Game Theory and Economics

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Module No. # 02
Strategic Games and Nash Equilibrium
Lecture No. # 06
Public Good Provision, Strict Nash Equilibrium

Welcome to the 6th lecture of module 2 of the course called game theory and economics. Before we start this lecture, let me try to briefly recapture it what we have done so far. We have defined what Nash equilibrium is - I have given the definition of Nash equilibrium. So far, we have been trying to analyze different game theoretic situations and how to apply the idea of Nash equilibrium and try to find out as to which are the outcomes or which are the actions that will be taken by the players and which will be steady state in the sense that they will be repeated over and over again. This steady state kind of outcome is something which is the basic cracks of Nash equilibrium.

They are a kind of social convention, given the fact that people have been playing a set of actions, they will continue to play that set of actions, because from each individual's point of view, he expects the other players will continue to play the actions that they have been playing. So, there is no reason for him to deviate and do something else and this is the idea of Nash Equilibrium.

The exercise that we were trying to figure out in the previous class was the case of public good game.

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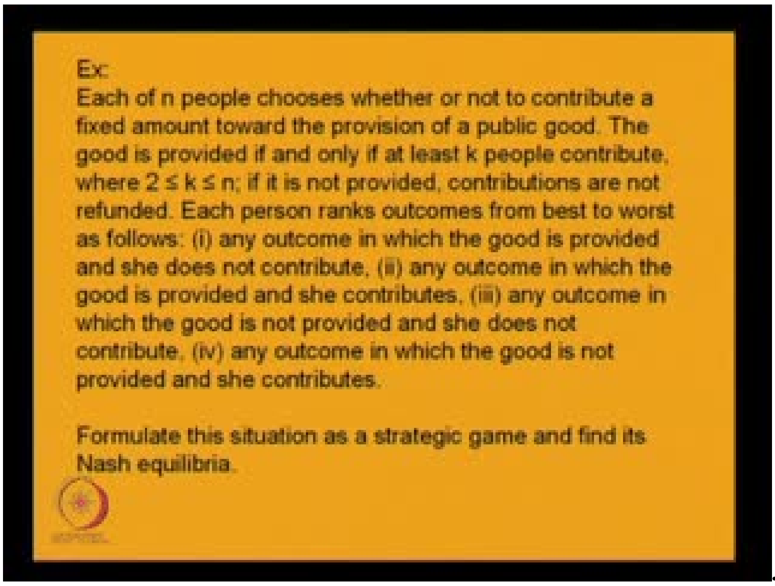

Here is the exercise once more to refresh our memory. Each of n people chooses whether or not to contribute a fixed amount toward the provision of a public good. The good is provided if and only if at least k people contribute, where 2 is less than or equal to k less than equal to n; if it is not provided, contributions are not refunded.

Each person ranks outcome from the best to worst as follows: First, any outcome in which the good is provided and she does not contribute; two, any outcome in which the good is provided and she contributes; three, any outcome in which the good is not provided and she does not contribute and four, any outcome in which the good is not provided and she contributes.

Question is, formulate the situation as strategic game and find its Nash equilibrium? Before, we start to find out the answer to this exercise let us be clear about what is a public good.

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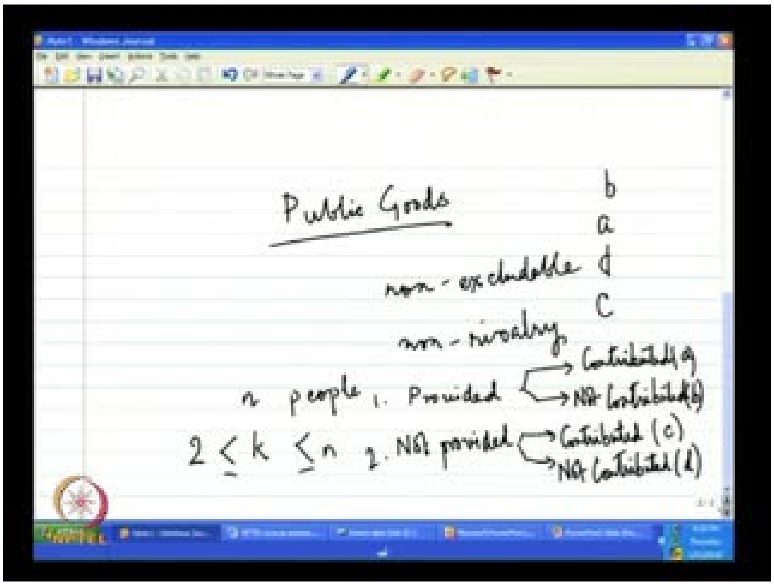

Public good in the economics has two characteristics: one is that this good serve what we call non-excludable which means that nobody can be debarred from using a public good. For example, take the case of a public park, if it is an open public park nobody can be told to just go off the public park, everybody has an equal rights to go to the public park. So, I cannot exclude anyone from using a public good or take the case of a government road, nobody can be debarred from using that road, so it is not non-excludable.

The characteristic of a public good is that non-rivalry and what it means is that if I am using a public good then, I do not come in the way of any other person to use that public good, so this is another important feature of a public good. Again, take the case of a road, if I am using that road is not that anybody else cannot use the road he or she, can obviously use the road but, this is different from a private good which is not a public good.

In case of a private good, if I am consuming a private good nobody else is consuming that; for example, if I am wearing a shirt, then it is obvious that anybody else is not wearing that shirt. So, a shirt is a good where rivalry is there whereas, a road is a good where rivalry is not there, so a road is a public good. The very fact that in public goods is people cannot be excluded that creates a problem, because look when I try to see, when I try to build a public good I need money.

