**Game Theory and Economics**

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**Module No. # 02**

**Strategic Games and Nash Equilibrium**

**Lecture No. # 05**

**Variants of Stag Hunt Game, Hawk Dove and Coordination Game**

Welcome to the fifth lecture of module 2 of the course called Game Theory and Economics. Before we start this lecture, let me recapitulate what we have done in the previous lecture 4.

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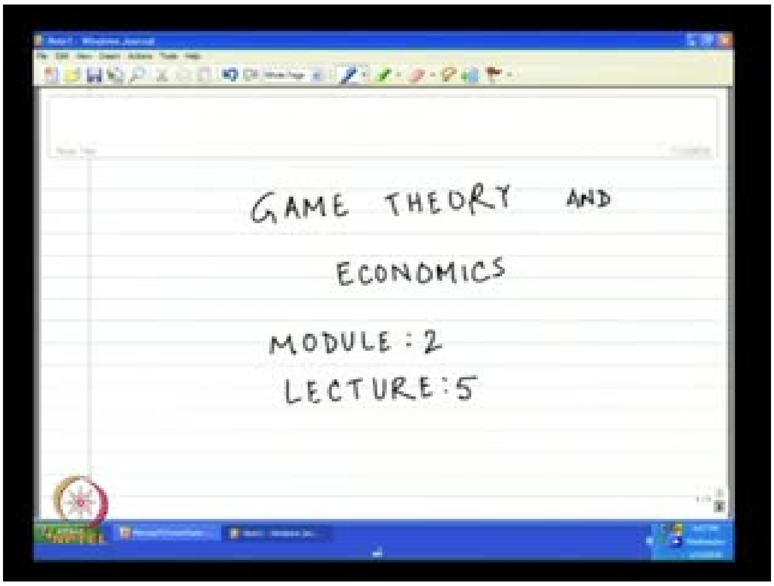


What we have done is that in the previous lecture, we have examined what is the implication of altruism in situations like prisoner's dilemma. We have already discussed the prisoner's dilemma game. Typically in a prisoner's dilemma game the players have an equilibrium, a Nash equilibrium, where the payoffs that they get is less then what they could get if they had cooperated with each other. But, they do not cooperate with each other and they try to free write and that is why since everyone is trying to free write they are pushing themselves into a situation and equilibrium situation nevertheless where out, where the payoff of each player is less compared to the case where they had not tried to free write.

So, this is basically coming from the fact that people are maximizing his or her individual payoff; we have tried to relax this assumption. We have seen that if there is altruism, what we mean by altruism is that an individual cares not only about his own payoff, but also the payoff of the other player. If that is the case then, we could get an equilibrium which is the best for every one that is the result that we got in the previous lecture.

Typically, we saw that if the degree of altruism is high enough if people care about the other player above a particular critical limit, then we can get that equilibrium which is the best possible outcome for everyone; but, if the degree of altruism is there but, it is little then, we get a prisoners dilemma kind of situation at the equilibrium is what we called Pareto inferior compare to other outcome.

We have already seen that though this situation is there, altruism could generate an equilibrium which is Pareto optimal that is, it is best for everyone concerned but, altruism per say is not good. I mean, there might be situations which are not like prisoners dilemma like situation, were altruism could in fact be harmful.

We have seen an example where in fact without altruism people were getting an equilibrium which is best for everyone but, if there is altruism we get back to the prisoners dilemma kind of situation where the equilibrium is an outcome, where people's payoffs are less then what they could get if they did not have altruism. So that is the main theme of the last lecture.

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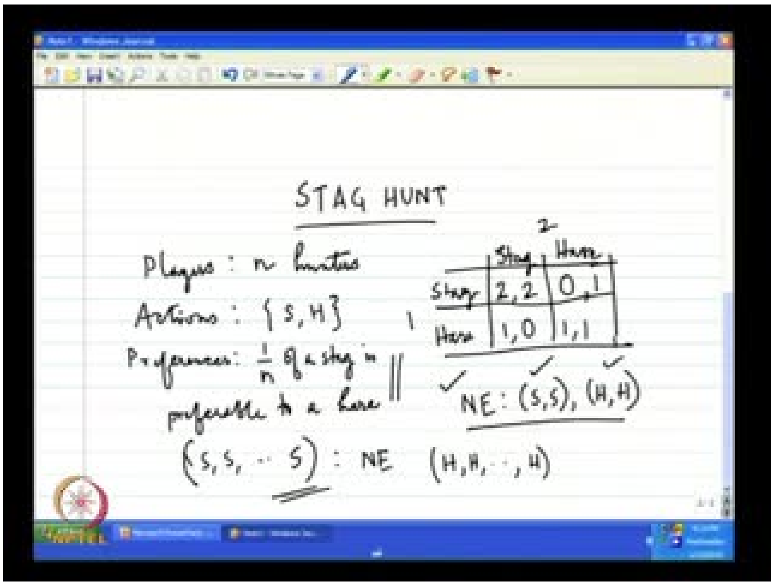


Today, what we are going to do is to consider some of the variations of stag hunt. If you remember, what the stag hunt was and what was the equilibrium in the stag hunt. So the Stag Hunt game again, there are two players and each player has 2 actions to choose from either to hunt the Stag or hunt a hare. There is a single stag in the forest which can be hunted if both the players cooperate and try to hunt that stag only. So, if both of them cooperate they get 2 each.

What is the alternative? Alternative is that, any player instead of going after the stag could go after a hare because there are plenty of hares in the jungle also; if he gets the hare, he is getting a hare so it is not bad but, it is not as good as a stag. Suppose player 2 instead of going after the stag goes after the hare he gets 1, whatever player wants. Well, it is such the case that, if the other player is not going after the stag a player 1 alone cannot be able to hunt the stag, so he gets 0, he gets nothing.

Similarly, if player 1 goes after the hare he gets 1, which is worse than the stag. In which case player 2 again will be left with nothing, he will get 0 and if both of them go after the hare they will get the respective hares, so this is 1 1. So this is the game and we have seen that if we apply the criteria now, Nash equilibrium in this game, there will be 2 Nash equilibrium.

One is s s that is both hunting the stag and the other is hare hare; that is both individually going after a hares. This is equilibrium because from stag stag that is both going after the stag, nobody will like to go after a hare because if one goes after a hare then that person, that player will be able to catch a hare but, that is worse than getting half of the stag. So this was the story.

