Game Theory and Economics

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Module No. # 02
Strategic Games and Nash Equilibrium
Lecture No. # 04
Altruism and Prisoner's Dilemma

Welcome to the fourth lecture of module 2 of the course called game theory and economics. Before we start the topic for today, let me take you through what we have discussed in the previous lecture.

What we have discussed in lecture 3, is that we have given the specific definition of Nash equilibrium which is a solution concept in game theory, often used, and we have given some examples in which Nash equilibrium can be found.

For example, we have talked about Prisoner's dilemma and we have looked at the Nash equilibrium - there was a single Nash equilibrium; we have to associate that in situations like battle of sexes, there are two Nash equilibria, so we have multiple Nash equilibria.

We have seen that in situations like matching pennies, which was a zero-sum game, in matching pennies there was no Nash equilibrium - in pure strategy. Also, we have talked about the stag hunt case and in stag hunt case, in that generic case, again there were two Nash equilibria - that is multiple Nash equilibria.

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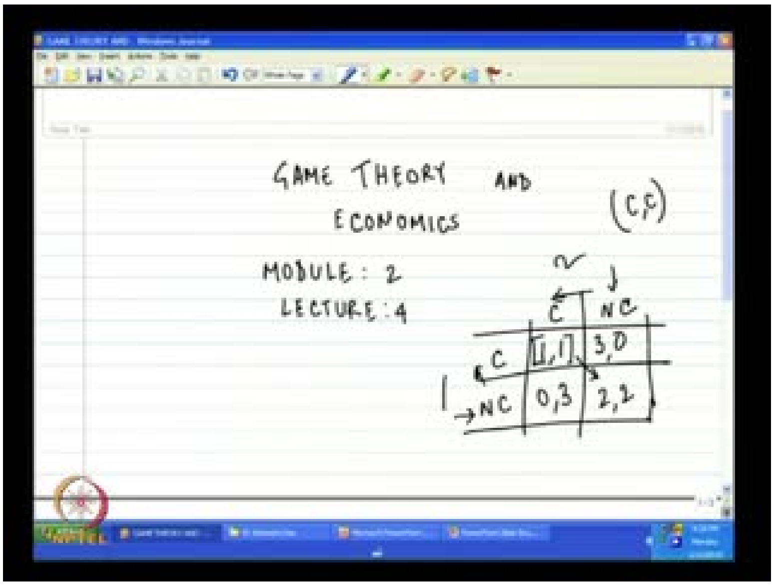

What we are going to do today and we have started doing this in previous lecture itself is that we are going to present a variation of the Prisoner's dilemma to bring out what the basic essence of Prisoner's dilemma situation is - what is it that is generating this bad kind of equilibrium; because, if you remember in Prisoner's dilemma, the game look like this - here is player 1, suppose C confess, NC not confess; similarly for player 2.

This was the game theory - the Prisoner's dilemma game - that we were discussing in the previous class and we saw that there was a unique equilibrium which is this one, C C which is a bad situation compared to this NC NC, because if both of the players had played NC then they could have got 2 each whereas in equilibrium they are playing 1 C and C and from which they are getting 1 and 1.

Why are 2 and 2 are not payoffs which they could get in Nash equilibrium? Because, if 1 knows the player 2 is going to play NC, in that case if 1 plays NC he gets 2 but if he shifts to C he gets 3 which is better, so he will shift; similarly for 2 also, if 2 knows that if 1 is going to play NC, 2 is not going to play NC - he will play C, so ultimately both of them will play C and 1 1 will be obtained.

The exercise that we are going to do today is trying to address this point; that in Prisoner's dilemma the players are concerned only about their own payoff and that is why may be they are trying to free-ride and we are getting an equilibrium which is at C

C, but which is not a good thing, I mean, they could have done - both of them - could have done better if they had played, not confess.

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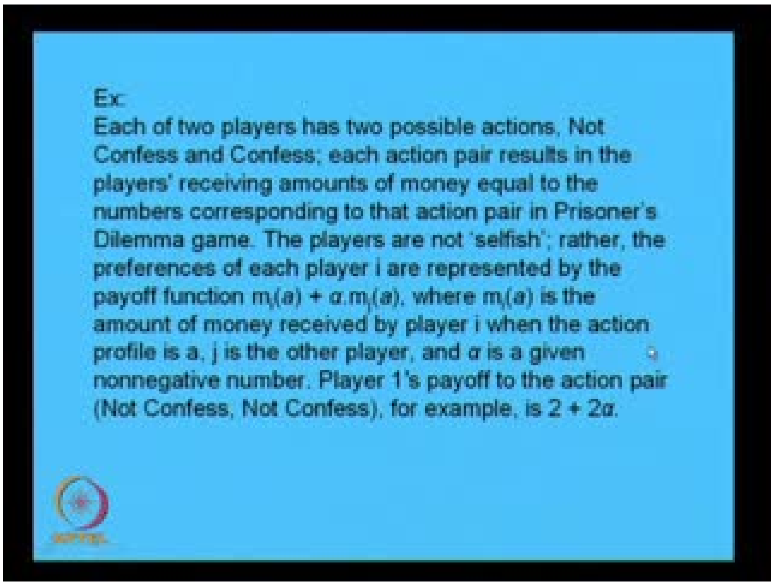

The exercise is the following, it is this: each of the two players has two possible actions, not confess and confess; each action pair results in the players receiving amounts of money equal to the numbers corresponding to that action pair in Prisoner's dilemma game. The players are not 'selfish'; rather, the preferences of each player i are represented by the payoff function m i a plus alpha multiplied by m j a where m i a is the amount of money received by player i when the action profile is a, j is the other player, and alpha is a given nonnegative number. Player 1's payoff to the action pair not confess, not confess, for example, is 2 plus 2 alpha.

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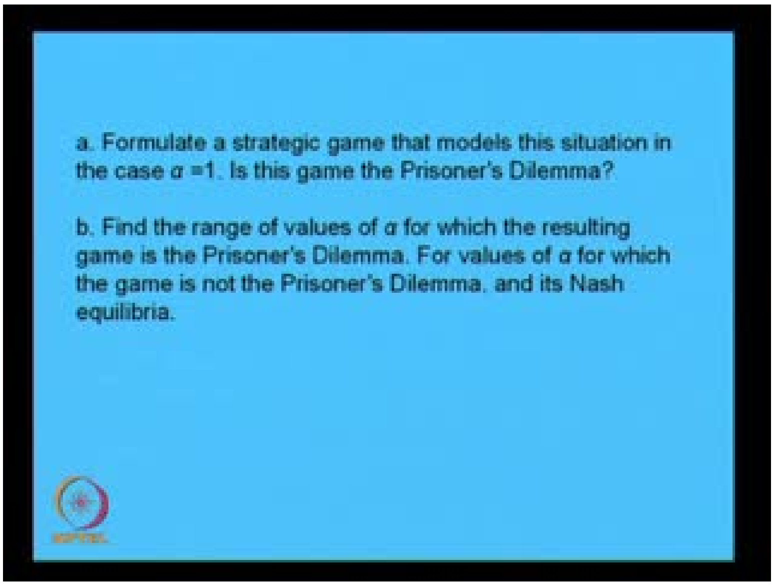

So first question that is being asked is that - formulate a strategic game that models this situation in the case of alpha is equal to 1. Is this game the Prisoner's dilemma?