Now, if I need money I have to collect that money from the users but, if I cannot exclude anyone from using that good, how am I going to say to that person, I cannot allow you to use it unless you pay, because I cannot exclude him. If I cannot exclude him then, I cannot post the threat to him that if you do not pay you will be excluded, because it is not excludable good. So, there is a very inherent problem in public good that people cannot be excluded and if people cannot be excluded, it will be difficult to make them pay. So that is a problem of provision of public good.

Now, this exercise is trying to capture that basic problem of a public good. What was the problem? Once again, there were n people and all of them can use this public good but, to construct this public good, to make it happen you need k contributions; contributions from not n people but k people, where k is greater than equal to 2 less than equal to n.

All of these people, who are agreeing to contribute are going to pay the equal amount of money. So, once I have decided that I am going to contribute the amount of money that I am going to contribute is fixed. There is a problem that suppose, I do contribute and it so happens that the number of total contributors is less than k, in that case, there is no sufficient amount of money. If there is no sufficient amount of money, the good will not be produced apart though the good is not being produced I am not going to get back that money, so that money is not being refunded.

There is a problem here, because depending on the total number of people I may not like to contribute and not only that remember if without me paying there are k contributors, there is no reason why i should contribute, because I am going to use that good anywhere it is a public good, I cannot be excluded. So, there are four possibilities basically, there are two possibilities but each of these two cases can be divided into two further cases from any individual's point of view. One is, good is provided and the other is not provided.

Now, if the good is provided again there are two cases from any individual's point of view. That individual has contributed and has not contributed, let us call it a, this is b (Refer Slide Time: 09:03). If it is not provided again the same thing will happen, if the good is not provided there are two possibilities, this individual that I am talking about any arbitrary individual has contributed or he may not have contributed, so there are four possibilities.

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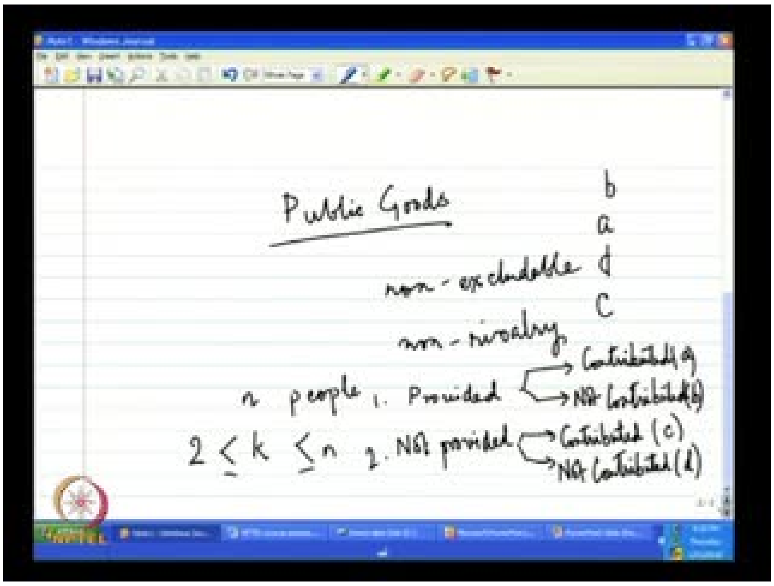

Possibility one that is provided, good is provided will happen only if the number of contributors is equal to k or greater than k. If the number of contributors is strictly less than k, the good is not going to be provided and what is the preference? I will always like the b possibility best that the good is provided but, I have not contributed. So, it is such a case that the number of contributors here is either k or greater than k but, I am not one of them. Here, I am free riding; I am not paying any money and using that public good.

What is the second best for me? Second best is which is that I have contributed and the good is provided. Third best is, I have not contributed and the good is not provided, so that is d; d is my third best. Last is, the worst possible case for me is c, where I have contributed but it so happens that the number of total contributors is less than k in which case the good is not provided and my money is gone, I am not going to be refund it, so it is a total loss for me. So, this is the ordering them b, a, d, c. Question is, what is the set of Nash equilibria in this game?

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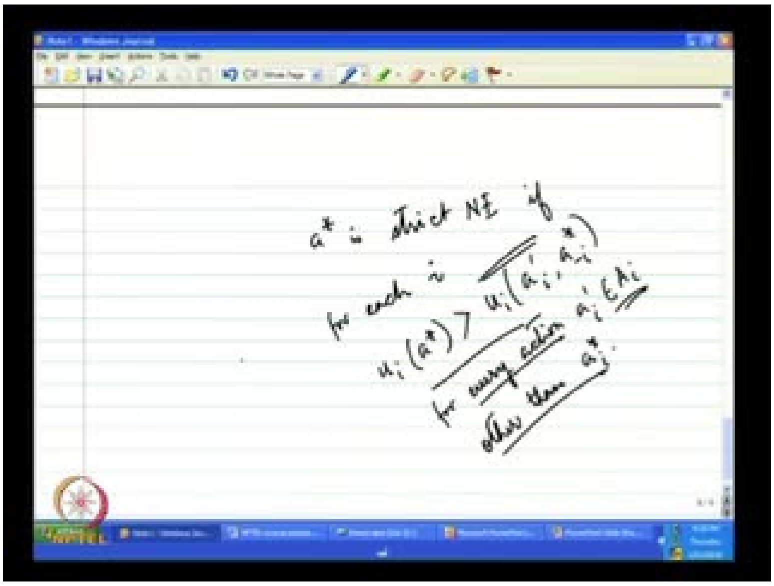

Now, let me introduce another definition here; this definition is the definition of Strict Nash Equilibrium. If you remember the definition of Nash equilibrium in general, it was that for every player i this must happen (Refer Slide Time: 12:08). That is, small a star, a star is a vector it is like a 1 star, a 2 star, somewhere here there is a i star the general term and a n star is the last term.

