Mathematics for Economics - I

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Lecture 16

Exponential, logarithmic functions

Welcome to another lecture of this course Mathematics for Economics Part 1. So, this particular topic that we have been covering since the last two lectures, it relates to continuity, differentiability, the idea of limits and we have talked about different properties of differentiability when a function is differentiable at a particular point, when is it continuous, those things we have dealt with. We have also dealt with the idea of series and sequences.

Not only that, we have also talked about the applications of these series and sequences, for example, evaluation of different investment projects. So, if you have multiple projects to consider and which one shall you pick up. Suppose you have to pick up only one or let us say two out of many investment projects that you can take up then what is the criterion or what could be the criteria that help you to pick up a particular project, investment project.

So, those things we have talked about as applications of the things that we have been discussing. Now, today, what we shall deal with is the last lecture of this particular topic. We are going to talk about a quite important tool as far as mathematical economics is concerned and this is exponential functions and logarithmic functions. As we shall see later on, that these kinds of functions are extremely useful in many analyses of economics.

For example, if you want to study the growth rate of income or population, then logarithmic functions, exponential functions or if you want to find out the present value, present discounted value and in the same thread if you want to find out the evaluation of investment projects, then you have to find out the present value obviously, as we have seen, then also these ideas are very important, the ideas of exponential function and logarithmic functions. So, without wasting time let us start with our lecture.

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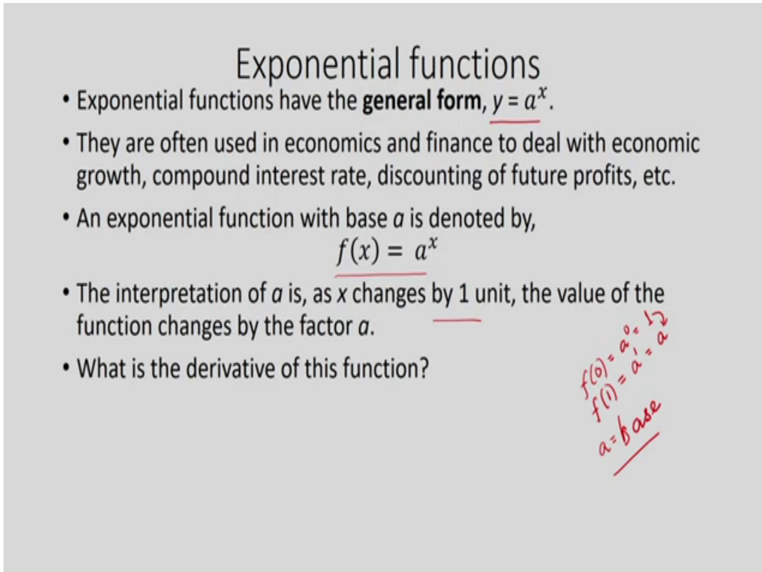

So, this is where we shall start today's lecture from. So, what are exponential functions? Exponential functions have the general form $y = a^x$. Now, here, obviously, it is a function so y is the dependent variable and x is the independent variable. What is this a? a is any fixed number. It could be a constant. But it is not a variable. So, it is given. As you can see, the independent variable is appearing here as an exponent that is it is a power on this given constant. So, it is a^x.

And where do you find such functions? They are often used in economics and finance to deal with economic growth, compound interest rate, discounting of future profits, etc. So, this is the form $f(x) = a^x$. The exponential function with base a is denoted by $f(x) = a^x$. So, a is called the base. This is the base of the function.

What is the interpretation of this base? The interpretation of base is as x changes by 1 unit the value of the function that is $f(x)$ changes by this factor a and that you can see. Suppose you take $f(0)$, $f(0)$ will be a^0, which is 1. Now, you take $f(1)$ that is x is changing by 1 unit, so it will be a^1 which is a. So, you can see that from 1 we are going to a. Therefore, the value of the function is getting multiplied by this quantity a. So, that is the interpretation of the a, the base.

It is the amount by which the value of the function gets multiplied when x changes by a single unit, 1 unit. Now, if a function is given, then we may want to find out what is the derivative of this function. So, that is the second thing that we are going to look at. The function we have defined $f(x) = a^x$, but what is $f'(x)$.

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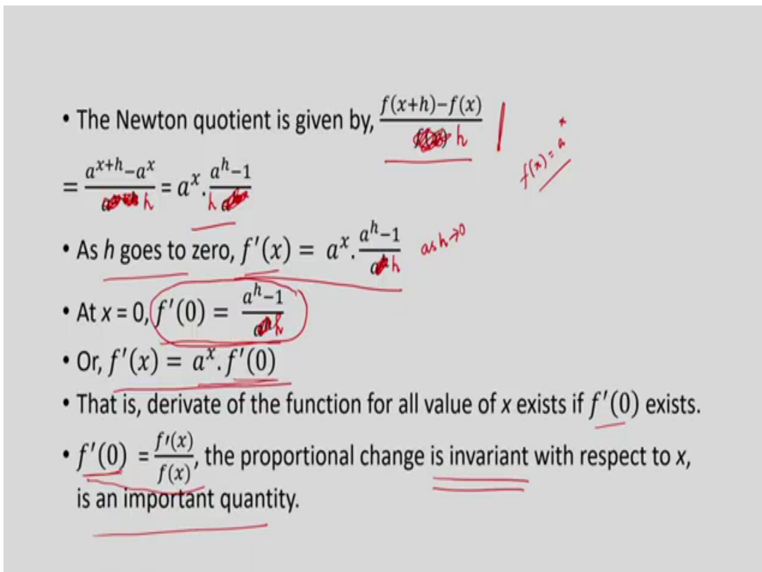

So, here is the Newton quotient. Newton quotient is given by this. Sorry, for this change, corrections that I have done here. So, Newton quotient is given by $\frac{f(x+h)-f(x)}{h}$, h is the change that is taking place, the increment that is taking place in x. So, we divide the change in the value of the function divided by that h.

Now, in this case since $f(x) = a^x$, so it will be $\frac{a^{x+h}-a^x}{h}$. And then the next step we take a^x common and so then we have this term, $a^x \frac{a^h-1}{h}$. Now, as h goes to 0, we know that we get the derivative that is the definition of derivative. So, therefore, $f'(x) = a^x \frac{a^h-1}{h}$.

Now, at $x = 0$, so I just substitute a^0 in case of x, therefore, on the left hand side I will get $f'(0)$. And on the right hand side, I will get $a^0 \frac{a^h-1}{h} = \frac{a^h-1}{h}$. So, this is the expression for $f'(0)$.

So, therefore, I find that $f^{'}(x) = a^{x}\frac{a^{h}-1}{h}$, which in fact is equal to $f^{'}(0)$. So, I substitute that here. So, I get this expression $f^{'}(x) = a^{x}\frac{a^{h}-1}{h}$.

Now, this is very important, this particular relation. And what it means is that the derivative of the function for all values of x exists if $f^{'}(0)$ exists, because $f^{'}(0)$ is appearing on the right hand side. As long as $f^{'}(0)$ is there, so $f^{'}(x)$ exists. Furthermore, I can see that if I divide both sides by a^{x}, then I will get $f^{'}(0) = \frac{f^{'}(x)}{f(x)}$. The proportional change is invariant with respect to x and this is an important quantity.

So, $f^{'}(0) = \frac{f^{'}(x)}{x}$ and what is the right hand side $\frac{f^{'}(x)}{f(x)}$. $\frac{f^{'}(x)}{f(x)}$ is the proportional change. And that always remains the same for this kind of function, the exponential, general exponential function. It is not only always the same and it is given by this quantity, it is given by $f^{'}(0)$.

