Mathematics for Economics - I
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Lecture 13
Differentiation: Properties
Approximations and Elasticities

Welcome to another lecture of this course Mathematics for economics part one. So, we have been discussing differentiation in the last few lectures. In particular, we have been talking about Higher Order Differentiation and different properties of Differentiation. So, in the last lecture, we covered what is known as Implicit Differentiation and Higher Order Differentiation. Today, we are going to deal with what is known as Approximation.

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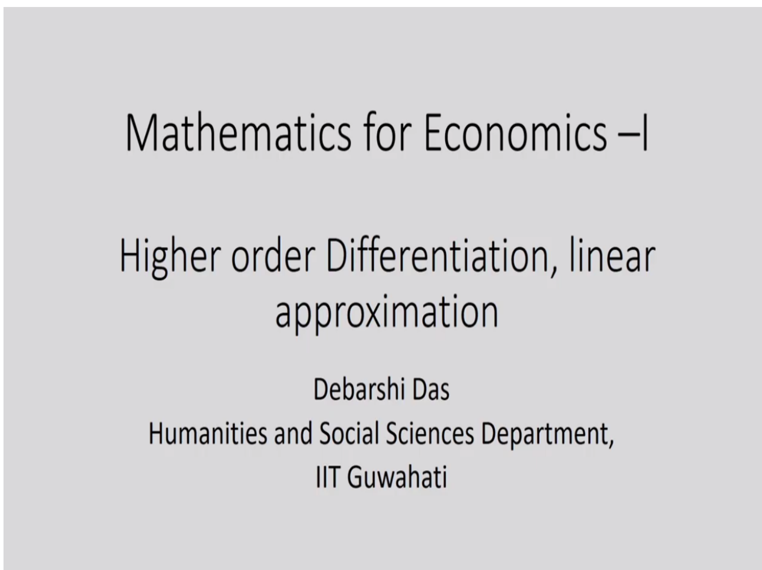

So, we are going to start with what is known as Linear Approximation, and then we shall see that there could be Nonlinear Approximation as well. And finally, we shall end the lecture with some applications in economics of differentiation, such as Elasticities we have talked about elasticities in some previous lecture, but in today's lecture, we are going to deal with elasticities in a more detailed manner.

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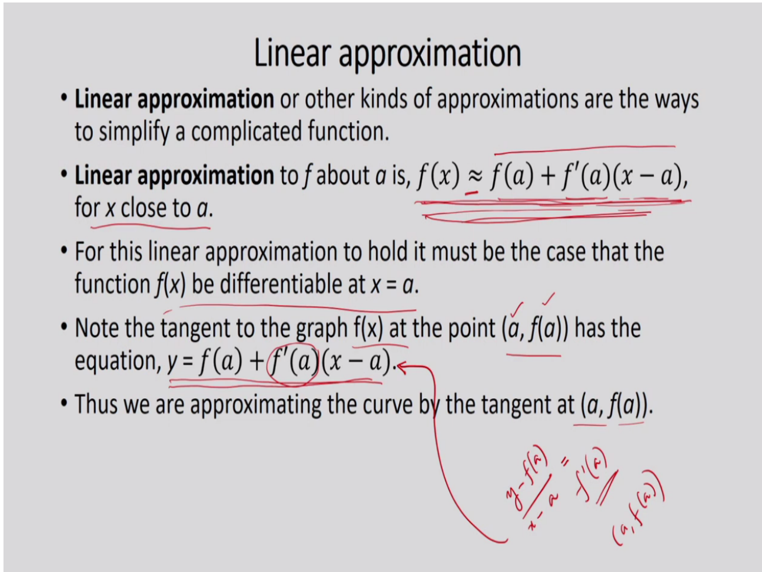

So, this is called Linear approximation, this particular topic. So, linear approximation or other kinds of approximations are the ways to simplify a complicated function. As we have seen that function could be of different nature, it could be very complicated in its properties, it could be having many twists and turns, but can we simplify a function through making it linear function, if it is a linear function, then it is very simple to deal with properties are very straightforward.

For example, if you have a linear function, then the slope remains the same irrespective of where you take the independent variable to be, the slope always remains constant if you have a linear function. So, therefore, if we have a complicated function, then we can linearize that function and then it is easier to deal with. So, we are going to start with linear approximation and here is an example, you see on your screen linear approximation of f, f is the function.

About a, a is a particular value of the independent variable is given by this $f(x) \approx f(a) + f'(a)(x - a)$. And this approximation holds if you have x that is the independent variable very close to this value small a. So, if you are in the close proximity of this particular value of x, which is small a, then you can predict the value of the function that is f(x) through this formula.

And again to make it clear, you have this approximately equal to sign which means that it is not exactly equal to $f(a) + f'(a)(x - a)$, but it is very close to that value and for this linear approximation to hold, it must be the case that the function f(x) be differentiable at x equal to a. So, we must be able to differentiate this function at x is equal to a otherwise you cannot find out $f'(a)$ obviously, this right hand side cannot be derived. So, therefore, you cannot linearly approximate the function about a.

Now, note that tangent to the graph f(x), right. Suppose just imagine this to be a graph in the 2 dimensional plane. Now think about the tangent to this graph at this point, (a, f(a)). And what is the equation to that tangent?? That equation is given by this, $y = f(a) + f'(a)(x - a)$. Now why am I saying this? Why did I get this particular form of the equation to the tangent? Well, the reason is very simple.

At point a, at x = a, the value of the function is f(a), so the slope of the function is what it is $\frac{y-f(a)}{x-a}$, if you have two points on a straight line, then the slope of the straight line is $(y1 - y2)/(x1 - x2)$. So we are applying the same thing $\frac{y-f(a)}{x-a}$. So that is the slope of the function. And that is exactly equal to $f'(a)$ that is the slope.

So from this identity, that is $\frac{y-f(a)}{x-a} = f'(a)$, you will directly get this particular form. And we know that this point, a, f(a) is a point on the graph, on the tangent. So therefore, we can use that fact. And we can use the fact that $f'(a)$ is known to us, and therefore, the slope of the tangent is given by this. So this is the slope to the graph of f(x) at point a, f(a).

Therefore, when I am writing that the function is given by this function is approximated by this, I am actually using the equation of the tangent to the graph at a particular point. So, it is as if the tangent is representing the function close to the point of the point of tangency. Thus we are approximating the curve by the tangent at a, f(a), this is what I just said that we are trying to get close to the value of the function.

That is what we are doing here, we are trying to get close to the value of the function by using the equation of the tangent.

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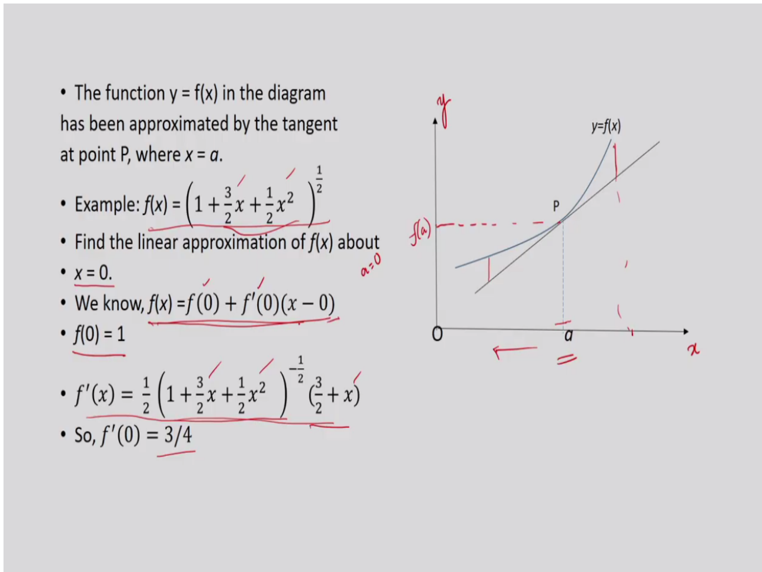

Here is the graphical representation of that. So, here you have along the x axis you have horizontal line and along the vertical axis, you have f(x) which is also written as y. So, the function y = f(x) in the diagram has been approximated by the tangent at point P, where x = a. So basically, what you have a x is equal to a at this point. So at this point, you have f(a).

And therefore, you draw a tangent through this point, capital P and the slope of the tangent is known to us it is $f'(a)$ and therefore, we approximate the curve around this point a by using the equation of the tangent. That is what we are doing. Now notice, if you are in the neighborhood of this point x = a, then the change in the value of the function is roughly equal to the change in the value of the line representing the tangent around the pointing.