Now, the idea is that by playing the stars, if everyone plays the stars actions then individual i is getting this much u i a star. Now, suppose everyone else goes on playing the star actions that is the Nash equilibrium actions and the individual i deviates and he plays some other action from his action set - capital A I is the action set of player i - from that he plays any arbitrary action. Then, the definition of Nash equilibrium is that by playing any other action he cannot be better off. He can be doing as good as he was doing by playing the Nash equilibrium action, he can be doing worse by playing the other actions but, he cannot be better off, so that is the definition.

In strict Nash equilibrium what happens is that if I play some other action not the Nash equilibrium action but, some other action then I am strictly worse off. Remember here, I am open to the possibility that the player plays some other action and the payoff is just equal to his Nash equilibrium payoff. So, he can deviate and be just as much satisfied as he was in the Nash equilibrium but, in Strict Nash Equilibrium if he deviates, he is sorely worse off. So that is the idea, so how do I write it mathematically? So, a star is strict Nash Equilibrium if for each i.

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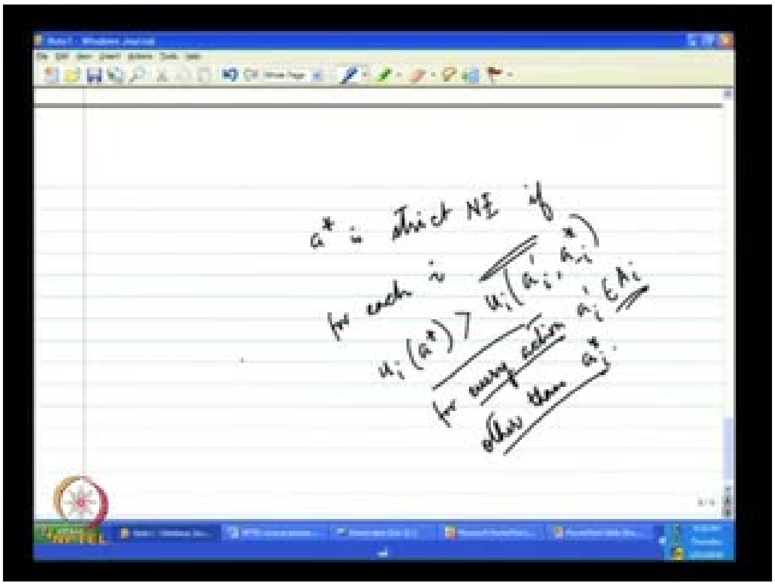

Now, important two differences are there, first I am taking the strictly greater than sign, this not greater than equal to sign. Secondly, I cannot include a i star when I am talking about a i dashed. If you have noticed in this definition of Nash equilibrium, for all a i dash in capital A I, so this a i dash could have been a i star also in which case there is a equality here, this will turn into an equality. I do not want to do that I want to consider only those actions which are not Nash equilibrium action. So, for every action a i dashed in capital A I other than a i star and for them it must be the case that the person is getting strictly less, this is strictly greater than.

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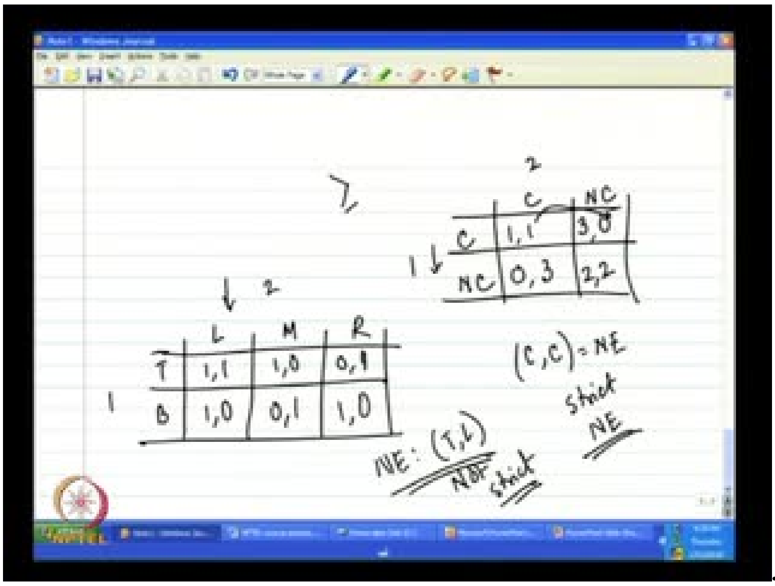

Now, what are the examples of strict Nash equilibrium? If you have noticed in all the games that we have considered so far, the equilibria that we have generated are in fact all strict Nash equilibria. Let us take any case of Prisoners dilemma for example, so this was the structure of Prisoners dilemma game and we found that there is a single Nash equilibrium at c c; c c is the Nash equilibrium.

From c c given the other player is playing c if anyone deviates, suppose player 2 deviates, he is getting 0. Now if c is less than 1, so from player one's point of view at least it is a strict Nash equilibrium. You will see that from players two point of view also if he deviates he is getting a strictly less than payoff, which means for both of them it is being satisfied and hence this is a Strict Nash Equilibrium.

What can be an example of a non-strict Nash equilibrium? Let me give you the example. Suppose this is the game, this is a payoff matrix, there are 2 players: player 1 has 2 actions and player 2 has 3 actions, what is the Nash equilibrium here?

If we examine the game carefully, we shall see that there is a single Nash equilibrium which is at T L. This is Nash equilibrium because if player 2 is playing l, player 1 by playing B is not better off but, he is not worse of either, because he is getting 1 in T by playing T. By B is playing again 1 and in Nash equilibrium, we had this sign greater than equal to sign.

