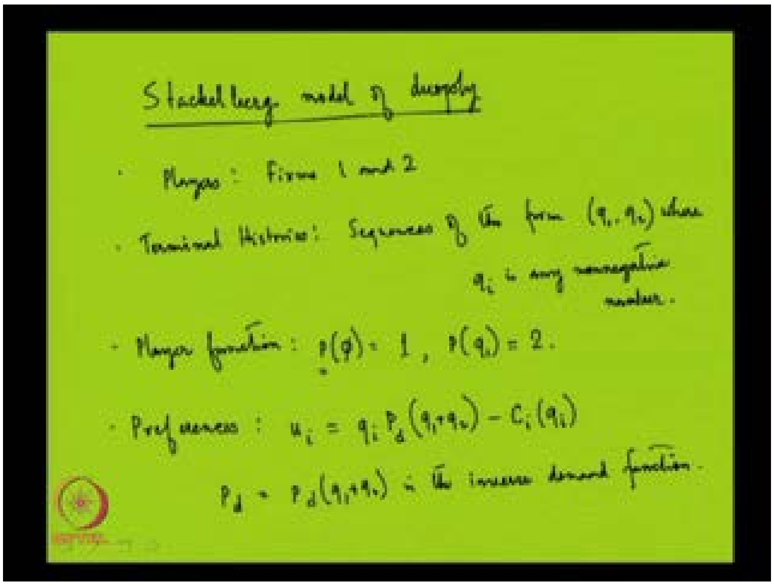

If I want to write entire thing in terms of the language of the game theory I have to specify four things, first 2 firms are there, so what are the terminal histories here? Sequences of the form q 1, q 2 where q i is any non-negative number.

First, firm 1 decide what is the value of output that it is going to produce, then firm 2 decides q 2 and there the game ends. Player function after the knowledge (( )) is firm 1 which is going to move, we shall denote it by 1. After firm 1 has decided the output level firm 2 moves, so this is P of q 1 is equal to 2. Preferences, so what is the thing that the 2 players want to maximize, obviously they want to maximize their payoff functions and the payoff functions are nothing but the profit functions.

How do I write that? Suppose, I want to write the utility or the payoff or the profit of the firm I then it is output produced by that firm multiplied by the price, so this q I multiplied by P d, P d is the inverse demand function. Remember, previously I was using the notation P not P d, but here I have P already, the player function, so I am using a different notation as P d, the demand function minus the cost which is given by C i q i.

The cost of firm I is depended on the output that it is producing. So, this is what a each firm wants to maximize where P d is given by this one, this function, is the inverse demand function. This price is expressed as a function of the total quantity, so this is what the player 1 to maximize. So, what is going to be the sub game perfect equilibrium in this game of Stackelberg duopoly?

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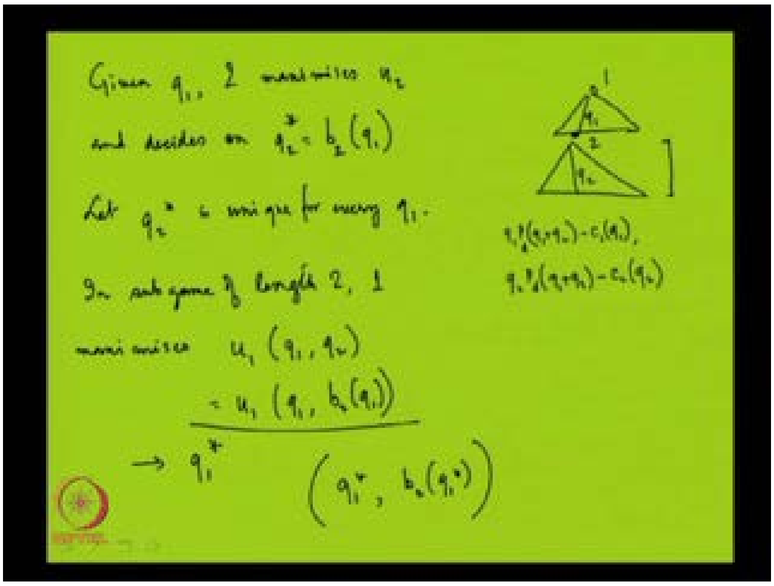

Like we have seen before, we have to start from the last sub game of length one. Here, in terms of game tree what is happening is that here is firm 1 and who is deciding on q 1, it is a continuous variable from 0 to infinity. After he has decided q 1 firm 2 moves and decides q 2 and then, the profits are obtained. So, profits are given by these; these are the profits of these 2 firms, so this is how it looks like.