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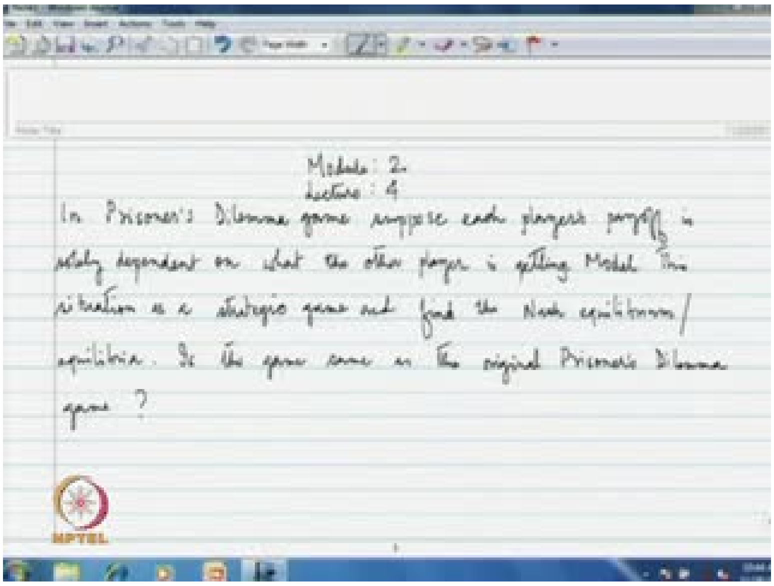

So this is the question; we are basically considering one variation of the Prisoner's dilemma game and the question is the following: in Prisoner's dilemma game suppose each player's payoff is solely dependent on what the other player is getting. Model this situation as a strategic game and find the Nash equilibrium or equilibria. Is the game same as the original Prisoner's dilemma game?

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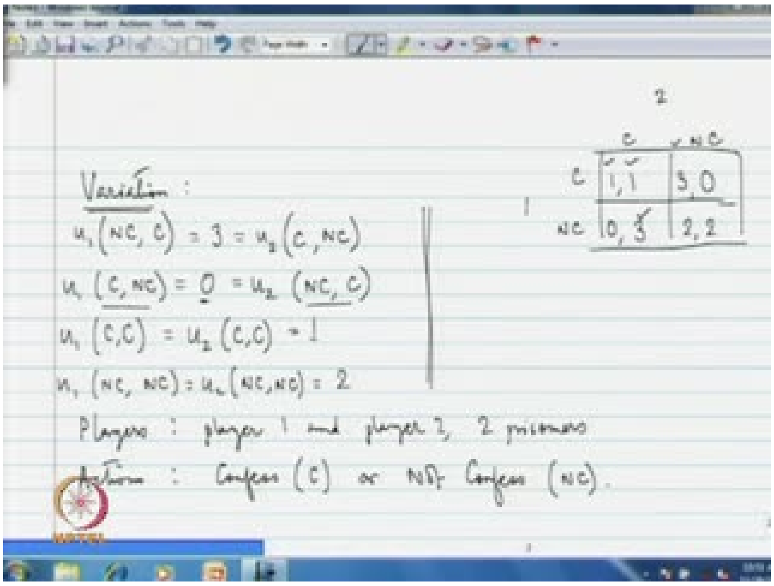

So, to solve this question let us first remember, what the basic structure of the Prisoner's dilemma was. Again I am writing down the payoff matrix, so there are these two players - player 1 and player 2 - 2 prisoners, and they have two actions to choose from, either to confess or not to confess.

Now, if both of them confess they get 1 and 1, represented by these numbers; if both of them do not confess then basically the get is not proven and they get better payoffs - 2 and 2.

If player 1 confesses and player 2 does not then player 1 is freed, so he is getting a higher payoff, but in that case player 2 goes to jail for a larger prison term, so player 2 is getting 0, the worst possible case for player 2. If player 1 does not confess and player 2 confesses then player 2 gets 3 and player 1 gets 0.

This was the original structure of the Prisoner's dilemma game. What we are interested in, in this variation is a following situation where player 1's payoff is solely dependent on what the player 2 is getting and vice versa - so player 2's payoff is not dependent on what he is getting but what player 1 is getting.

So in the variation what we shall have is the following, we shall have u 1 which is payoff to player 1 if he does not confess, but player 2 confesses, then we know that in the original game player 2 is getting 3 here and that is the same thing that player 1 will get in the new game, just 3.

This 3 will be obtained by player 2 if player 2 does not confess but player 1 confesses, because we know that if player 1 confesses, player 1 is getting 3 which is same as player 2 getting in the variation.

So this is one, then we have just the opposite case here. What is happening is, the player 1 is confessing, so in the original game he should have got 3, but since player 2 is getting 0 in the original game in this particular action profile - that is what player 1 gets in the variation game, player 1 gets 0 here.

Similarly if player 1 does not confess, but player 2 confesses then player 2 is the player who is getting the most out of the situation which is basically 3, but player 1 is getting 0, so player 2 will get 0 in this variation.

These are the symmetric cases that when C C is played - that both the players are confessing, then each player gets the same payoff which is 1 in the original game and which remains the same in the variation also; similarly if both the players do not confess, both the players get the same payoff and if you flip that payoff vector, it remains the same, 2 2.

So, that is why u 1 NC NC is equal to u 2 NC NC is equal to 2. So these are the payoffs that will be there in the variation and we have to specify what the other elements are of this particular game, so the players are same as before - player 1 and player 2, basically 2 prisoners, and actions we have to specify same as before, so confess C or not confess NC.

This is the basic setting of the variation, so let us now try to draw the payoff matrix of this variation. What we have to remember by looking at this structure is that when the players are taking the same action, that is, C C or NC NC, their payoffs in the variation will be same as their payoffs in the original game.

So, we shall have 1 1 as the C C action profile payoffs, and 2 2 as the payoffs of the NC NC action profile.

What will happen is, that if 1 player plays C and other player plays NC, then the payoffs will just flip compared to the original game because my payoff depends on your payoff in the original game, so my original payoff in the original game does not matter, it is the payoff of what you are getting in the original game that matters. So, how will this look like?