Similarly, for player 2 also by deviating from L to M he is strictly worse off. If he deviates from L to R he is doing just as fine as he was doing by playing L. So, this is a Non-Strict Nash Equilibrium, because by deviating it is possible that you are doing just as well as, you were doing in the Nash equilibrium. It is not that you are doing strictly worse than you were doing in Nash equilibrium.

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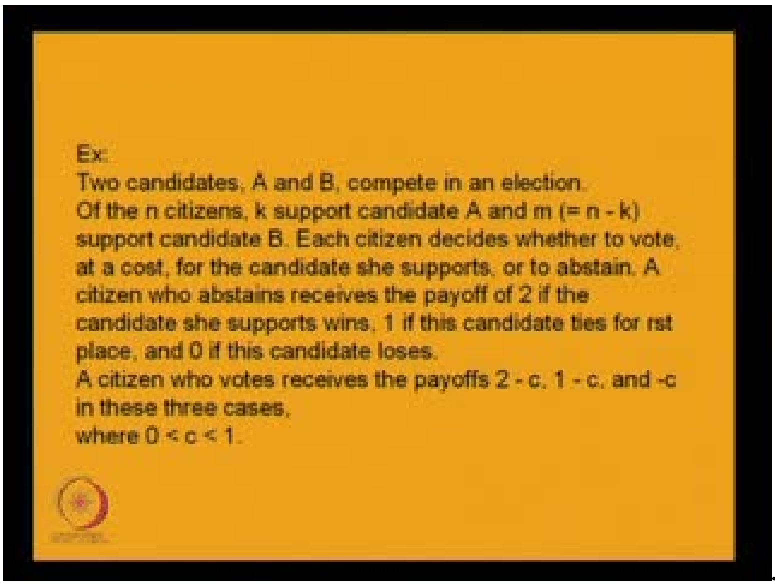

We are going to consider some other exercises. This is an exercise which is little difficult from the exercises that we have seen so far. Here the number of actions of each player is more than 2 or 3; there are many actions in fact. Now, if there are many actions then it will be difficult for construct a payoff matrix and find the Nash equilibrium. Then, we have to look for patterns and try to figure out if for different patterns thus a Nash equilibrium exists or not?

Here is the question; this is a question about voting, so as we have said before that in literature of political science also the tools of game theory are used. So, this is one example in which we can use game theory in political science. Two candidates are there A and B, they compete in an election. Now, there are n number of citizens are there, out of them small k support candidate A and m number of citizens, where m is n minus k support candidate B.

Each citizen decides whether to vote, at a cost, for the candidates she supports or to abstain. So, there are two actions which are available to any voter, he can decide to vote and if he votes, he bears a cost or he may choose to abstain, he may choose not to go to the voting booth at all. So, there is an incentive not to vote because if you go to vote you are bearing a cost.

A citizen who abstains receives the payoffs 2 if the candidate she supports wins, 1 if the candidate she ties for the first place and 0 if this candidate loses. A citizen who votes receives the payoffs 2 minus c, 1 minus c, and minus c in these three cases, where c is a number between 0 and 1, strictly less than 1 and strictly greater than 0.

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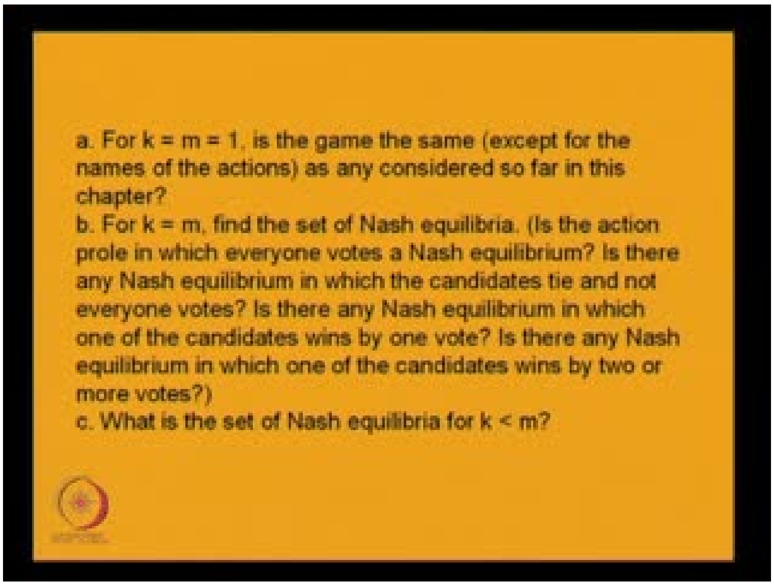

What is the question there are three parts to the question. First part, part a; for k is equal m is equal to 1 is the game same as any considered so far.

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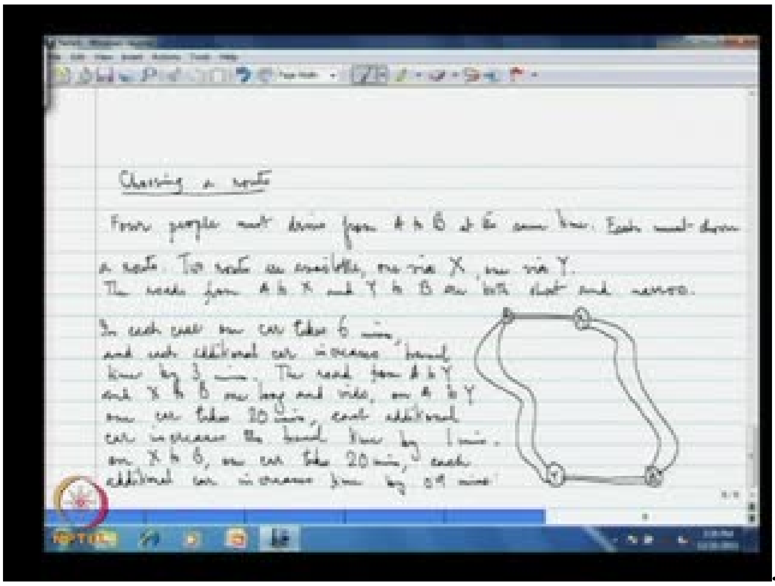