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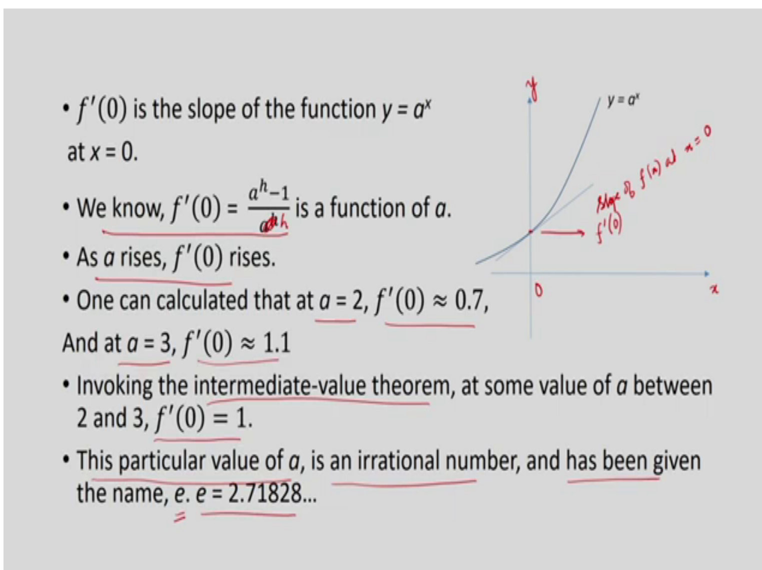

And what is $f^{'}(0)$? I said this is an important quantity. But what is the interpretation of $f^{'}(0)$. Well, here I have drawn the function. So, you have x along the horizontal axis and y along the vertical axis. And this function is going to look like this. It is an upward rising function. And it is going to intersect the y-axis at a positive intercept. And that point where the intersection is taking place, at that point, I have $f'(0)$, the slope.

This is the slope of $f(x)$ at $x = 0$ and this is given by $f'(0)$. And therefore, basically, if I draw a tangent to that graph at this particular point of intersection, then the slope of that tangent is $f'(0)$. You might ask, why do I draw this function like this? It is an upward rising function, because as x goes on rising, obviously a^x will go on rising.

And we also know that as x goes on rising, y will go on rising at an increasing rate that we shall, let us see, let us consider that for future discussion. It will come just shortly after this. That as x goes on rising, y rises, but it rises at an increasing rate. Therefore, you have a function which is rising at an increasing rate. Now, we know that $f'(0) = \frac{a^h-1}{h}$ and this is a function of a.

So, as a changes, this thing will also change. $f'(0)$ is this. So, $f'(0)$, as we know, it is the slope of this function at this particular point. So, for example, suppose $a = 2$, then one can calculate that $f'(0) \approx 0.7$. That is the slope of this line. This line, if you take $a = 2$, is equal to 0.7, is close to 0.7.

And if you take a higher value of a, suppose $a = 3$, then $f'(0)$ actually rises and it is approximately equal to 1.1. That is what is meant by this: that a rises the slope of this function at $x = 0$ rises. So, basically, this tangent is getting steeper and steeper. Now, invoking the intermediate-value theorem, at some value of a between 2 and 3, $f'(0) = 1$.

And I have talked about the intermediate value theorem, $f'(x)$ or $f'(0)$ is a continuous function of a, as a goes on rising, we have seen that this $f'(0)$ goes on rising. So, at some point between 2 and 3, it must be the case that $f'(0) = 1$, because at 2 $f'(0) \approx 0.7$ and at 3 it is very close to 1.1. So, therefore, there will be some value between 2 and 3, maybe very close to 3, rather than 2 and that point $f'(0) = 1$.

And this particular value of a is important. It is an irrational number. And it has been given the name e. And the exact quantity of e cannot be specified because it is an irrational number, but it is very close to 2.71828 and it goes on like that. So, the main thing that we are getting here is that I have defined a quantity which is e and this is very close to 2.71818. And what this e does, is that at this particular value of e if I take a is equal to this particular value then $f^{'}(0) = 1$. The slope becomes equal to 1.

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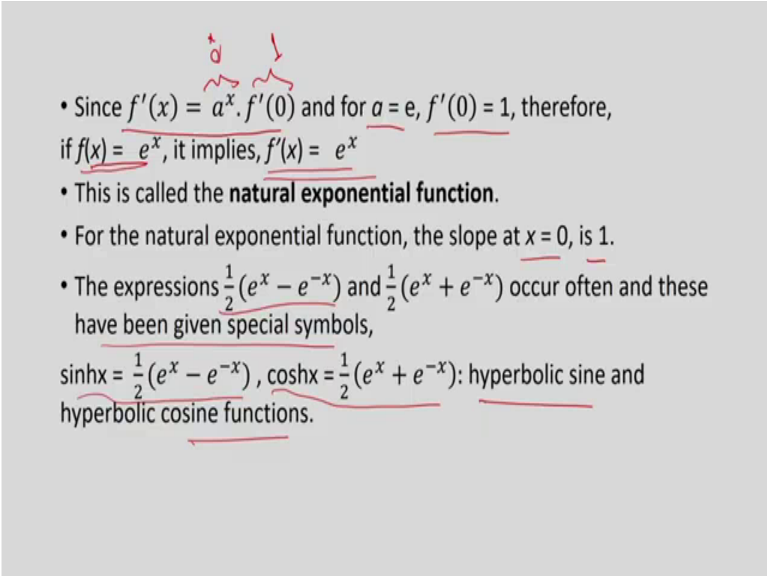

Now, this is known to us $f'(x) = a^x. f'(0)$. And for $a = e$ we have just seen that $f'(0) = 1$. Therefore, if I put $f(x) = e^x$then $f'(x) = e^x$. So, because this part is becoming 1 and this part is becoming e^x. So, therefore, $f'(x) = e^x$.

Now, this function is very often encountered in economics. This is called the natural exponential function, this function, $f(x) = e^x$. For the natural exponential function the slope at $x = 0$ is 1 that we have just seen. That is how it is defined actually. The expressions $\frac{1}{2}(e^x - e^{-x})$ and $\frac{1}{2}(e^x + e^{-x})$ occur often and these have been given some special symbols.

These symbols are given as this, sinhx this is called hyperbolic sine function and $sinhx = \frac{1}{2}(e^x - e^{-x})$ and $coshx = \frac{1}{2}(e^x + e^{-x})$. This is the cosine function. So, this sine and cosine functions are actually functions of natural exponential functions.

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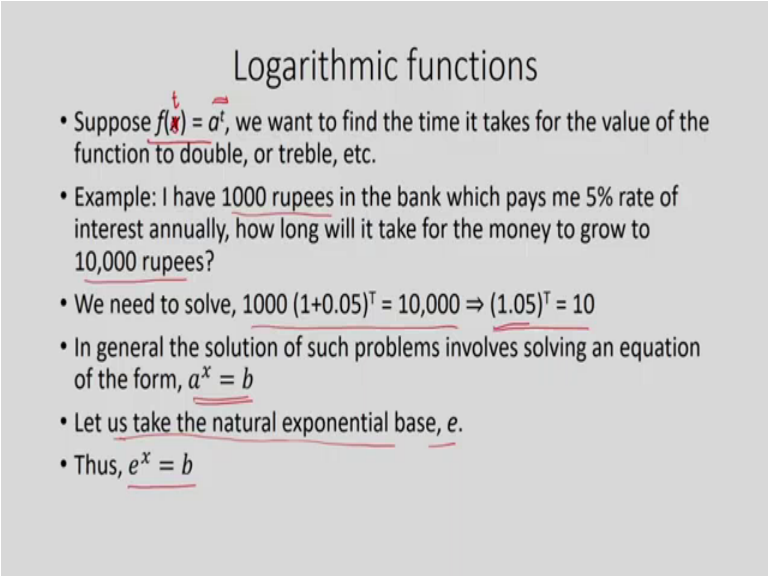

Now, relatedly there is this logarithmic function. So, what is the idea of a logarithmic function? Suppose $f(t) = a^t$, because it is a function of time, suppose t is time, and we want to find the time it takes for the value of the function to double or treble etc. So, time is changing. So, therefore, this right hand side will go on changing. It is going to go up. And if that is true, then at what time the value of the function will double or it will treble or it will rise by 100 times etc, etc.