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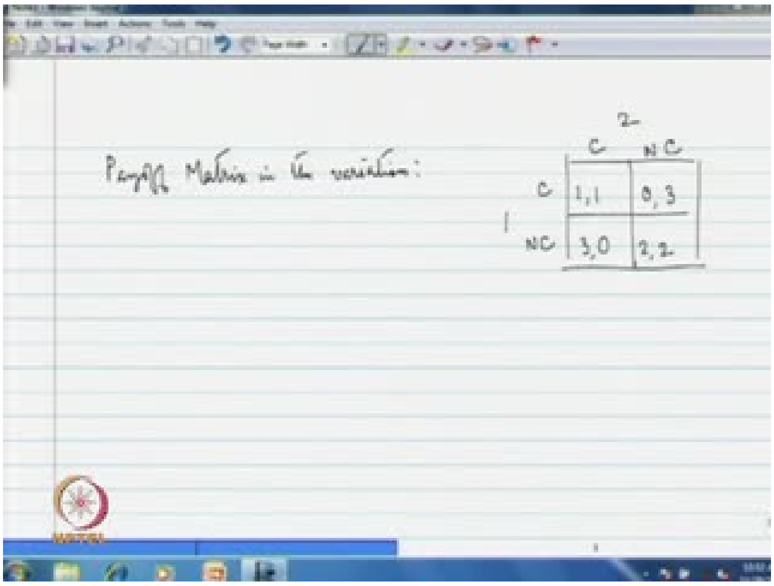

So C C payoffs are 1 1, NC NC payoffs are 2 2; C NC payoff in the original game was 3 0, now it will become 0 3, and NC C payoff which was 0 3 in the original game will become 3 0, so this is the payoff matrix.

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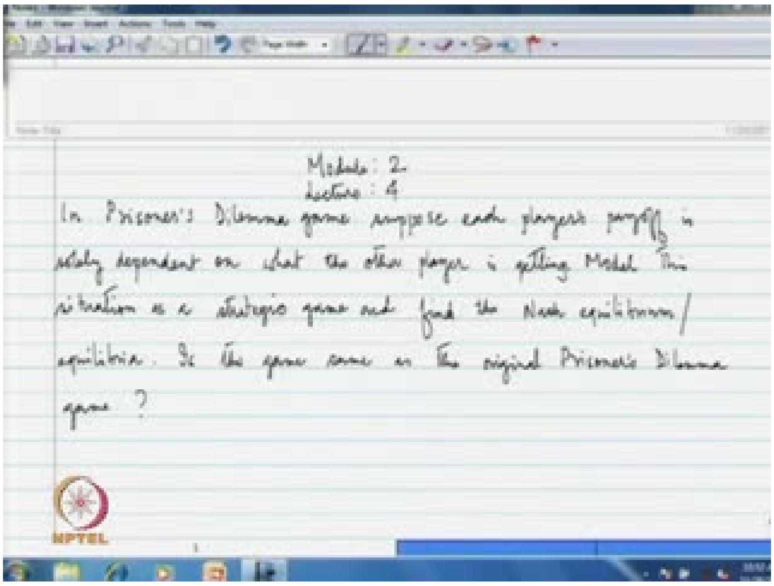

Now, please remember what the original question was - model the situation as a strategic game and find the Nash equilibrium or equilibria, we have to find the Nash equilibrium. Is the game same as the original Prisoner's dilemma game?

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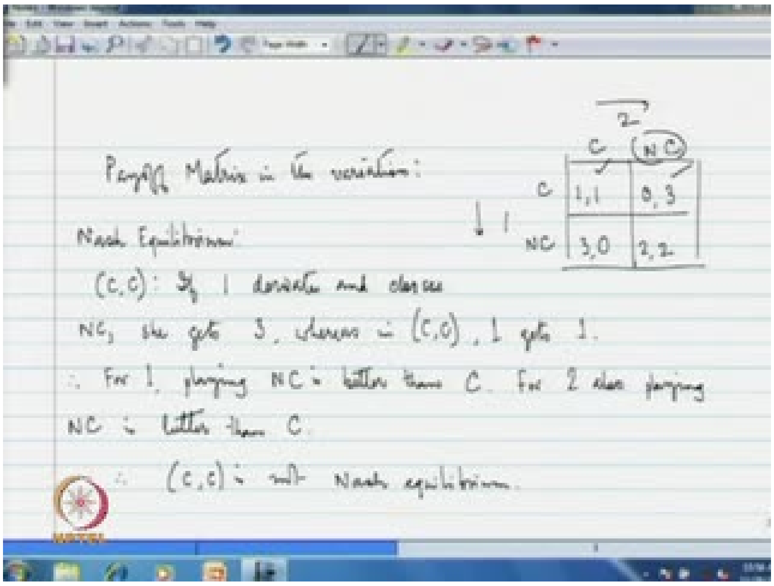

Now, Nash equilibrium - so basically, we are considering Nash equilibrium in few strategies. There are four action profiles that we have to consider and considering those action profiles we have to see whether deviation by any player can give that player a better payoff. If it can give the player a better payoff, then that action profile is not Nash equilibrium.

Let us consider all action profiles one by one.

C C, from C C action profile we can see that both the players have actions which are better than what they are playing in this C C profile. If player 1 deviates and chooses NC she gets 3 whereas in C C 1 gets 1, so for player 1 playing NC is better than playing C and this is true for the other player also. For player 2 also, we can see that if player 2 plays NC she is getting 3 whereas in C C she is getting 1, so deviation is profitable for player 1 also. So, C C is not Nash equilibrium that is what this exercise of C C is telling us.

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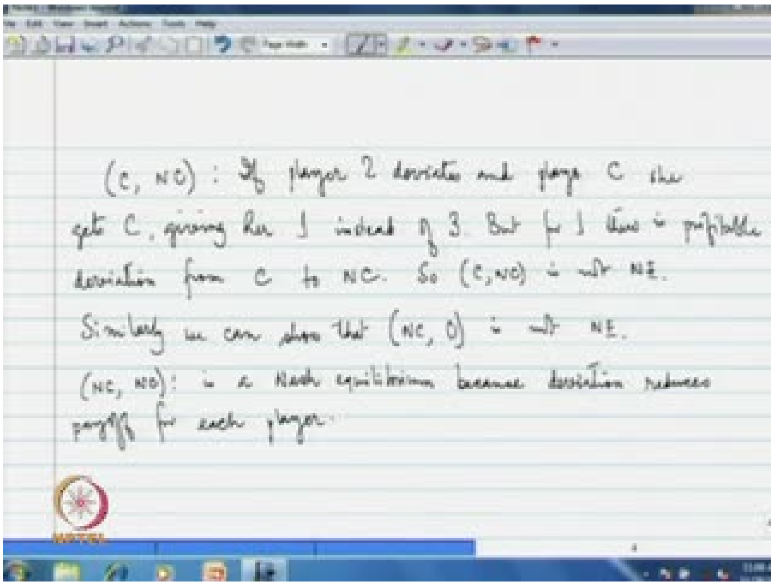