Question is, how do we solve this game? What we do is to start with sub game of length 1, which is this sub game, and find out what is the best response of player 2 given the value of q 1. Given q 1, 2 maximizes u 2 and decides on q 2, which we shall write as b 2 of q 1, because as q 1 will go on changing for different values of q 1 the optimal value of q 2 will go on changing also. So, it is a function of q 1 the optimal q 2 is a function of q 1, so let us write it as q 2 star this is the optimal thing that player 2 is choosing. Let us suppose, let q 2 star is unique for every q 1. For every value of q 1 there is a unique value of q 2, which maximizes u 2, the payoff of player 2.

Now, this is the optimal strategy of player 2 for every possible sub game, remember there are infinite number of sub games, for each value of q 1 there is a different sub game.

Now, given this optimal action of player 2 in each of these sub games, player 1 has to decide what q 1 he is going to produce, which maximizes his profit. In sub game of length 2 player 1 maximizes u 1. Now, I know u 1 is a function of q 1 and q 2, in this case it is a function of just q 1 because q 2 is given by this. And maximizing this, he will get a unique value of q 1, suppose, this is called q 1 star. So, the sub game perfect equilibrium will be the following that in period 1, player 1 that is firm 1 will choose q 1 star, which maximizes this function u 1 which is the function of q 1 and b 2 q 1.

In stage 2, the output that firm 2 will choose is the given by nothing but this (Refer Slide Time: 32:15). So, this is going to be the terminal history in the sub game perfect equilibrium of this game. After q 1 star has been chosen q 2 star that is b 2 q 1 star is chosen, we are going to have the profits which are going to be on by these 2 firms. So, this is going to be the general structure of the sub game perfect equilibrium. This is the general setting of this game and the sub game perfect equilibrium.

What we are going to do now is to take up a specific set of functions just as we had done in case Cournot equilibrium and see how this equilibrium is compared with the Cournot equilibrium.

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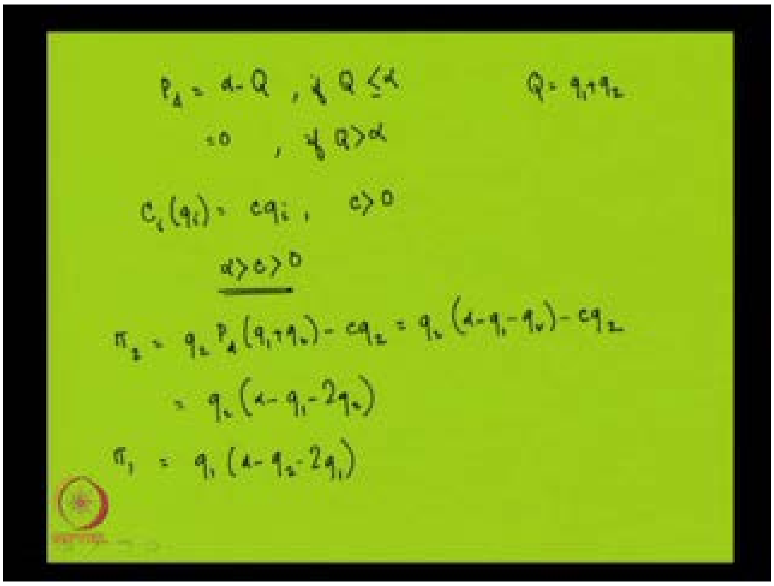

We are going to take the following specific functions, suppose, p t is given by alpha minus q like before, where q is less than or equal to alpha is equal to 0. If q is greater than alpha, so this is the inverse demand function, it gives me the equilibrium price for different values of total output in the market, q is the summation of q small q 1 small q 2. We are going to assume the following cost function, this is going to be the cost function where c is positive, alpha is greater than c which is positive, this an assumption we are going to maintain.

Now, with the setting what is the profit function of each firm? Suppose, I am writing the profit function of firm 2. So, this the profit function of firm 2 likewise, the profit function of firm 1 will be the following, this is going to be the profit function of firm 1 (Refer Slide Time: 34:00). As we have just said, how we solve for the sub game perfect equilibrium of this game? We look at the best response function of firm 2 given the value of q 1. So, given firm 1 has produced a value of q 1 firm 2 will maximize it is profit in the sub game of length 1 and decide on the optimal q 2.