Now, notice however, that this will rise if you have a to be greater than 1. Now, here is an example, suppose, I have INR1000 in the bank, which pays me 5 percent rate of interest annually. Now, how long will it take for the money to grow to INR 10,000. So, the money is going to go up by 10 times and the rate of interest in the bank is 5 percent. I wanted to find out what is the time it will take.

Now, basically to find that out I have to solve this equation. It is $1000(1 + 0.05)^T = 10000$. So, I have to solve for capital T. Now, this boils down to this simpler expression $(1.05)^T = 10$. In general, the solution to such problems involves solving an equation of the form $a^x = b$.

Here a, in this particular case, $a = 1.05$ and $b = 10$. And we take the natural exponential base which we know is e that was our natural exponential base. So, instead of small a I am replacing the small a by small e. So, I get $e^{x} = b$.

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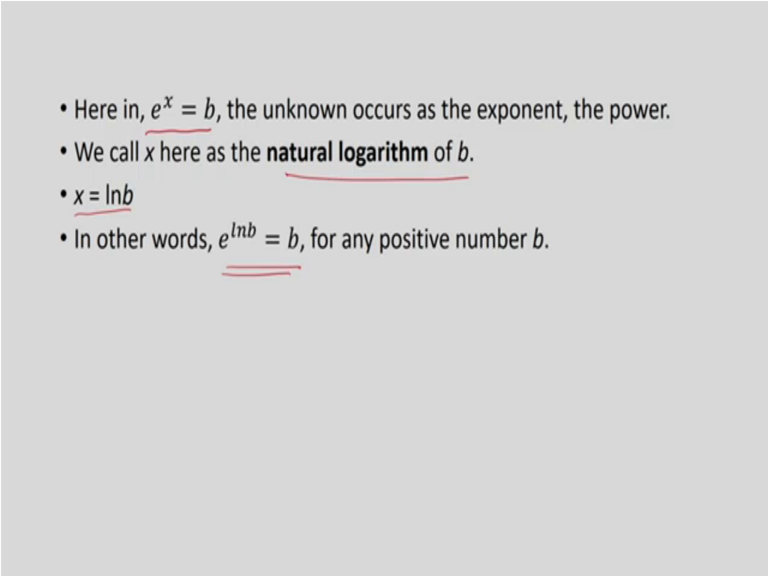

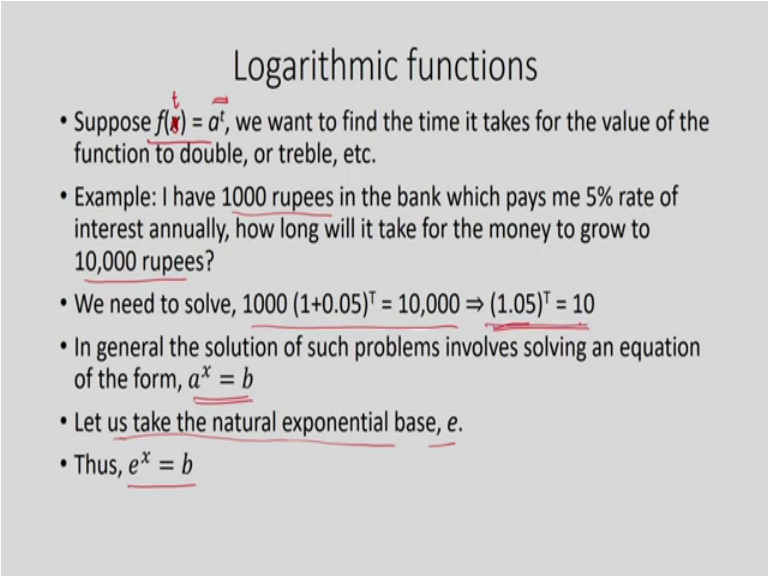

Here in $e^{x} = b$, the unknown occurs as the exponent. So, x is the unknown, e we know and b is also known to us. What is not known is that small x. And that is appearing as the exponent. So, it is the power. So, here, for example, the T has to be found out, so T is appearing as the exponent. We call x here as the natural logarithm of b, and we write this as $x = lnb$, log natural b.

So, $x = lnb$, and if I substitute back that in this expression what I get is $e^{lnb} = b$. So, in other words, $e^{lnb} = b$ this is true for any positive number b. So, that is important. b has to be a positive number.

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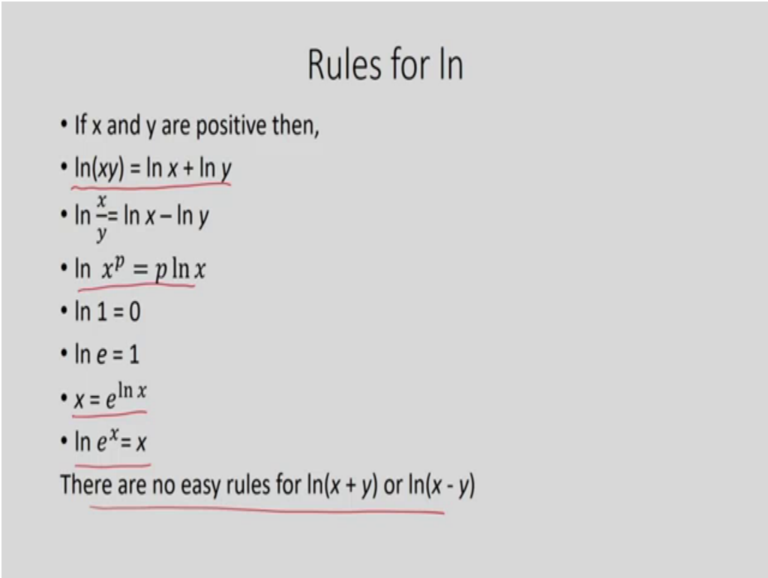

Now, here are certain rules for natural log or ln. If x and y are positive, then $ln(xy) = lnx + lny$; $ln(\frac{x}{y}) = lnx - lny$; $ln\, x^p = p\, lnx$; $ln\, 1 = 0$; $ln\, e = 1$; and $x = e^{ln\, x}$, that we have just seen; and $ln\, e^x = x$.

However, we can see that $ln\,(x + y)\; or\; ln\,(x - y)$, they do not have any easy rules, like, for example, $ln(xy)$. So, it is not the case that $ln\,(x + y) = lnx + lny$.