Now, if I generalize this module little bit and consider that the number of players is instead of 2 hunters, suppose there are n hunters and his actions as before either to go after the stag for each player this is the case, one can hunt try to hunt the stag or one can try to hunt a hares; whatever the preference is, preference will change a little bit.

So it is the case that one-nth of a stag is supposed is preferable to a hare. Now if this is the case, look this is similar to the 2 player game because in the 2 player game also half of the stag was preferable to 1 hare. Here n is equal to 2 in that case but, here we are telling the general case where n could be any number, any positive integer. Now here, what will be the Nash equilibrium?

Let us from this 2 player game, if we try to get some hint maybe we should try for these two sorts of profile, where everyone is going after the stag and where everyone is going after the hare. So, is it true that in this n player game everyone going after the stag is the equilibrium. Let us see, this is equilibrium, to see that what we need to do is to see if there is a profitable deviation from this (Refer Slide Time: 09:19). What is meant by profitable deviation is that from this action profile can someone deviate and be better off.

Now, deviation in this particular game can be only of one type; either if everyone is going after the stag then you can deviate and go after catching a hare. Now obviously, this is not profitable because if one goes after a hare he gets a hare but, getting a hare is worse than getting one-nth of a stag; which is happening if everyone goes after the stag. So, this is Nash equilibrium (Refer Slide Time: 10:00).

Similarly, if I consider the other profile which might be a candidate everyone goes after the hare, is it Nash equilibrium. Again, we have to consider deviation for any player here everyone, every player is similar. So, if I consider any player suppose player I and he deviates, supposes he deviates and he tries to catch a stag, and is that beneficial for him? The answer is no, because if he goes after the stag he will not be able to catch anything because to catch a stag it must be the case that everyone tries to catch the stag; so alone he will be getting nothing, it will be a futile effort.

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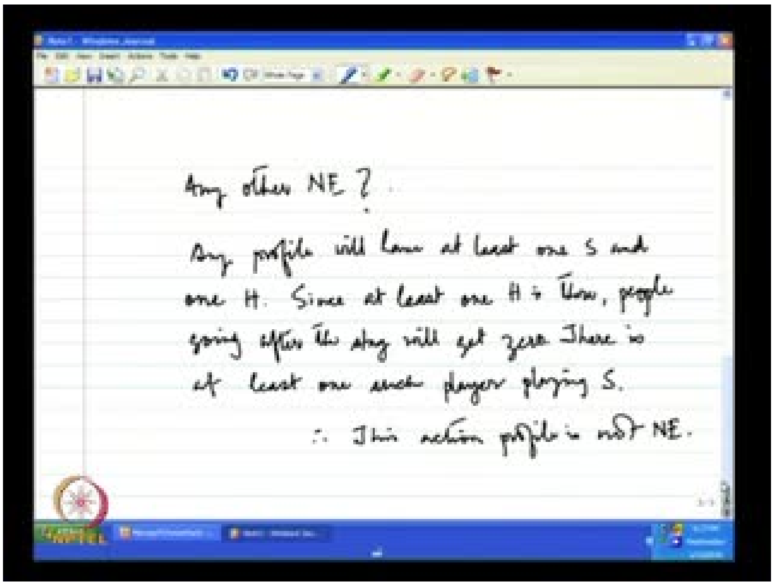


So, deviation is not profitable in fact it is harmful; that is why this is again Nash equilibrium. We have 2 Nash equilibriums, is there any other Nash equilibrium? So, any other Nash equilibrium is it possible? The answer is no, because if I consider any other Nash equilibrium then, how shall that action profile look like. That action profile must have a mixture of s and h some players must be going after the stag, at least 1 player should be going after the stag and at least one player should be going after the hare.

Now, if there is at least 1 player going after the hare, then that means the other players will not be able to catch the stag because to catch a stag, you need every one going after the stag. In that case, it is futile for all these other players whose number is at least 1, who are going after the stag, their effort is futile because they could go after the hare and be better off.

Any profile will have at least one s and one h, since at least one h is there people going after the stag will get zero and there is at least one such person, one such player playing s that is going after the stag. His action is a suboptimal; he could do better by going after the hare. So, this is not Nash equilibrium. This profile that we are considering here, we are thinking about whether this is Nash equilibrium or not. It is not Nash equilibrium because that person at least 1 person is there who could change his action from s to h and could be better off.

So therefore, this action profile is not Nash equilibrium. So that is the proof that there is only 2 Nash equilibrium in the game where the number of hunters can be anything it can be any positive integer, it can be more than 2. What we are going to do after now is that we can consider another a little complicated version of the same stag hunt case.

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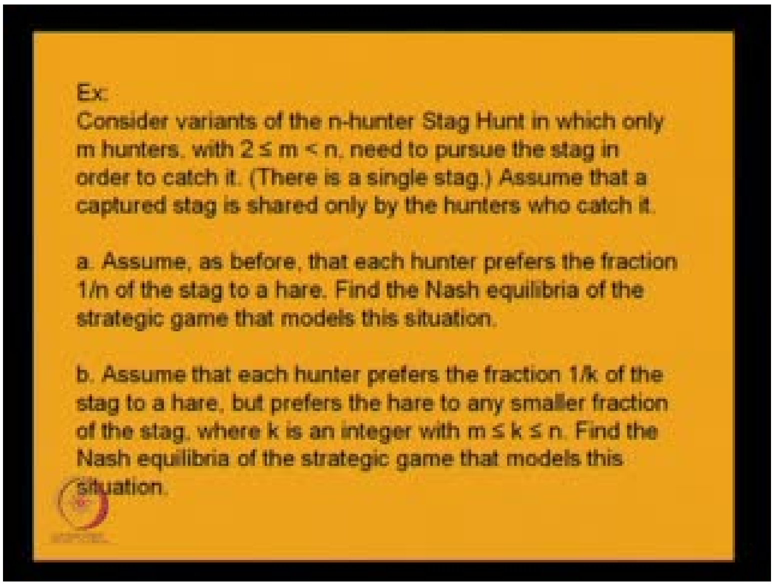


Let me tell you the question here. So here is the variant, let me read out the question first. Consider variants of n-hunters stag hunt in which only m hunters, where m is greater than equal to 2, strictly less than n need to pursue the stag in order to catch it, that is single stag. Assume that is captured stag is shared by the hunters who catch it.