This is an exercise about Nash equilibrium and how to see the Nash equilibrium changes as the conditions change. So, this exercise is called choosing a route. Let me write down the exercise first then, we shall try to solve the exercise. Before we go further, let me draw a diagram to show how the game looks like. There are two places: one place is A the other place is B and there are 4 people who are thinking of driving down from A to B. Now, there are 2 routes to go from A to B, one passes through x and the other passes through y.

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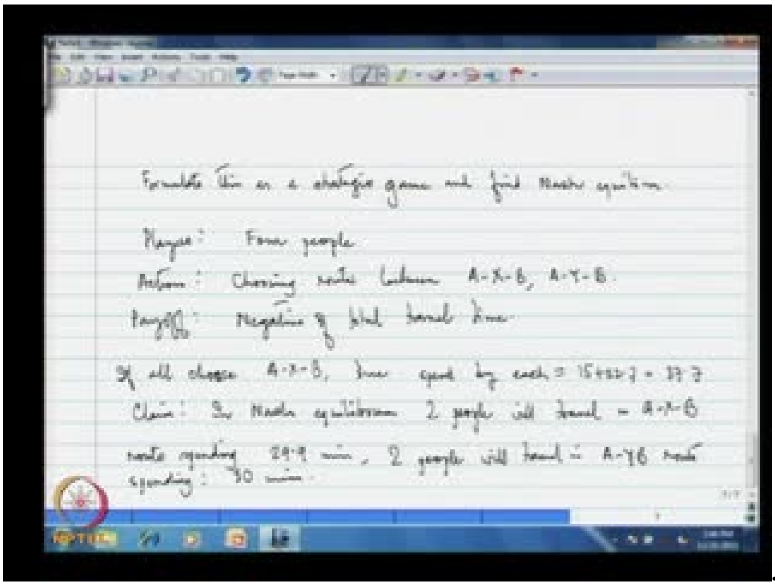

Here is how it looks like; the routes from A to X and from Y to B are both short and narrow. ((No audio from 27:00 to 30:30))

So, this is the first part of the exercise, we shall talk about the second part once we have done the first part.

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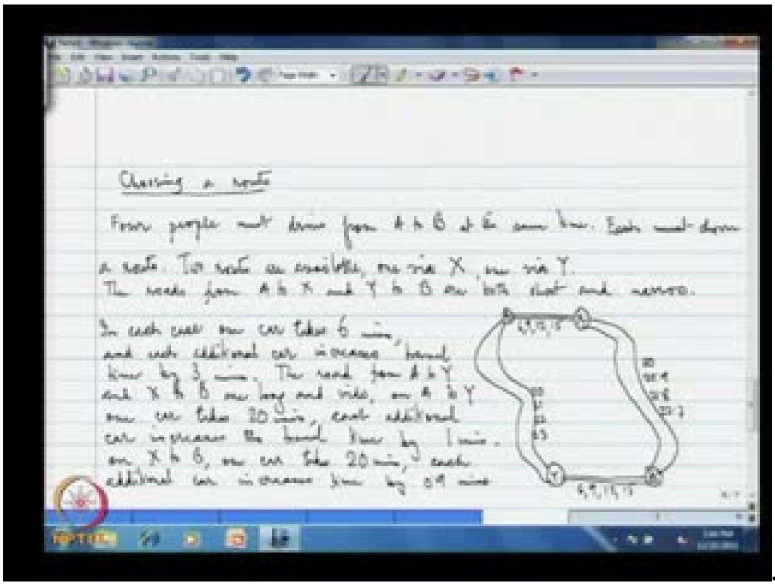

As it is written here the road from A to X is short and narrow therefore, as more cars enter into that road the time it takes to travel from a to x rises. These are the numbers that shows the time that can that will be taken by cars, if the number of cars goes on raising 6, 9, 12, 15 similarly, here 6, 9, 12, 15. If there is a just a single car to travel from A to X it will take 6 minutes but, if there are 2 cars each of them will take 9 minutes and like that it goes on. From A to Y or X to B the road is wide but, it is a long road so the time that will be taken will be like this. So, this is the game we have to formulate this situation as a proper strategic game and find out the Nash equilibrium.

Now, in a strategic game we know that we have to specify players. Here 4 people are the players; actions, choosing routes, it could be between A-X-B or A-Y-B and payoff, negative of total travel time, they want to minimize the travel time obviously more time is spend on road its worse for you. We have to find out who is choosing what route and that is the exercise.

Now, it can be easily shown that it is not likely that all of the 4 people are choosing A-X-B because if all of them choose A-X-B then, what is the time that each is taking. Time spent by each is what? It is given by 15 plus 22.7, which is 37.7. Now, is this the best thing that they can do? Obviously not, because from this situation if anyone of them goes to the other route, which is the other route? The other route is A-Y-B; if anyone goes for the other route he will make the journey in 20 plus 6 26 seconds, sorry 26 minutes.

So, all of them choosing the same route are not the optimal thing to do therefore some will deviate and my claim is that in Nash equilibrium 2 people will travel in A-X-B route spending how much time? If they travel in the A-X-B route then each will be spending 9 plus 20.9 which is 29.9 minutes and 2 people will travel in A-Y-B route spending, if they 2 people travel in the A-Y-B route then the time that they are going to spend is 30 minutes and that is, I am claiming is a Nash equilibrium why? Because deviation, let us consider deviation.

