Humanities and Social Sciences

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Module No. # 02

Strategic Games and Nash Equilibrium

Lecture No. # 03

Examples of Nash Equilibrium

Welcome to the third lecture of module 2 of this course called game theory and economics. Before we begin this lecture let me just recapitulate what we have discussed in the previous lecture. In the previous lecture we have finished with the giving of examples of different basic sorts of games, we have discussed two additional games, one was matching pennies and the other was stag hunt.

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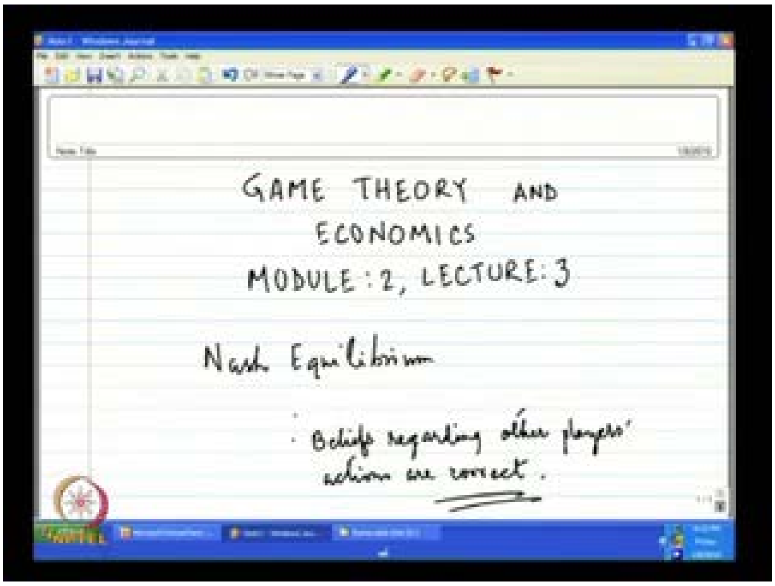

And after that we have started with the first solution concept and which is an very important solution concept of game theory which is called the solution concept called Nash equilibrium. Because this concept will be used in this class and later classes also, let me just briefly go over the idea. The idea is that this Nash equilibrium concept uses or it depends on the fundamental theory of rational choice which says that every player should take that action which is best for him according to his preference.

Now, the point is that in game theory we have a strategic situation which means that what action is best for me depends only, depends on not only my action, but also the action of others. Now, so if that is the case then to say which is best for me I have to have some belief regarding what the other players are doing, what action they are going to undertake. And what is important in this context, in the context of strategic games is that people are taking their action simultaneously.

So, it is not that one action has been taken by one player and saying that any player is taking that action which is best for him. It is the case that the actions are being taken simultaneously. So, beforehand any player cannot say what action that other player will take. But what he can do and this is the justification that we are going to offer in this context is that he can have a belief regarding what the other player is going to do. So, before the start of the game, before the play of the game he has a belief regarding other player's action. And this this is the second component. The second component is saying those beliefs are correct because if the, my belief regarding other player's action is not correct.

So, which means that my calculation is wrong and I am taking an action which is may not be best for me. Then in future I will change my action, I will not continue with that action which I took on a false belief. So, if I have to give a notion of equilibrium, if I have to have a notion of equilibrium then this sort of situation cannot recur, a situation where the beliefs of the players regarding other player's actions are proven false again and again. So, this is the second component of Nash equilibrium. The first component is the theory of rational choice; the second component is that players beliefs regarding other players actions are correct.

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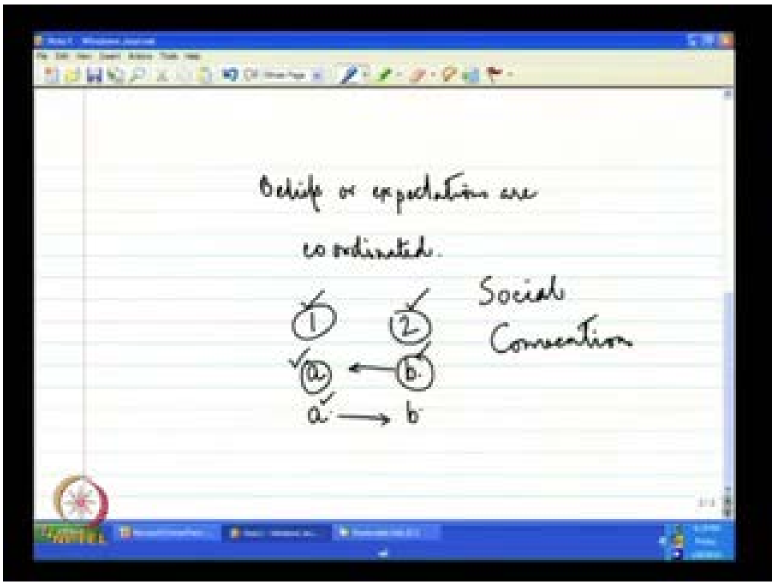

And to probe, if we probe this matter it will further then it also means as we have discussed in the previous class is that the beliefs or expectations. Here expectations means expectation regarding other players actions are coordinated which means that if there are two players in the game. Suppose, one and two. One beliefs that player two will take the action b and that is why he takes the action a which is best for him, given his preference. So, this is his expectation. One is expecting that two will take the action b. Now, this will have to be true or this will have to be synchronized from the point of view of two also. Two must have the belief that one is going to take the action a, this is his expectation and that is why his best action is b.

So, this is leading to this, this expectation leading to this and this expectation is leading to this and you see that they must match with each other. This a must be equal to this a and this b must be equal to this b. So, that is what we mean by the concept that the beliefs or expectations have to be coordinated. They must reinforce each other, they must support each other. If they do not support we cannot have a state of rest what which is the state of equilibrium.

Now, seen in this, seen in this fashion in this slide it is easy to see Nash equilibrium or a situation of Nash equilibrium as a situation of a an established social convention. Why I am saying this social convention, establish social convention because how their beliefs are formed. Beliefs are formed because player one has seen that whoever there is in player two's place has been playing b. So, if player two has been or the likes of player two has been playing b he has enough enough belief or enough support to his belief that player two is going to play b when it is player one's turn to play with player two.

So, that is experience. Player one has an experience that other player is going to play b and that is why he expects that player two is going to play b. And similarly, from the point of view player two, player two sees that whoever there is in player one's place that person has been playing a. So, player two finds it sufficiently credible that when it is his turn that that is player two's turn to play the game whoever there is in player one's place is going to play a and if it is a then the best option for player two is to play b.

So, things had been being repeated some some some set of actions are being repeated over and over again and their best responses to one another. For example a is best response to b and b is best response to a. And since their best responses to each other and since these actions had been played in a particular play of the game both player one and player two just repeat that set of actions. So, this is the idea. That is why we call that this is going to be repeated and that is why it is like a social convention. So, this entire thing can be written in a very concise way if we take the help of mathematics.

