

Mathematics and Economics – 1 Sets and Set Operations

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Module Name: Sets

Lecture 3: Definitions

Welcome to the third lecture of this course called Mathematics for Economics. My name is Debarshi Das, I teach at the Department of Humanities and Social Sciences at Indian Institute of Technology, Guwahati. This course we have already covered one module, which was about Mathematical Logic. We also talked about the real number system, the fact that there are different kinds of numbers, rational numbers, irrational numbers, integers, negative numbers, positive numbers, etcetera, etcetera.

And we also talked about what is known as Mathematical Logic, how to prove anything? What are the different kinds of proofs that one can think of in mathematics? And the same kinds of strategies of proofs are used in economics also. For example, proof by contradiction, or direct proof, those things we talked in the last module. Now, today we are going to talk about something else, we are going to start with a new topic.

This topic is the part of the second module, and you can see it on your screen. It is called sets and set operations. So, here we are going to actually be detailing about some of the things that we talked about in the first module itself. For example, we talked about Venn diagrams, when we talked about Mathematical Logic, how to represent Mathematical Logic.

Now, in this module, we are going to be a little bit more elaborate, as far as Venn diagrams are concerned, and we are going to talk about what are known as sets and these things will be extremely crucial as far as the basic analysis of economics is concerned. So, let us start without further delay. So, here is the first slide you can see on the screen.

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Sets

- A collection of objects is called a **set**. The objects contained in a set are called the **elements or members** of the set.
- For example, the vowels in English alphabet form a set, say A .
- $A = \{a, e, i, o, u\}$ ✓
- We use the second brackets { and } to denote a set. The elements are written inside the brackets.
- Elements of a set can be numbers.
- Example: set of numbers for which $x(x - 1)(x + 2)$ is zero.
- This is given by, $S = \{0, 1, -2\}$.
- Two sets are equal when each element of the first is contained in the second, and each element of the second is contained in the first.
- We write $A = B$.

$\{ \dots \}$
 $x(x-1)(x+2) = 0$

So, we are defining what is known as a set, what is a set? A set is a collection of objects, these objects must be satisfying some property otherwise, there is no reason to put them together. So, there must be something common among the objects of a particular set and that particular property distinguishes this set from all other kinds of sets. The objects contained in a set are called the elements or the members of the set.

So, you have the set which is the collection and within the collection there are different elements and these elements are members of that set that we are talking about. For example, the vowels in English alphabet could form a set and we generally use the capital letter of the English alphabet to denote a set this is just a convention, we could be using something else as to denote a set also.

So, for example, here A is suppose the set of vowels in English alphabet. So, A could be written as you can see, $A = \{a, e, i, o, u\}$, these are the 5 vowels in the English alphabet and there are no other sixth vowel in English alphabet. So, we have included all of them in this particular set. The set is called capital A . We use the second brackets you can see that.

We use the brackets, this. So, these brackets are called the second brackets and within the brackets the elements of the set are listed. We use the second brackets and to denote a set, the elements are written inside the brackets. Now in this example, the elements of the set are just alphabets, which are vowels. But the elements could be numbers as well. For example, set of numbers, for which $x(x - 1)(x + 2) = 0$.

So, the elements of the set that we are talking about must be satisfying this equation. We have talked about what are known as variables in our previous module. So, here x is a variable, and for certain values of x only, this equation will be satisfied. So, if you take any arbitrary value of x , it is not necessary that this equation will be satisfied. So, in this case, only those numbers for which this equation is satisfied those numbers could be the members of a particular set. So here, that set is denoted by capital S .

And as you can see, what are the elements of this set capital S , there are actually 3 elements in the sets $S = \{0, 1, -2\}$. So, if you put $x = 0$, then the left-hand side becomes 0, because the first element is 0, and 0 multiplied by anything is 0. Similarly, if you put $x = 1$, then the second element becomes 0.

If you put $x = -2$, then this third element becomes 0. So, for each of these 3 values of x , this equation gets satisfied. So, therefore, in this set, there are 3 elements, and we have listed them all. So, we have defined what is a set and elements of any set. Now the question that might arise is, if you have two sets, then under what condition or conditions, these two sets could be said to be equal and here is the definition.

Two sets are equal when each element of the first is contained in the second. And each element of the second is contained in the first and we write $A = B$. So, this is the, this is the property that must be satisfied by two sets, if they have to be equal sets, that if I pick up any element from the first set, then that element is there in the second set as well.