This is the set up that you need not have all the n hunters going after the stag to catch it. If the number of hunters is less then n, suppose this is m, where m is greater than equal to 2, then that will be sufficient to catch the stag. Now, what is the question; question is that under part a, our assumption is that as before each hunter prefers the fraction one-nth of the stag to hare, find the Nash equilibrium of the strategic game that models this situation.

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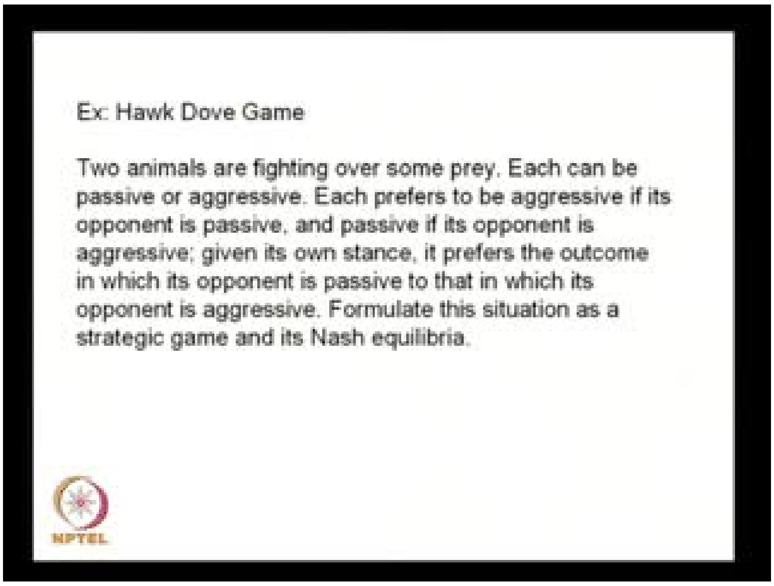


Let me take you to another sort of game which is similar to some of the games that we have seen. It has 2 Nash equilibriums; this is known as a Hawk Dove Game. This is given as an exercise in this power point. Here is the story; two animals are fighting over some prey, each can be passive or aggressive. So number of players are 2, two animals each can be passive or aggressive, so these are the actions.

One can be passive or one can be aggressive, each prefers to be aggressive if its opponent is passive and passive if it is opponent is aggressive. Given its own stunts, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game and find its Nash equilibria.

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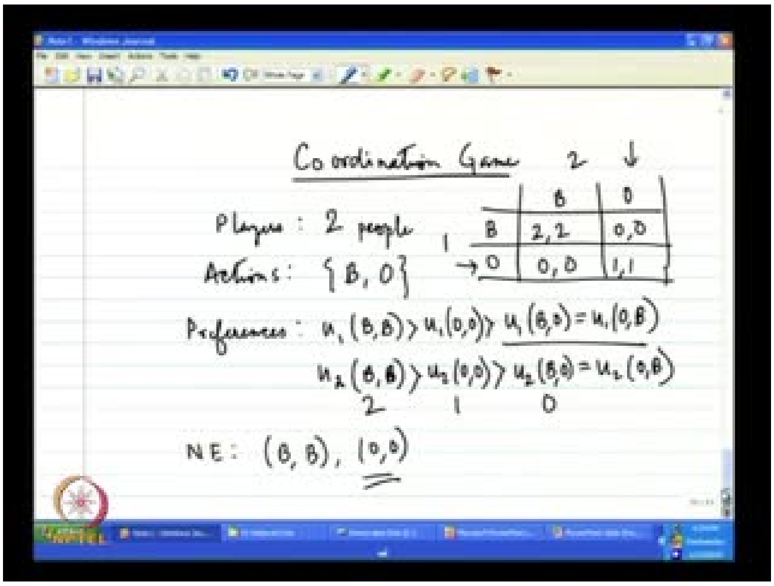


Let me discuss another game before we finish this lecture. This is known as coordination game. Coordination Game can be thought of as a variant of the battle of sexes game. Here again like the battle of sexes game, there are 2 players. So players, 2 people let us suppose and like the coordination battle of sexes game, they are planning to go either to a boxing match or to an opera house. How about the actions are, though the actions are like this, preferences are little different.

Here suppose in battle of sexes, the husband and wife the 2 people differed on which is the better one to go. One player is thought that boxing match is preferable to the opera and the other player thought that the opera is preferable to the boxing match.

Here suppose both of them think, both of them prefer the boxing match to the opera and both of them want to be together that is remaining constant as in the case of battle of sexes. So it is like this that in O B or B O the last 2 they are not together, so that is the worst possible. For player 1 going to the boxing batch together is better than going to the opera house together (Refer Slide Time: 19:09).

However, this is the same case for player 2 also; his preference is not different. The numbers we attach to them will be like before 2 1 0 then, what is the sort of game that we get? The sort of game that we get is known as the coordination game. So B B is better for both of them, O O is worse and the worst possible is O B and B O.

Now in this game, what is the equilibrium? The Nash equilibria in this game if I examine this game carefully I will find that there are 2 Nash equilibria one is B B and the other is O O. That is both of them going to the boxing match together is a Nash equilibrium and both of them going to the opera house together is also a Nash equilibrium.

Though there are these 2 Nash equilibria, it is clear from this game itself that one Nash equilibrium that is B B is better than the other Nash equilibrium which is O O but, does that mean, this O O Nash equilibrium which is the worse Nash equilibrium which we defined as a Pareto inferior state and this is a Pareto superior state. Does that mean that Pareto inferior state is not going to prevail, the answer is no.

If I follow the definition of Nash equilibrium of course, there can be a stable steady state at O O each of the players is going to play O, though they both of them know that this is a worse situation than the case were both of them were playing B B and the reason for this is that I cannot make the other people other player act according to my whishes I have command over what I am going to do but, unified action is not considered in non-cooperative game theory which we are doing right now.

So if I know that player 2 is now going to play O then, the best possible thing for me to do is to play O, there is no other way and vice versa.