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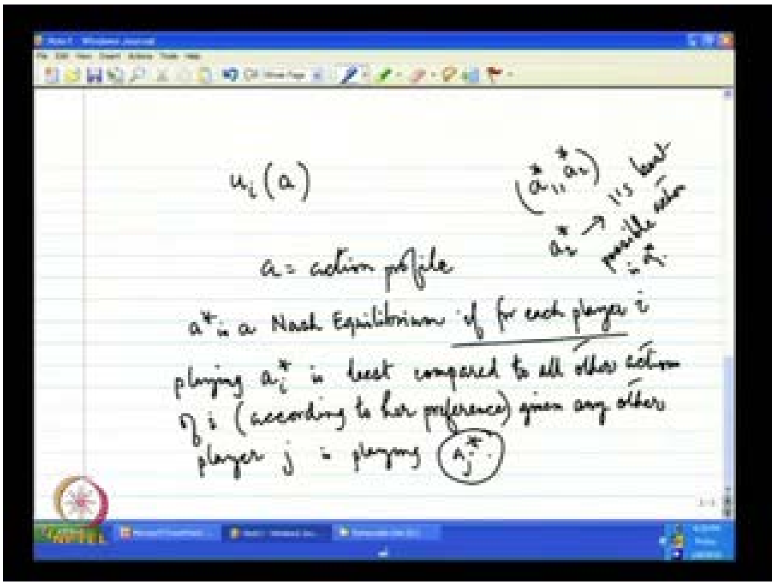

If we remember u i was representing the payoff function of player i and this is defined over these set of action profiles. Now, so see a is an action profile. It is a vector of actions taken by all the players. It is a list of actions. Now, I shall call a star which is an action profile, a particular action profile. In Nash equilibrium is a Nash equilibrium if for each player i playing a i star is best compared to all other actions of i according to his preference, his or her let us say her preference given any other player j.

J is any arbitrary other player which means it can be, it can be representative of all the other players. If any other player j is playing a j star. So, let me see what this definition is saying. So, I have defined something known as Nash equilibrium. It is an action profile; it is just, it gives me a vector of actions. And what is the property of this Nash equilibrium? It must happen that given what all other players are doing and one all the others players are doing is that they are playing a j star.

So, it is like that if I have two players and these are the actions, a 1 has been taken by player one, a 2 has been taken by player two. So, a 1 star and a 2 star they are some specific actions of one and two. Given two is taking a 2 star action one's best possible action is a 1 star and vice versa which means because it is for every player. So, it must be the case for that for player two also given one is taking the action a 1 star, a 2 star is the best action that two can take. So, that is the idea.

If there are n players from the point of view of player one if a 2 star is the action of player two, a 3 star is the action of player three blah, blah, a n star is the action of player n his best action is a 1 star. Now, here best action means that if he takes any other action, in his action set if he takes any other action then he cannot be better off. He cannot improve his payoff from what he is getting by playing a 1 star. It is possible that if he takes any other action suppose a k star then his payoff is as same, the payoff that he is getting out of a k star.

Whereas, other players are taking a 2 star a 3 star etcetera etcetera that payoff is just equal to the payoff that he is getting from this, from a 1 star. But it is never the case that by taking some other action he is going to improve his payoff which means that there can be more than one best possible actions for him given what the other players are playing. And so this can be written in another different way more precisely.

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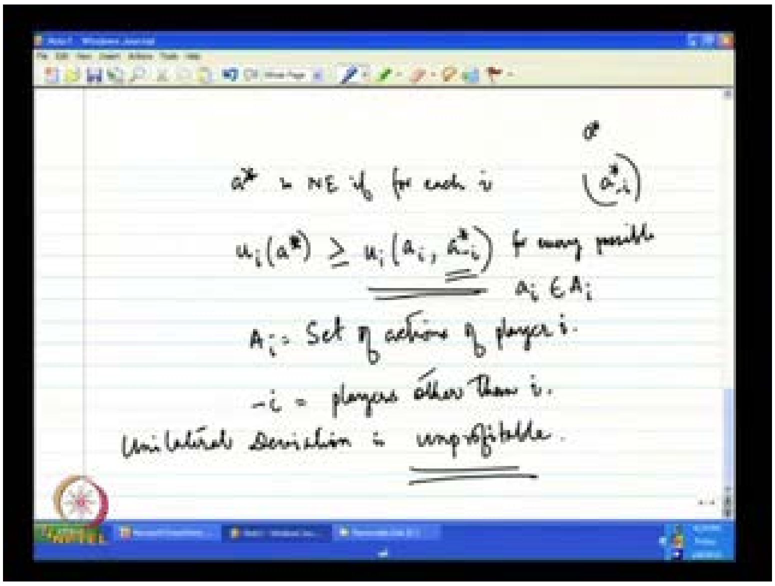

If for each, each i, A i is the set of actions. So, this is another very short way to write what is a Nash equilibrium. We say that this small a star, the action profile a star is a Nash equilibrium if for every player i, u i a star which is the payoff player i is getting by playing a one's a i star whereas, other players are playing their stars is greater than or equal to a u i a i. Where A i can be any action belonging to his action set. So, he is, he can take any action, possible action, but and other players are taking their action.

Here, minus i what is written as minus sign and then i is meaning any player who is not i, players other than i. So, all the players who is not the i th player is taking this action a i, a naught i star and given that they are taking their star actions which is the Nash equilibrium action. If player one takes any action from his action set his payoff cannot be more than what he is getting, when he is playing his a i star, that is the idea. It can happen that they are equal which means that you deviate, you deviate from your star action, that is the Nash equilibrium action and you are doing as good as you were doing when you are taking here your star action.

But it can never happen that you are deviating and being better off. So, this is another way of, there is a another of writing this which is known as unilateral deviation is unprofitable which means that if someone moves only and he only moves which is means that, which means that there is unilateral deviation, unilateral deviation from this star actions from a star. If there is a Nash action profile which is a star from there if there is some unilateral deviation, if someone individually moves away and does something else, takes some other action then that is not going to improve the payoff of that player, it is not going to be profitable. He can do as well as he was doing in Nash equilibrium profile, but he cannot improve his payoff. That is the idea.

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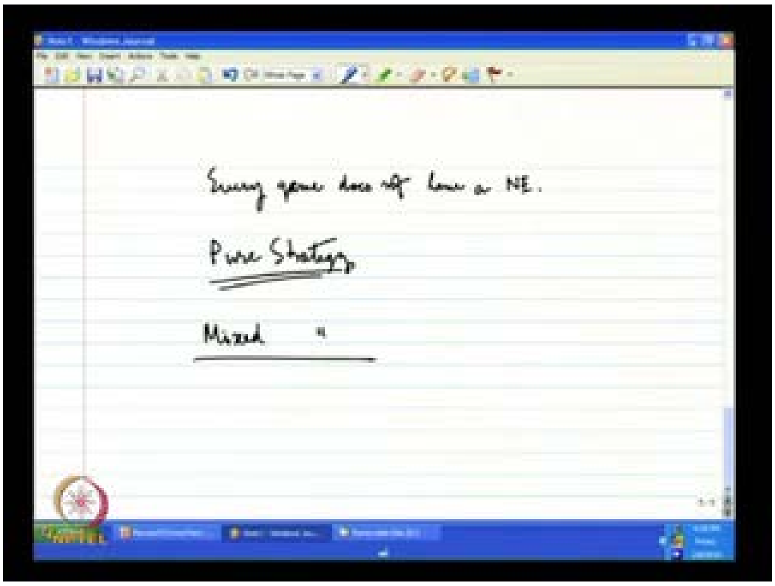

Now, from this it does not mean that, it does not mean, what does, what it does not mean is that every game does not have a Nash equilibrium. So, it means that it is not necessary that if you give me any game and I will find a Nash equilibrium. There might be games which will not have any Nash equilibrium and we shall see such games. But here I should mention one thing is that here at present we are considering games with what is known as pure strategy.