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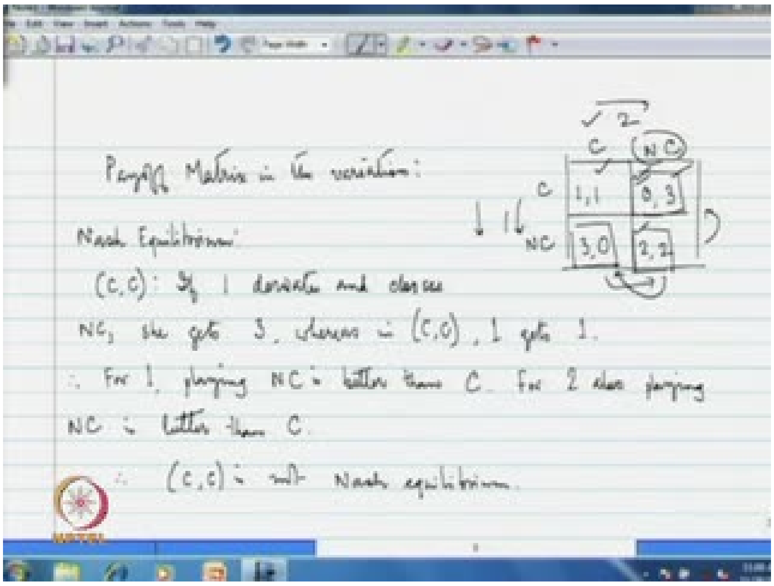

Let us consider another one, C NC. If we consider C NC, basically we are talking about this payoff, vector. Here we can see that from C NC player 2 does not have any profitable deviation because what player 2 can do is that player 2 can play C instead of playing NC, but if he plays C he is going to get 1, which is worse than 3; so, for player 2 there is no profitable deviation from C NC. So player 2 will not deviate, but for player 1 there is profitable deviation from C to NC. Let us see this, if player 2 is playing NC and player 1 is playing C, player 1 is getting 0, but if player 1 deviates and plays NC then player 1 will be getting 2, which is better. There exists a profitable deviation and therefore C NC is not Nash equilibrium.

So, C C got ruled out, now C NC is getting ruled out. Similarly, we can show that NC C is not Nash equilibrium.

NC C is this one. Here player 1 does not have any profitable deviation but player 2 does have a profitable deviation from C to NC. Therefore, NC C is not Nash equilibrium.

The last one we are left with is NC NC. What about NC NC? It is not necessary that every game should have Nash equilibrium in pure strategy; it might very well happen that there is no pure strategy Nash equilibrium. Let us check whether NC NC is a Nash equilibrium?

We are talking about this cell here, NC NC, giving each player 2. Now if player 1 deviates and plays C then player 1 will be getting 0, if player 2 deviates to C then also player 2 will be getting 0 instead of 2. So deviation from NC NC by each player reduces that player's payoff and therefore, this NC NC that we are considering here is a Nash equilibrium; deviation, in fact, reduces payoff for each players, so that is the answer of the question.

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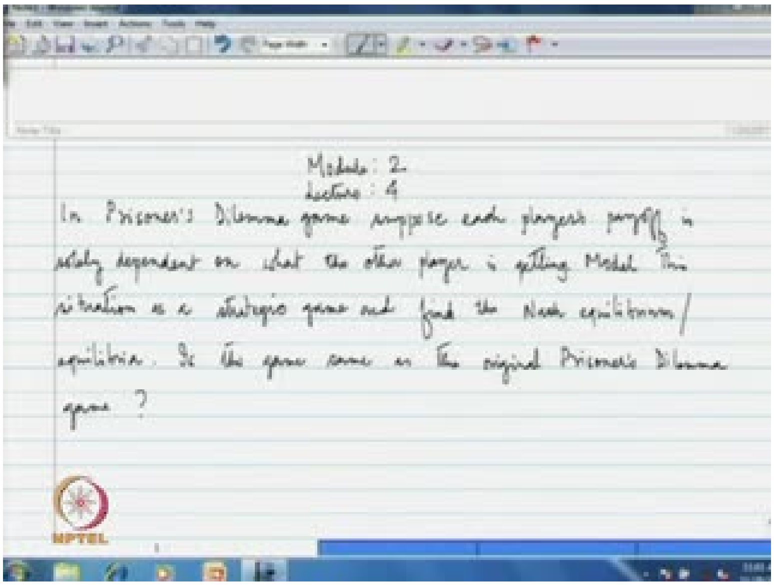

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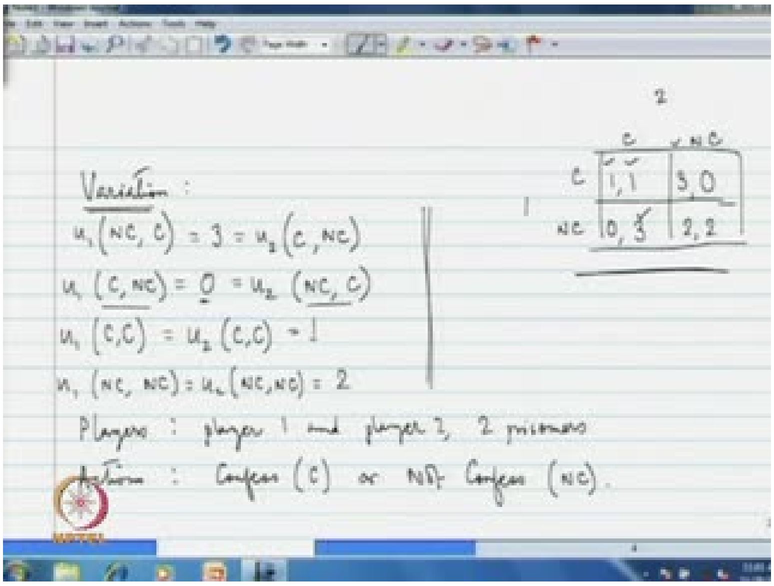

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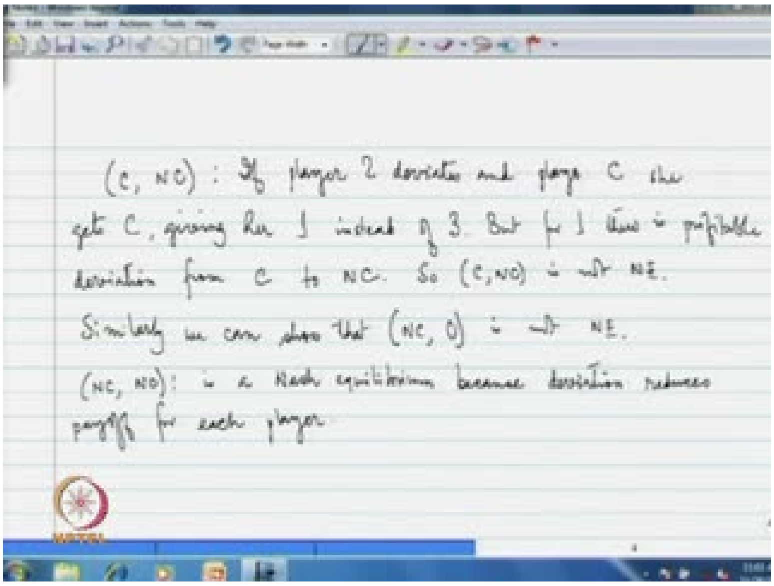