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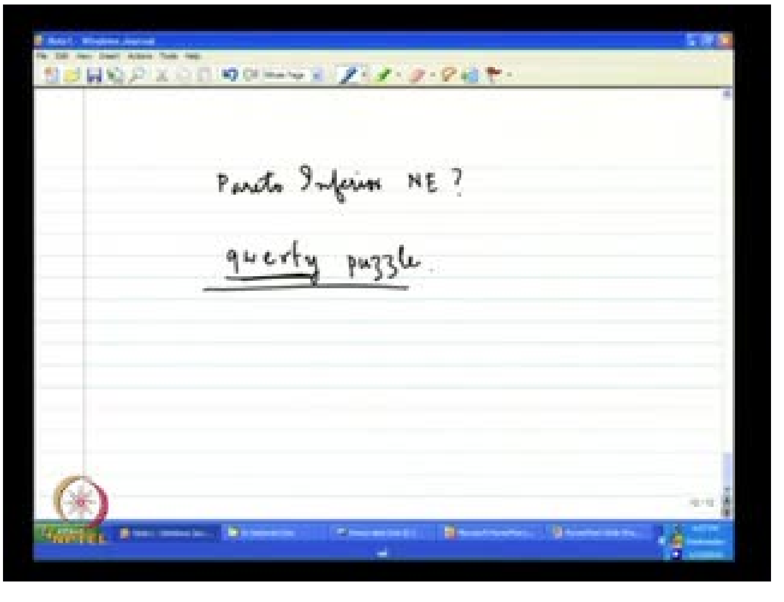

Now, can this situation that we have discussed here that is Pareto Inferior Nash equilibrium, is this realistic? Do people get in to this kind of trap where there in equilibrium but, this not desirable. The answer is perhaps yes, if we look in to real life cases, we all many a time find situations where people are stuck with actions which none of them wants to take, but still they do because others are taking that action.

One could be this which we call QWERTY puzzle. What it means is that this qwerty is nothing but, the keys of a typical key board, may be computer key board or may be type writer key board; q w e r t y these letters come on your left hand on the top. Now, this kind of key boards that we use is not the best sort of key board that can be devised, can be designed.

In fact when this sort of key board was designed, it was to reduce the speed of the type writers people who use to type, their speed is to reduce because of this sort of, this design of type writers. This sort of key boards one reason could be that most of the peoples are right handed but in this key board, the latest which we use most for example, e a r s all of them come under left hand, so this is not the best key board that can be designed.

This is in fact suboptimal key board but still people have continued to use this key board and why is that? Because everyone insist doing that. So, each company which may be makes this computer machines and key boards finds that everyone is buying this sort of key board.

If I do something else which is might be better if everyone stuck to that better key board everyone would have been better off but, I as an individual if I try to do something else, my keyboard may not be sold in the market. So I still stick to the one of the main key board, which is an inefficient key board and which is the q w e r t y key board.

So the point is that, if there is a Nash equilibrium which is a Pareto inferior Nash equilibrium, there is no way to rule that out as a steady state, it is Nash equilibrium by its own, right. We are going to do an exercise on this topic of stag hunt; this is an extension of stag hunt. Let us first write down the exercise and then we shall try to solve this exercise ((no audio from 27:00 to 31:00)).

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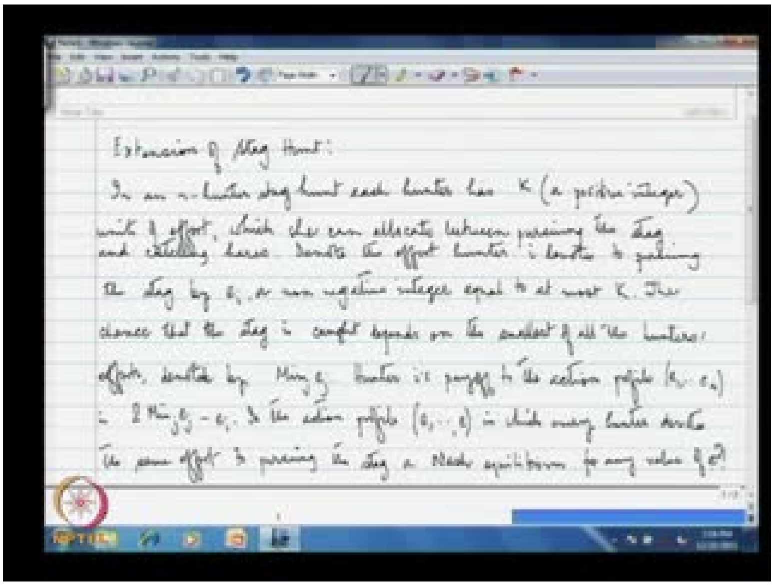

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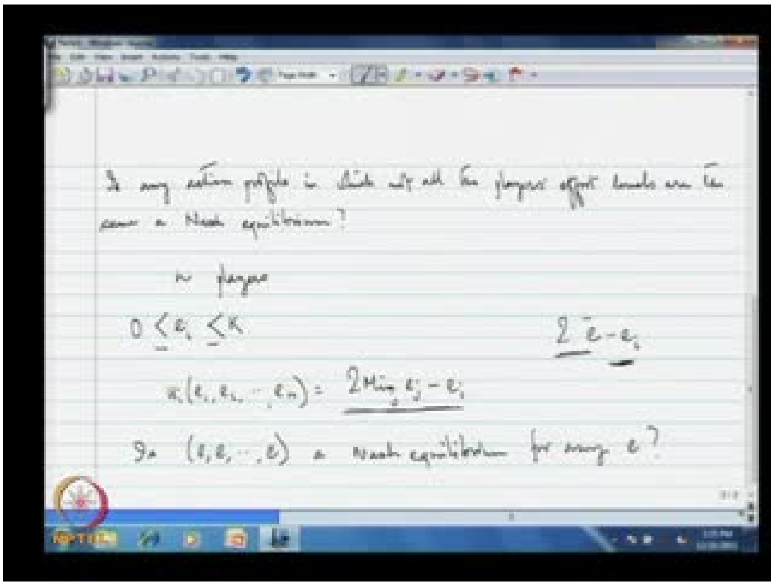

This is the question; let me read out the question. This is an extension of the stag hunt game and the question is the following: In an n hunters stag hunt, each hunter as capital K, a positive integer units of effort which we can allocate between pursuing the stag and catching hares. Denote the effort hunter i devotes to pursuing the stag by e i and non-negative integer equal to at most capital K. The chance that the stag is caught depends on the smallest of all the hunters efforts denoted by minimum j e j hunter i's payoff to the action profile e 1 e 2 …. e n is two minimum e j minus e i.