But if you take a point very different, very far away from this point a, then this gap is rising. So if you are taking a point here, the gap will be too much. That is why if you are in the neighborhood of x = a, then this approximation is valid work if you are far away from this point a, then this approximation cannot be valid. So that is the geometry of it. Now, let us take an example.

Suppose fx is given $f(x) = (1 + \frac{3}{2}x + \frac{1}{2}x^2)^{\frac{1}{2}}$. So this is seems to be a little bit complicated function. And what I need to do is find the linear approximation of f(x) about x = 0. So here, basically small a is given to be 0, around the point 0. Now to do that, what we have to use is this particular form, $f(x) = f(0) + f'(0)(x - 0)$, because a = 0.

So this is the form we have to use. Now here, what we do not know is $f'(0)$, that we need to find out. So this one, we need to know an f(0) that also we need to find out. And if we put those things on the right hand side, then we are through. Let us put x = 0 in this particular form, if we do that, it becomes this part drops out this part drops out. So 1 to the power half, 1 to the power half is 1. So you have f(0) = 1.

Then we need to find out $f'(0)$? So I have to first find out what is $f'(x)$. And then I can substitute x for 0, and I will be getting $f'(0)$. And I will get this particular expression $\frac{1}{2}(1 + \frac{3}{2}x + \frac{1}{2}x^2)^{-\frac{1}{2}} (\frac{3}{2} + x)$. Because we have to differentiate the term inside with respect to x, and if we do that, I will get $\frac{3}{2} + x$.

Now this is the form for $f'(x)$. And so here, I am just going to put x = 0, so this term will drop out, this term will drop out, this term will drop out. So ultimately, I am going to get $\frac{1}{2} X \frac{3}{2} = \frac{3}{4}$. So I have got everything that I need. And now I will just plug them in this expression, and I will get f(x).

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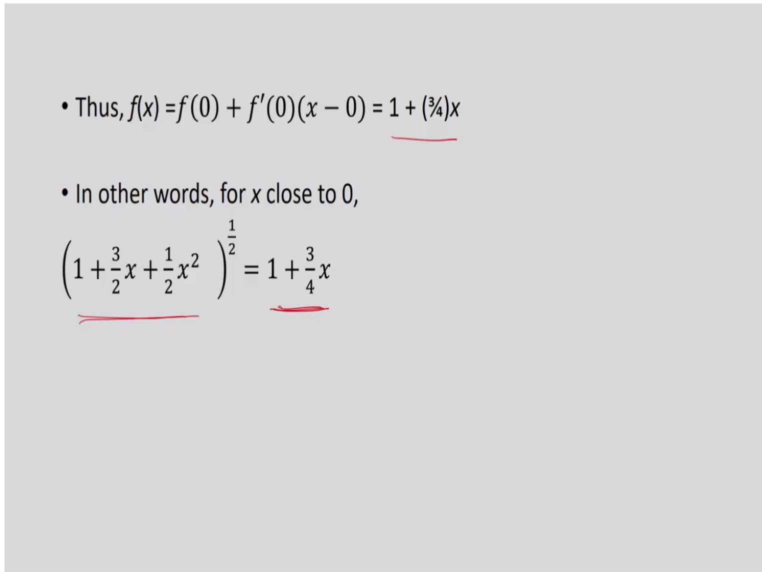

So $f(x) = f(0) + f'(0) = 1 + \frac{3}{4}x$. So, this is the particular expression that I needed to find out. In other words, if x ≃ 0 remember, we are approximating around the point 0. So, therefore, if x is close to the point 0, then this function can be approximated by this particular form. And you can see that on the right hand side I have a linear expression of x is not very complicated.

On the left hand side, the expression is much more complicated. It is not a linear form of x worked on the right hand side I have linearized it I have made it very simple. So that is the usefulness of a linear approximation. And actually, now, if you put different values of x which are close to 0, you can just put them there and you will get the value of the function and so this could be very useful.

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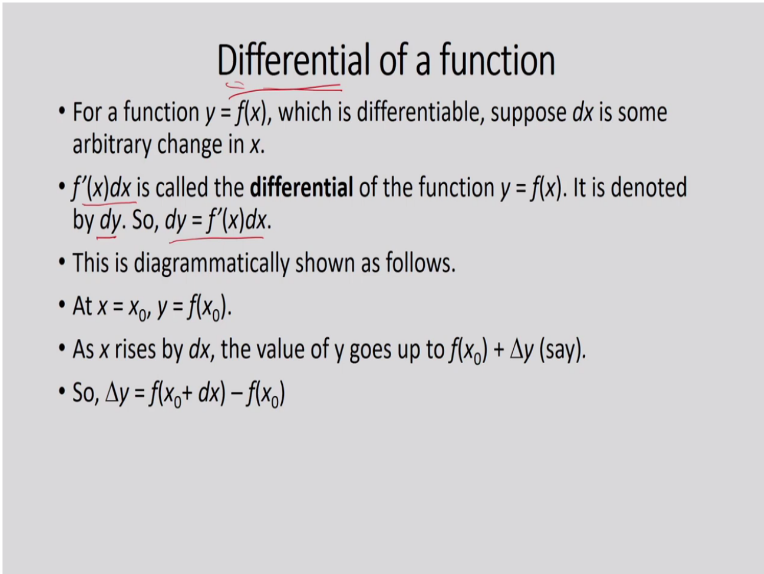

Now, close to the idea of linear approximation is the idea of differential of a function. So, this will be used many times in economics, it is helpful to have sort of discussion about this differential. Suppose your function y = f(x), which is differentiable and suppose the x is some arbitrary change in x so dx is the change in x, x is the independent variable. Then $f'(x)$dx is called the differential of the function y = f(x). It is denoted by dy. So, $dy = f'(x)dx$. So, this is diagrammatically shown as follows. So, diagram is in the next slide.

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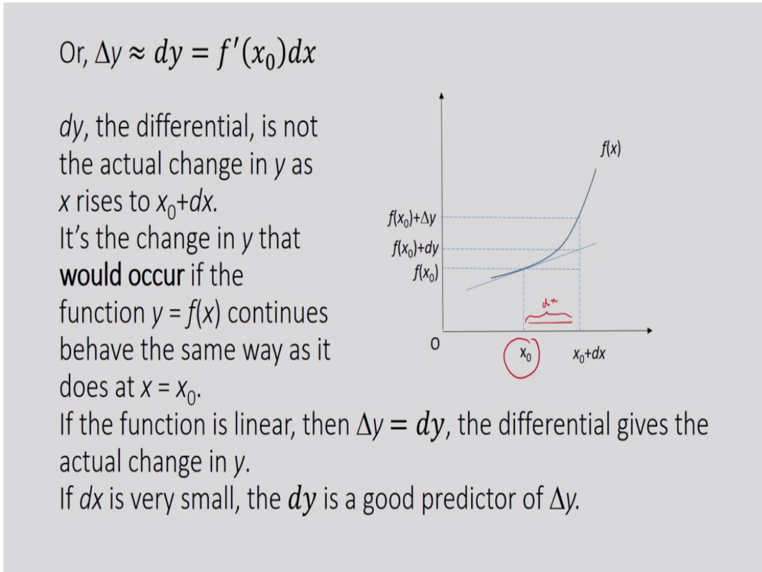

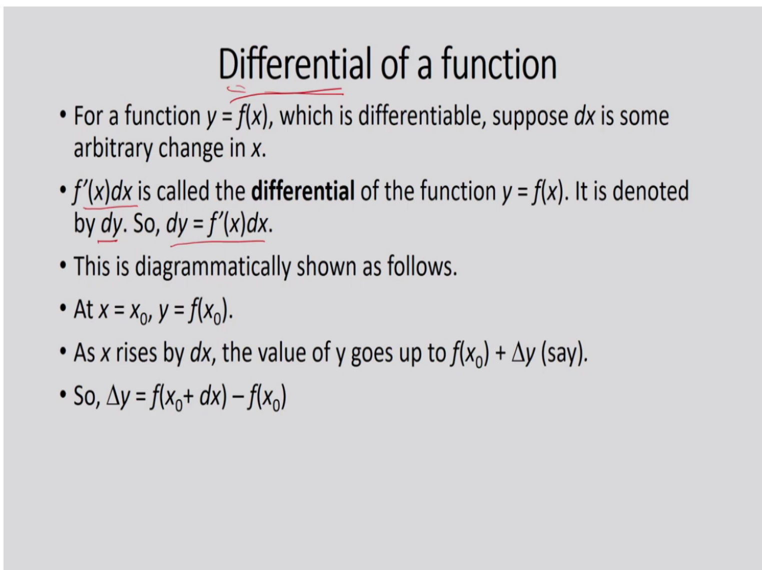

So, here you have f(x) function and you have taken a point x = 0 here and dx is some change in x, so, this portion is dx. So, here dx is positive, so, therefore, the value of x is rising because dx is positive it becomes $x_0 + dx$. So, at x = 0 value of the function is $y = f(x_0)$.