And vice versa, which means if I pick up any arbitrary element of the second set, then that element is present in the first set as well. And then we write $A = B$, where A and B are 2 different sets.

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- It does not matter if the order of the elements in a set is changed.
- $\{a, b, c\} = \{b, c, a\}$
- Or, if an element is written multiple times.
- $\{1, 2, 3, 4\} = \{1, 2, 1, 4, 3, 3\}$
- Examples from economics:
 - Set of inputs a producer needs to produce goods,
 $S = \{\text{labour, capital, land}\}$
 - Set of prices a producer can set, $P = \{p: p \geq 0\}$.
- The possible prices can be zero at the minimum or it can take positive values. Notice, we are not writing all the elements of the set P .

In this case, actually, they become equal, when we write the equal sign. It does not matter if the order of the elements in a set is changed. So, remember, in a set, I am writing all these elements as comma, comma, comma, but it does not really matter in which order I am writing the elements. So, here is an example.

So, on the left-hand side, I have written the elements as $\{a, b, c\}$. And on the right-hand side, I have written a set where the order has changed. I have written $\{b, c, a\}$. But the point is that it does not really matter if I just changed the order as long as the elements are same in both the sets, then the sets are equal. So, that basically gives us also another sort of property of a set that take any set you change the order of the elements then the it does not matter the set has remained the same.

And furthermore, it also does not matter if an element of a set is written multiple times. In that case also, we say that the set has remained the same. So here is the another example. On the left-hand side you have no repetition, you have $\{1, 2, 3, 4\}$, these are like what these are the, this is the set of first 4 natural numbers. Now, on the right-hand side, what I have done is that I have repeated 1 you can see 1 has been repeated, 2 has not been repeated, but 3 again has been repeated once.

So, there are 3 appearing twice, 1 appearing twice, but the point is that even if some elements are repeating in the second set, the set has remained the same. Now, then, I come to some examples, the examples that I gave just now could be said to be examples of mathematics, I

just took some examples, arbitrary examples, but not particularly, they were examples of economics.

Now, I take some examples from economics. So, for example, I am talking about set of inputs a producer needs to produce goods. So, this is the set of inputs and I write this set as capital S. In this case, I take the set of inputs as $S = \{\text{labour, capital and land}\}$. So, as any, elementary student of economics will find out that these 3 elements, these 3 inputs are generally mentioned as the most critical inputs.

And sometimes, we also include entrepreneurship as the fourth important input, but these are the 3 most basic inputs labour, capital and land. So, these 3 elements form a set and that set is capital S. Another little bit different kind of example, is the set of prices, a producer can set, all possible prices, a producer can set. What does a producer do? The producer produces goods and sells the goods in the market.

Now, this capital P is the set of prices, and look how I am writing this $P = \{p: p \geq 0\}$. So, what is meant by this is the what is written below. The possible prices can be 0 at the minimum or it can take positive values. So, think about price of any commodity, prices are not negative, the minimum value the prices can get to could be 0, but that is also a very extreme case, prices are generally positive.

In this example, when I have written this capital P, so, I have just mentioned the property of any price, I have not written all the members of this set P, I have not written I have not enumerated all the members of this particular set P. I have just written that p, small p, which is an element of capital P, the small p can be positive and the minimum value it can take is 0.

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- Sometimes it becomes difficult to specify a set by writing all its elements. For example, the number of elements could be infinite. These sets are called **infinite sets**.
- In economics, **budget set** is an infinite set.
- The budget set contains all the commodity bundles a consumer can buy by spending her income M .
- Prices of the two goods x and y she can buy are p_x and p_y respectively.
- Here there is a budget constraint: $x.p_x + y.p_y \leq M$, which specifies all combinations of x and y she can buy by spending her money M .

Handwritten notes:
 $x.p_x + y.p_y \rightarrow$ Spending
 income $\rightarrow M > 0$
 prices $\rightarrow p_x, p_y > 0$

So, this is what is being explained here in this next slide. Sometimes it becomes difficult to specify a set by writing all its elements. For example, the number of elements could be infinite these sets are called infinite sets. So, we just had an example of an infinite set. This is an infinite set. Here, think about price to be a variable which is continuous. So, if it is continuous, even in the interval 0 to 1 it can take infinite number of values.