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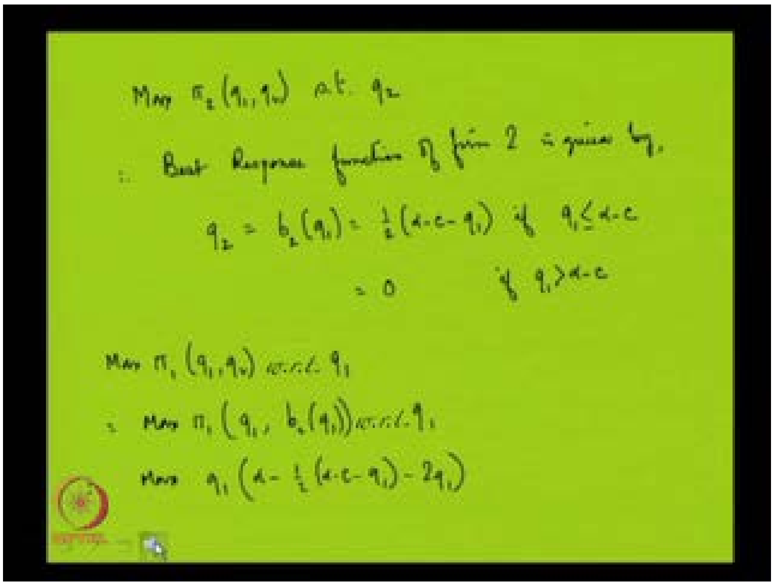

It is going to maximize pi 2 which is a function of q 1 and q 2 subject to q 2. This is something which we have done before, we have found out the best response functions of each of the firms when we discuss the Cournot duopoly model. We have seen that the best response of firm 2 is given by… this was given by half alpha minus c minus q 1, if q 1 is less than equal to alpha minus c equal to 0, if q 1 is greater than alpha minus c (Refer Slide Time: 36:24). So, I am not going to derive that this best response function it is very easy to derive, but since we have already derived this in pervious lectures, so this we know is the best response function.

Now, since, this is the best response function of firm 2, what firm 1 will decide is that right at the beginning of the game he will take this as given and maximize its own profit. So, maximize I am writing pi 1 which is nothing but u 1 subject to q 1. So, basically in terms of our specific function it is the following.

Maximize q 1 multiplied by alpha minus this function which is half of alpha minus c minus q 1 minus 2 q 1. So, this going to be a maximized by the first firm with respect to q 1 and if it is maximized then, we shall get the following value of q 1 from the first order condition.

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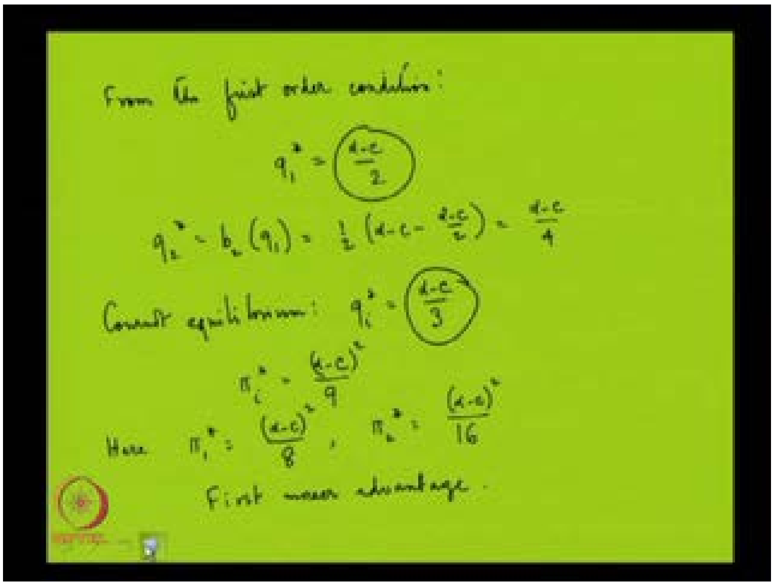

We shall get the following value of q 1 which we are writing as q 1 star as alpha minus c divided by 2. So, this going to be the equilibrium value of firm 1 output. Given q 2 star which is b 2 q 1 I know the value of firm 2 output half of alpha minus c minus alpha minus c divided by 2, it is going to be alpha minus c divided by 4. These are equilibrium outputs of these 2 firms in the sub game perfect equilibrium. If we want to compare this with Cournot equilibrium under similar assumptions and similar functional forms, they both the firms were producing the same level of output and it was given by alpha minus c divided by 3.

Now, we see that compared to the Cournot equilibrium in this sub game perfect equilibrium in the Stackelberg model, firm 1 is producing more output and firm 2 is producing less output, so that is one conclusion. What about the profits? In the Cournot equilibrium what were the profits? It was alpha minus c whole square divided by 9, so this is something which we have seen before, whereas here, firm 1 profit if we plug this q 1 star and q 2 star in firm 1 profit, we shall get the following expression alpha minus c whole square divided by 8. Similarly, firm 2 profit will turn out to be alpha minus c whole square divided by 16, which means that compare to the Cournot model, where there was a symmetry, both the firms were taking the actions simultaneously, there was no first mover or second mover. Firm 1 is profit has gone up, from alpha minus c whole square divided by 9 it has gone up to alpha minus c whole square divided by 8 whereas, firm 2 is profit has gone down, from alpha minus c whole square divided by 9 it has gone down to alpha minus c whole square divided by 16.