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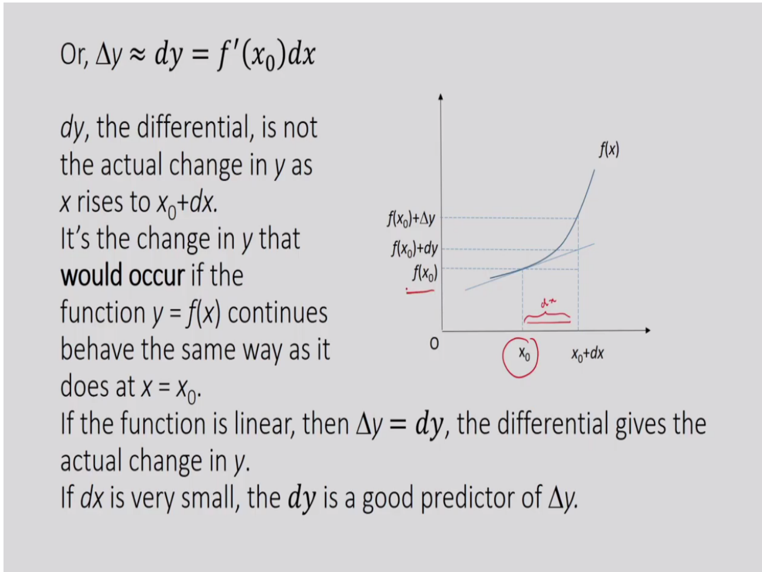

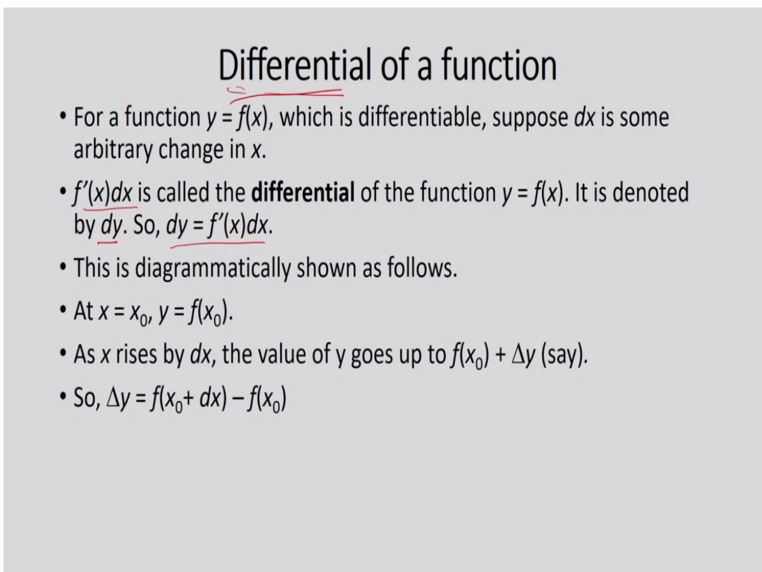

So which is given by this point here. As x rises by dx the value of y goes up to $f(x_0) + \Delta y$.

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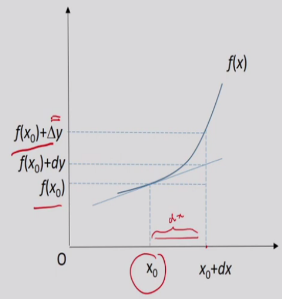
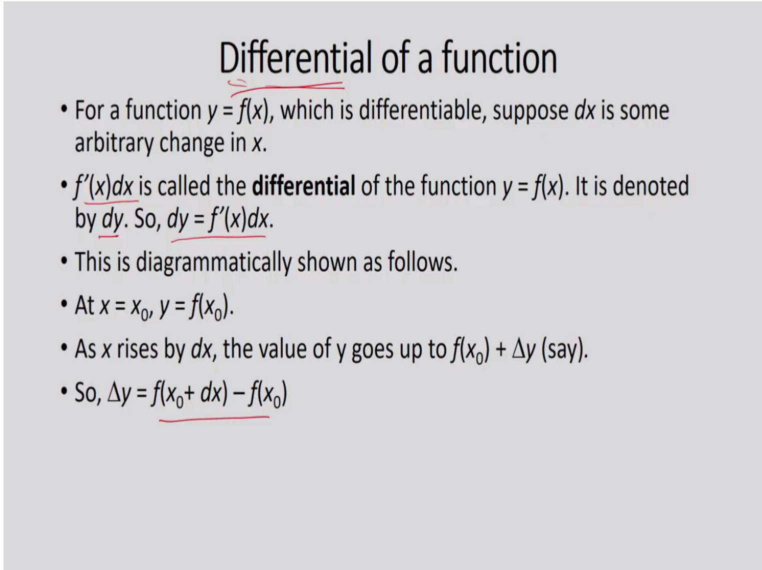
Or, $\Delta y \approx dy = f'(x_0)dx$

dy, the differential, is not the actual change in *y* as *x* rises to x_0+dx.
It's the change in *y* that **would occur** if the function $y = f(x)$ continues behave the same way as it does at $x = x_0$.



If the function is linear, then $\Delta y = dy$, the differential gives the actual change in *y*.
If *dx* is very small, the dy is a good predictor of Δy.



So, suppose Δy is the change. So, here you have that value of x, $x_0 + dx$ corresponding value of y, is this value. So, the value of the function has gone up. So, a change in the value of y is denoted by Δy . So, the new value is $f(x_0) + \Delta y$. So, Δy I can straight away write $= f(x_0 + dx) - f(x_0)$, that is very obvious.

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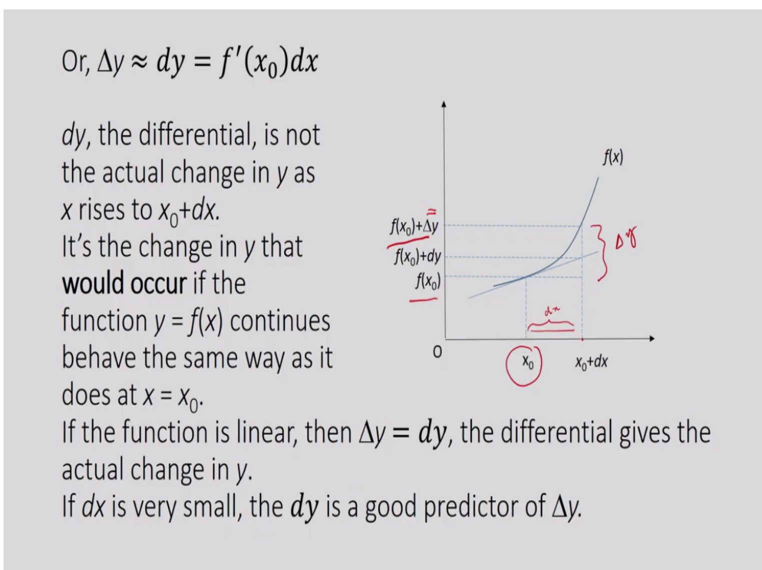

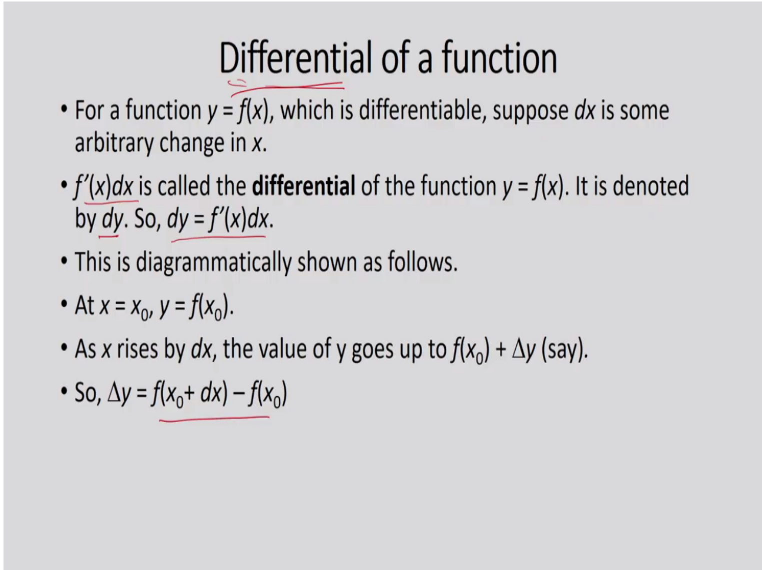

So, Δy is this much, this is Δy and this is $= f(x_0 + dx) - f(x_0)$, this is what I have written here.