Because it can take all kinds of fractions and irrational numbers. And moreover, here we are not talking even of a particular interval, we are talking about the any finite interval, we are talking about the entire range from 0 to infinity, positive infinity. So, here obviously, there are infinite number of elements and in this case, it is not possible to write all the elements. So, we just write this in terms of the property that a typical element will satisfy. These sets are called infinite sets.

In economics budget set is an infinite set. What is the budget set? So, let me try to explain what is a budget set. The budget set contains all the commodity bundles a consumer can buy by spending her income M . So, there is any arbitrary consumer whose income is M , it could be 100 rupees 200 rupees, that is his income and prices of the 2 goods that she can buy are given by P_x and P_y .

So, prices given by these two quantities and we are implicitly assuming that all these are positive. Because prices cannot be negative, we have seen that and it can be 0 in very exceptional cases, but generally we can assume that the prices are positive here there is a

budget constraint. So, what is a budget constraint? So, budget constraint is written like this $X \cdot P_x + Y \cdot P_y \leq M$, and what is the meaning of this budget constraint?

This constraint is specifying all combinations of x and y , that is commodity x and commodity y that the consumer can buy by spending her income M . So, think about the word budget in a very loose sense in a family context not in the government context, in a family context family has to tailor its spending according to the family income, and let us suppose in a particular month, the family cannot borrow money from anyone else.

So, whatever the money that it spends has to be bounded by the income that the family has, and it does not have any savings, let us suppose. Then the total spending of the family is what? This is the total spending, the left-hand side, the left-hand side is the total spending. And this budget constraint is telling me that the spending must be less than or equal to the income that the family earns. That is what the budget constant is. But I have not so far defined what is the budget set, I am coming to that.

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- The budget set, the set of commodity bundles (x, y) she can buy, is given by

$$B = \{(x, y) : x \cdot p_x + y \cdot p_y \leq M, x \geq 0, y \geq 0\}.$$

- Here the constraints $x \geq 0, y \geq 0$ ensure that the consumer can not buy negative quantities of the two goods.
- In this convention of denoting a set, within the brackets first a typical element of the set is specified; (x, y) , in this case specifying the quantities of the two goods a consumer can buy.
- Next, after a colon sign, all properties that the typical element must satisfy are mentioned. Thereby the set is specified.

The budget set the set of commodity bundles x and y she can buy is given by this. So, this is the set. $B = \{(X, Y) : X \cdot P_x + Y \cdot P_y \leq M, X \geq 0, Y \geq 0\}$. Because we are talking about sets here. So, within the budget set, what I have written is that budget set, a particular element of that budget set is a arbitrary combination of x and y . So, x and y . So, here, any element of the set has actually 2 components within it, x and y that is an element, that pair.

But that pair has to satisfy certain conditions. First is the budget constraint, we have seen this, but there are 2 other constraints also, $X \geq 0$ and $Y \geq 0$. Here the constraints $X \geq 0$ and $Y \geq 0$ ensure that the consumer cannot buy negative quantities of the 2 goods. And this makes perfect sense but because, it is absurd that you buy negative of certain goods.

So, that has to come with the budget constraint. And now we have 3 sort of conditions and you put them together and you get the budget set. In this convention of denoting a set within the brackets, first a typical element of the set is specified. So that I am just trying to define how or try to elaborate how this particular specification of a set is done. So, within the brackets, we are first specifying a typical element of this set. In this case, the typical element is x, y .

So, specifying the quantities of 2 goods the consumer can buy. So, consumer can buy x and the consumer can of course also buy y . This is a combination or we also call it a bundle. Next, after a colon sign, all properties that the typical element must satisfy are mentioned. So, these are the condition or conditions in this case that x, y must satisfy. Thereby the set is specified.

This is what is the way in which sets are written if you cannot write all the elements of the set. Because, think about it, if M is positive and if you think about this budget set, it has infinite number of elements. So, you cannot possibly write all the elements within this second brackets. So, therefore, we use this convention of writing this set.

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- This kind of specification of a set is called the set builder form.
- It can be used for finite sets also.
- Membership of a set: when an object is an element of a set it can be symbolically written in the following manner:
 - $x \in S$.
 - In words this reads as “ x is an element of the set S ”. Or, “ x belongs to S ”.
 - An object may not belong to a set. In that case it’s denoted by, $x \notin S$.
 - Example: $S = \{a, e, i, o, u\}$;
 - $u \in S$, but, $v \notin S$.