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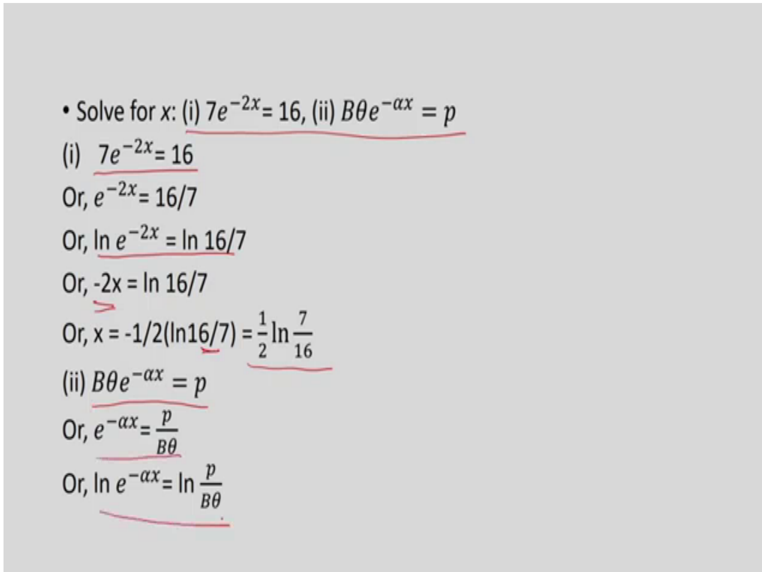

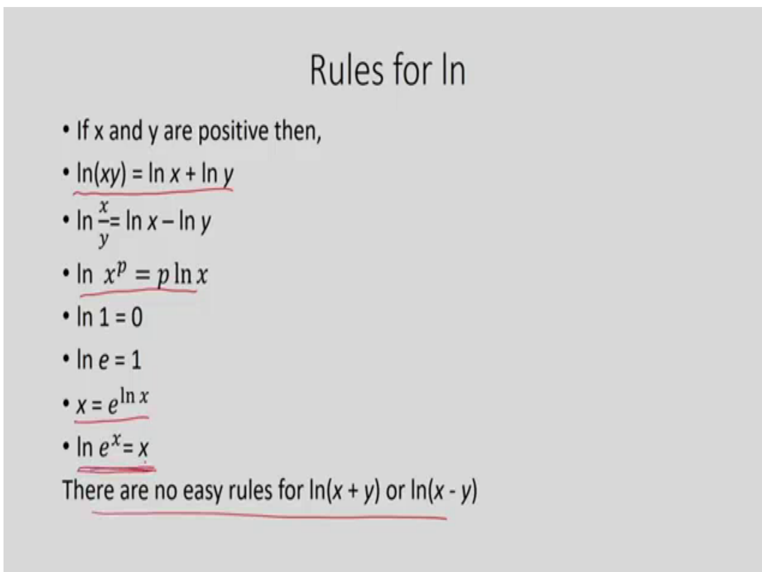

Here are some examples to practice. Now, suppose we have to solve for x for these two problems. And the first problem is this, $7\,e^{-2x} = 16$,. So, from this I divide both sides by 7, I get $e^{-2x} = \frac{16}{7}$, and then I take natural logarithm of both sides. Now, if I take $ln\ e^{-2x} = ln\,\frac{16}{7}$, then it simply becomes - 2x. So, how do I know that?

Here, this is the rule that I am using here, $ln\, e^{x} = x$. So, $ln\ e^{-2x} =- 2x$. And on the right hand side, the expression remains the same. So, therefore, $x = (-\frac{1}{2})\, ln\, \frac{16}{7}$. And this turns out to be $\frac{1}{2}\, ln\, \frac{7}{16}$. So, I just have the reciprocal of $\frac{16}{7}$, which is here, and it becomes $\frac{7}{16}$.

And why this is so because I have here used this rule, the quotient rule. Now, the second one, let us look at that , $B\theta e^{-\alpha x} = p$. So, we have to solve for x. So, I take these constants $B\theta$ to the right hand side, and then I take the log natural log of this e, $ln\, e^{-\alpha x} = ln\, \frac{p}{B\theta}$.

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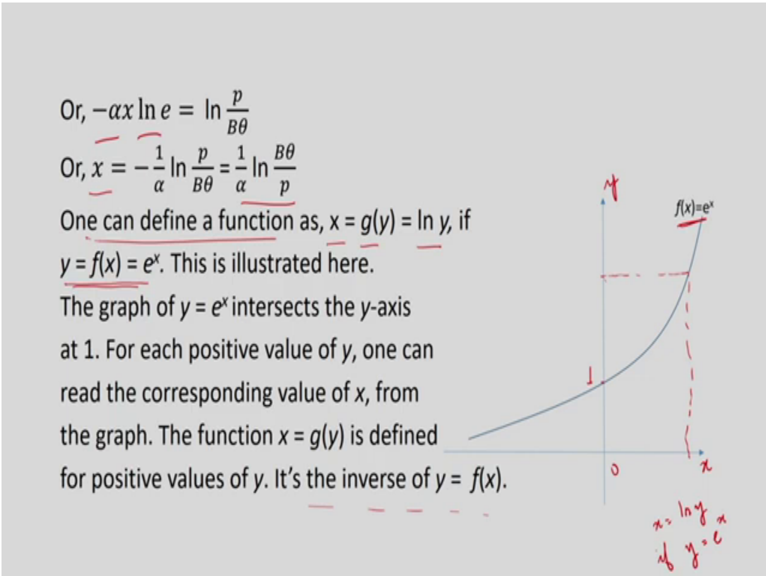

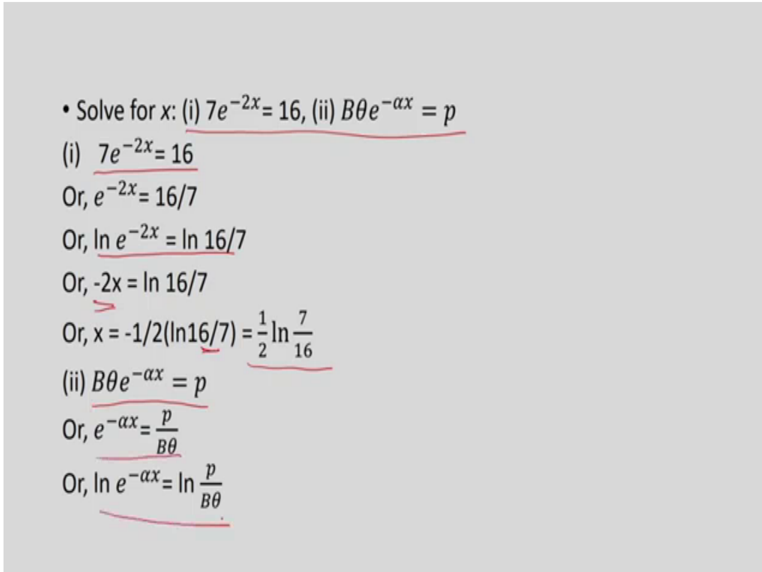

And this will simply become $-\ \alpha x$, because $ln\, e\ =\ 1$ and on the right hand side, I have $ln\ \frac{p}{B\theta}$. And so I get $x\ =-\ \frac{1}{\alpha} ln\ \frac{p}{B\theta}$. And, again, I use the rule for quotients. And I get $x\ =\ \frac{1}{\alpha} ln\ \frac{B\theta}{p}$. So, this is how we solve for x.

Now, we define a function, one can define a function as $x\ =\ g(y)\ =\ ln\, y$. So, x is a function of y. If $y\ =\ f(x)$like this, $y\ =\ f(x)\ =\ e^{x}$. And this is illustrated here. The graph of y, here $y\ =\ e^{x}$, the exponential, natural exponential function. It intersects the y-axis at 1 that we have seen before.