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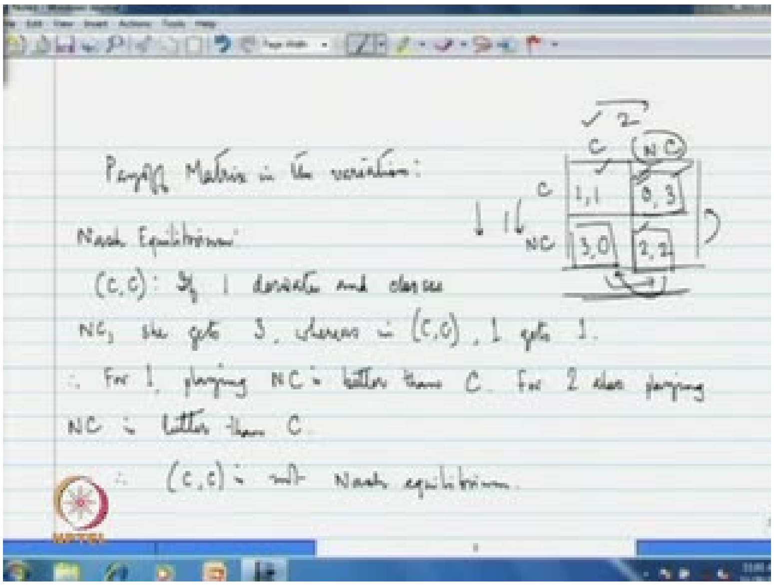

The last part of the question was - is the game same as the original Prisoner's dilemma game? Now this is a very easy question to answer; it is not same as the original Prisoner's dilemma game. This was the original Prisoner's dilemma game, and here what we are getting is this; what we are getting in this changed game is that the payoffs in NC C and C NC in these two cases have changed and because of the change in the payoff in these two cells, that is, C NC and NC C, the game does not remain the same, obviously, because the payoffs have changed. Moreover, the Nash equilibrium of the original game no longer remains the Nash equilibrium in the changed game.

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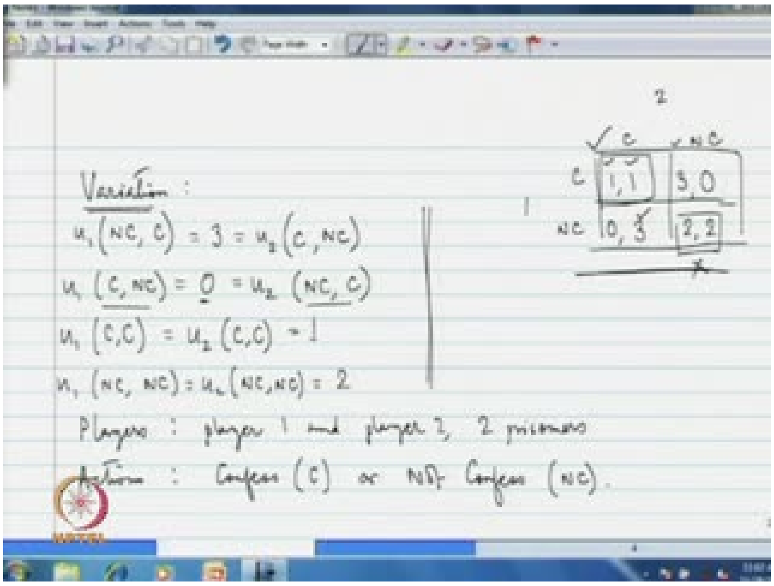

In the original game this was the Nash equilibrium - C C, this was not the Nash equilibrium. We have seen that this is a kind of Nash equilibrium which is sub optimal - for both of the players they are getting less than what they could have got if both of them had moved to NC NC.

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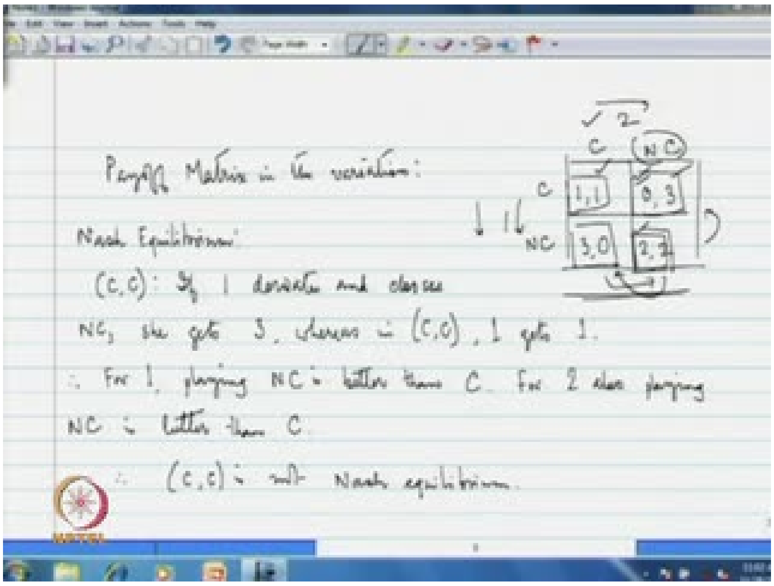

Here in the variation, this C C - the sub optimal outcome, is no longer the Nash equilibrium, what we are getting is that NC NC is the Nash equilibrium.

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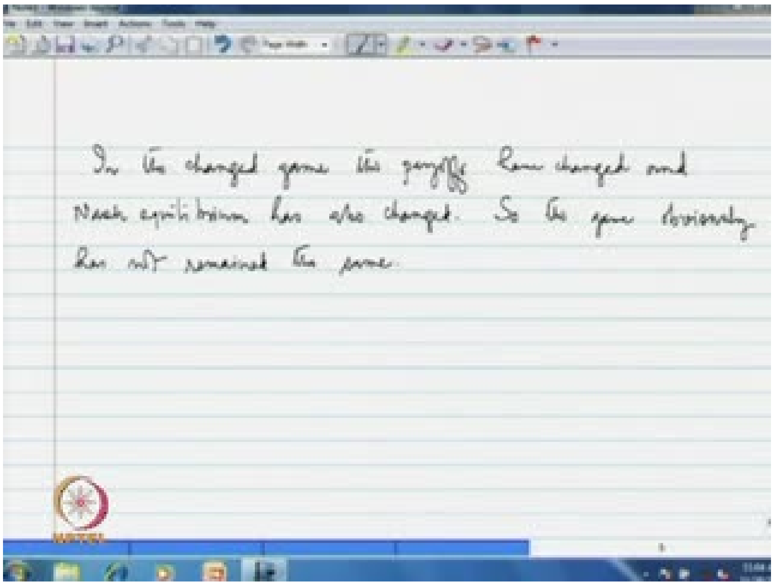

So, in the changed game the payoffs have changed and Nash equilibrium has also changed; therefore, the game obviously has not remained the same.