I do not want to spent a lot of time explaining this because we are going to spent time on this anyway later on in this course. But what it means is that in pure strategy game that we are, we are considering now any player takes any action with probability 1. He says that I am going to take this action or he says that this action I am not taking. So, it is 1 0. This is called a pure strategy game where the probabilities attach to any action can either be 0 or can be 1.

Whereas there can be in general mixed strategy games. We shall see how the mixed strategy games look like. There it is a more general case where it may happen that I have two actions, possible actions. I play the first action with probability one-fourth and the second action with probability three-fourth. So, 0.25 and 0.75, in that case it is not very clear which action I am exactly going to take. With some probability I am going to take this action, with some other probability I am going to take the other action.

Now, in this general case it is a general case of the pure strategy case. If we have some pure strategy games then definitely we are going to have at least one Nash equilibrium. So, you give me any game there must be one, at least one Nash equilibrium. So, that it is a very powerful theorem. So, but right now we are considering only pure strategy and in pure strategy it might happen that I might have games which do not have any Nash equilibrium. And what this theorem does not say, what this theorem also does not guarantee. This theorem of Nash equilibrium, it does not guarantee that I shall have, if I have Nash equilibrium it will be unique. It might have I have two or three Nash equilibrium.

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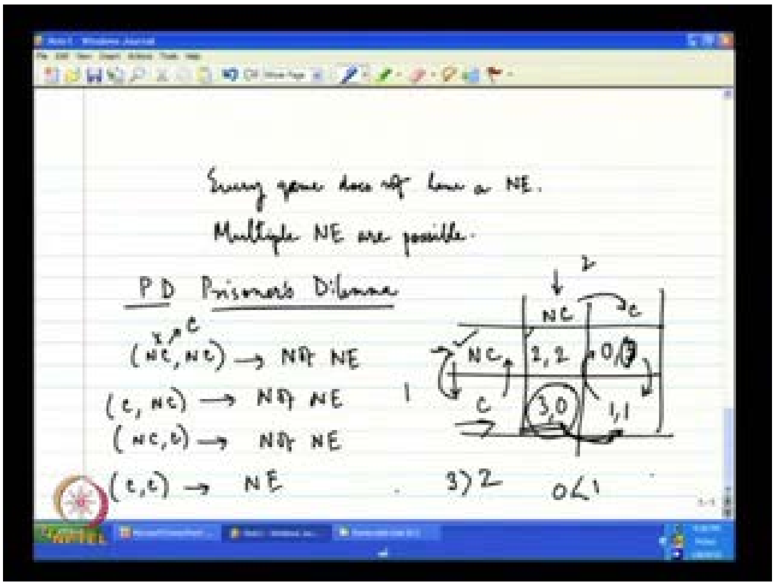

So, multiple Nash equilibria are possible. And we shall see such games that we are having a game, what they are Nash equilibria and they are more than one in number. So, this is more or less the theory part. The definition part of Nash equilibrium. Question is how do we apply this Nash equilibrium? If we, I am given a game then can I find out what exactly are the Nash equilibria which is other way which can be said in another way is that if I am given a game can I say that this outcome or this combination of actions player one is doing this, player two is doing this etcetera etcetera.

These set of actions is going to be repeated over and over again. This is going to be what is known as a steady state set of actions. So, can you predict that and the answer is yes. Given the games that we have discussed so far, we have basically discuss four categories of games. Now, let us take the first one that we have discussed the prisoner's dilemma and see if there is a Nash equilibrium there or if there are more than one Nash equilibria.

So, this was the game, player one and player two, two prisoners. They could either not confess or confess for both the players. If they confess they get 1 1 which is worse than in the case if they do not confess. If someone confesses, suppose player two confesses, player one does not confess, player one gets more. No, it is just the opposite. If player two confesses, player one does not confess then player one is worse off. So, player one gets 0, player two by confessing getting 3 and if player one confesses, player two does not, player one gets 3, player two gets 0.

Now, what we need to do to find out the Nash equilibrium on this game is to check each and every action profile. And from each and every action profile we have to check whether any at least one player can he or she deviate and be better off. If I can find such a deviation then obviously that action profile that we are considering is not a Nash equilibrium. So, to say that some action profile is not Nash equilibrium is easier, relatively easier. I have to just find one profitable deviation, but to say that this is a Nash equilibrium then it is more harder. It is, I have to say that, I have to check whether each and every deviation that can be considered if they are profitable or not profitable. If none of them is profitable then it is a Nash equilibrium.

So, let us first start with N C, N C which is this one and players are getting 2 and 2. Now, if player two is playing N C then it is, it is very clear that player one by deviating that is not by playing N C, but playing C suppose, he can improve his payoff. He can get 3 instead of 2 and 3 is greater than 2. If 3 is greater than 2 then there is no no reason why player one will stick to this action of N C, he will shift to C. Similarly, if player two knows that player one is going to play N C he will similarly, shift to C. He can again get 3 which is greater than 2.

So, this N C, N C is not Nash equilibrium because both of them has an incentive to deviate if any of them knows that the other player is going to play N C. So, this is not Nash. Let us see if C, N C is Nash which is this one. Now, from C, N C I can, I have, if I have to show that this is not a Nash equilibrium then I have to basically show one profitable deviation. Here the profitable deviation that I can construct or I can visualize is of player two because player one, from player one's point of view it is obvious that he is getting 3 and if he deviates he can deviates only in one direction which is that he is playing N C.

If he plays N C he is worse off, he is getting 2. So, there is no reason why player one should deviate. If he knows that the other player is not going to confess, he is going to confess. So, he will be free which is best for him. But this cannot be a stable outcome because if in one play of the game this happens, in the next play of the game player two will know that player one is going to play C and if player one is going to play C then it is not in the interest of player two to play N C. He is going to play C also.

So, both of them will confess and both of them will get 1 each. So, player two has a profitable deviation which means again this is not Nash equilibrium. Similarly, as N C, C it is just the opposite case of C, N C. From N C, C here player two will not deviate because he is confessing whereas the other player is not confessing. So, he is getting 3 which is high, which is the highest in fact. But player one will deviate now. So, again since the deviation is profitable for player one it is not a Nash equilibrium.

We are left with the last one. Let us see if that is Nash equilibrium C, C that is both of them are confessing. If both of them confessing then it is true that there is no deviation, which is profitable because if the other person is confessing what do I get by not confessing. I go to jail for 5 years. So, this is the the absurd that the other person is confessing and I hesitated, choose not to confess. Therefore, C, C is a Nash equilibrium. You can see it from here itself. If anyone deviates from the C, C action for example, if one deviates he gets 0. 0 is less than 1. So, he is not going to deviate.

Similarly, for player two, he is not going to deviate and play N C. So, in prisoner's dilemma game we have got one Nash equilibrium which is that both of them will confess. And this is an interesting situation then. What the theory of Nash equilibrium now is telling us is the following. That if we have a prisoner's dilemma like situation then the only outcome that is going to prevail over and over again is the outcome where people are getting 1 1 and it is obvious that both of them could have got 2 2 if they had not confessed, if they had somehow not confessed.

If I see for example, the other person is not confessing over some periods and I expect that he is not going to confess in the next period also then I shall not confess. I may think that, but in this case it is not going to happen because if the other person is not confessing in the best for best action for me is to confess. But if I confess in there is no reason why the other person will go on not confessing. So, we end up being here. So, the point I was trying to make is that if both of them do not confess then it is better for both of them, but if I know that the other person is not going to confess I try to free ride. I try to confess and get out of the jail and both of them are trying to do the same thing and we are back to a situation which is worse for both of us, 1 1 which is less than 2 2.