Why, because if you put e^{0}, it becomes 1. For each positive value of y, one can read the corresponding value of x. From the graph, the function $x\ =\ g(y)$ is defined for all positive value of x and it is the inverse of $y\ =\ f(x)$. So, here, $x\ =\ ln\, y$ if $y\ =\ e^{x}$. So, one is the inverse of the other.

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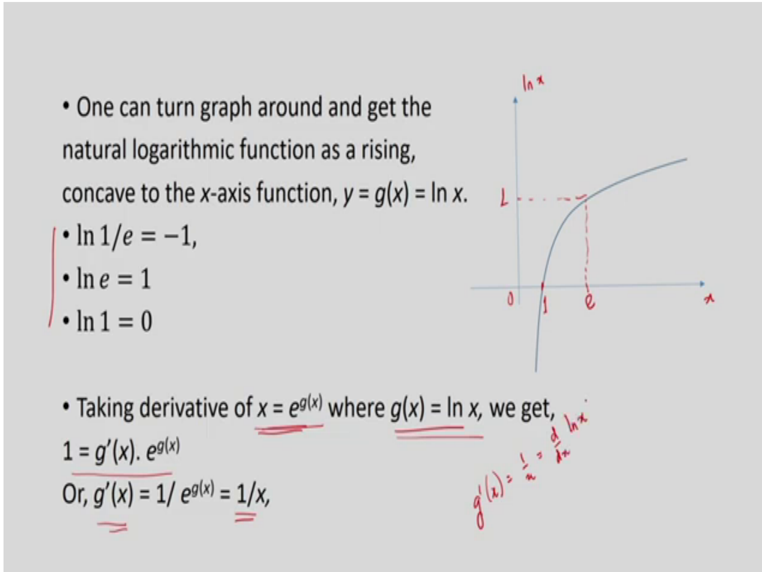

So, this is what we have done here. I have turned this figure around. Now, along the vertical axis, I have the log. So, this is log. So, instead of writing it $ln\, y$, I am writing is $ln\, x$, because x is generally taken in the horizontal axis. So, here, $y = ln\, x$. That is what we are getting here. And here are some basic properties of log function.

If you take $ln\, 1/e = -1$, because $ln\, 1/e = ln\, e^{-1}$ and that becomes - 1, $ln\, e = 1$, and $ln1 = 0$. So, basically, this point is 1. And if you have some point here, which is 1, then this should be e. Now, whatever the derivative, if you have $x = e^{g(x)}$, where $g(x) = ln\, x$, then we get, I am taking the derivative of this function, so with respect to x, I will get $1 = g^{'}(x). e^{g(x)}$.

I am just using the formula for the derivative of an exponential function. And therefore, I will get $g^{'}(x) = 1/e^{g(x)}$. But what is $e^{g(x)}$, $e^{g(x)} = x$. So, $g^{'}(x) = 1/x$ and that is what we wanted to find out. So, $g^{'}(x) = 1/x$ and this is nothing but $\frac{d}{dx} ln\, x$. So, that is the formula for derivative.

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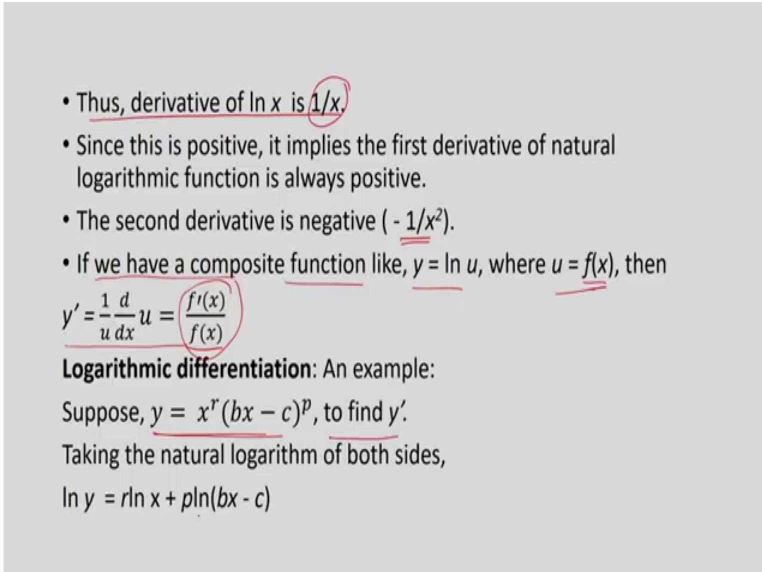

So, derivative of log of x, that is natural log of x, is equal to 1/x. Now, 1/x is a positive number, because x is always positive. Since this is positive, it implies the first derivative of natural logarithmic function is always positive. But if you take the derivative of this, then you will get the second derivative, then it becomes a negative quantity $- 1/x^2$. So, the second derivative is negative of a logarithmic function.

Now, if you have a composite function, so what happens if you have a composite function like $y = ln\, u$, where u itself is a function of x, $u = f(x)$. Then how do I find the $y^{'}$, that is the derivative of y with respect to x, I use this formula, and I will get $\frac{f^{'}(x)}{f)x)}$. This is the proportional change in u with respect to x. And that is equal to $y^{'}$.

Logarithmic differentiation, there is something called logarithmic differentiation, I start with an example. Suppose, $y = x^r(bx - c)^p$, I have to find out $y^{'}$. So, what I do is that I take the natural logarithm of both sides of this particular function. And if I do that, I will get $ln\, y = rln\, x + pln(bx - c)$.

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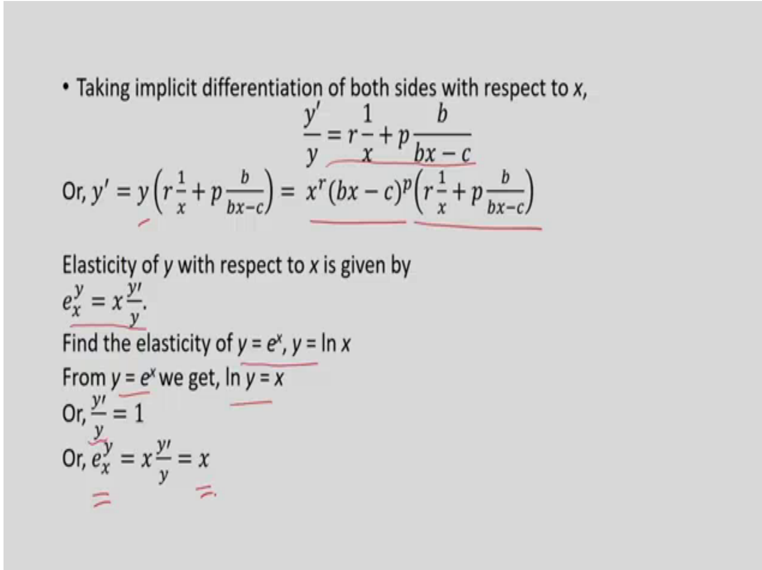