*b, k, A
e ∈ A*

A = {x : x is a vowel of English alphabet}

This kind of specification of a set is called the set builder form, this second bracket and then you mention the any element typical element and then write the properties of that element this specification form is called the set builder form. It can be used for finite sets also it is not necessary that it is used only for infinite sets. So, for example, take that first set I talked about the set of vowels.

I could have written it like this, that is a finite set. So, x , x is a vowel of English alphabet. So, if I say this I do not have to write a, e, i, o, u . I have just mentioned the property and that will itself generate this corresponding set. Membership of a set. When an object is an element of a set, it can be symbolically written in the following manner. x and you have a sign and this sign stands for what?

It looks like the epsilon sign in the Greek alphabet and this sign denotes something in words this reads as x is an element of the set S or x belongs to S , $x \in S$. So, for example, $e \in A$, A is the set of alphabets if you remember. So, this is how we denote the fact that a particular object is an element of a set, but it can happen that an object is not an element of a set and then what do I write?

I write x does not belong to S $x \notin S$. So, this sign \notin means does not belong to. For example, $b \notin A$, b means small b is not an element of the set of vowels. So, I have used similar examples here also, suppose S is the set of vowels, then u belongs to S $u \in S$, but v does not belong to S $v \notin S$.

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- **Subsets:**
- Suppose there are two sets A and B , such that every element of A is also an element of B . Then A is called a subset of B .
- Example, $A = \{x, y, z\}$, $B = \{w, x, y, z\}$
- $A \subseteq B \Leftrightarrow [x \in A \rightarrow x \in B]$
- Is a subset a smaller set? Not always, because the two sets may be equal.
- If they are not equal, then A is called a **proper subset** of B .
- $A \subset B$ (A is a proper subset of B)

$\supset \supset$
 $\supset \supset$

Now, we come to another part of the set theory. Subsets, suppose, there are 2 sets A and B such that every element of A is also an element of B then A is called a subset of B. So, you have 2 sets and if you take the first set and take any arbitrary element of A then that element is an element of B also, in that case A is called a subset of B. Remember, A is itself a set, but at the same time it is a subset.

So, here is an example A has 3 elements $A = \{x, y, z\}$ and $B = \{w, x, y, z\}$ and you can see that, if you pick up x which belongs to A, then x is there in B also. Similarly, for y and z, so, therefore, the property is satisfied and we say that $A \subseteq B$. \subseteq this is the sign, this is a symbol. So, this \leftrightarrow we have introduced this symbol before. So, this is if and only if symbol. So, $A \subseteq B \leftrightarrow [x \in A \rightarrow x \in B]$.

Now, is a subset a smaller set? It might give you the impression that if a set is a subset of another set, then the first set is a smaller set and here actually A the smaller set than B, A has less elements. But this is not always correct. The reason being that 2 sets could be just equal sets. In that case also this property will be satisfied. I mean think about this that suppose I consider B as this set it does not have this element w, in that case also A is a subset of B but A is not a smaller subset anymore.

A and B are just equal. In the case that they are not equal then A is called a proper subset of B, $A \subset B$ (*A is a proper subset of B*). So, this was in this case you have a proper subset when w is there, but if you do not have w it is not a proper subset, A is no longer a proper subset. So, this symbol \subset is sometimes reserved for proper subset. So, you do not have the sign below the horizontal sign below this sign. So, it is like a greater than and greater than equal to. So, here you have this sign and this sign,

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• **Set operations:**

Union of sets:

$$A \cup B = \{x: x \in A \text{ or } x \in B\}.$$

The elements contained in the set $A \cup B$ belong to either A, or of B, or both.

Intersection of sets:

$$A \cap B = \{x: x \in A \text{ and } x \in B\}.$$

The elements contained in the set $A \cap B$ belong to both A and B.

$$\begin{aligned} A &= \{1, 2, 3\} \\ B &= \{2, 4, 6\} \\ A \cup B &= \{1, 2, 3, 4, 6\} \\ A \cap B &= \{2\} \end{aligned}$$

Subsets, over. Now, we come to another very important part of set theory which is called the set operations. So, we start with what is known as union of sets. So, you have two sets A and B, $A \cup B$ is another set and what is this set? I have written that on the right-hand side $A \cup B = \{x: x \in A \text{ or } x \in B\}$. So, $A \cup B$ is a set and what is the property of any element of that set is that if you pick up any element of that set then that element belongs to A or it belongs to B.