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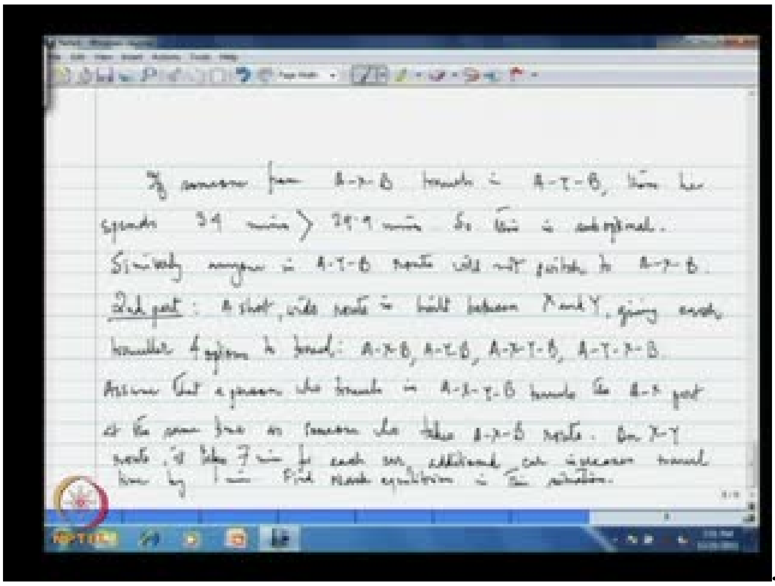

If someone forms A-X-B route travels in A-Y-B then, he spends how much time? If he travels in the A-Y-B route, then there are 3 people in the A-Y-B route and the total amount of time that he spent is 22 plus 12 which is 34 minutes, more than the time that he was spending in the A-X-B route which is 29.9 minutes.

There is no point for anyone travelling in the A-X-B route to switch to the A-Y-B route. Similarly, this is suboptimal. Similarly, we can show that anyone in A-Y-B route will not switch to A-X-B, so that is why this is a Nash equilibrium.

There is a second part to it. Suppose a relatively short wide road is built between X and Y. ((No audio from 39:00 to 41:40)).

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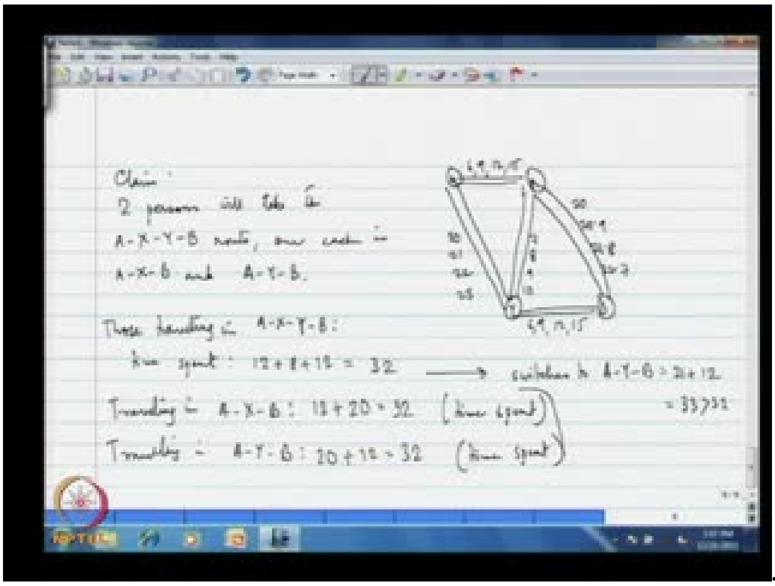

Here, I have a change situation, we have X as before and we have Y, X and Y as before, but what has changed is that between X and Y there is now a new road. So, besides A to Y, Y to B, A to X and X to B I have a new route between X and Y and the time to travel here is 7, 8, 9 and 10, so it is a wide road.

Time for the other routes remains the same which is basically 6, 9, 12, 15 these are the times, we have to find out what is the Nash equilibrium in this changed situation. My claim is the following. 2 persons will take the A-X-Y-B route one each in A-X-B and A-Y-B, so this is my claim. How do I prove this? Let us find out if this is the situation, then what is the time that is spend by each person those travelling in A-X-Y-B - time spent in A- X -Y-B.

Now, I am proposing that there are 2 people who are travelling in this route, so what is the time that they will be spending while travelling. First, when they are going from A to X there are 3 people on this part, so 12 plus 8 when they are moving from Y to B again there are 3 people whereas, between X and Y there are 2 people. So, 12 plus 8 plus again 12 which is basically 24 plus 8 is 32, so this is the time that they are spending.

What is the time spent by the person travelling in the route A-X-B? Now, the person is travelling from A-X-B is spending 12 minutes in the first part. In the second part, A-X-B he is spending 20 minutes, so the total amount of time spent is 32. The person travelling in the A-Y-B route, time spent in the A Y portion he is spending 20 but, in the last portion he is spending 12, so he is spending 32.

Now, can someone be better off by switching routes, if these people who are travelling in the A-X-Y-B route, if any of them switches the route and let us say, takes A-Y-B route then the total amount of time that he will be spending is how much? It is 21 plus again 12 and that is 33 which is more than 32.

So, switching to A-Y-B route is not profitable. If he switches to the A-X-B route then what happens? If he goes to the A-X-B route, he will be spending 12 plus 20.9 which is 32.9, which is greater than 32. Again, switching the routes is not profitable; therefore, this is the equilibrium. We can show the same thing for those people who are travelling in the A-X-B or A-Y-B route also.