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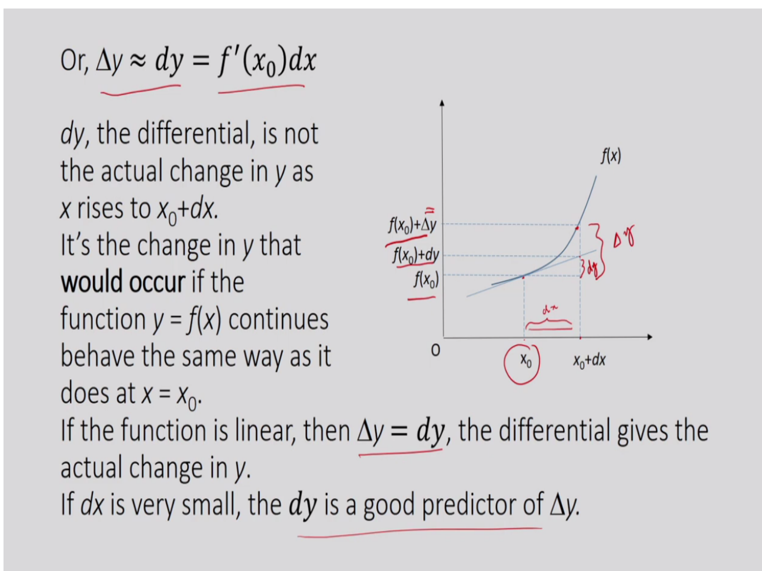

So, $\Delta y \simeq dy = f'(x_0)dx$. dy is called the differential, is not the actual change in y as x rises to $x_0 + dx$. So, as x is rising, what is the change in y it is here this is the new value of y what is dy? dy is not that one $dy = f'(x_0)dx$. So, geometrically how we can understand that is that dy is this particular value.

This is the value of dy and this is I have written that here also because this value is the center value is $f(x_0) + dy$. So, that is why I have written that dy the differential is not the actual change in y as x rises to $x_0 + dx$. dx, dy and delta y are different. They are close but they are not equal. So, what is dy? dy or the differential it is the change in y that would occur if the function y is equal to f(x) continues to behave the same way as it does at $x = x_0$.

So, at $x = x_0$ there is a point on the function, this is the that point. And at this point if I draw a tangent, this is that tangent. And if the function actually had behaved like the tangent, then the value of the function at $x_0 + dx$ would have been $y_0 + dy$. But as we know the function is not a straight line. So, therefore, the functions value actually is exceeding that particular value.

If the function is linear, then $\Delta y = dy$ the differential gives the actual change in y and if dx is very small, so, you are taking x as which are in the neighborhood of x_0, then the dy is a good predictor of Δy and this is again something we have referred to earlier that in the immediate neighborhood of this particular value of x, the value of the tangent is a very good predictor of the value of the function.

But if you take the x to be very different from x_0, then the value on the tangent is not a very good predictor of the value of the function, if the function is not a linear function and he had the function say general function is not a linear function. So, that is the idea of differential, differential which is given by dy, it gives you the change in y around a particular point it has to be evaluated at a particular point if the function behaves like the tangent at that particular point.

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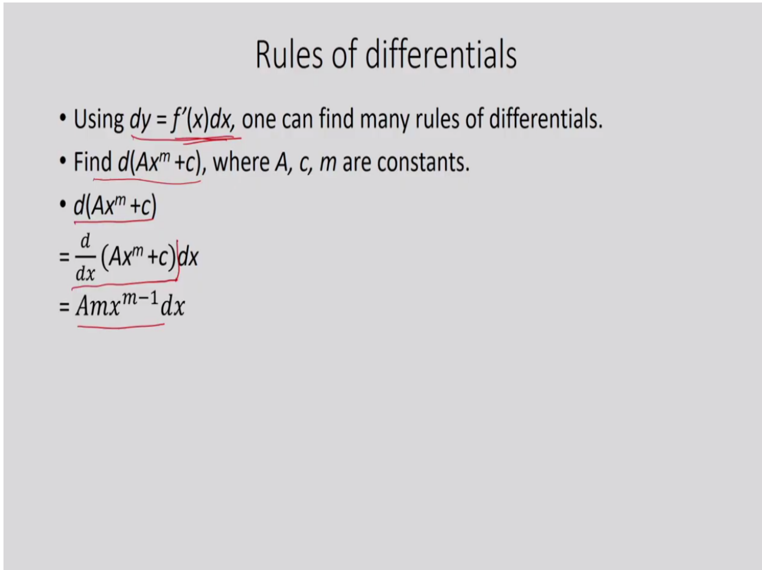

So, that is about differentials and here are certain rules of differentials. So, we know that differential formula is given by this that suppose y = f(x), then dy which is the differential is written as $f'(x)dx$. So, this is the general definition, but now we can use that to get different rules of differentials. So, for example, suppose you have to find out what is $d(Ax^m + c)$, where $A,\ c,\ m$ are constants. Now, this can be actually simplified. So, I am just using the formula here, which is $f'(x)dx$. So, we have $f'(x)$ is this part and dx is this part, and then I just use the power rule and I get $Amx^{m-1}dx$ that is the expression that I was looking for.

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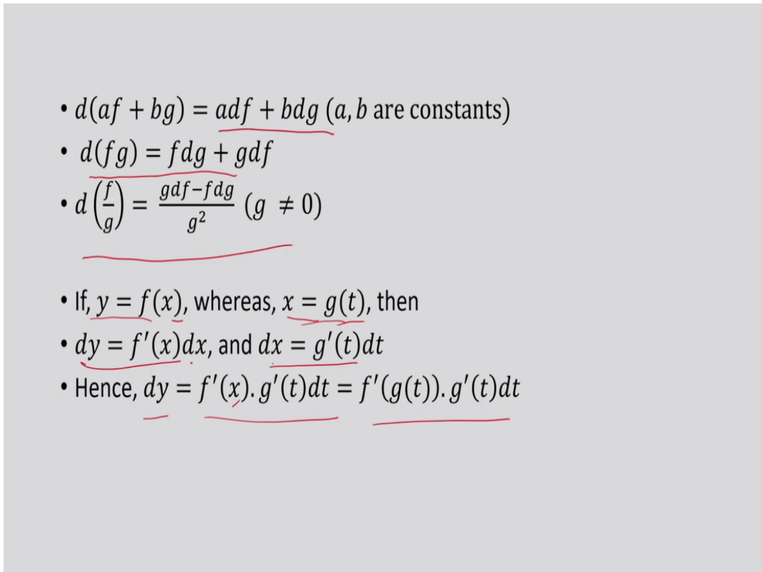

And here are certain other rules of differentials. So, suppose a and b are constants, then what is $d(af + bg)$, where f and g are functions of x, let us suppose. So that is given by $adf + bdg$. So, the a and b are constants, so, they are coming out and f and g are not constant. So, df and dg, and in the next stage, maybe you can if they are f and g are functions of x, then you can write them as $f'(x)dx$ and $g'(x)dx$.

If you have f and g multiplied together and take the differential of that, then it becomes $f(dg) + g(df)$. So, it is like the product rule of differentiation and then you get the quotient rule kind of thing here, $\frac{d(f)}{g} = \frac{gdf - fdg}{g^2}$. So, these are the rules of differentials.

Now, there is a composite function here, suppose, so $y = f(x)$ and f itself is a function of t, $f = g(t)$. Then I can use the rule of differential what is dy? $dy = f'(x)dx$. And I can use the rule of differential on this second function. So, $dx = g'(t)dt$. Now, I substitute this dx to here and I get the expression for dy and it turns out to be this one, $f'(x).g'(t)dt$ and to express everything in terms of t, I substitute $x = g(t)$ here so, this is the final term I am getting $f'(g(t)).g'(t)dt$.