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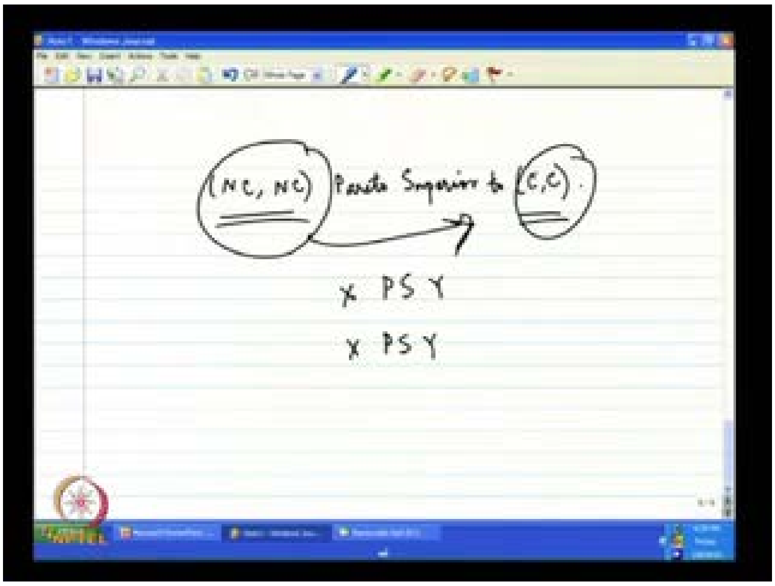

And this N C, N C in economics this is known as Pareto Superior to C, C. What is meant by Pareto Superior? Pareto Superior when we try to apply this idea it it basically invokes, it basically tries to say that we have two states, may be two states in society. X is Pareto Superior to Y if compared to Y in X at least one person is better off whereas, I am not decreasing the the well being of other persons. So, if I can make at least one person better off whereas I am not reducing welfare of other people then I can say that X is Pareto Superior to Y. You can see here both the people are better off. It, so it is definitely Pareto Superior state. I can make both of them better off 1 and 2 in N C, N C compare to C, C because in C, C they are going to jail for 3 year, in N C, N C they are going to jail for 1 year.

But even then the Pareto Superior state which could have made each of them better off is not a Nash equilibrium. So, it might happen that people, both of them will like to have a state like this, but their individual rationality that is I want to free ride, that basically leads to a situation which is an Nash equilibrium situation C, C, but which is not a very desirable situation because both of them are worse off. That is the idea. We have while discussing the prisoner's dilemma given other sorts of situation which is, which are like the prisoner's dilemma situation.

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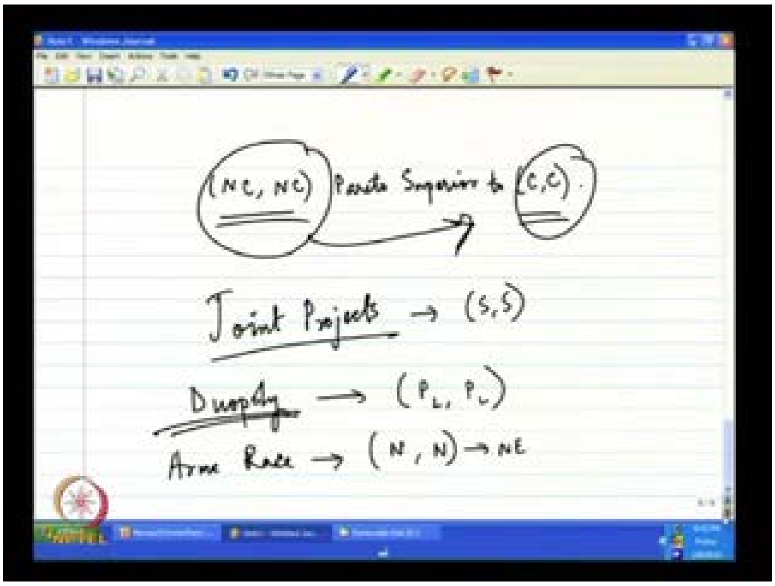

We talked about joint project for example, while where the two students are trying to finish a project and they have the options of working hard or just shirking their jobs. I mean they shall not working very hard. In that case like the prisoner's dilemma case the only Nash equilibrium that they, that will prevail when that both of them will shirk which means that the project quality will be poor. And which is like what we have discussed so far that the Nash equilibrium in the prisoner's dilemma case that they could have done better. Both of them could have worked better and both of them could have been better off, but that is not going to be the Nash equilibrium.

The stable steady state outcome is that both of them will shirk. We have talked about this duopoly game. Two forms were there and they were planning whether to charge a high price or low price. So, here both of them will charge the low price and if both of them charge low price, the profits of both of them will be low. So, in it is a Pareto Inferior state compare to the case where both of them charge high price.

We have talked about arms race and this is this was again like the prisoner's dilemma case and the Nash equilibrium that that it will that prevail, that will prevail in the arm race will be that both of them will built nuclear facility. None of them will refrain. So, if both of them had refrained that would have increased the equality of both of the countries, it would have been a peaceful world, but nevertheless this is going to be the Nash equilibrium.

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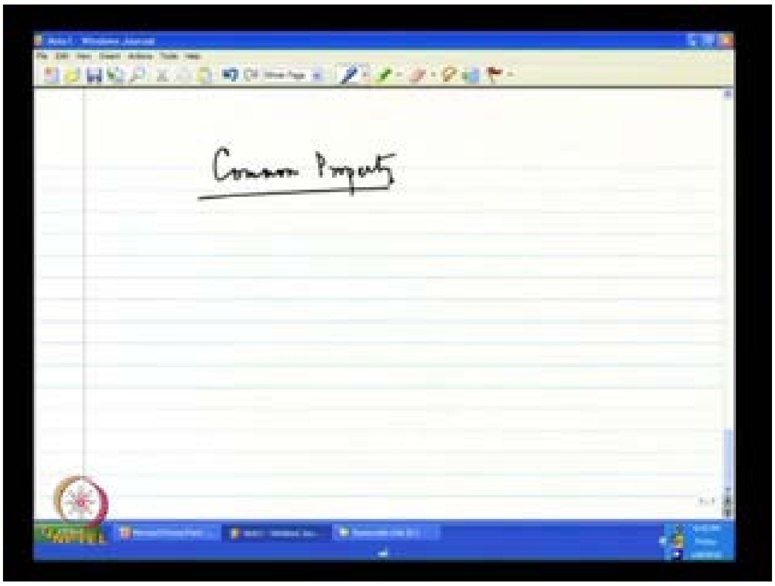

And finally, we talked about this problem of common property. The problem of over grazing, there also the Nash equilibrium will be that situation where all the villagers will let their sheep graze the village pasture by a large amount. So, that after a point of time it will get exhausted, the field will not able to generate enough grass and may be the sheep will starve. So, this is how it occurs. In each of this prisoner's dilemma like situation what we are ending up with is a Nash equilibrium which is not a very good state of affairs because both of the players or all of the players, in case of common common property they were more than two players.

All the players are in a situation from which they could have done better. Each of them could have done better, but that better state is not a Nash equilibrium and because people try to free ride. Now, let us come to this second state of, second set of games that we have discussed which is known as the battle of sexes.