Is the action profile e e e for there are n e’s in which every hunter diverse the same effort to pursuing the stag and Nash equilibrium for any value of e. This is the first question and there is second part to the question which is that is any action profile in which not all the players effort levels are the same on Nash equilibrium.

So to put the story in a succinct way there are n players and what they are choosing is e i that is effort level that hunter i puts to hunt the stag and e i is less than or equal to capital K. We know e i can be an integer and capital K is also an integer and e i is greater than equal to 0. This is their action and what is the payoff? Payoff is given by the following: so u i e 1 e 2 e n is equal to this is the payoff to player i.

Now before we go further, let us look at this payoff function more considerate way here. What we are having is 2 of minimum of e j minus e i, which means that given the effort levels put by all the hunters we take up the minimum of those efforts levels. This minimum effort level is let us say any value e bar and then their player once payoff putting e i is twice e bar minus e i.

The underlying story behind this is that if the minimum level of effort put by all hunters goes down then the stag might escape their hunt and in that case, they will not be able to catch the stag, which means that the payoff of this particular hunter goes down from putting any effort for the stag hunt.

So that is why e e bar which is the minimum of e j’s is coming as a positive entry here; as the minimum goes up there is higher chance that the stag will be caught. Then there is a negative entry here, which is minus e i and which is quite intuitive more effort is put on the hunt of stag; it is worse, the hunter is worse off, in the sense that he has now less effort level left to pursue the hares. So that is why minus e i is coming as a variable in this payoff function.

So this is the intuition behind the payoff function, we have to answer two questions. First is, action profile e e e dot e there are n e’s in which every hunter diverse same effort to pursuing the stag a Nash equilibrium for any value of e. So that is the first question and our answer is the following that for all e’s all integral values of e less than equal to k e e e is a Nash equilibrium and the reason for this is simple.

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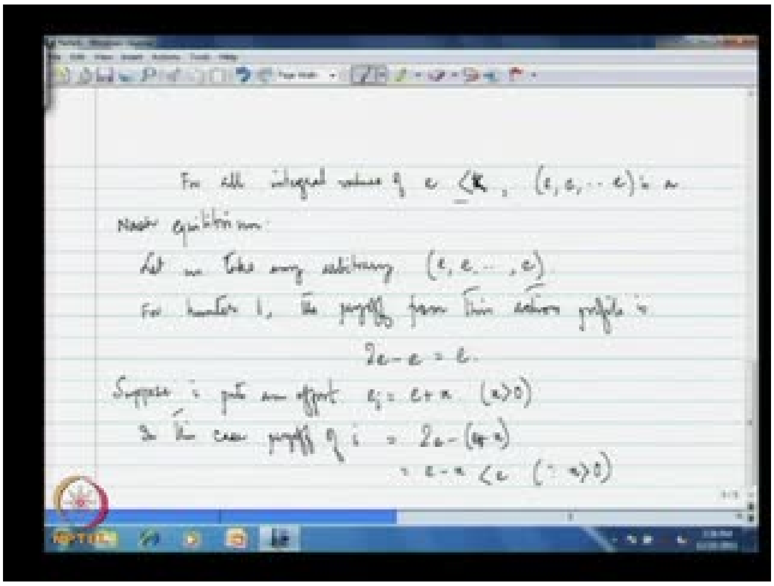

Let us take any arbitrary such profile this profile now for hunter i the payoff from this action profile is what is the minimum of all e's it is e itself, so 2 e minus his effort which is again e and which is e.

So this is the payoff that he is getting, any hunter is getting, from this action profile e e e. Now to say that this is Nash equilibrium, we have to consider deviations. For any player i, there could be two kinds of deviations. One is putting an effort more than e or putting an effort less than e.

Suppose, i puts an effort e i which is e plus let us say x, where x is positive and n x has can take only integral values. Now in this case, what is the payoff of i? It is 2 of minimum of all these e's which is again 2 e minus e minus, minus of e plus x and which is less than e because this is nothing but, e minus x. This is less than e because x is positive. So that is why deviation upwardly or putting an effort more than the other, what the others are putting is unprofitable; so upwardly deviation is unprofitable.

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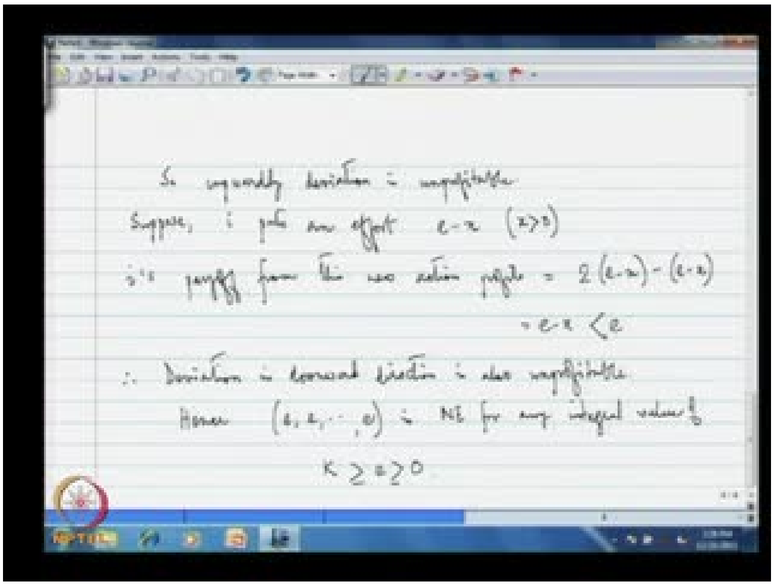


Now, let us consider downward deviations. Suppose, i puts an effort e minus x, where x is again positive, so what is i’s payoff? i’s payoff from this new action profile will be how much? It is minimum of all these which is 2 of e minus x minus of e minus x which is e minus x, which is again less than e.

So, downward till the deviation in downward direction is also unprofitable hence e e e e is Nash equilibrium for any integral value of e greater than equal to zero less than equal to k. This is the first part, there is a another part second part which is saying that is any action profile in which not all the players effort levels are the same in Nash equilibrium, so this is the second part.