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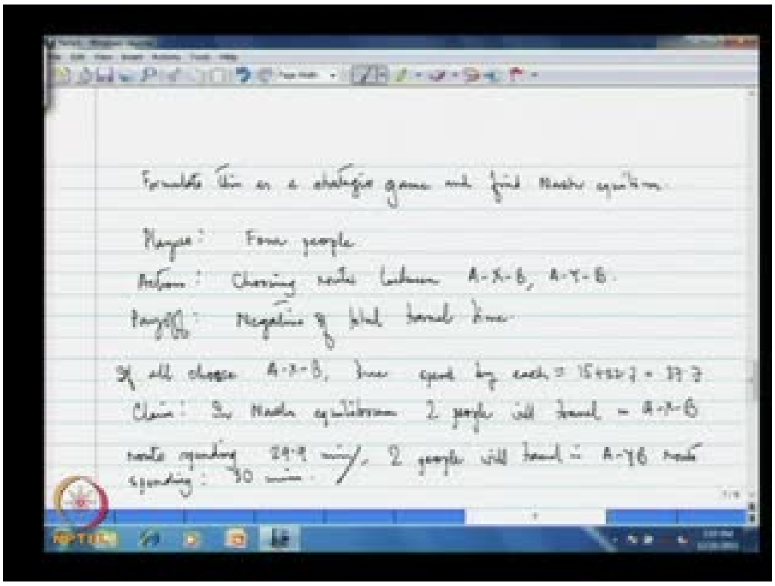

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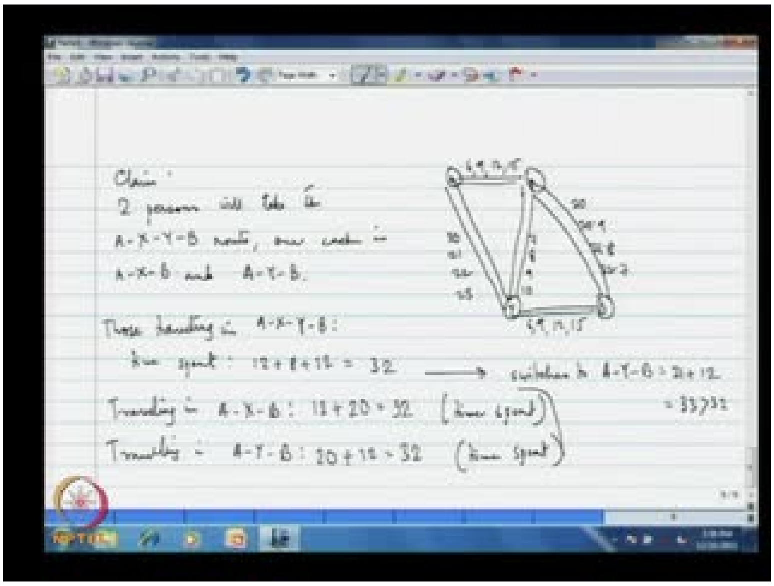

Now, the funny thing about this result is that when this X Y stage was not constructed in the original game, each person was taking less time 30 and 29.9 minutes but, when this route is constructed apparently it was there to reduce the travel time but, instead of reducing the travel time it is increasing the travel time for each, because now each person is taking 32 minutes to travel from A to B. Whereas, previously when this X Y route was not constructed each was taking either 29.9 minutes or 30 minutes. So, this is a kind of interesting and paradoxical result that may be building new roads does not help in reducing the time of travel.

Let me wrap up this lecture by just try to recapitulate what we have done. We have defined one new concept which is the concept of strict Nash equilibrium. Besides that, we have seen two exercises, in one exercise there was the provision of public good, were we have seen that in provision of public good there can be more than one Nash equilibrium. If the number of people required to build the public good is less than the total number of people.

We have seen another exercise where the number of actions for each player is quite a large number it is not two or three. So, that in this case we cannot draw the payoff matrix and find out by comparing what is the Nash equilibrium, which we have to look at the pattern of the actions and try to find out if there is a possibility of any existence of Nash equilibrium, thank you.

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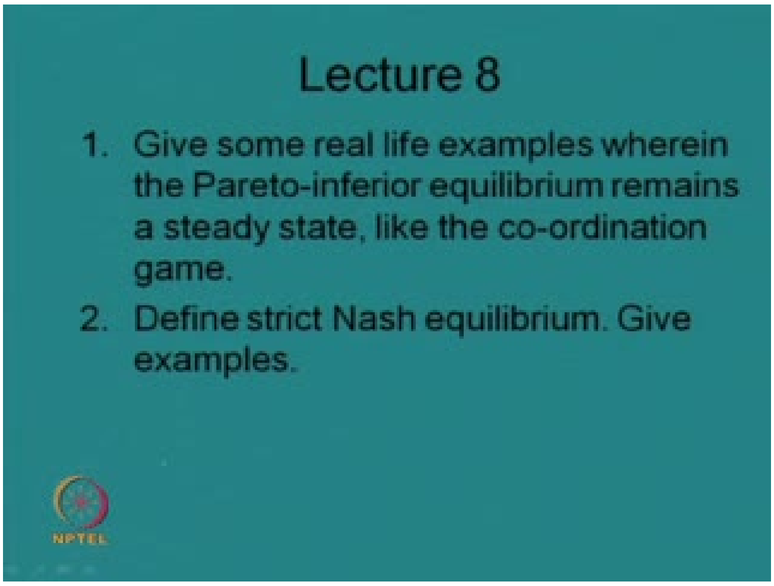

Give some real life examples wherein the Pareto-inferior equilibrium remains a steady state, like the coordination game.