So, what it basically means is that firm 1 by deciding it is own output in the beginning of the game is having some advantage over firm 2, compared to the case where they were taking their actions simultaneously and this is what is known as the first mover advantage. The player who is going to move first is getting some advantage because, he can see what is the optimal action of the second player in the stage 2 and taking that into consideration he maximizes his own profit, this advantage is denied to the other player, so this is how it looks like. Can we show this story, this comparison between the Cournot equilibrium and the Stackelberg equilibrium in terms of a diagram?

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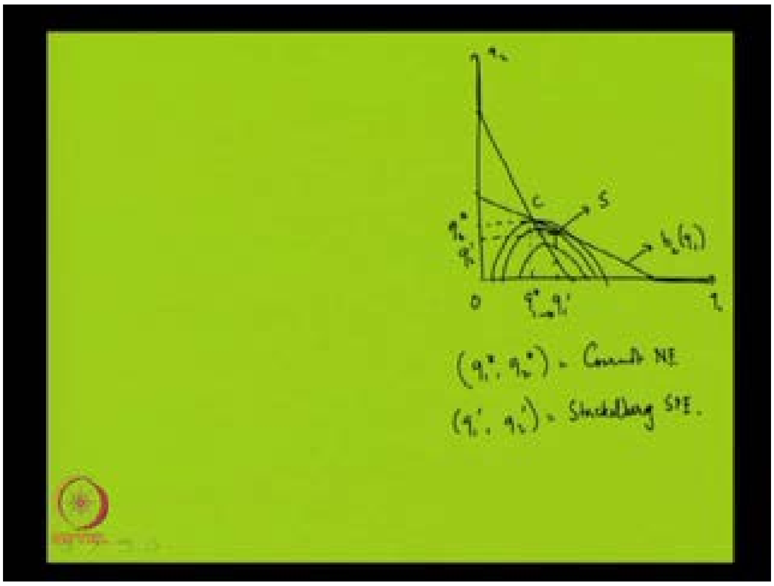

Diagram will look like the following; this is the familiar diagram which we have seen before. This is the best response function of firm 1 and this is the best response function of firm 2 and then, we draw what is known as the isoprofit curves or the equal profit curves. We know this the point of Cournot equilibrium, so let us call it q 1 star and q 2 star this point is suppose c.

Now, we have seen this before that we can draw what are known as isoprofit curves or equal profit curves of both these firms. Suppose, I want to draw the isoprofit curves of firm 1, so they will look like the following, it is a concave curve and the point where it is meeting or interesting the best response function of firm 1, the curve is having a slope of 0.

There are infinite number of such curves, what I claim is that c is the point of Cournot equilibrium where the best response functions are intersecting with each other and this point let us call it as s the point of Stackelberg equilibrium. What is the characteristic of this point? This is the b 2 q 1 line, the best response function of firm 2 with respect to different values of q 1, this line and then we have this horizontal portion, and this vertical portion and this downwards sloping is the b 1 line. The characteristic of s is that at s, a particular isoprofit curve of firm 1 is having a tangency point with p 2.

So, what is happening is that at the point s firm 2 is on his best response function; so given q 1, he is doing his best. At the same time firm 1 is maximizing his profit taking into consideration that firm 2 will be on his best response function. This is nothing but the Stackelberg equilibrium, firm 1 knows that firm 2 will be on the on his best response function, he will be producing on p 2 given this knowledge firm 1 is maximizing profit.

How can his maximize his profit? By choosing isoprofit curve or equal profit curve which is closest to the horizontal axis and a isoprofit curve can be closest to the horizontal axis, at the same time I can have a point on the best response function of firm 2, if I have a tangency of the isoprofit curve with the best response function and that is how s is defined. So that is why we are saying that s is the Stackelberg equilibrium.

From is I can draw these two perpendiculars, let us call it q 1 dashed and q 2 dashed, so q 1 star to the Cournot Nash equilibrium and q 1 dashed q 2 dashed is the Stackelberg, sub game perfect equilibrium. From the diagram also it becomes very clear as to what is happening to the output level of each firm and what is happening to the profits also.