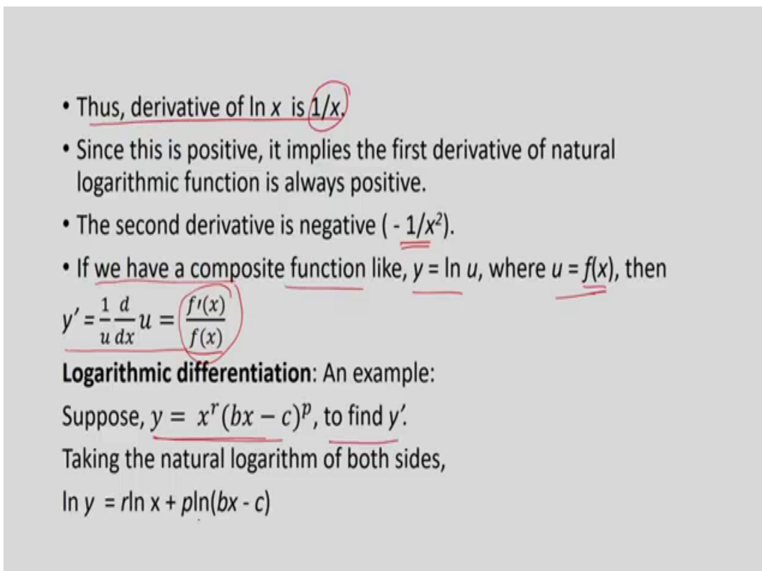

Now, I take the implicit differentiation of both sides with respect to x. So, what I will get is $\frac{y'}{y} = r\frac{1}{x} + p\frac{b}{bx-c}$. So, that is what we are getting here. And from this, I get y', because I have to find out y' at the end of the day, so y' will be y multiplied by this expression on the right hand side, so it gets $y' = y(r\frac{1}{x} + p\frac{b}{bx-c})$.

And then I expand the y. This is the y. So, I get this expression, $x^r(bx - c)^p(r\frac{1}{x} + p\frac{b}{bx-c})$ as my y'. Now, I come to the idea of elasticity. Remember, I introduced the idea of elasticity. Now, we shall see that with logarithmic function the elasticity is somewhat easy to find. Elasticity of y with respect to x is given by, as we know, it is given by $e^{y}_{x} = x\frac{y'}{y}$. Now, suppose I have to find out what is the elasticity of $y = e^{x}$ or $y = ln\, x$, these two functions are given, I have to find the elasticity of y with respect to x.

Now, from $y = e^{x}$, if I take the log of both sides, I will get $ln\, y = x$. And then I differentiate both sides with respect to x. So, through implicit differentiation, I will get $\frac{y'}{y} = 1$. And so I know the formula of elasticity is $e^{y}_{x} = x\frac{y'}{y}$. And $\frac{y'}{y} = 1$, so this becomes simply x. So, elasticity is equal to x for this first function.

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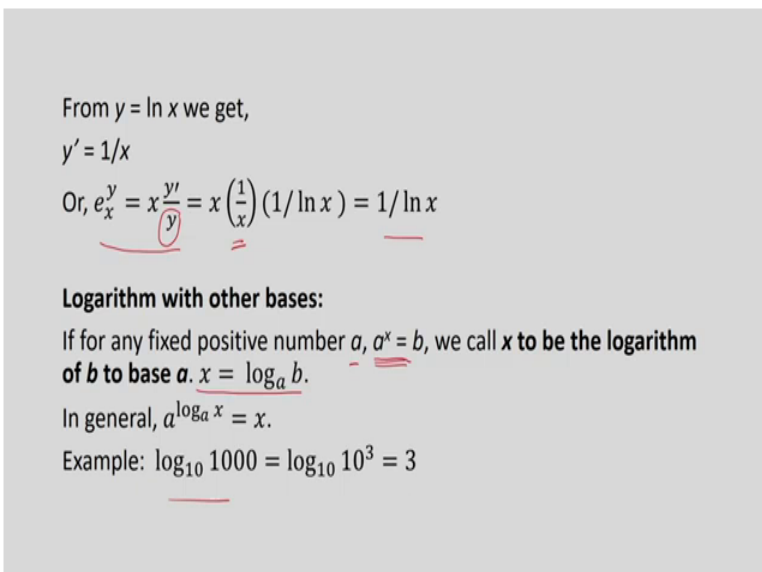

For the second function, $y = ln\, x$, what I do is that I take the derivative of both sides with respect to x, then it becomes $y' = 1/x$, because I am taking the derivative of log x. And what I know about the elasticity, I use the formula. I have already found out y'. I substitute that here. This is $(1/x)(1/ln\, x)$, log x because y is there, and this becomes equal to $1/ln\, x$.

Now, all these were logarithmic functions with base e, natural exponential. Now, if I take log with other bases then also it can be defined. For any given fixed positive number small a, if I have $a^{x} = b$, we call x to be the logarithmic of b to base a. So, write as $x = log_{a} b$. And therefore I can substitute this back here.

So, $a^{log_{a} x} = x$. So, here are some examples. Suppose you have $log_{10}1000$ then you can express $1000 = 10^{3}$ and this will become simply equal to 3, because 3 will come in front and $log_{10}10 = 1$.

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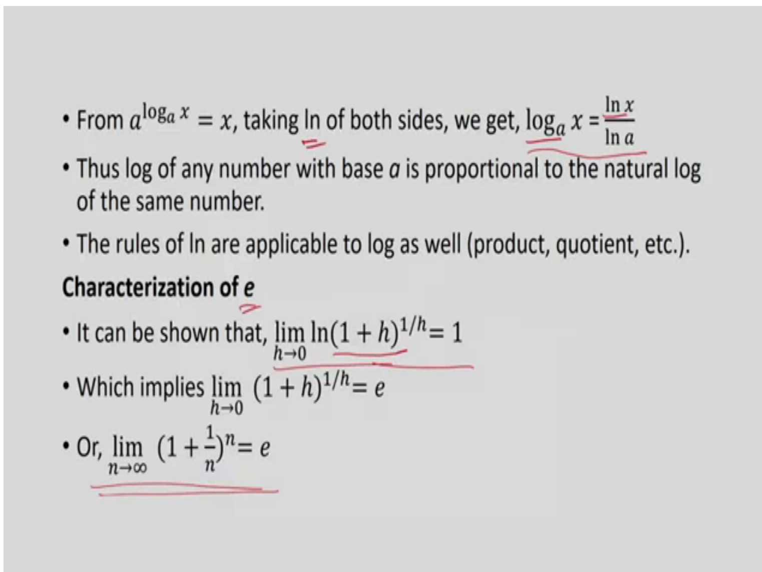

From $a^{log_{a} x} = x$ taking natural log of both sides I will be able to get this $log_{a} x = \frac{ln\, x}{ln\, a}$. Now, natural log of a is a constant. Thus log of any number with base a is proportional to the natural log of the same number, because this is equal to $ln\, x \frac{1}{ln\, a}$.

So, it is proportional to log natural log x. And rules for log natural are applicable for log in general as well that is the product rule, quotient rule etc. are applicable. Now, characterization, I come to characterization of small e, because we know we have found out the small e, but this can be characterized in a different manner also apart from what we have done before.

Now, it can be shown that if you take the $\lim_{h \to 0} (1 + h)^{1/h} = 1$. This becomes equal to 1. Which means that forget about the log natural, this particular quantity, this will go to e.

That is why log of this thing goes to 1. Or limit of this, so instead of h I am taking the reciprocal of that, suppose that is n. So, I am replacing h by 1/n and if h goes to 0, then n goes to infinity. So, this thing should be correct that $\lim_{n \to \infty} (1 + \frac{1}{n})^{n} = e$. So, this is another way to understand the e.