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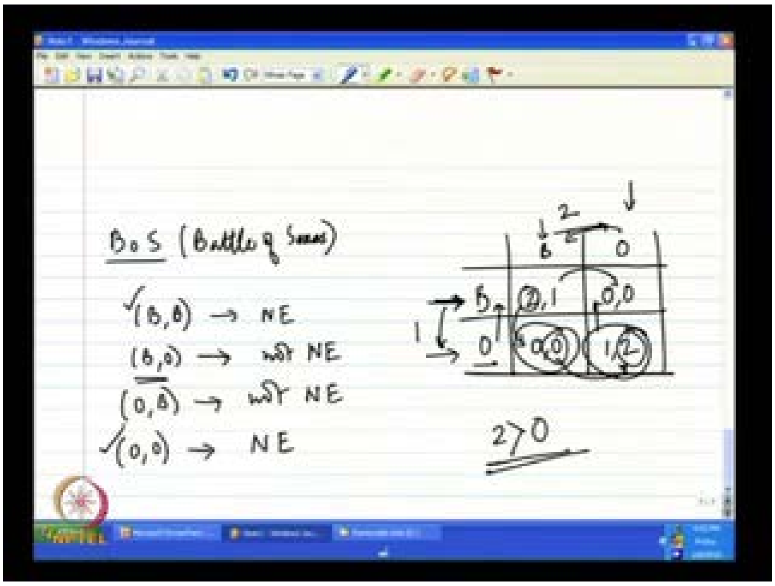

And this is how the payoff matrix looks. Two players are there, either boxing match or opera and these are the payoffs. Now, again if I have to find out what is the Nash equilibrium action profile I have to check for each action profile if there is some profitable deviation. If there is no profitable deviation for every player then this is a, this action profile that we are considering is a Nash equilibrium action profile. So, let us consider B, B which means that the husband and the wife they are going to the boxing match together.

You see from here nobody can move and take some other action and can be better off. If the wife goes to the boxing match sorry to the opera house she gets 0 instead of 1, if the husband deviates she get, he gets 0 instead of 2. So, unilateral deviation by none of the players is profitable which means that this is indeed a Nash equilibrium. What about B, O which means that the husband is going to the boxing match, the wife is going to the opera. From B, O each of them has a tendency to deviate because if the wife is going to the opera and the husband knows that then he is not going to the boxing match. He is going to shift to opera also.

So, that he gets 1 which is greater than 0. So, profitable deviation exists for the husband. However, it at this, if there is profitable deviation for the husband there is profitable deviation for the wife also. Because from B, O it might happen that the wife expects that the husband is going to the boxing match. In which case instead of going to the opera house she might as well decide to go to the boxing match which will give her give her 1, better than 0.

So, this B, O is not Nash equilibrium because for both of the players profitable deviation is possible. What about O, B? Similar, from O, B which means this one, this is the action profile we are considering. Husband is going to the opera house; the wife is going to the boxing match. From here both of them can deviate and be better off. The husband can instead go to the boxing match and get 2. 2 is greater than 0. Similarly, the wife instead of going to the boxing match now goes to the opera house and instead of getting 0 she gets 2. Again 2 is greater than 0. So, again there is or there are profitable deviations for both the players and there that is why O, B is not a Nash equilibrium.

Finally, what about O, O? That is this one. From O, O also if I consider deviation by each player, for each player there can be only one deviation. If the husband instead goes to the boxing match then there is a basically miss match. He is going to the boxing match, the wife is going to the opera house and the husband is getting 0 which is less than 1. So, there is no profitable deviation, but I need to check from the wife's points of view also. The wife will not, definitely not deviate because she is going to her prefer, preferred destination which is the opera. If she goes to the boxing match instead and she is alone there obviously she is going to get 0.

So, for none of the players from the action profile O, O that is opera house there exists any profitable deviation. So, O, O is a Nash equilibrium. So, this here we have an example where the game can contain more than one Nash equilibrium. Here I have two Nash equilibrium, this and this.

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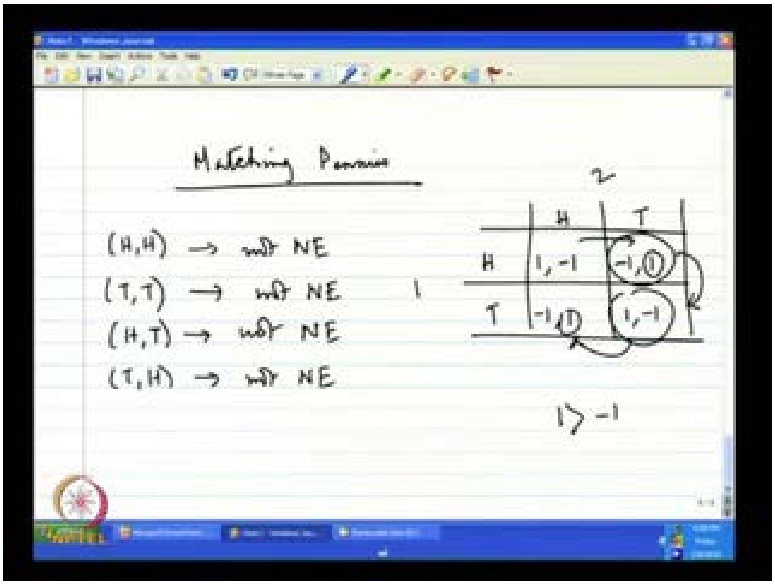

Let us now look at the third class of games which is matching pennies and the game look like the following. So, this is how the payoff matrix looks like in matching pennies. You can see by examination that in this game there is no Nash equilibrium. I can check it out by considering the action profiles one by one. If I consider H, H, from H, H is there profitable deviation? H, H is the case where both the players are showing heads which means that the pennies are matching. If the pennies match player one gains at the cost of player two.

Now, if that is the situation there is no reason why player one should deviate because it is best situation for him. But what about player two? Player two, for player two it is not a very good situation. So, he will deviate and I can see that from here. If he instead of H plays T, does not play H that is does not show head, but shows tail then there is a miss match. Then he is getting 1 which is greater than minus 1. So, from here profitable deviation is possible. H from H, H player two will deviate profitably. So, this is not Nash equilibrium.

Similarly, just take the opposite case of T, T where both of the players are showing tails. Again from player one's point of view it is the best possible situation he is getting 1. But what about two? Again player two will like the pennies to miss match. So, he is going to do what he knowing that player one is playing T, if that is has been the convention from player one's point of view player two will now play H and he will get 1 which is greater than minus 1.

So, this is not Nash equilibrium. H, T, so this is H, T. Here just the opposite thing will happen. Here player two is gaining because the pennies are in fact not matching to each other which means player one has to give 1 rupee to player two. In this case it is now in the interest of the player one to deviate and he, if he deviates then the pennies will match. If he deviates and he plays T like this then he is gaining 1 rupee. So, again there is profitable deviation. So, h t is not Nash equilibrium.

Similarly, T, H is not Nash equilibrium because again one will deviate and play H. So, that the pennies match and he gains 1 rupee. So, this is, you have one example where there is no Nash equilibrium. Nash, the definition of Nash equilibrium does not guarantee that every game has a Nash equilibrium. And let us look at the last class of games that we have discussed which is stag hunt.