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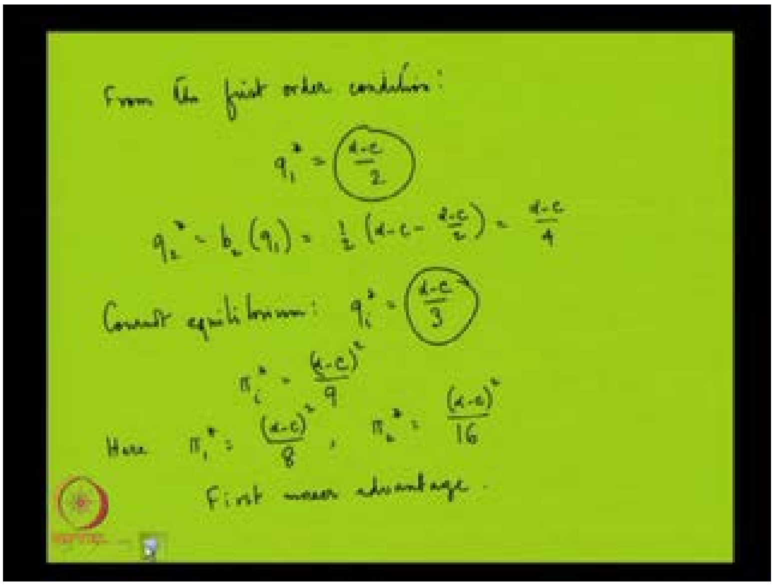

Firstly, we can see that firm ones output has gone up which is confirmed by our calculation here, it had gone up from this to this. Firm 2 output has gone down from q 2 star to q 2 dashed, this is also confirmed from the movement from this to this (Refer Slide Time: 47:35). What about the profits? Again, the profits are easy to see that the profits of firm 1 have gone up, because closer the isoprofit curves are to the horizontal axis, they denote higher level of profits. This profit curve which is passing through c is further away from the horizontal axis whereas, the curve which is passing through s is nearer to the horizontal axis, which means firm 1 profit has gone up.

Similarly, if I had drawn the isoprofit curves of firm 2, I am not going that because it will be very messy, I can see that isoprofit curve of firm 2 which passes through s are further away from the vertical axis than the isoprofit curve which passes through c. At s the firm 2 is earning less amount of profit than it was earning in the Cournot Nash equilibrium.

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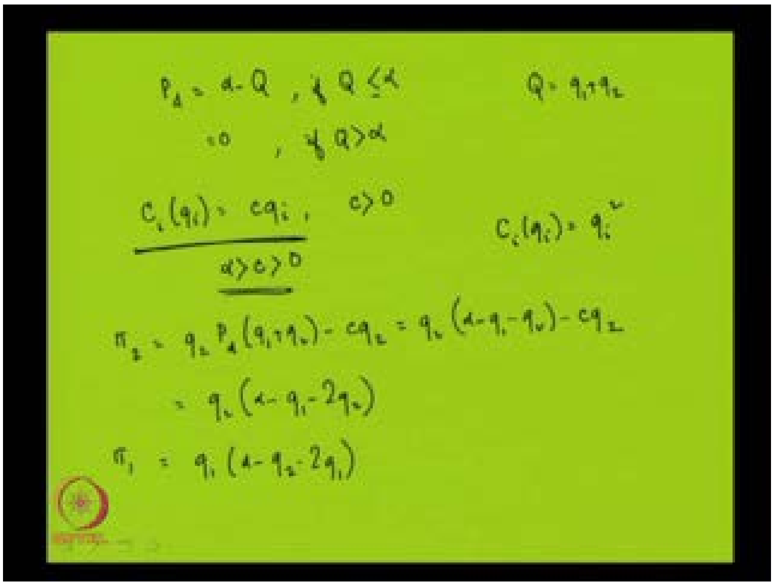

So, these are some of the conclusions that we can draw from this exercise that in the Stackelberg equilibrium firm once profit goes up, its output goes up whereas, firm 2 is profit and output both go down. Is this cost function that we had assumed - remember, we assumed a very simple kind of cost function, linear cost function crucial for this result.

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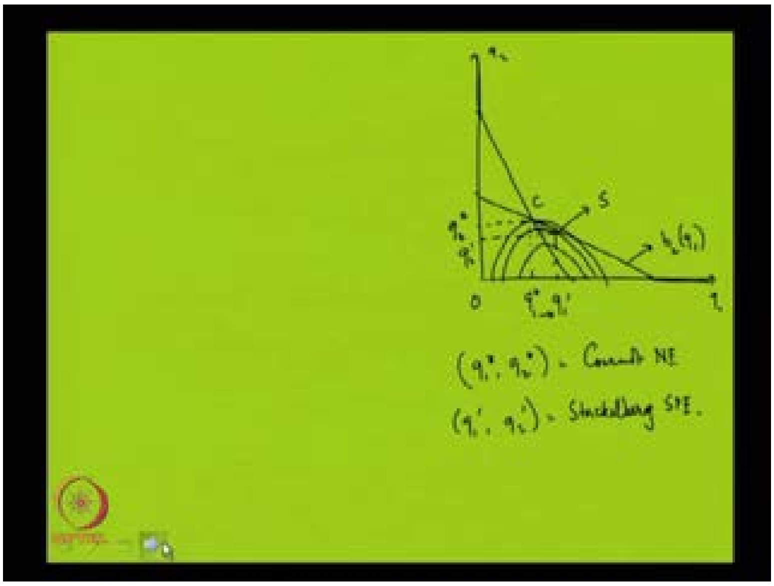