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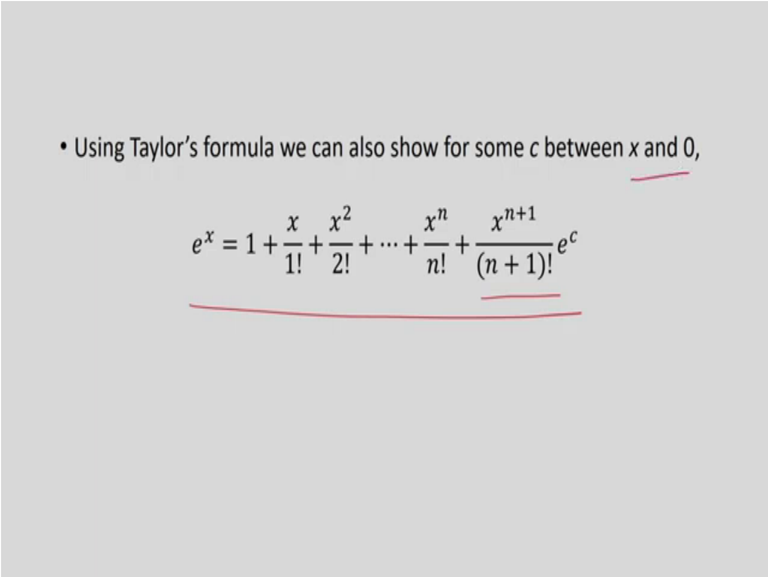

Taylor's formula can also show for, using Taylor's formula, we can also show for some c between small x and 0, $e^{x} = 1 + \frac{x}{1!} + \frac{x^{2}}{2!} + ... + \frac{x^{n}}{n!} + \frac{x^{n+1}}{(n+1)!}e^{c}$.. So, ex can be found out through this Taylor's formula as well.

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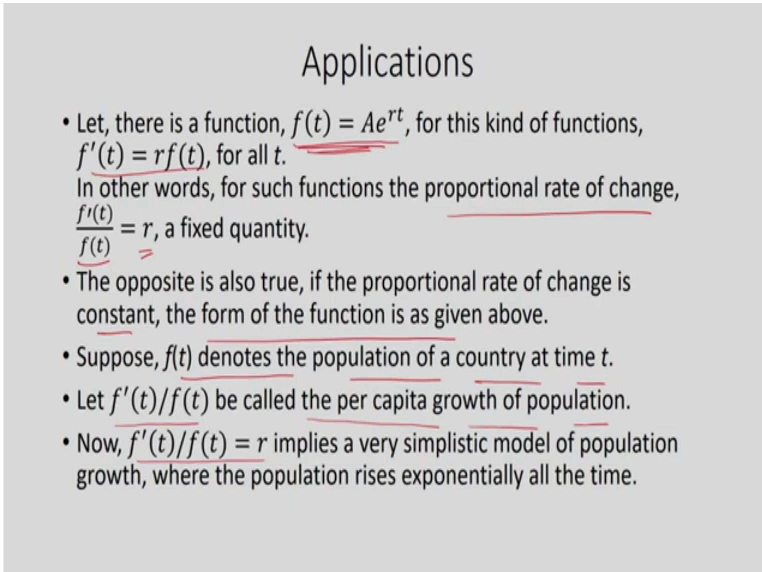

Now, I come to applications. Maybe I shall not be able to cover all the applications in this lecture itself. So, some parts will be left out to be covered in the next lecture. Applications, suppose, there is a function like this $f(t) = Ae^{rt}$. So, here capital A is constant, r is constant, e we know, it is that natural exponent and t is the variable here, t is the time. For this kind of functions, if I take the derivative of both sides with respect to small t, I get $f'(t) = rf(t)$and this is correct for all values of t.

In other words, for such functions, the proportional rate of change that is $\frac{f'(t)}{f(t)} = r$. And r is a constant as we know, so it is a fixed quantity. That is for these functions the proportion rate of change is given. It is constant. And the opposite is also correct. In fact, that, if the proportional rate of change of a variable is constant, then the form of the function is like this, this particular form.

Now, suppose $f(t)$ denotes the population of a country at a particular time t. $f(t)$ as we have seen it is a general function. So, I can interpret $f(t)$ in any way we like. So, let us suppose $f(t)$ is the population at a particular point t of a particular country. Now, what is $\frac{f'(t)}{f(t)}$. We can call this as per capita growth of population or per capita rate of change of population. I am taking the

$f^{'}(t)$ which is the change per unit of time and I am dividing that by the original population. So, that will give me the per capita growth rate of population.

Now, $\frac{f^{'}(t)}{f(t)} = r$, suppose that is the formula that is applicable for a particular country, that per capita growth rate of population is constant is given by r. But if that is correct, it implies a very simplistic model of population growth, where the population rises exponentially all the time, because behind this we know there is this function $f(t) = Ae^{rt}$. But this is an exponential function where r is constant.

So, it basically means that the population is rising all the time at an exponential rate of growth. And that may not be correct for a long period of time. It may be correct for a short duration of time, but may not be for longer time durations.

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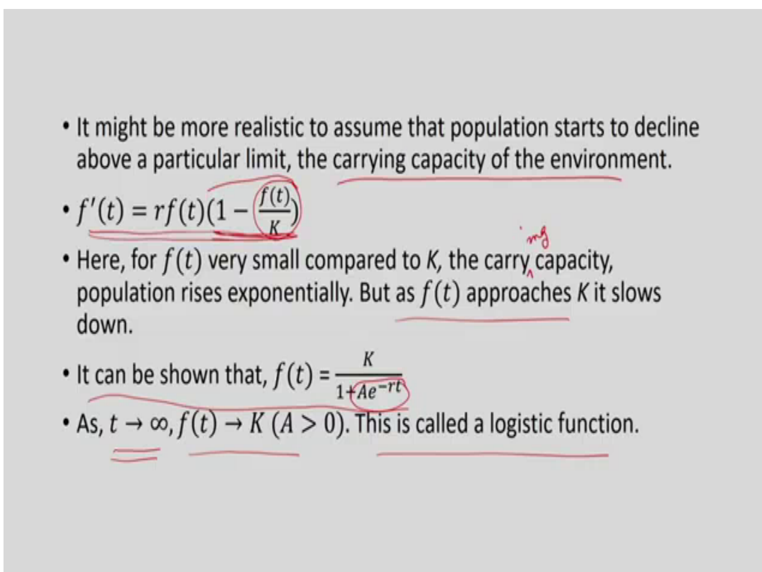

It might be more realistic to assume that the population starts to decline above a particular limit, which is the carrying capacity of the environment. So, the idea is that as the population rises to very high levels, it is possible that the environment will not be able to sustain that high amount of population. So, behind this is the old mathematician idea that population is limited by the natural resources of the country.

And with the growth of pollution and environmental hazards and different diseases, we are also seeing the return of that kind of idea to the mainstream that the environment is very important. Population can just go on rising, but after the point of time it can hit a upper limit and after that population might decline in fact. And this more realistic, but a little bit complicated picture is captured by this equation $f'(t) = rf(t)(1 - \frac{f(t)}{K})$, but then there is another factor here $(1 - \frac{f(t)}{K})$.

Now, as you can see that as $f(t)$ becomes higher than K, then this term becomes negative and if this term is negative then $f'(t)$ is negative which means the population is going down. So, basically $f'(t)$ becomes negative if $f(t)$ exceeds capital K. Here for $f(t)$very small compared to K the carrying capacity. This is carrying capacity. Population rises exponentially. So, this part becomes equal to 1, roughly equal to 1 if $f(t)$ is very small compared to capital K.

So, in this case, there is no environmental problem. Population has a lot of margin to grow. But as $f(t)$approaches capital K it slows down. So, imagine$f(t)$ not equal to K, but very close to K. So, then this term will be very close to 1. In that case, 1 minus term which is very close to 1 then this factor becomes close to 0. So, in that case, $f'(t)$ approaches 0. So, that is what is meant by this carrying capacity, the idea of carrying capacity, capital K. It sets an upper limit.