So you have a composite rule for differentials.

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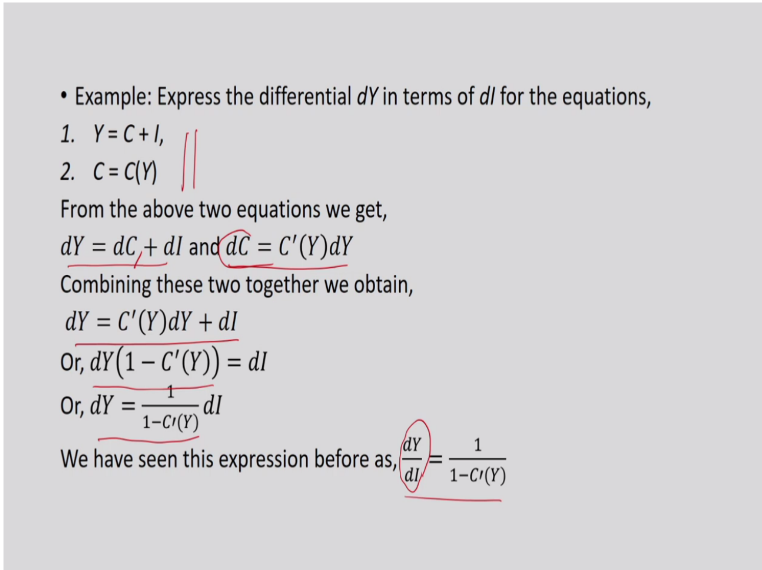

Here is an example of how to use differentials. Example, express the differential dY in terms of dI for the equations. So these two equations are given $Y = C + I$ and $C = C(Y)$. So, if you remember this is an, an example of macroeconomics, where Y is the output of the economy is equal to consumption plus investment. And consumption function is given it is a function of the output C(Y).

So, we combine these two, 1 and 2 together, first I take the differential of the first equation, I will get $dY = dC + dI$ from the second one, I will get $dC = C'(Y)dY$. And then I will use dC from here in this first expression. So, this is what I will get $dY = C'(Y)dY + dI$ and I will take all the dY terms together on the left hand side. So $dY(1 - C'(Y)) = dI$ and therefore, the $dY = \frac{1}{1-C'(Y)} dI$.

And interestingly we have seen this expression before this side say that this is like the investment multiplier in the Keynesian model , but there I expressed this like this form dY/dI here I did not differentiate Y with respect to I, I took the differentials and I got this expression, similar expression.

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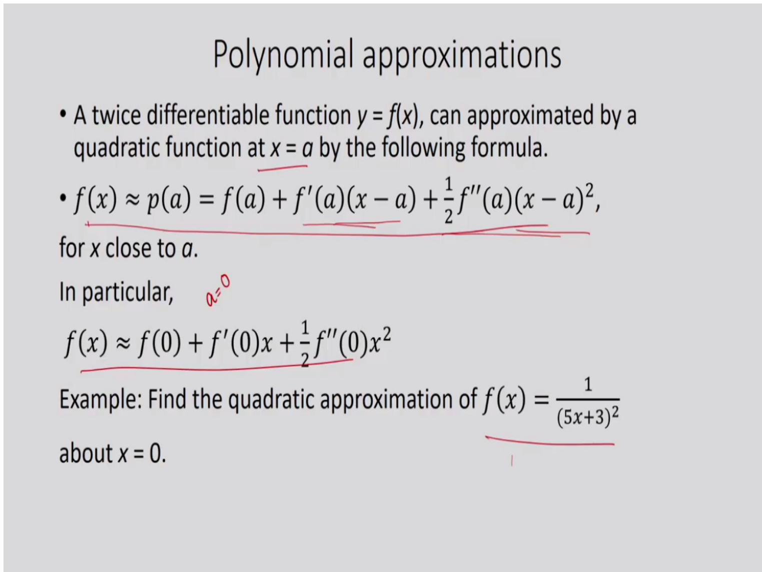

We have talked about linear approximation of a function, but there could be approximation of higher orders. Suppose you have a twice differentiable function $y = f(x)$, twice differentiable means you can differentiate it two times successively. Then it can be approximated by a quadratic function at $x = a$ by the following formula. So, here instead of a linear function, we are approximating a particular function, but now, the resultant function that is approximated function will be a quadratic function that is a function polynomial of degree 2.

So, this is how we are writing it. So, f of x is equal to I am writing, I am expressing it as just p of a, $f(x) = p(a)$. So, $p(a) = f(a)$, so, you are evaluating the function at that particular point a $+ f'(a)(x - a)$. Remember this term was there in the linear approximation also but now, you have another term which is $\frac{1}{2}f''(a)(x - a)^2$ half of f prime of a multiplied by x minus a whole square.

So, from here you will get x^2 that is, this expression becomes a quadratic function and again like before this will hold in the neighborhood of the point a, it is not valid if you take x very far away from a. In particular, if you take $x = a$, $a = 0$ the point around which you are approximating it if you take that point to be the point 0, $x = 0$, then the terms become a little bit easier to deal with.

It becomes $f(x) = f(0)$, last $f'(0)x$ because a is $0 + \frac{1}{2}f''(0)x^2$. So it looks much more cleaner if you take a is equal to 0. Here is an example. Find the quadratic expression of f(x)? f(x) is given by this about x = 0. So, as you can see this f(x) is kind of complicated function because the power of $5x + 3$ is -2. So we have to make this a quadratic function of x.

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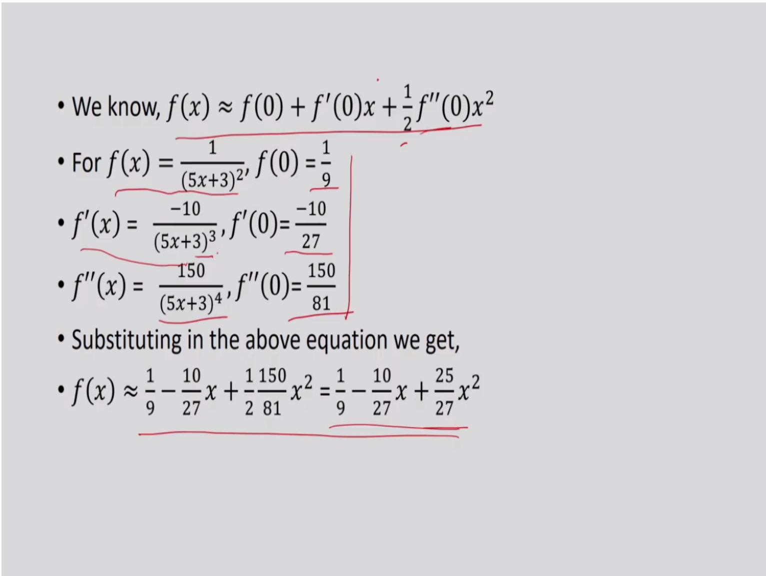

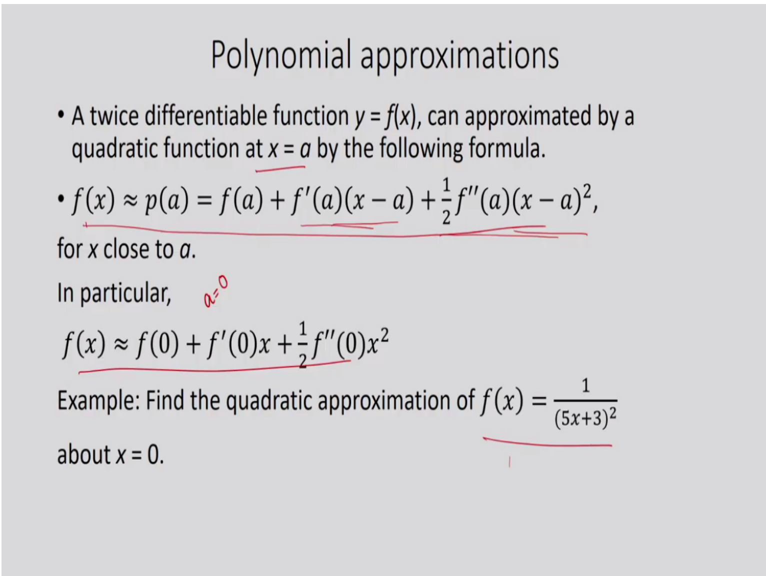

Now we first use, we are going to use this formula $f(x) \simeq f(0) + f'(0)x + \frac{1}{2}f''(0)x^2$. So, you can see that I have to take the second derivative, which is the reason why we have to have this function to be twice differentiable. Now, the function is this, so I need to find out what is f (0), I take x = 0, it will just become $\frac{1}{9}$.