In other words, if I had assumed different kind of cost function, suppose quadratic cost function, would the result still hold? Suppose, I had c i q i is equal to q i square, put the same result hold, I mean, in the Stackelberg compare to the Cournot model, the first mover has more profit and more output and second mover has less profit and less output. This could be one exercise, one can do as a home work, that if even if we take quadratic cost functions the same result holds, that the first mover is having some advantage over the second mover, because he can guess what the best response of the second mover is going to be in the second stage and a plan accordingly unless his or her profits. So, this is what it is the Stackelberg model compared to in comparison to Cournot model.

Before we end this lecture, let me recapitulate what we have briefly done in this course. What we have done is that we have basically covered in terms of themes three topics: one is the very idea of Nash equilibrium in strategic games where the actions are taken simultaneously. We have looked at the definition and we have looked at the various facts of that Nash equilibrium, we have defined what are known as dominant actions.

Then, we looked at what are the well known applications of Nash equilibrium in different real life situations, for example, we have looked at markets. And there were basically two models of markets that we have looked at Cournot model and Bertrand model.

We have also looked at the application of Nash equilibrium in elections, how we can analyze elections in terms of the idea of Nash equilibrium. We have looked at for example, an auction which is again related to what is known as auction theory. We have also looked at other things like laws, how laws can be form which could be efficient by using the idea of Nash equilibrium.

The second topic that we have basically covered is the case of mixed strategy Nash equilibria, where it is assume that players can randomize their actions. It is possible that players do not take their actions for sure, but take each action with some probability which could be less than 1. So, there are some uncertainties regarding what action if particular player will take.

In that case also we have seen that the idea of Nash equilibrium gets generalized and an important result in that theme is that for every game where the number of actions of the players are finite a Nash equilibrium surely exists in terms of mixed strategy a Nash equilibrium will surely exists. Then, lastly we have looked at the last theme which is the case of extensive games, where the actions are taken not simultaneously, but in stage by stage one by one, one after another.

In this case of sequential actions taking also we have seen that the idea of Nash equilibrium could be applied, but there is a problem of interpretation of that idea. The idea of Nash equilibrium in the case of sequential game is not very robust. So, we have refined the idea and we have talked about what is known as a sub game perfect equilibrium. We have looked at different aspects of the sub game perfect equilibrium how it could be found out by a convenient method called the backward induction. And we have looked at some applications of this sub game perfect equilibrium in real life. So that is the end of this course, I hope you have benefited from this course, thank you.

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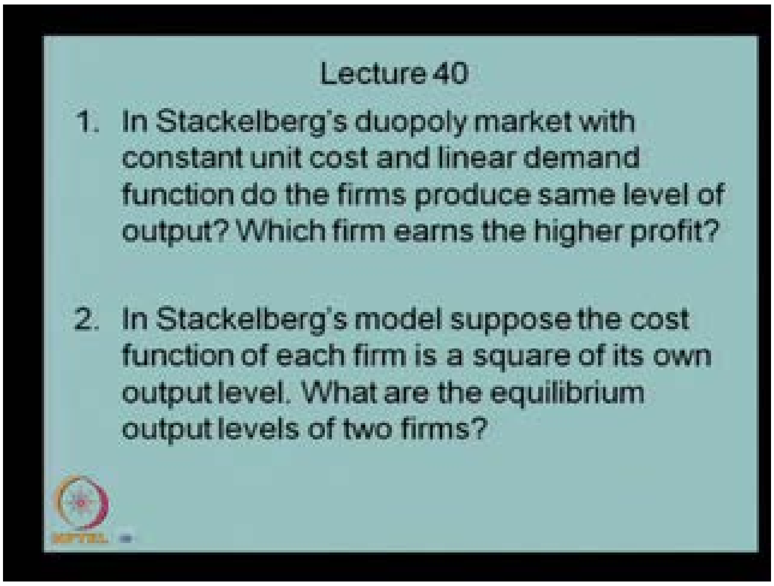

This is exercise for lecture 40 in Stackelberg's duopoly market with constant unit cost and linear demand function do the firms produce same level of output? Which firm earns the higher profit?