So, it means that it belongs to at least A or B at least to one of them. Now, this also means that it might belong to both actually. So, you have suppose I am giving you an example so, the set of let us say first 3 natural numbers $A = \{1, 2, 3\}$ and the set of first 3 even numbers $B = \{2, 4, 6\}$. Suppose, this is A and this is B. Now, what is $A \cup B$? $A \cup B = \{1, 2, 3, 4, 6\}$. So, it will contain some elements and which are these elements 1, 2, 3.

So, these are the elements from A and what are the elements from B? 2 is there, but 2 is already there in A so, I do not have to write that again I just write the other elements which are not part of A but part of B. So, here you see this 2 is an element which is there in a as well as B and that will be part of A union B.

I guess that is not very difficult to understand A union B when we take the union of two sets, then this new set is such that any element of this new set belongs at least to one of the sets. Intersection of sets and this is related to the first idea. Here $A \cap B$; this is, this looks like

inverted U is a set such that any element of that set satisfies this property that any particular element of this set belongs to A as well as B, $A \cap B = \{x: x \in A \text{ and } x \in B\}$.

So, and, and means it belongs to A but it also belongs to B. The elements contained in the set $A \cap B$ belongs to both A and B. So, here if I take the same example, then A intersection B will be what? That element which is there in A as well as in B and there is a single element which satisfies this property, which is 2, $A \cap B = \{2\}$.

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Minus of sets:
 Contains the elements of the first set, but not of the second set.
 $A \setminus B = \{x: x \in A \text{ and } x \notin B\}$

Example:
 $A = \{a, b, c, d\}$, $B = \{d, e, f\}$ then,
 $A \cup B = \{a, b, c, d, e, f\}$
 $A \cap B = \{d\}$
 $A \setminus B = \{a, b, c\}$, $B \setminus A = \{e, f\}$

• Set operations:

Union of sets:
 $A \cup B = \{x: x \in A \text{ or } x \in B\}$.
 The elements contained in the set $A \cup B$ belong to either A, or of B, or both.

Intersection of sets:
 $A \cap B = \{x: x \in A \text{ and } x \in B\}$.
 The elements contained in the set $A \cap B$ belong to both A and B.

Handwritten examples:
 $A = \{1, 2, 3\}$
 $B = \{2, 4, 6\}$
 $A \cup B = \{1, 2, 3, 4, 6\}$
 $A \cap B = \{2\}$
 $A \setminus B = \{1, 3\}$

Minus of sets contains the elements of the first set but not of the second set. So, here you have A minus B, $A \setminus B$ sometimes it is written as just the minus sign of arithmetic. So, here this is defined as the following that x is a typical element of this new set and x satisfies the

property that it belongs to A but it does not belong to B, $A \setminus B = \{x: x \in A \text{ and } x \notin B\}$. So, if I take the previous example and if I want to take A minus B then what will this be?

It will be just 1 and 3, $A \setminus B = \{1, 3\}$. So, both 1 and 3 satisfy the property that they belong to A but at the same time they do not belong to B. We have taken another example here. So, capital A, $A = \{a, b, c, d\}$ and capital B is another set, it consists of, $B = \{d, e, f\}$ then A union B is just all the elements, which are there either in A or in B; $A \cup B = \{a, b, c, d, e, f\}$ and A intersection B, is there any intersection element? $A \cap B = \{d\}$

Yes, there is this d, which is common to both the sets. And then we talk about $A \setminus B$. So here, I am just going to take out that element of A, which is there in B, which is d. So, if I take that out, I get, $A \setminus B = \{a, b, c\}$. And B minus A, so, here I am going to take out that element of B, which is there in A which is D. And so, the elements that I am left with are only e and f, $B \setminus A = \{e, f\}$.

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- **Example:**
- Suppose A is the set of positive even numbers, B is the set of positive prime numbers. What is $A \cap B$?
- $A = \{2, 4, 6, \dots\}$
- $B = \{2, 3, 5, 7, 11, \dots\}$
- $A \cap B = \{2\}$, it has a single element. This is because all other elements of B cannot be elements of A . By belonging to B they must not be divisible by 2, whereas all elements of A are divisible by 2.