Then I take the first derivative, $f'(x)$, and I am skipping some steps here. So, it will be just $\frac{-10}{(5x+3)^3}$. And if you put $x = 0$ it becomes $\frac{-10}{27}$.

Similarly, if you differentiate this once again, $f'(x)$, you are differentiating it once again, then you are going to get $f''(x)$ which is $\frac{150}{(5x+3)^4}$ and again you put $x = 0$ you are going to get $\frac{150}{81}$. So, all these values are now going to be put here and this is what you are going to get it will be $\frac{1}{9} - \frac{10}{27}x + \frac{1}{2}.\frac{150}{81}x^2$. So, if you simplify this, this becomes this expression. So, the main point that one is getting that this particular function which seems to be a little bit complicated, because the power of $5x + 3$ is -2.

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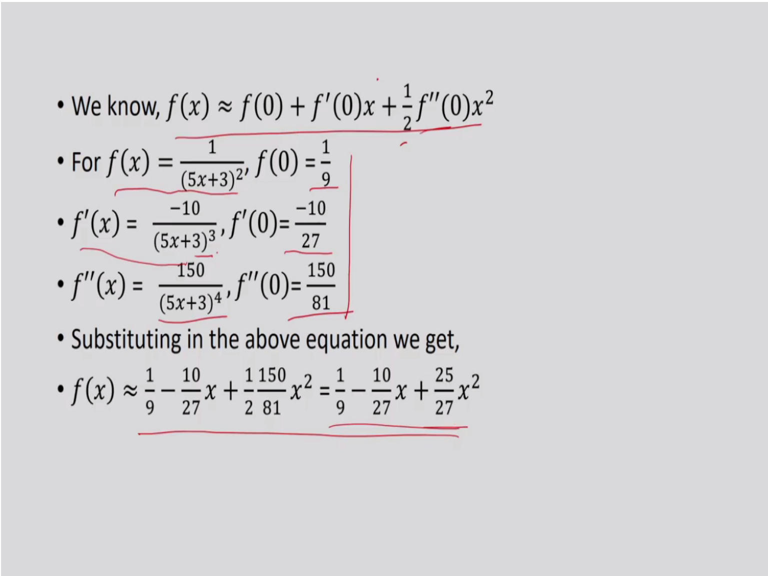

Which is not very easy to deal with, can be approximated by this relatively simpler looking function, quadratic function of x around the point x = 0 that is very important. This is true not everywhere, but around this point x = 0.

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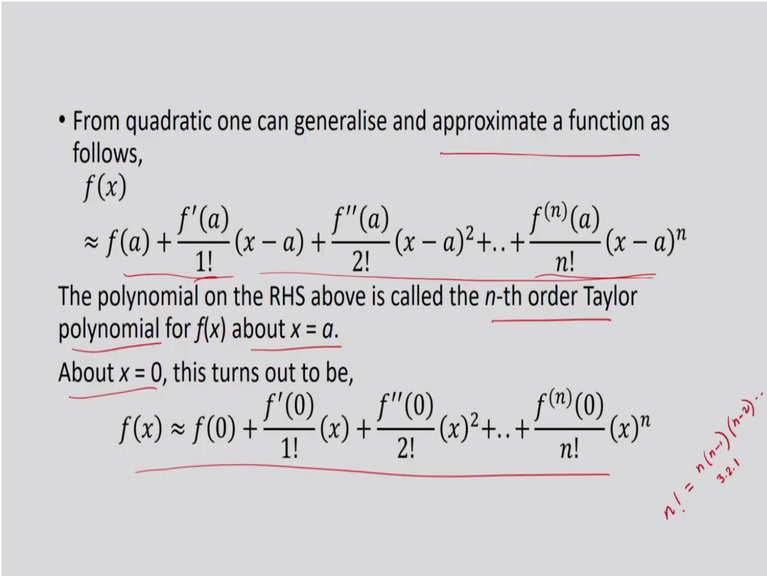

Now, we have discussed about quadratic expression, quadratic approximation. Now, 1 can generalize more and approximate a function as follows $f(x) = f(a) + \frac{f'(a)}{1!}(x - a) + \frac{f''(a)}{2!}(x - a)^2 +.... + \frac{f^{(n)}(a)}{n!}(x - a)^n$.

If you have forgotten your high school mathematics and wondering what is factorial? Factorial is this expression, suppose you are taking n!, so this will be n(n - 1)(n - 2). So the numbers are going down by 1 by 1. So the last terms will be 3, 2, 1. So all these terms are getting multiplied together. Now, this term on the right hand side is obviously it will be a polynomial of degree n because the power of x is n.

The polynomial on the right hand side above is called n-th order Taylor polynomial. For f(x), about x is equal to obviously we are taking the approximation around a particular point, that point is x = a. So about x = 0, that is, if you take $a = 0$, then this term on the right hand side becomes a little bit easier to deal with a little bit simpler. Then it becomes $f(0) + \frac{f'(0)}{1!}(x) + \frac{f''(0)}{2!}(x)^2 +... + \frac{f^{(n)}(0)}{n!}(x)^n$.

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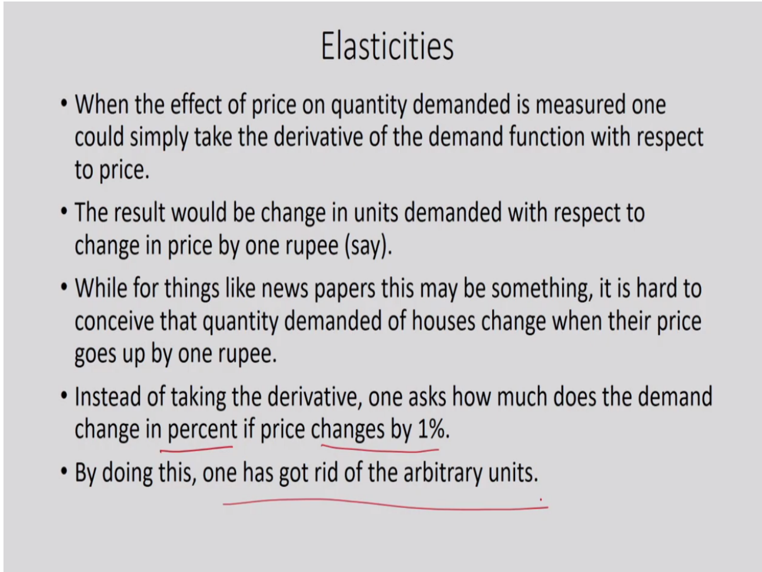

Finally, before we wrap up, here is a little bit of discussion about Elasticities. What are Elasticities? When the effect of price on quantity demanded is measured, one could simply take the derivative of the demand function with respect to price. So, suppose you want to find out if price changes, then how much does the quantity demanded changes. So, how much are people responding to the change in prices?

How much are they buying more or buying less, that is what you want to find out. So, the first thing that comes to mind is that you take the derivative, derivative gives you the instantaneous change in the dependent variable with respect to change in the independent variable by 1 unit. Now here, if you take the derivative, the result would be change in units demanded with respect to change in price by 1 rupee. So, suppose you are talking about cars.

So, if the price changes by 1 rupee, how much is the demand changing how many more cars or how many less cars are people buying? Now, here the problem is this, while for things like newspapers, these may be something, it is hard to conceive that quantity demanded of houses change when their price goes up by 1 rupee. So, think about it, this is valid for cars also. So, if the car price rises by 1 rupee, because we are talking about Indian context.

So, minimum unit of money, they just take that to be 1 rupee. So, if the car price is rising by 1 rupee, then the derivative will give you how much less cars are people demanding. Now, that will be negligible. In fact, that could be 0 in fact, because people are not really bothered about change of the car price by 1 rupee, because car itself is an expensive thing. Similarly, the houses these are expensive things, if the price changes by 1 rupee, you will not get any result as far as derivative is concerned, it might be 0.