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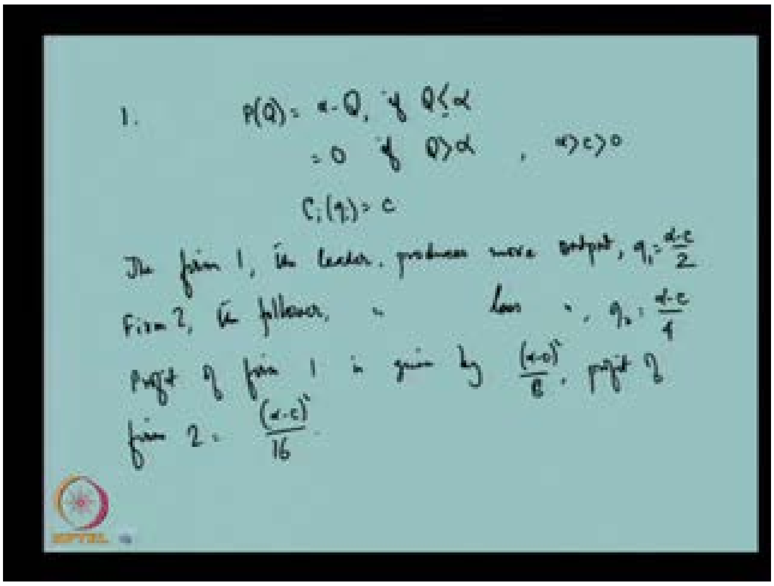

This is a question about Stackelberg duopoly model, if we remember the linear demand function or rather the inverse demand function was given by this (Refer Slide Time: 54:31). We know alpha is greater than c this is greater than 0, where c i q i is small c the cost function. In this case, the firm 1, the leader produces more output it is given by q 1 alpha minus c divided by 2.

Firm 2 the follower produces less output given by alpha minus c divided by 4. So, there are differences in output levels, which firm earns higher profit? - this is question number 1- Profit of firm 1 is given by alpha minus c whole square divided by 8, profit of firm 2, it alpha minus c divided whole square divided by 16.

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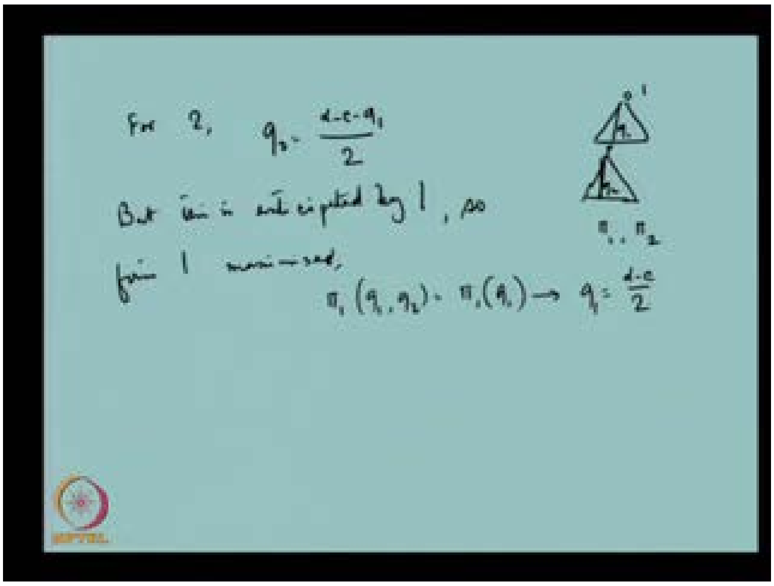

The reason why this is happening why there are differences in output level and profit level is the following. That here, firm 1 is sitting and he is deciding q 1 and then, firm 2 is deciding q 2 and after q 1 and q 2 are decided we are getting this pi 1 and pi 2, these are the payoffs of the 2 firms.

For firm of 2 he is going to play according to the best response function which is given by q 2 is equal to alpha minus c minus q 1 divided by 1, but this is known by firm 1, this is anticipated, let us say, by firm 1. So, firm 1 maximizes says pi 1 which is a function of q 1 q 2 but which basically turns out to be function of q 1 only and that gives him the output this (Refer Slide Time: 57:00). Here, player 1 that is the firm 1 has some advantage, because he can anticipate what is the reaction of firm 2. Therefore, he earns more profit also and produces more output.