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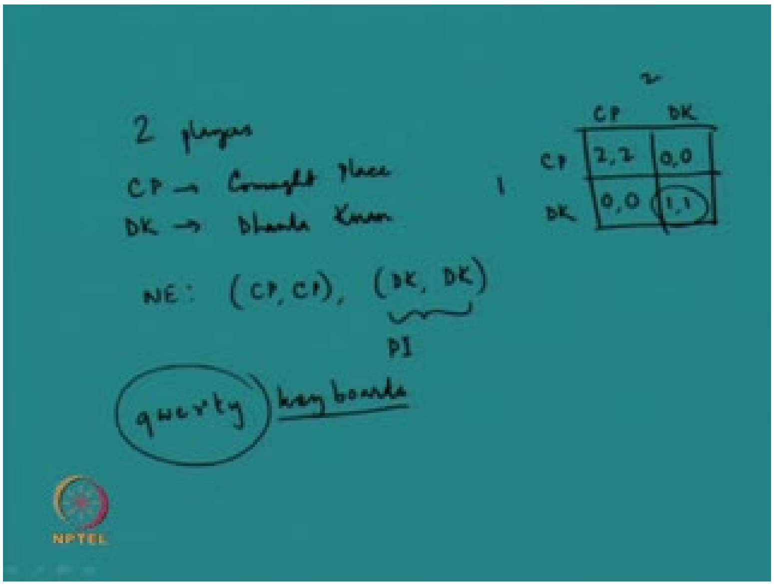

First let us see, what is a co-ordination game? Co-ordination game is the following game - I am giving one such example of co-ordination game. Suppose, there are 2 players and they want to meet each other by either going to C P, this is like Connaught place or suppose, they can go to D K - Dhaula kuan.

Now, if they do go to different places of course, they cannot meet. So, they have to go to the same place so that they can meet. However, going to the same place does not give the same benefit to both of them. Meeting at C P apparently is beneficial for both of them compared to meeting at Dhaula Kuan. Here, there are 2 Nash equilibrium you can verify, one is at C P, C P the other is at D K, D K, but here D K, D K is an equilibrium which is a Pareto inferior, because they are getting 1, 1 whereas, had they made at C P they would have got 2 each.

Question is how does it still remain Nash equilibrium D K, D K? We have several examples in real life where D K, D K's remains a equilibrium, one example is the case of qwerty keyboards. So, most of the computer keyboards which are used are called qwerty keyboards because q, w, e, r, t, y are those keys which appear on the left hand side on top of the keyboards that we use, but this keyboard is not the most efficient keyboard that can be designed.

In fact this is very uncomfortable for any beginner because if you remember s the letters s, a, e all come from the left hand of the person who is using the keyboard and most of the people are in fact right hand people, so this keyboard is not a very efficient keyboard but, nevertheless this has become the steady state keyboard. The reason is that if some manufacturer tries to device a better keyboard, he may not be able to sell that keyboard because the customers may not buy that keyboard by just observing weird kind of design. So, an inefficient keyboard has remained in the market and people are buying it also this is a kind of D K, D K equilibrium that we have stuck with.

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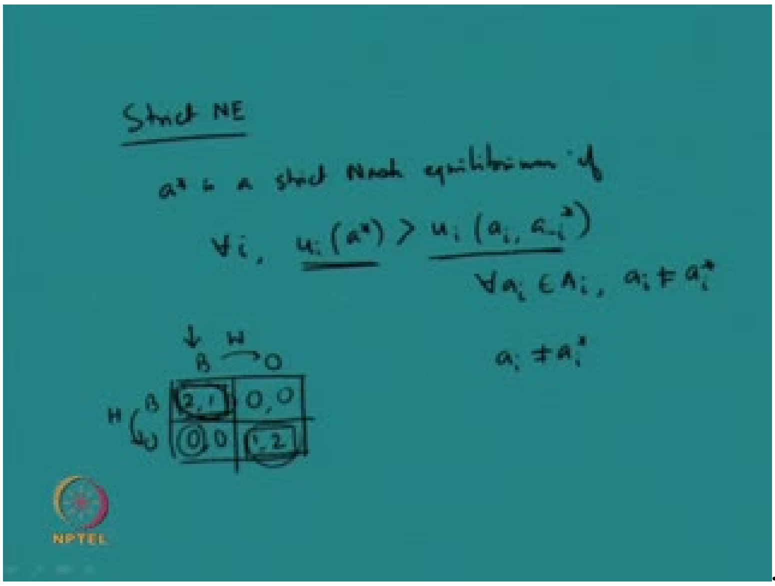

Define strict Nash Equilibrium, give examples. A Strict Nash Equilibrium is an equilibrium, where for all players of all i, this is the payoff in the equilibrium if everyone is playing star actions, if i deviates, so this a i belongs to his action set but, this a i is not equal to a i star. If this player deviates from the equilibrium action he takes a i which is not equal to a i star then he is strictly worse off and this should happen for each and every player. This means, that in a Strict Nash Equilibrium the players are getting payoffs which are best for them given what the other players are doing. If they do something else they are strictly worse off.

What are the examples? The most of the examples that we have gone through are in fact examples of strict Nash equilibrium. Let me just repeat one such example, which is the battle of sexes example, if you remember. This was the structure of the game, this was the Nash equilibrium, this was the Nash equilibrium and given player wife is playing B. If player H deviates, he gets 0 which is strictly worse than 2.

Similarly, for player 2 also that is, the wife also if she deviates she is getting a payoff which is strictly less than 1 but, this was not necessary for this to be a Nash equilibrium it could have been one also but, if it is one then this is not a strict Nash equilibrium, so that is the point. In this case, both this and this are examples of strict Nash equilibrium.