You do not actually have to reach that. Even if you are very close to that carrying capacity, the population starts to slow down. It can be shown that if I start from this particular form, I can go to this particular form, $f(t) = \frac{K}{1+Ae^{-rt}}$. Now, this is called a logistic function.

And in this function suppose you take t approaching infinity, and if you take t approaching infinity what we can show that this function, the value of the function $f(t)$ approaches capital K if capital A is greater than 0, because this term as t goes to infinity this term goes to 0. So, therefore, $f(t)$ becomes very close to capital K. So, geometrically, how does this look like?

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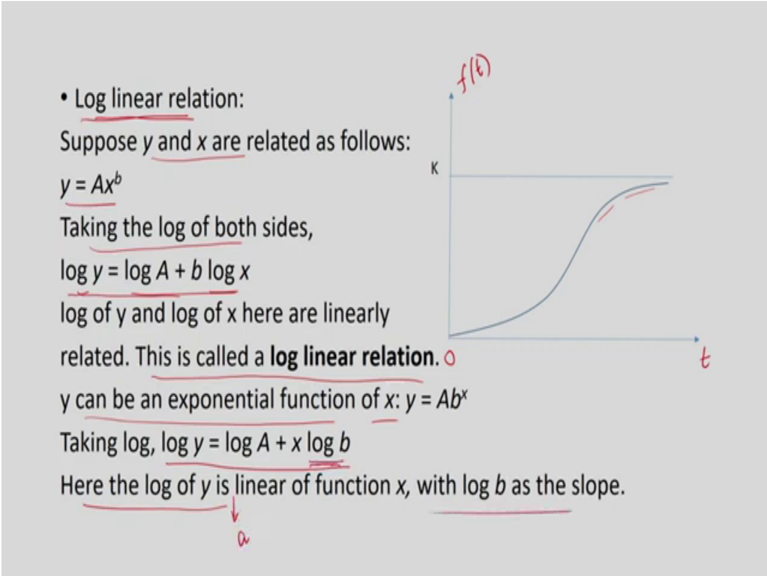

Geometrically, this is how it looks like. You have time here on the horizontal axis and on the vertical axis we have the population which is $f(t)$. So, the population starts to rise at a very steep rate first, then it is sort of slows down and you can see that it approaches capital K, but it will not be able to reach capital K ever. As t goes to infinity it goes on approaching capital K. So, this was an application of logarithmic function, exponential functions in case of population growth and what does it have to deal with environment.

Now, we take up another topic which is log linear relation, because again this is used often in economics. Suppose y and x are related as follows, $Y = Ax^b$. So, here A and b are constants, small x and small y are variables. Now, we take the log of both slides. This could be log with any base.

I am not specifying the base. So, $log\, y = logA + b\, log\, x$. Now, from this I can see that $log\, y$ and $log\, x$, they are related in a linear manner. $log\, y$ you can think of that to be another variable altogether and $log\, x$ is another variable altogether, so maybe capital Y and capital X. Then capital Y and capital X are linearly related to each other.

Now, this kind of function is called a log linear relationship. It is a linear relationship, but log of the variables have been taken first. Now, we take y to be an exponential function of x, $y = Ab^x$.

So, it is an exponential function of x. Now, if I take the log like before of both sides then I get $log\, y = logA + x\, log\, b$.

Here also we are getting kind of linear function here, but in that linear relationship $log\, y$ can be considered one variable and x is the other variable. So, one of the two variables is a log function, the other is not. Here $log\, y$ is a linear function of x with $log\, b$ as the slope. $log\, b$ is the slope here.

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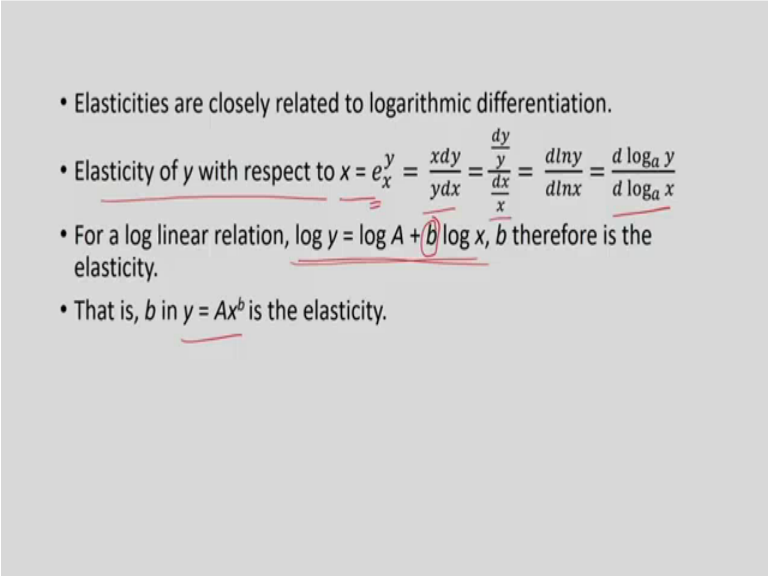

Now, elasticities, as we have seen, are closely related to logarithmic differentiation. So, suppose you have elasticity of y with respect to x, this is given by e^y_x. Now, we know the formula for that is this, $\frac{xdy}{ydx}$ So, I rewrite it like this, $\frac{dy/y}{dx/x}$. And dy/y is nothing but $d\, lny$. $d\, lny$ is differential of log y. And therefore I get this form that this becomes $\frac{dlny}{dlnx}$. The base could be anything. It could be natural logarithm. It could be non-natural logarithmic.

For a log linear relation, $log\, y = logA + b\, log\, x$, b therefore is the elasticity, because if you take this and you take the $dlny/dlnx$ then it becomes b and $dlny/dlnx$ we have seen that is the elasticity, so b is the elasticity. Therefore, in the relation $Y = Ax^b$, b is actually the elasticity of y with respect to x. So, this is an important relationship which is often used in econometrics.

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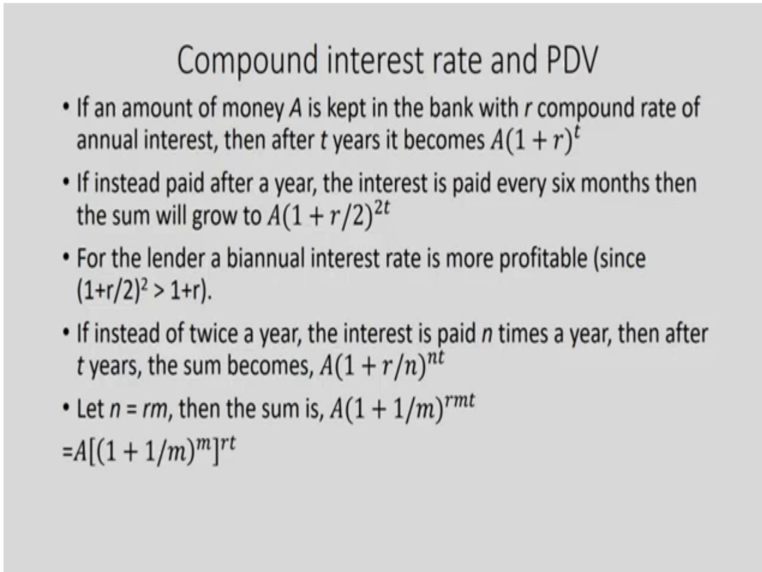

I think I will stop here and take up this particular topic in the next lecture. And in the next lecture if we have time, we are going to start with a new topic as well. So, I will call it a day. And thank you.