The game has not remained the same - the Prisoner’s dilemma game that was there in the original case has changed in these changed circumstances, where each player’s payoff is dependent solely on what the other player is getting, and curiously what we are getting here is that in the original game when players were only considering their own payoffs, we had a sub optimal Nash equilibrium at C C - where the payoffs are not the maximum payoffs what they could have got. When they are being sort of all twisty - when they are considering other player’s payoff as their own payoff and they are ignoring their own payoff, in that case, the Nash equilibrium that is emerging is that outcome where payoffs of both the players are maximum.

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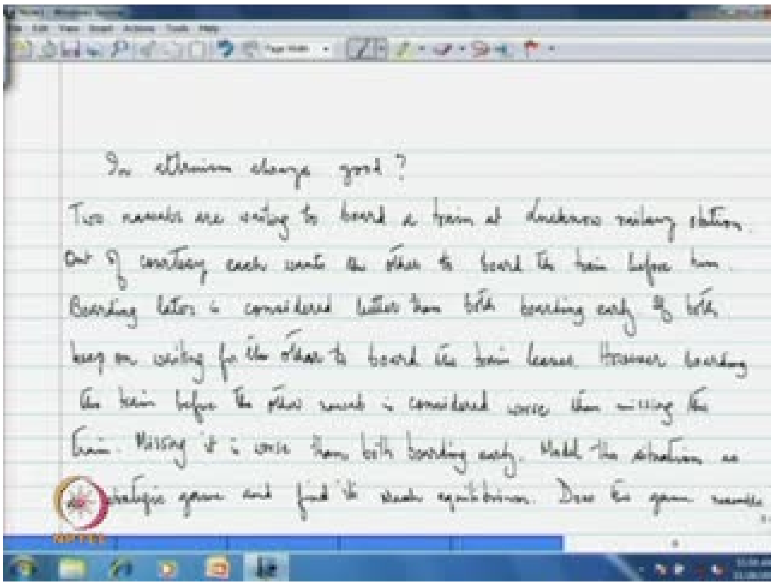

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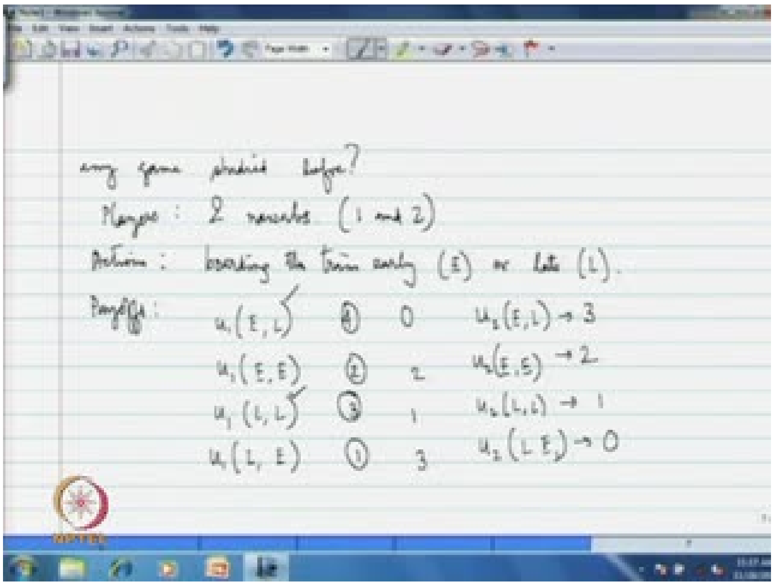

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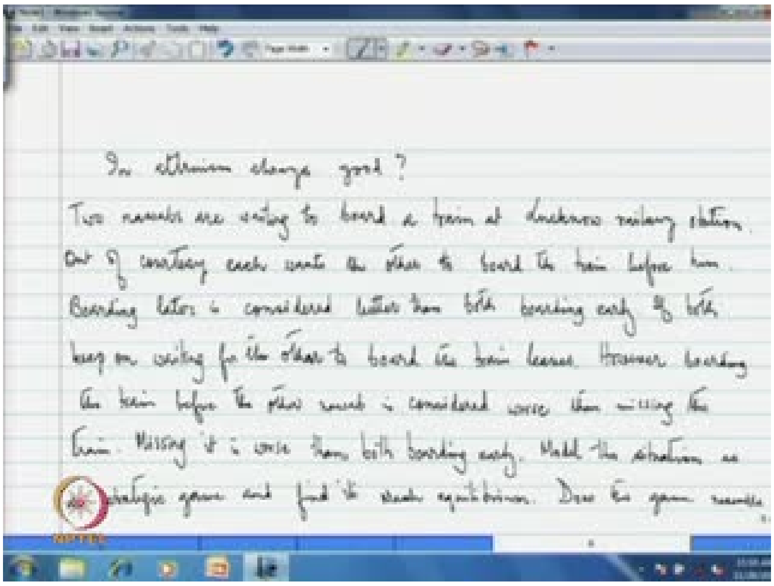

This is an exercise where we are trying to answer this question - is altruism always good? Let me first write down the question and then we shall try to solve this question and try to see whether altruism always gives us the best outcome. So this is the question, let me read out the question before we try to solve this.

Two Nawabs are waiting to board a train at Lucknow railway station. Out of courtesy each wants the other to board the train before him - boarding later is considered better than both boarding early. If both keep on waiting for the other to board, the train leaves. However, boarding the train before the other Nawab is considered worse than missing the train; missing it is worse than both boarding early. Model the situation as a strategic game and find its Nash equilibrium. Does the game resemble any game studied before?

So, what we are having here is that there are 2 Nawabs - so players are 2 Nawabs, let us say 1 and 2 - player 1 and player 2. Actions, what are they doing here - either boarding the train early- we shall call it, let us say E, or late - we shall call it L, so there are two actions. Whatever the payoffs, here we have to be careful, what is best for, let us say, player 1? Let us look at the question once more. Out of courtesy each wants the other to board the train before him - so if I board it later than the other player that is good for me, boarding later is considered better than both boarding it early, so this is the other thing that if both are boarding early, that is, E E - that is worse.

However, boarding the train before the other player, other Nawab, is considered worse than missing the train all together. Basically, if both of them are waiting for the other to board it early, because basically both of them are choosing late, then the train leaves the station; so that is another outcome, but that outcome is considered better than boarding the train before the other Nawab.

Basically what we are saying is, here we have four outcomes: one is E L - first player is choosing early and the second player is choosing late, another could be both of them are choosing early, another could be both of them are choosing late, and another could be that the first one is choosing late and the second one is choosing early.

For player 1 what is best is this one - this is rank 1, because he is choosing late the other is choosing early, so that is the pure courtesy which is shown by player 1.

Second one is, both of them are choosing early, so this is 2; third one could be both of them are choosing late, when in that case the train basically leaves the station none of them can board the train.

The fourth one, which is the worst possible for player 1 is, he has boarded the train early and the other Nawab has boarded the train late out of courtesy, he considers this to be even worse than this - that if I board it early than the other Nawab, that is considered worse than both of them missing the train all together, because in the latter case when both of them miss the train all together, at least, I have shown courtesy to the other player, so that is considered very worth wide; so this is how the outcomes are ranked by player 1 and let us give them some numbers, so here I have 3 2 1 0.