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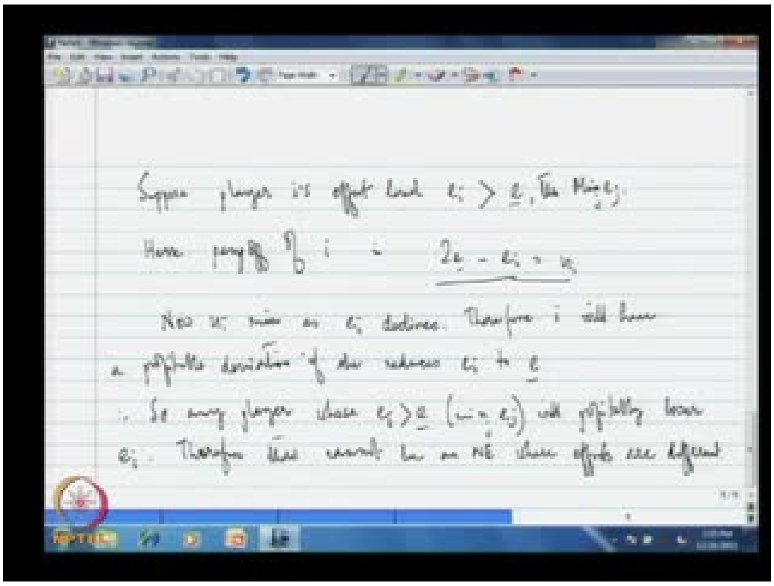

Suppose player i player i's effort level e i is greater than let us call it e lower bar the minimum of e j. So player i is a player whose effort is more than the minimum of all the efforts put by all the players, can that situation be sustained as Nash equilibrium? We have to show that if that cannot be sustained as Nash equilibrium then perhaps, players putting different effort levels cannot be Nash equilibrium.

So here, payoff of i is how much? It is 2 e lower bar minus e i because minimum of e j is e lower bar that therefore, we have 2 e lower bar minus is own effort which is e i. Now this is u i, u i rises as e i declines; as e i goes on declining this u i will rises because we have a minus e i term here. Therefore, i will have a profitable deviation if she reduces e i to e bar because this will happen at this payoff function is valid as long as e lower bar is less than e i. So, as e i goes to e lower bar this payoff is going to rise.

Any player whose effort level, whose e is more than e lower bar the minimum will profitably lower e i; therefore, there cannot be Nash equilibrium where efforts are different. The only Nash equilibrium that there can be in this game is where efforts put by all the players are same. So that is the exercise, we shall do 1 example and then close this lecture.

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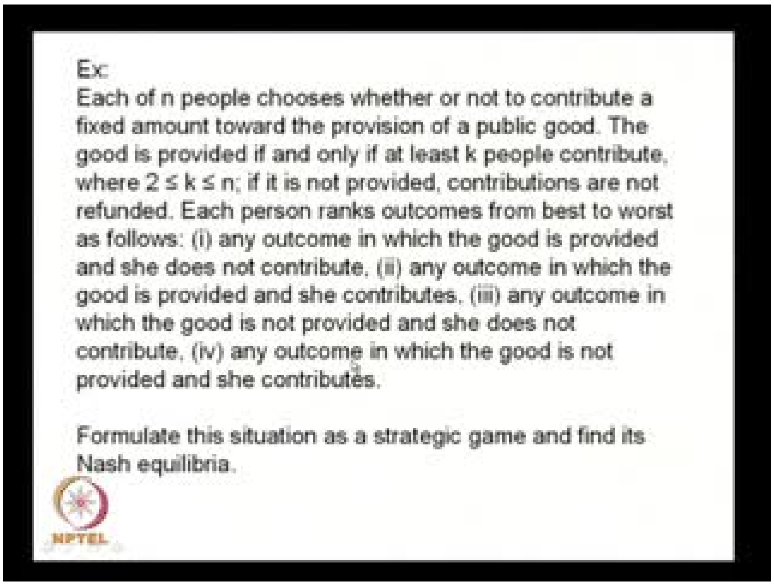


The example is the following; this is also known as public goods provision game. What is a public good? Public good is something which has been produced and provided to the public and I cannot restrict any one from consuming that good. So what happens is that suppose, there is a road which is not crowded and in that road obviously, if it is constructed every can take make use of that road but, you need money to construct a road. Now, if I contribute to the construction of a public good, there is no way I can exclude someone from using that.

Here, we have a typical case of free riding that I contribute but, everyone is using that of course, I am also using that road I am getting some benefit out of it but, I cannot restrict any one from using that particular facility. Now if that is the case then, will people contribute; under which conditions people will contribute or can there be situation where nobody contributes.

So this is called the provision of public and the problems related to that. So question is the following. Let me read out the question; each of n people chooses whether are not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where k is less than equal to n and k is greater than equal to 2; if it is not provided contributions are not refunded. So which means that, if I am contributed for the construction but, suppose not many people have contributed so in that case the good will not be produced; no public good is there but, I am not going to get back the money if it is not provided the contributions are not refunded each person ranks the outcome from best to worst as follows.

One: any outcome in which the good is provided and she does not contribute, so this is the best possible for any person. Second is any outcome in which the good is provided and she contributes. Third is any outcome in which the good is not provided and she does not contribute and fourth is any outcome in which the good is not provided and she contributes. Formulate the situation as strategic game and find its Nash equilibrium. This is public good game.

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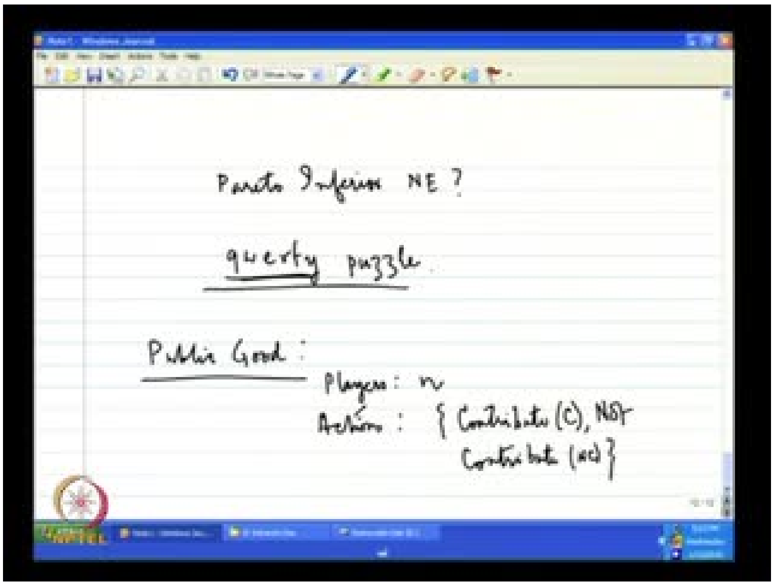


Players are n in number, actions - what are the actions? Either to contribute are not to contribute - so contribute let us call it c and not contribute c and whatever the preferences where this is the interesting part.