Whereas, those goods whose prices are not very large for example, newspapers or let us say matchsticks, if the price rises by 1 rupee that might affect the demand. So, there is a problem in this case, if you want to take the derivative, then in some cases, you might get some interesting result. In other cases, it might be ineffective. So, instead of taking derivative one asks, in elasticities, one asks, how much does the demand change in percent?

If the price changes by 1 percent, we are not talking about units as such, we are not talking about 1 rupee or 1 pound or 1 dollar. Here one is saying that, if it changes by 1 percent, then how much by what percent does the quantity demanded is changing. So, in both cases, if it is the price or if it is the quantity one is not depending on the units, one is depending on the percent. By doing this 1 has got rid of the arbitrary units.

So, another benefit of doing this apart from the fact that there is a difference between cars and cigarettes or let us say matchsticks. So, that is why you need percent rather than just units. Another benefit of doing only through percent is that now you have got rid of units. So, you can compare across countries. So, even if you are measuring elasticity in USA, the elasticity will be a pure number, it will not be in terms of pounds for dollar or something like that, in India, it could have been kg per rupees.

So, you see the units if they are there, then it is difficult to compare between countries. But since we are now talking only in terms of percentages, then it becomes unitary. Does one get the price elasticity of demand measured as at a particular price. That is also important that when you are measuring the price elasticity of demand it is being evaluated at a particular price.

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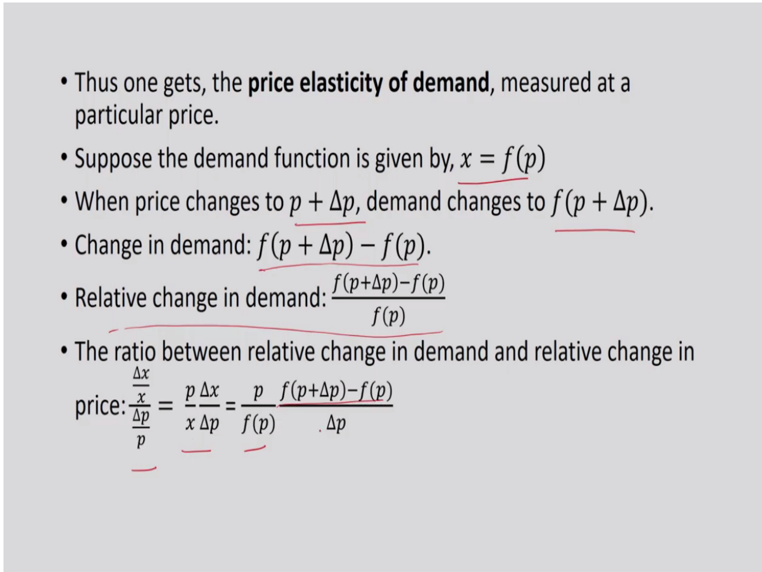

So, it is not a general expression of how much does quantity change with respect to price , it is evaluated only at a particular point of reference. So, here is a concrete demonstration, suppose the demand function is given by $x = f(p)$, p is the price, x is the quantity demanded. Now, suppose the price changes to $p + \Delta p$, and since the price has changed, the demand will change to $f(p) + \Delta p$.

So, what is the change in demand, the absolute changes in demand is given by this $f(p + \Delta p) - f(p)$, the new demand minus the old demand that is the change in demand. And what is the relative change in demand, it is given by this $\frac{f(p+\Delta p)-f(p)}{f(p)}$, this is the relative change in demand. The ratio between the relative change of demand and the relative change in price is given by this $\frac{\Delta x}{x} / \frac{\Delta p}{p}$ and this turns out to be this $\frac{p}{x}\frac{\Delta x}{\Delta p}$.

And now, I am going to just use this thing here. $x = f(p)$ and what is change in demand? Change in demand is this.

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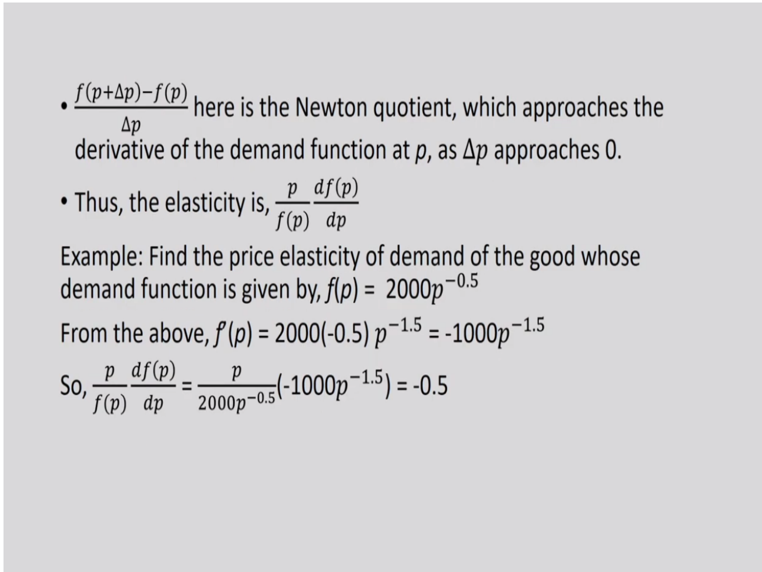

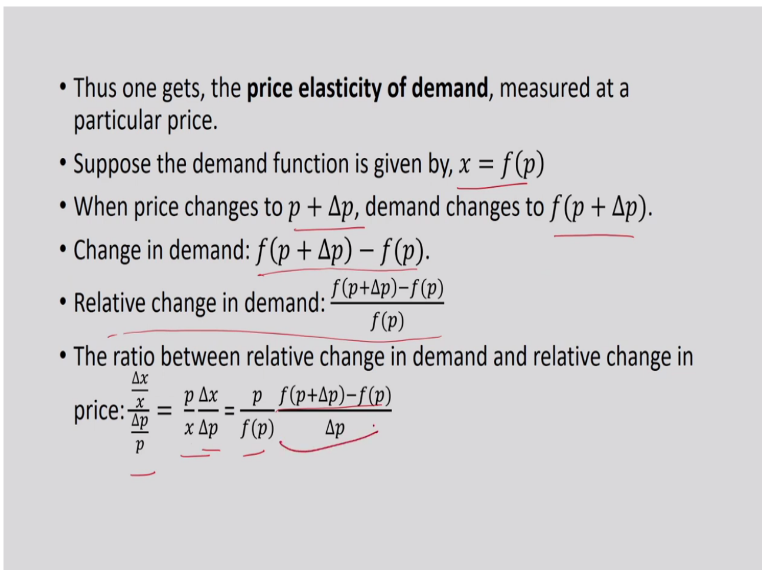

Now, this second term here is nothing but the Newton quotient.

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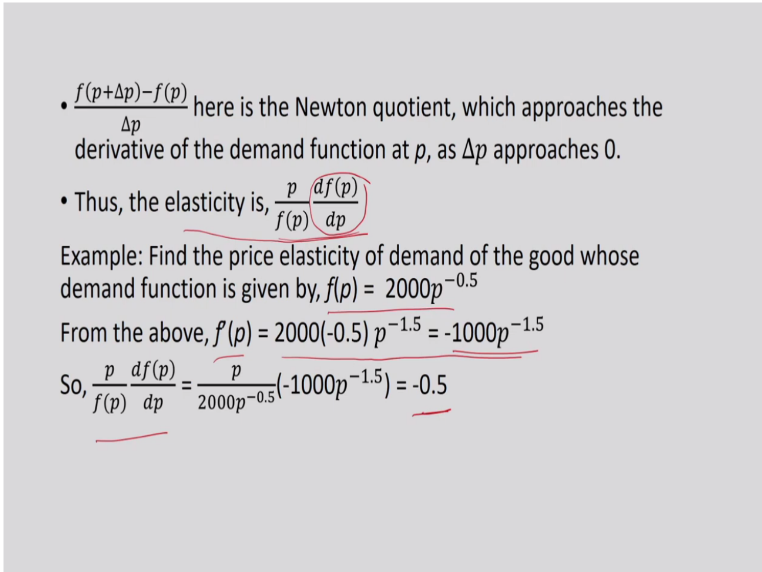

$\frac{f(p+\Delta p)-f(p)}{\Delta p}$ here is the Newton quotient, which approaches the derivative of the demand function at p as Δp approaches 0. So, changing the price you take, that as small as possible Δp approaches 0, thus, the elasticity becomes this $\frac{p}{f(p)} \cdot \frac{df(p)}{dp}$. So, this is nothing but the derivative of the demand function with respect to the price.