So this is for player 1, player 1 is considering that outcome the best where he boards it late and the other Nawab boards it early. The second best is both of them are boarding it early, third is both of them boarding it late in which case the train leaves and fourth is he boards it early but the other Nawab boards it late, that is worst, this is for player 1.

What about player 2? For player 2 obviously, it will be just be the opposite - for player 2, boarding it late than the other Nawab is the best so he will give E L the highest payoff; the second highest payoff will be when both of them board it early, the third highest is both of them boarding it late, and the fourth highest is that he is boarding it early and the other Nawab is boarding it late - this courtesy shown to the other prisoner.

So these are the payoffs; let us try to look at the payoff matrix and try to find out the Nash equilibrium.

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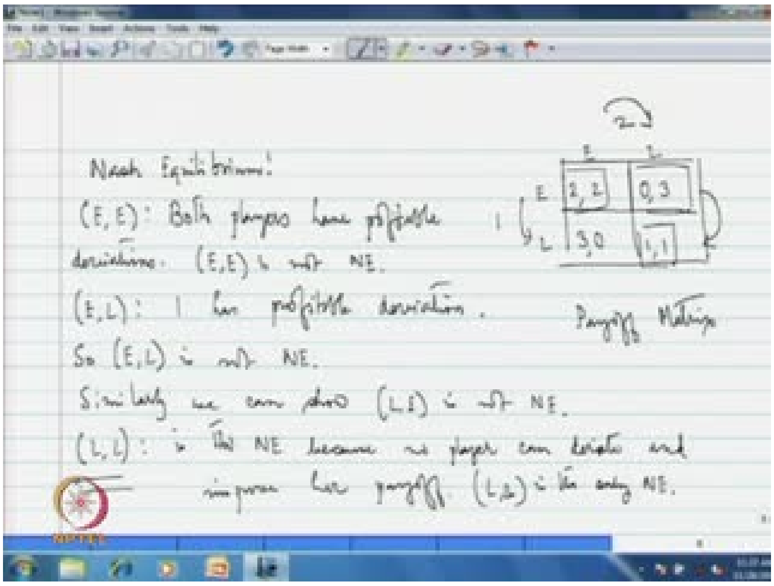

This is the payoff matrix, for player 1 - the best is I am late and the other player is early and the worst is I am early the other player is late. Second best is, both of us are early and the third best is, both of us are late and the opposite for the other Nawab. So this is the payoff matrix in this game.

In this game what is the Nash equilibrium - that we have to find out. So we follow the usual method of finding the Nash equilibrium, we start with any arbitrary action profile let us say E E.

From E E, is deviation better? Obviously it is better; both players have profitable deviation - this is easy to see, because if player 1 deviates to L he is getting 3, if player 2 deviates to L he is also getting 3 instead of 2. So, deviation by each player is beneficial, therefore, E E is not Nash equilibrium.

What about, let us say E L, so we are looking at this action profile E L. Here player 1 has a profitable deviation. Essentially, what is happening is, player 1 is choosing early, player 2 is choosing late, so player 1 is being discourteous here and that is why he is getting 0. In this case what he will do is that rather than boarding the train early he will also wait for the other player to board and so he will get 1, but obviously in this case both of them will miss the train, so E L is not Nash equilibrium.

Similarly, we can show L E is not Nash equilibrium and finally what is left is L L; L L is a Nash equilibrium because from L L nobody tries to deviate, because if anyone deviates and boards the train early, he is being discourteous and that is worse, therefore, L L is Nash equilibrium. In fact, it is the Nash equilibrium, because no player can deviate and improve her payoff. So, this is the only Nash equilibrium.

The third part to the question - does the game resemble any game studied before? Well yes, this game is just like the Prisoner's dilemma game because what we are having is the Nash equilibrium where each player is getting less than what he or she could have got, if they had played some other action profile, which is like 2 E E - in this case each player would have got 2 2 but that is not a Nash equilibrium. E E is not a Nash equilibrium we have seen that, so they are basically left with a Nash equilibrium which is worse for both of them.

So let me conclude here, thank you.

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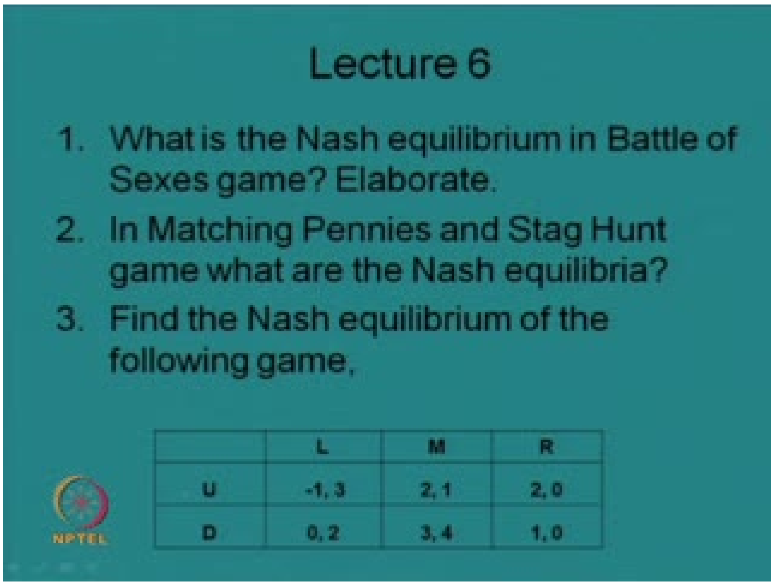

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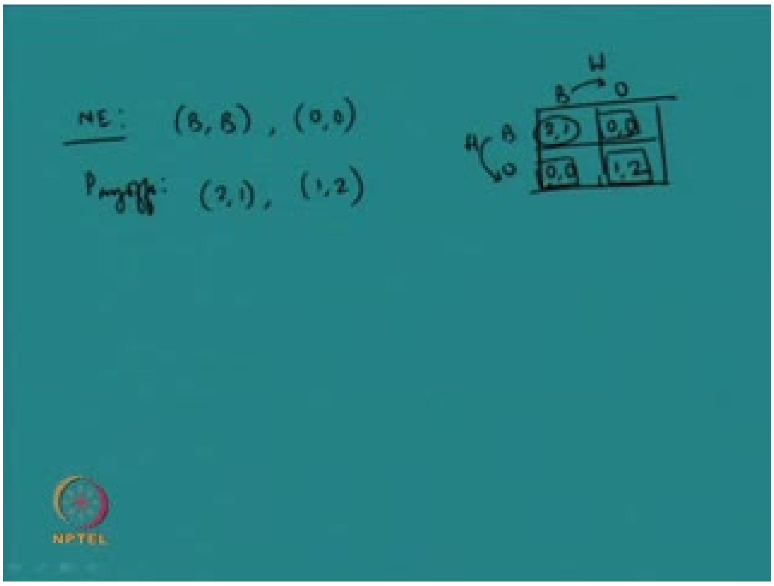

What is the Nash equilibrium in the battle of sexes game? Elaborate. Let us go back to the payoff matrix of battle of sexes game. This is what the payoff matrix looks like; and what is the Nash equilibrium? We shall see that there are two Nash equilibria here; one is B B and the other is O O; payoffs - it is 2 1 and 1 2.