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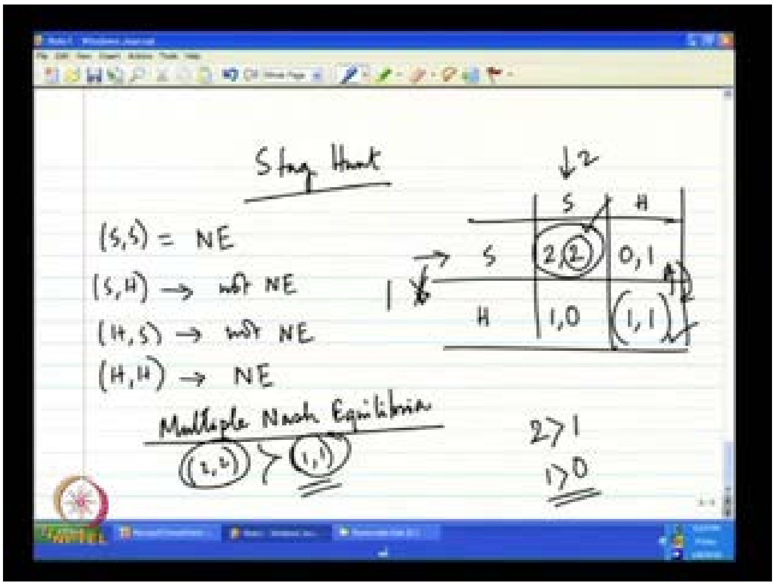

And here the payoff matrix was as follows. There were two actions for each player, one is to concentrate on the stag and the other action was to catch a hare running after a hare. And we saw that the preferences are such that if both of them just concentrate on their stag hunt then each of them will get 2 which is the best possible that they can get. However, if player one concentrates only on the stag, but player two goes for the hare then player one is unable to catch any stag. So, he gets 0 whereas, player two since now, he is going after the hare he will be able to catch the hare and he will get 1.

Similarly, if I have H, S that is player one going after the hare and player two going after the stag then player two is getting 0. If both of them go after the hare each of them will get 1 that is each of them will be able to catch a hare. Again I can apply my definition of Nash equilibrium and look for the possible set of Nash equilibria. Let us take the profile S, S. From S, S nobody can deviate and be better off because if player two has been playing S that is he has been concentrating only on the hunt of stags then it is best for player one to continue hunting stag also. He he is not going to deviate. This deviation is unprofitable.

He is going to get 1 if he deviates. So, this is going to be Nash equilibrium because the similar logic can apply for player two also. If player one goes on hunting the stag then player two is not going to deviate and go after the hare because if both of them can get the stag they are getting 2 and 2 is greater than 1. What about the others? Let us look at S, H. S, H is not a Nash equilibrium because here what is happing is that player one is going after this stag whereas, player two is going after the hare which means that player one is getting nothing. He is not able to catch the stag because without your your partners cooperation you cannot catch a stag.

So, if player one knows that player two is going to go after the hare it is best for him to go after the hare because 1 is greater than 0. So, profitable deviation exists at least for for player one and that is sufficient for me to say that this is not Nash equilibrium. Similarly, if I consider H, S that is player one is going after the hare, player two is going after the stag then there will be a profitable deviation from the point of view of player two because he is getting 0 here H, S because he is going after the stag and he is not getting the stag.

So, which means that instead of going after the stag in a futile way he will like the player one go after the hare. But is going after the hare by both a Nash equilibrium? Let us see. So, here we are considering this one H, H. And this is indeed seems to be a Nash equilibrium because from here if any one deviates he gets 0. If I know my partner my friend is going to go after the hare there is no reason why I also should not go after the hare. Why? There is no reason why I should go after the stag because I will not able to catch the stag.

So, if player one deviates he gets 0. 0 is less than 1. So, there is no reason why he should deviate and it is the logic is similar for player two. Player two will also not go after the stag if he knows the player one is going after stag. So, that means I have a Nash equilibrium at H, H also. There are two Nash equilibrium, one is here and the other is here. One is both of them are going to catch the stag and the other is that both of them are going to catch the hares.

So, here we have again a multiple Nash equilibrium case. And another important and interesting thing of this game of stag hunt is that I can compare between these two Nash equilibria. I can compare between 2, 2 and 1, 1. I mean these are the payoffs; these are not exactly the Nash equilibrium. These are payoffs in Nash equilibria. As it is obvious that if the Nash equilibrium is at S, S that is both of them going after the stag it is beneficial for both of them compared to the case where they are going after the hare.

So, it I can write it like this. This is we can, this sign means preferred to. So, 2, 2 will be preferred to 1, 1 for both the, for both the players. But that does not mean that this Nash equilibria S, S is has, will occur this Nash equilibrium at H, H will not occur. I cannot rule out the fact that H, H can also occur for the simple reason that if both the players had been playing H, H that is both of them had been playing, going after the hare then there is no reason why any player should deviate and go after the stag because that will be suboptimal as we have seen in their case, he does not catch anything.

So, what we have done so far is that we have given, we have discussed the definition of Nash equilibrium and then we have looked at how this Nash equilibrium can be applied in different situations in the four generic games that we have discussed so far. We have seen that it is not necessary that one has a Nash equilibrium in every game. It may happen that you have a game which has no Nash equilibrium and there can be games where the number of Nash equilibrium is more than 1. We have also seen that there can be games where the number of Nash equilibrium is more than 1 at the same time one can say that this Nash equilibrium is better than the other Nash equilibrium like we had in the stag hunt case.

But nevertheless I cannot rule out that this worse Nash equilibria that can; that we can find out from that game will not prevail. There is no reason why i will not rule out the occurrence of such Nash equilibrium. So, that is more or less it. In this lecture from time to time we shall be doing some exercises. These exercises have been collected from many sources. So, one exercise let me try to give to you.

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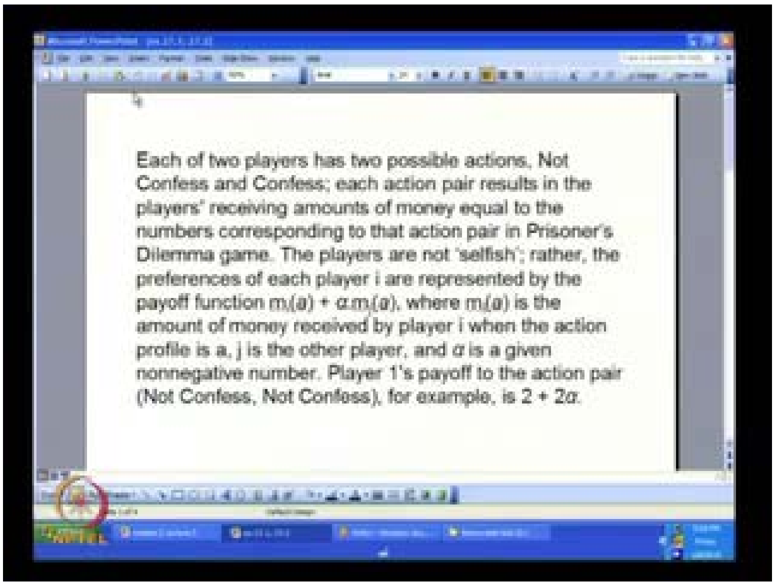

This exercise is a variation of the prisoner’s dilemma game and the exercises are the following as you can see in the monitor. Each of two player has two possible actions not confess and confess. Each action pair results in the players receiving amounts of money equal to the numbers corresponding to that action pair in prisoner’s dilemma game. But the players are not selfish, rather the preferences of each player i are represented by the payoff function m i a plus alpha multiplied by m j a where m i a is the amount of money received by player i when the action profile is a, j is the other player and alpha is a given non negative number.