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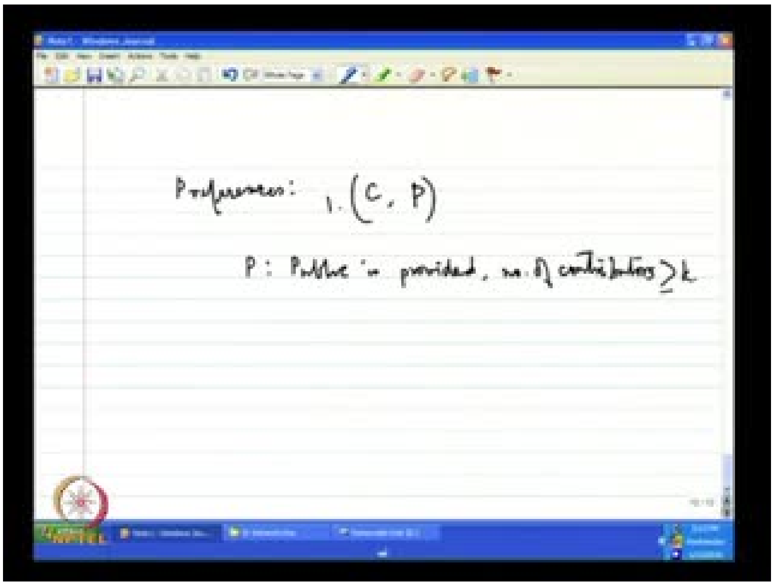

First option is that I am free riding over others; that is other peoples have contributed the public good has been produced. So, there is the number of the contributors are more than or equal to k. There is no (( )) of contributions but, I have not contributed, so I am getting some benefit without contributing anything.

So c that is I have contributed, and let us call this p; p is standing for public good is provided. That is number of contributors is greater than equal to k, so this is the first outcome that I shall prefer, this is the best outcome for me. We are more or less running out of time, so I shall finish this lecture right now and we shall discuss this problem in the next lecture, so that is all for today. Thank you.

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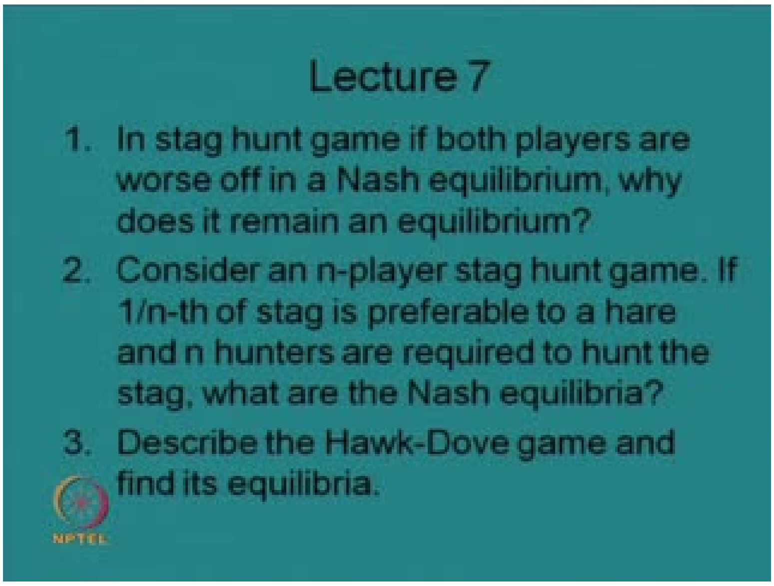


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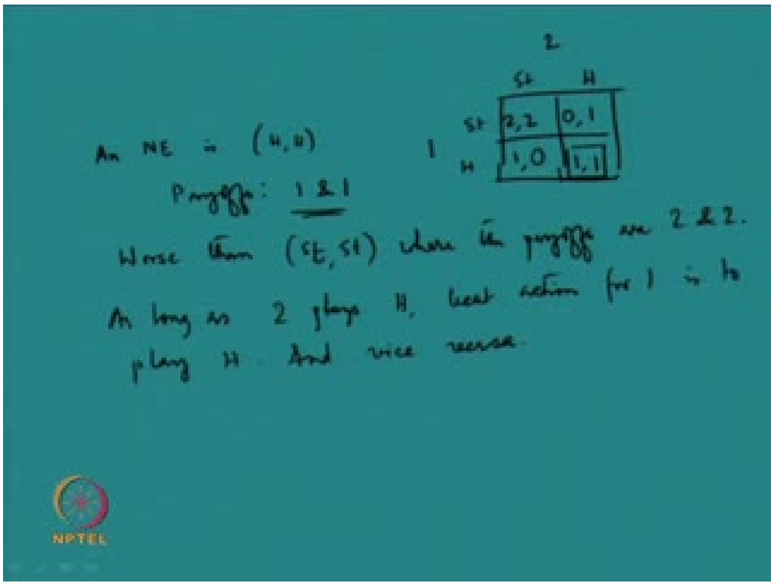


The first question is in stag hunt game, if both players are worse off in a Nash equilibrium why does it remain an equilibrium? The basic idea of Nash equilibrium is being question here. So in stag hunt let us remember, how does it look like; there are 2 players, 1 and 2, 2 hunters. What is being asked is that suppose there is a Nash equilibrium which is Pareto inferior where both players are worse off in Nash equilibrium then why does it remain Nash equilibrium.

We have a Nash equilibrium here, which is H H payoffs are 1 and 1 it is here the players are worse off than say s t s t where the payoffs where 2 and 2 but, still this is a Nash equilibrium by its own, Why? Because go back to the very definition of Nash equilibrium as long as player 2 is playing H, as long as 2 plays H best action for 1 is to play H and vice versa.