Why is this, the Nash equilibrium? Let us consider B B, from this profile if player - the wife deviates to O she gets 0, if the husband deviates to O again she gets 0; so there is no reason why they should deviate. Similar logic applies to the O O profile as well, so both of these are Nash equilibria. We can similarly show that neither B O nor O B is Nash equilibria because from them we can have what is known as profitable deviation.

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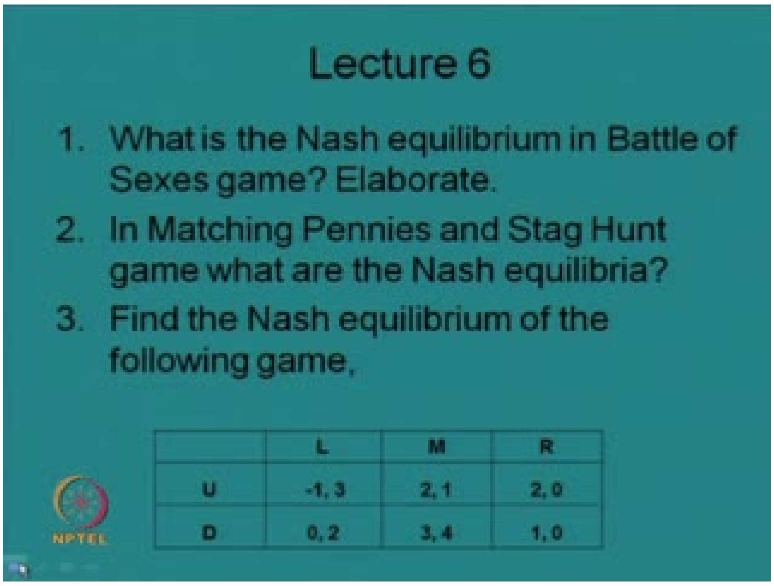

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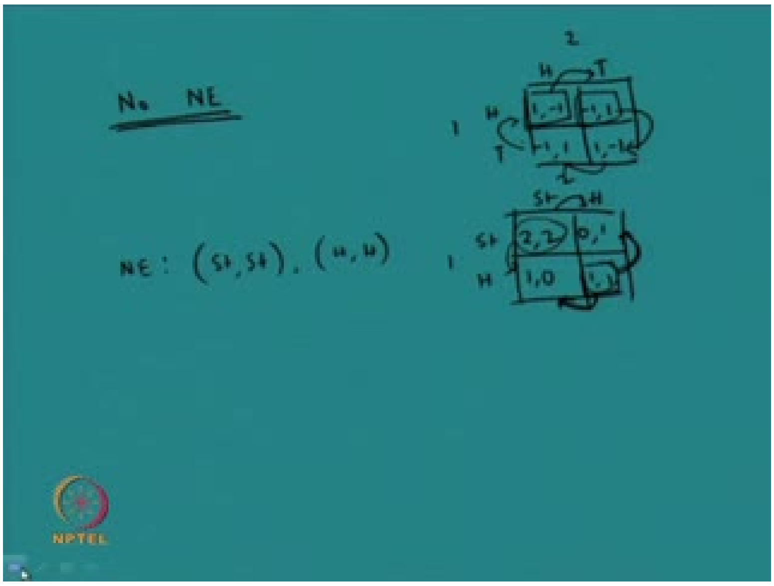

In matching pennies and stag hunt game what are the Nash equilibria? In matching pennies game, this was the payoff matrix and we can see that there is no Nash equilibrium in the matching pennies game.

For example, let us start with this H H, from here player 2 will have a profitable deviation because player 2 is getting minus 1 in H H profile whereas if she plays T she gets 1 which is greater than minus 1; but, is H T Nash equilibrium? No, because player 1 will then have a profitable deviation here because he will get 1 here whereas he is getting minus 1 here. From here again player 2 will have a profitable deviation and from here player 1 again will have profitable deviation, so basically, there is no Nash equilibrium.

What about stag hunt? In stag hunt there are two hunters; this was the game and we shall see that Nash equilibria are there; there were in fact two Nash equilibria and these are St St and H H. From St St there is no profitable deviation - neither is profitable. And similarly from H H also there is no profitable deviation; because if someone deviates, for example, if player 1 deviates she is indifferent between playing St or H, in either case she is getting 1.

Similarly for their sorry in this case player 2 is deviating, if player 2 deviates she gets 0 which is worse, in this case player 1 is deviating and if she deviates she gets 0 which is bad, which is worse; so this is not a Nash equilibrium, neither is this a Nash equilibrium. So, this 1 1 is a Nash equilibrium and 2 2 is also the payoffs in the Nash equilibrium profile.

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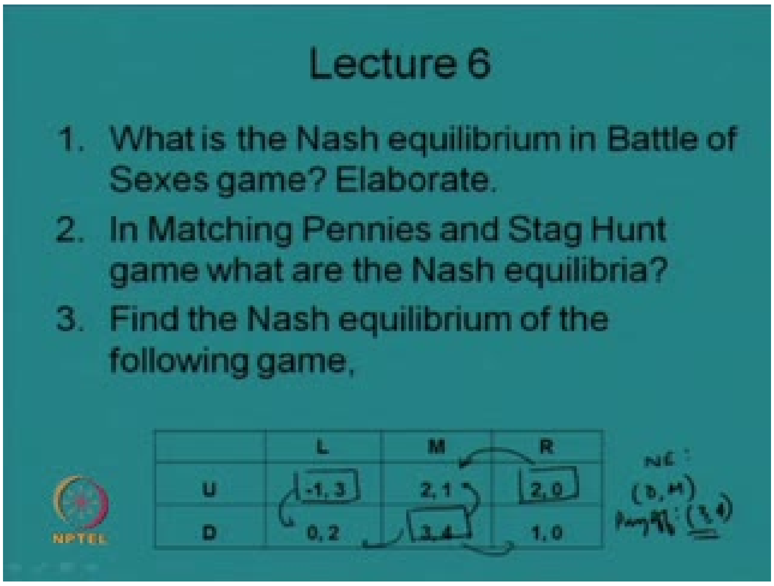

Third question - find the Nash equilibrium of the following game. If we look at this game carefully we shall find that there is a single Nash equilibrium, which is here, which is basically D M. Why? Because, from M if player 2 deviates she can deviate to R, where she is getting 0 which is worse; she can deviate to L which is again worse, it is 2. If player 1 deviates, she can deviate to U and she is getting 2 here which is worse than 3, so no profitable deviation is there. We can verify that for other profiles there are profitable deviations, for example, let us take this; from here player 2 can deviate profitably, from here - let us take this profile U L - from here obviously player 1 will deviate, so the only Nash equilibrium is D M where the payoff is 3 4.

So that is the end of the exercise, thank you.