Player i’s payoff to the action pair not confess not confess for example, is 2 plus 2 alpha. So, this is the, this is the background of the question. We have to answer two questions.

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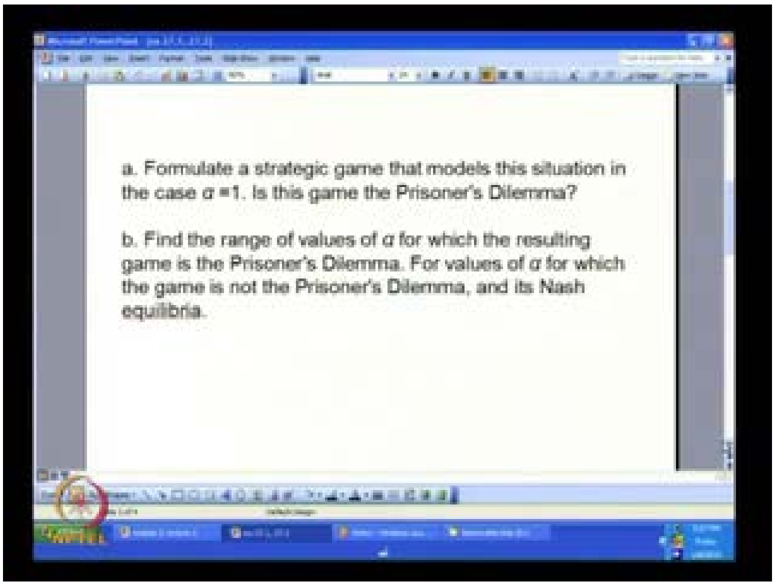

Formulate a strategic game that models this situation in the case of alpha is equal to 1, is this game the prisoner's dilemma game? So, the story is the following.

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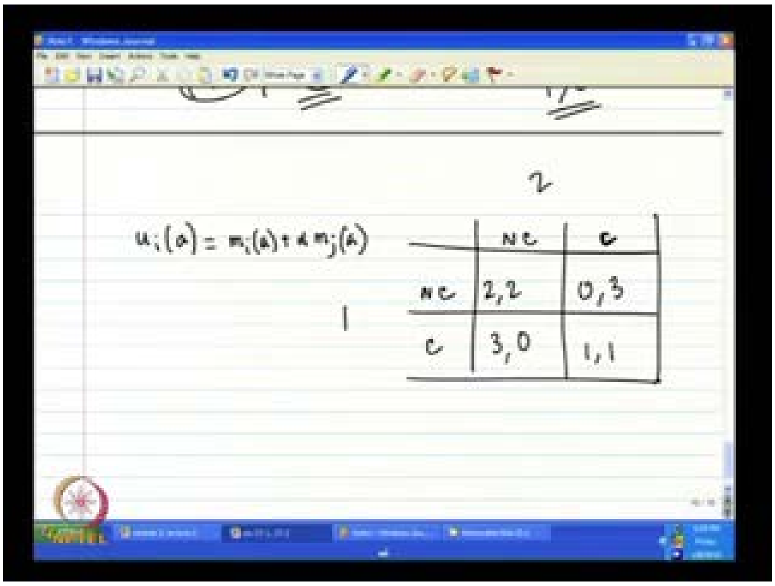

What has been asked is that suppose we have this prisoner's dilemma game. This was the prisoner's dilemma game. So, players have two actions each, confess and not confess. Now, we are going to consider variation of this game. The variation is the following that suppose it is the case that u i that is player i's payoff is not simply these numbers, but variation of these numbers in the sense that this is m i a plus alpha multiplied by m j a where a is the action profile. And m i a and m j a are these numbers. m i a is the payoff that i that is player i is getting from the action profile a.

And likewise m j a is the payoff that player j, the other player is getting from the action profile a, alpha is any non negative number, it can be 0 and positive. The question that is being asked is that formulate a strategic game that models the situation in the case alpha is equal to 1, is this game the prisoner's dilemma? We shall talk about the ramifications of this exercise and what it means in the next class.

Thank you.

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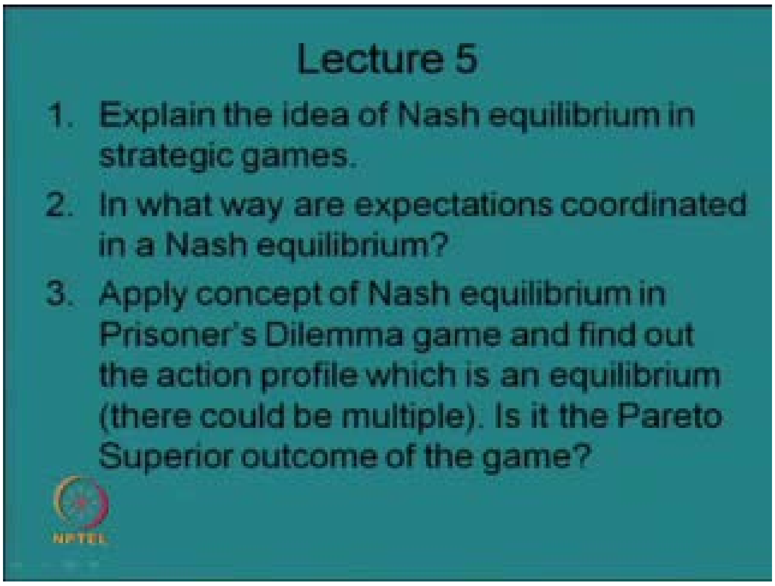

So, this is the exercise for a lecture 5. Explain the idea of Nash equilibrium in strategic games.

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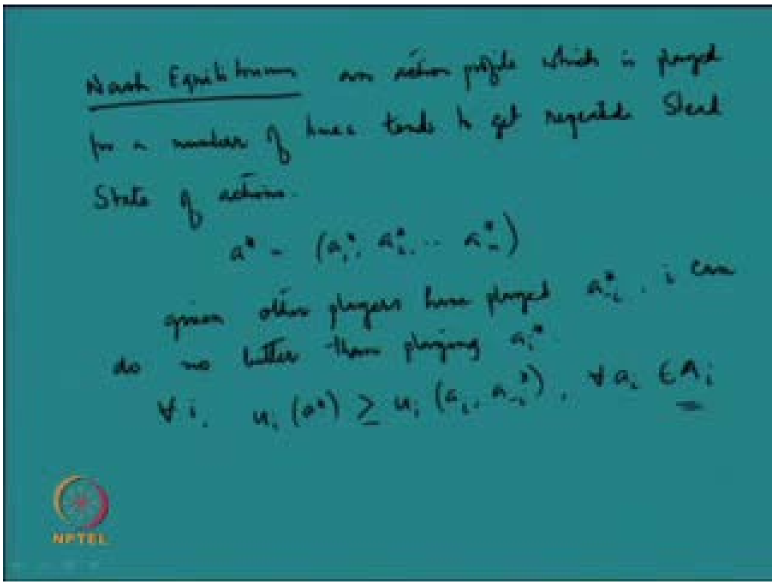

Nash equilibrium, in short Nash equilibrium is an action profile which if played for a number of times tend to get repeated, tends to get repeated. So, the idea that we have is a, it is a kind of steady state of actions. In other words suppose a star is an action profile it looks like the following. a 1 star, a 2 star, dot, dot, dot, a n star where there are n players, then given other players have played a naught i star I can do no better than playing a i star. In other words in terms of signs here capital I is the set of actions that i has. Mind you this is greater than equal to.