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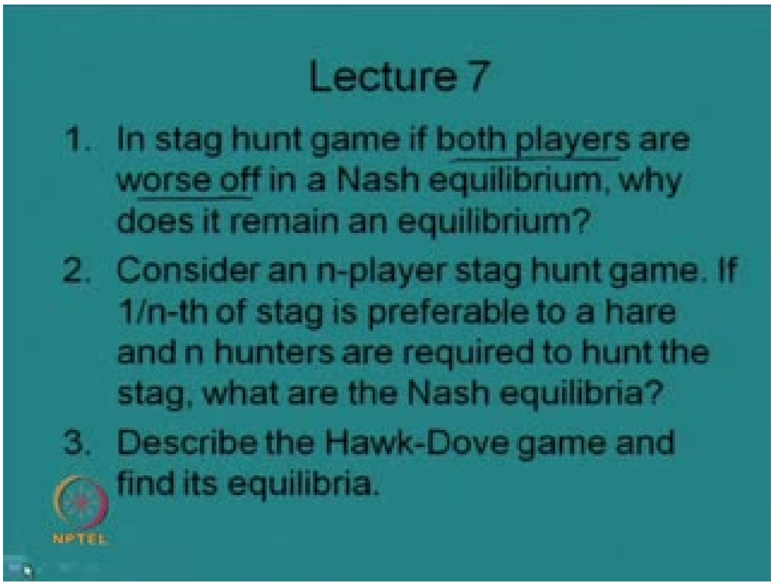


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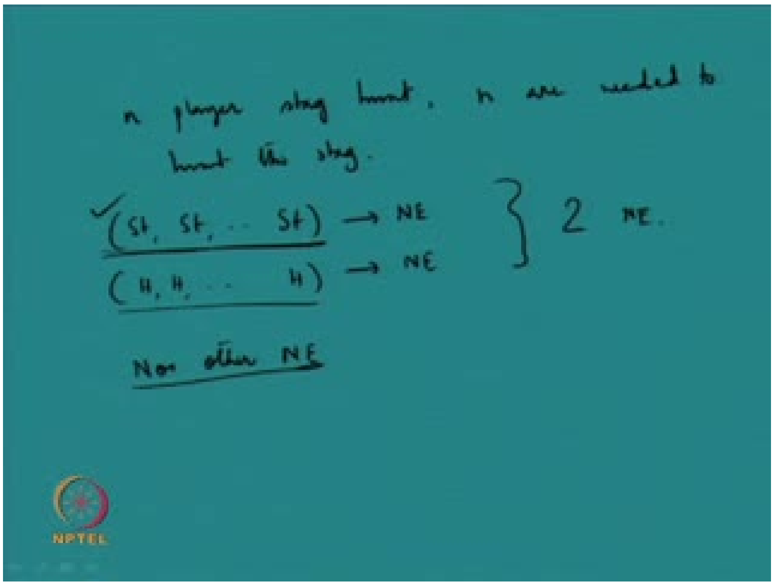

Though here both players are worse off, it is Nash equilibrium. Consider an n player stag hunt game if one-nth of stag is preferable to a hare and n hunters are required to hunt the stag, what is the Nash equilibrium?

So n players stag hunt and n are needed to hunt the stag. What I proposed is that following are the Nash equilibrium: this is Nash equilibrium and this is also Nash equilibrium and in all 2 Nash equilibrium. The reason is that if everyone goes after the stag there is no reason why any individual player will go after the hare, because one hare is worse than one-nth of a stag.

So, that is why this is a Nash equilibrium. Why everyone going after the hare is Nash equilibrium? Because, from here if any one starts to go after the stag she will not able to catch the stag, so she will get 0. Here, at least she is getting 1 hare, which is better than getting nothing.

Both are Nash equilibrium, is there any other Nash equilibrium? No other Nash equilibrium, Why? Because in any other profile where at least 1 player is going after the stag right there could be more than 1 players going after the stag. In such cases, the stag will never be hunted because you need all the n players to go after the stag but, that is also ruled out, because this is taken care of here.

The number of stag hunters is here more than 0 but, less than n which means that these players were going after the stag, will not able to catch the stag. So it is better for them to go after a hare that is why any other profile is not Nash equilibrium.

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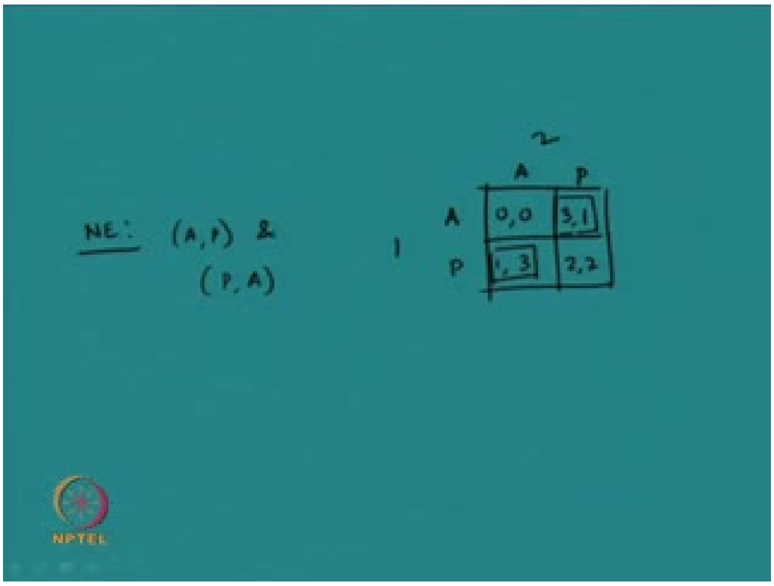


Describe the Hawk Dove game and find its equilibria.

The Hawk Dove game is the following game, I am just drawing a payoff matrix; as usual there are 2 players and their actions are being aggressive or being passive. If both of them are aggressive they get 0 0, both of them passive they get 3 1. If both of them are passive they get 2 2, if one is aggressive - the first player is aggressive, the second player is passive they get 3and 1 and vice versa.

What is the Nash equilibrium here and what is the Nash equilibrium here? There are in fact 2 Nash equilibria here, one is aggressive passive and the other is passive aggressive; that is this and this (Refer Slide Time: 58:58). You can check that there is no profitable deviation for either of the players from this two equilibrium profiles.