So, here is an concrete example, find the price elasticity of demand of the good whose demand function is given by this. So, as you can see $f(p) = 2000p^{-0.5}$, we are going to use this particular formula for that, I need to find out what is the derivative $f'(p)$ and this is by using the power rule this turns out to be this $-\ 1000p^{-1.5}$.

So, I put this in the elasticity expression and if I simplify this actually it becomes a pure number $-\ 0.5$. So, it no longer remains a function of p, it is a simple number $-\frac{1}{2}$.

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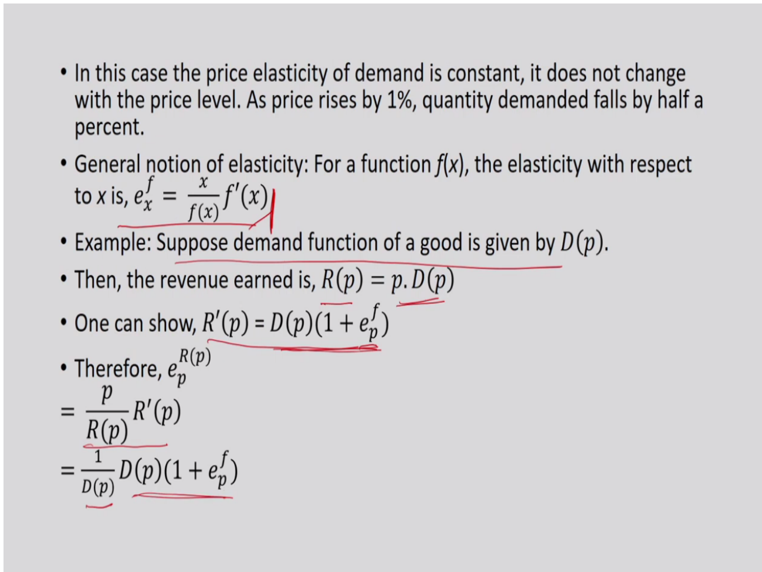

So, in this case the price elasticity of demand is constant. But I must caution that this is not a general case in many cases, it could be a function of p. It does not change with the price level as price rises by 1% quantity demanded falls by half a percent. That is what it means $-\frac{1}{2}$ means if price rises by 1% quantity demanded falls by $\frac{1}{2}$%. Now, this is a general notion of elasticity.

So, if you are just ignoring the fact that we are talking we are talking about only price elasticity of demand, there could be a general notion of elasticity for a function f(x), it is a function of x, the elasticity with respect to x is given by suppose this $e_x^f = \frac{x}{f(x)} f'(x)$. Here is an example. Suppose, the demand function of a good is given by D(p). So, the revenue earned, revenue earned means total amount of money that the sellers are getting is $R(p) = p.D(p)$.

And if you take the derivative of R(p) with respect to p, you will get this expression. So, I am going to skip some steps. You can show that this particular right hand side you can get it, if you take the derivative of this with respect to p and use the formula of price elasticity of demand which has been used here. Therefore, $e_p^{R(p)}$ that is the elasticity of the revenue with respect to price is given by this particular expression.

I am going to I am just using this particular formula here. And now you simplify this by using $R'(p)$ is given by this so I am substituting this here. And what is R(p)? $R(p) = p.D(p)$, so p and p will get cancelled. So you are going to get $\frac{1}{D(p)} D(p)(1 + e_p^f)$. So Dp, Dp will cancel.

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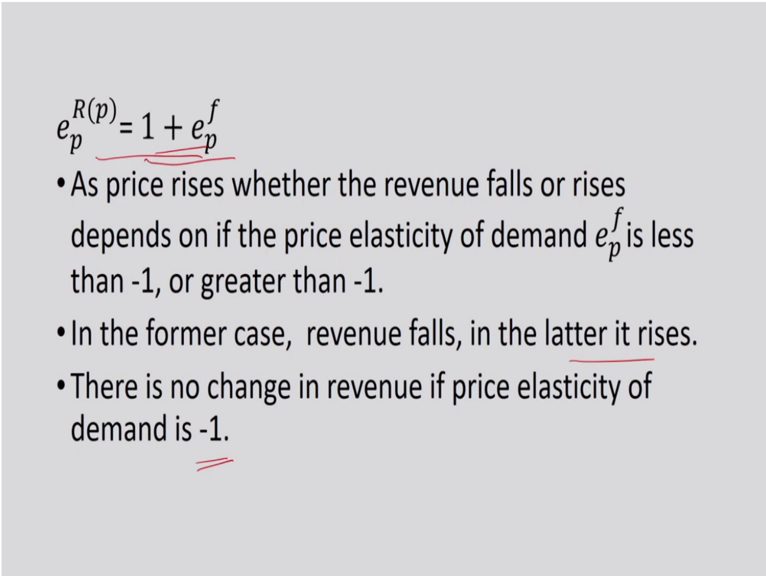

And therefore, the elasticity of revenue with respect to price is given by $1 + e_p^f$. What is e_p^f? It is the price elasticity of demand. So as price rises, whether the revenue falls or rises, depends on if the price elasticity of demand is less than -1 or greater than -1. In the former case, that is, if the price elasticity of demand is less than -1, then this whole expression becomes negative. So therefore, the revenue falls as the price rises.

And if it is the other case that if it is greater than -1, then the latter rises, that is the revenue rises as the price rises. If there is no change in the revenue, if the price elasticity of demand is exactly equal to -1, in that case, this right hand side becomes 0. So as the price rises, the revenue does not change at all.

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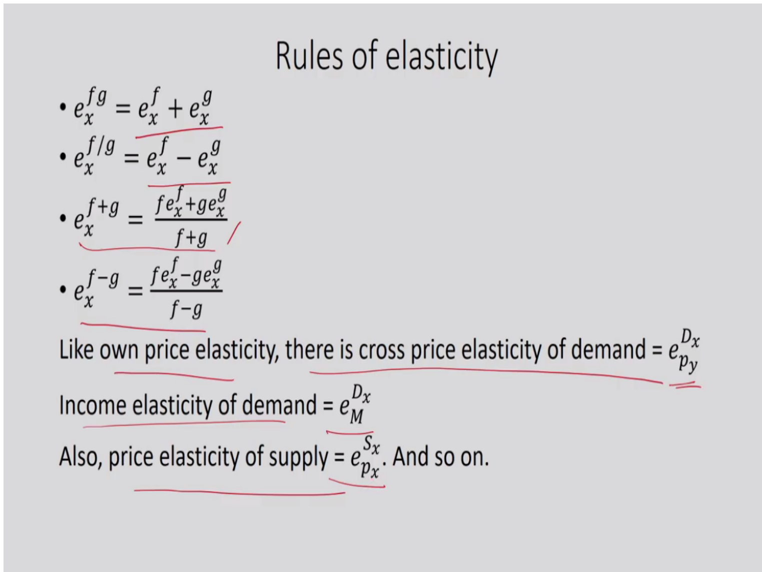

And we are going to end with some rules of elasticity. So if you take f multiply g and take the elasticity of that with respect to x, $e_x^{f.g}$, you are going to get $e_x^f + e_x^g$, so it is the sum of the elasticities. And if you take the quotient of them, $e_x^{\frac{f}{g}}$ then it is the difference of them $e_x^f - e_x^g$. If you take the summation of f and g e_x^{f+g} then it becomes a little bit complicated expression f of $\frac{fe_x^f+ge_x^f}{f+g}$ and similarly if you take the difference, it becomes this expression.

Now, we have talked about own price elasticity before, so you take a quantity demanded and you change the price of the same good that is called Own price elasticity. Now, there is something called a Cross price elasticity also, which is suppose price of some other good changes, then the quantity demanded our particular good can change. Well think about tea leaves and let us say sugar, if the price of sugar changes, it is conceivable that the demand for tea leaves can change.

So we can measure that also. So, this is given by $e_{p_y}^{D_x}$. Here the price and the quantity are two different goods. You can find out income elasticity of demand also if income changes, what is the change in the quantity demanded and that is called Income elasticity of demand. And all these things were from the point of view of demand there could be elasticities from the point of view of supply also.

So as the price changes from the suppliers point of view, how much do they respond to that, that is called the Price elasticity of supply. So this is given by $e_{p_y}^{S_x}$, and so on. There could be many more other kinds of elasticities that one can explore. I think we shall call it a day today. And I will see you in the next lecture. Thank you.