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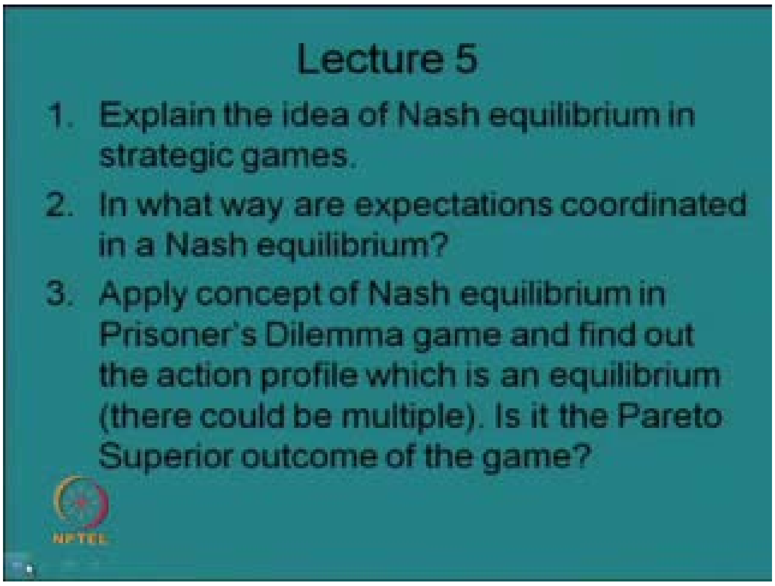

In what way are the, in what ways are expectations coordinated in a Nash equilibrium?

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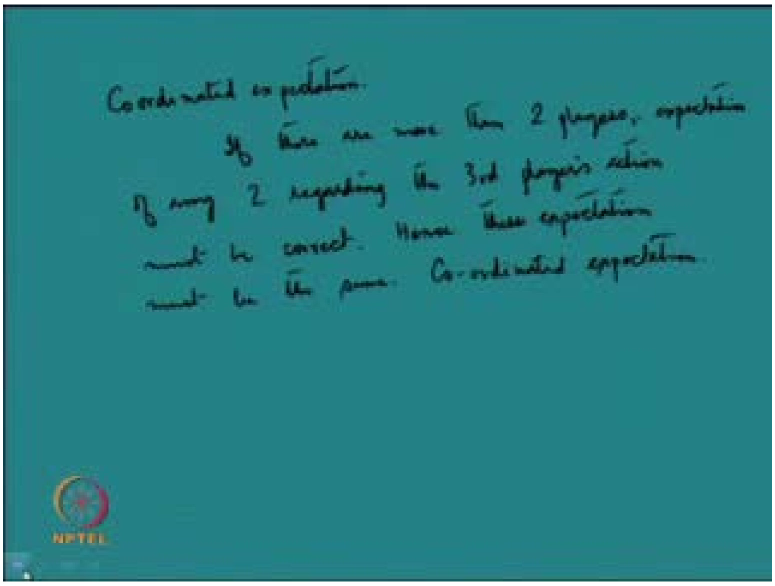

Expectations, if there are more than two players then what should happen is that expectations of any two players regarding the third player must proved to be true, expectations of any two regarding the third player's action must be correct, must be proved true. Hence, these expectations must be same this is called coordinated expectation.

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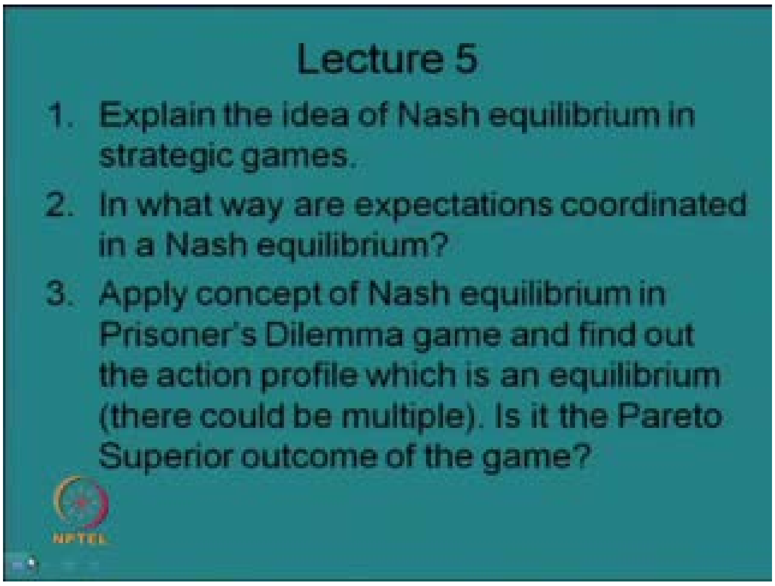

Apply concept of Nash equilibrium in prisoner's dilemma game and find out the action profile which is an equilibrium? Is it the Pareto Superior outcome of the game?

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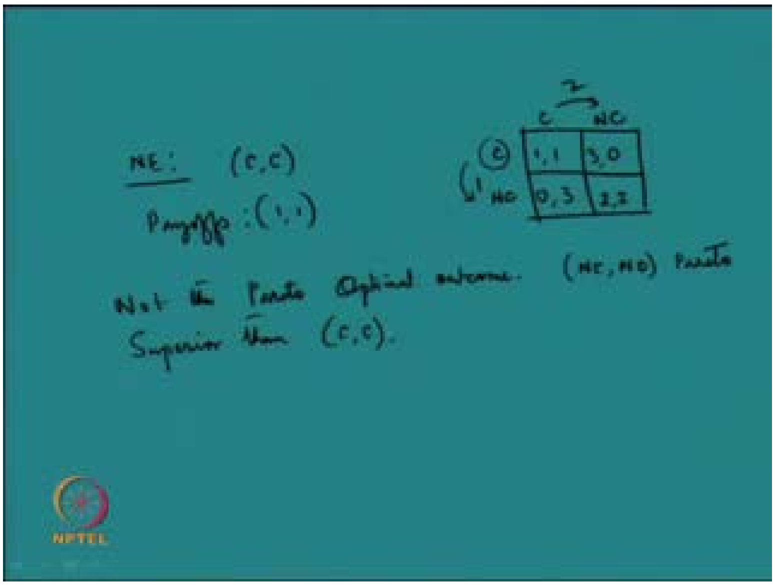

So, let us go back to prisoner's dilemma game. So, this was the prisoner's dilemma game. What we find here is that Nash equilibrium here is a single Nash equilibrium is there which is C, C. The payoffs are 1, 1. Why this is Nash equilibrium because given player one is playing C player two cannot deviate and be better off because he will get 0 and given player two plays C player one cannot deviate and be better off because again he will get 0.

And we can verify that no other profile is Nash equilibrium. Is this the Pareto Superior outcome? No, this is not the Pareto Superior outcome or what is known is that it is not the Pareto Optimal outcome. N C, N C is in fact Pareto Superior than C, C because both of the players are better off in N C, N C compared to C, C. And that is the idea of Pareto Superiority that you are making someone better off without making anyone else worse off. Here in fact both of them are better off in the C, C outcome.

Thank you.