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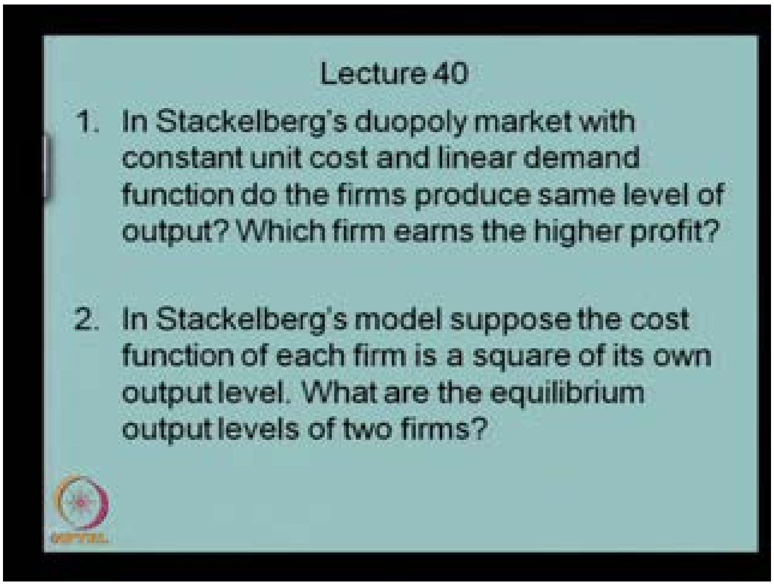

In Stackelberg's is model suppose the cost function of each firm is a square of it is own output level. What are the equilibrium output levels of 2 firms?

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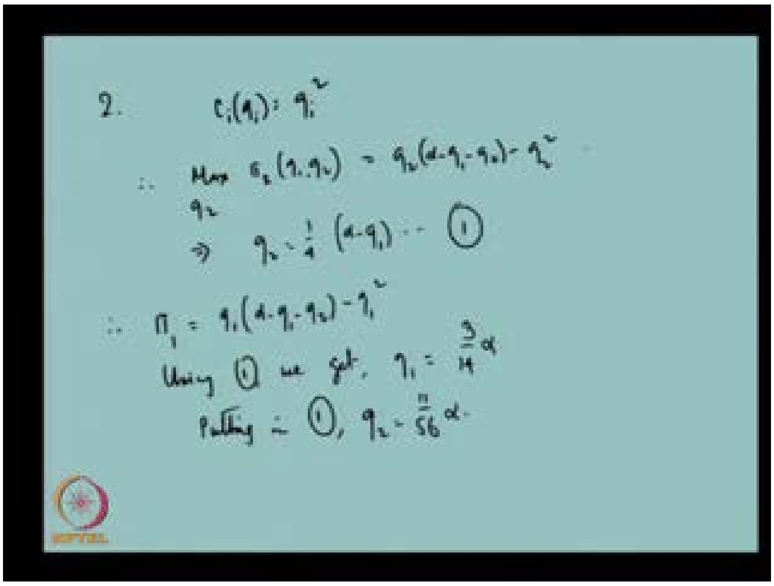

Here, I have q i square, so that the problem remains more or less the same, here also first we go through backward induction, first maximize pi 2 which is given by this with respect to q 2. So, this will give us q 2 1 4th of alpha minus q 1, let us call it 1. Now, we have going back and going to the beginning of the game and maximizing by 1.

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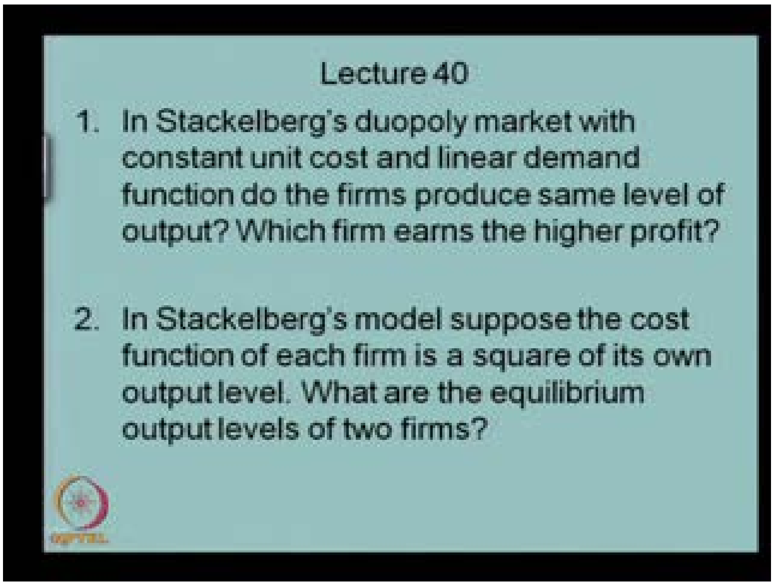

Using 1, we get, q 1 is given by 3 by 14 alpha and putting it in 1, we get q 2 which is given by eleven by 56 alpha. There will be differences in the profit level also firm 1 will earn more profit firm 2 will earn less profit. The sort of qualitative result we got for constant unit cost remains the same even if we have quadratic cost of production. That the firm 1 is producing more earning more profit firm 2 is producing less earning less profit, thank you.