Another example, if I am considering infinite sets, then how does it look like? I mean does the problem become more difficult? Suppose A is the set of positive even numbers B is the set of positive prime numbers, what is A intersection B . So, first let us try to write these two sets. So here A will be set of positive even number so, $A = \{2, 4, 6, 8, 10, \dots\}$ and B set of prime numbers, $B = \{2, 3, 5, 7, 11, 13, 17, 19, \dots\}$

So, what is $A \cap B$. And interestingly, there is only of course, these two sets are in finite sets. So, you might, think that then we a lot of sets, a lot of elements in the intersection set, but here, it has only 1 element, it is just 2, it has a single element, $A \cap B = \{2\}$. The reason being that this is because all other elements of B cannot be elements of A intersection means what? This any element, which is in the intersection set must be in both the sets.

So, they must be in B if they are to be in the intersection set. But except 2 all other elements of B cannot be element of A , by belonging to B they must not be divisible by 2. Because that is the definition of prime numbers. They should not be divisible by any other numbers, except 1. So, all these elements of B are actually not divisible by 2, whereas all elements of A are divisible by 2. So, except the number 2, there is no intersection.

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- **Example:**

- In a village 50 people don't own a bicycle, 25 don't have a pucca house, 10 have neither.
- A and B denote the sets of people who don't own a bicycle and don't own a pucca house respectively.
- $A \cap B$ is the set of people who own neither a bicycle nor a pucca house.
- $A \cap B$ has 10 elements or people

Now, I am going to take a more real-life kind of example, this kind of examples one encounters in economics. In a village, 50 people do not own a bicycle. So, 50 people are there who do not have a bicycle and 25 do not have a pucca house. Mind you, we are not saying 25 have a pucca house, we are saying 25 do not have a pucca house. And 50 people do not own a bicycle.

And 10 people are there who have neither. So, they neither have a bicycle nor they have a pucca house. A and B denote the sets of people who do not own a bicycle and do not own a pucca house respectively. Suppose A and B are these 2 sets. So, A has 50 elements that is 50 people and B has 25 elements. $A \cap B$ is the set of people who own neither a bicycle nor a pucca house.

So, this is the intersection set. This is the set of people who are in both the sets. So that means they do not own a bicycle because they belong to A at the same time they belong to B also so that means they are very poor. They do not own a pucca house also. So, how many people are there in this intersection set, $A \cap B$, there are 10 elements, it is mentioned in the problem itself.

Now, where do you encounter these kinds of problems in economics is when we measure something called poverty or deprivation in the villages, and who should get the government benefits? For example, how in India BPL families, BPL is below poverty line, how in India

BPL families are identified, who will get the cheap ration from the government. So, they have to satisfy certain criteria.

And these are the kinds of criteria that have to be satisfied. And if someone meets let us say, 20, out of 20, they meet 15 of the criteria means they do not do not own bicycle, do not have pucca house, do not have a TV, do not have electricity connection, etcetera. So, they are meeting a lot of criteria at the same time, then they are more eligible to get those government benefits. So that is how these things are actually done in India, practically.

So here in this particular bookish example, we have identified set A people who do not own a bicycle, and B, they do not have a pucca house and there is a set which is A intersection B means they are quite deprived in the sense that they neither have a pucca house nor have a bicycle and there are 10 elements, 10 people of that nature.

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- $A \cup B$ = set of people who either don't own a bicycle or don't have a pucca house. The set $A \cap B$ is a subset this set.
- $A \cup B$ has 65 elements/people. ✓
- $A \setminus B$ = set of people who don't own a bicycle, and own a pucca house.
- It has 40 people.

Handwritten notes:

- $\rightarrow 50 - 10 = 40$
- $\# A - \# A \cap B$
- $A \cap B \subset A \cup B$

• **Example:**

- In a village 50 people don't own a bicycle, 25 don't have a pucca house, 10 have neither.
- A and B denote the sets of people who don't own a bicycle and don't own a pucca house respectively.
- $A \cap B$ is the set of people who own neither a bicycle nor a pucca house.
- $A \cap B$ has 10 elements or people

$$\begin{aligned} \# A &= 50 \\ \# B &= 25 \\ \# A \cap B &= 10 \\ \# A \cup B &= 50 + 25 - 10 \\ &= 65 \end{aligned}$$

What is $A \cup B$ here? $A \cup B$ = the set of people who either do not own a bicycle or do not have a pucca house. So, this consists of those people who satisfy at least one of these two criteria A or B , the set $A \cap B$ is a subset of this set. So, we can write it like this, that $A \cap B \subset A \cup B$. And you can see this quite logically that people who belong to A intersection B , they do not have a pucca house at the same time they do not have a bicycle. So, they are satisfying both the criteria.

So obviously, they will be satisfying at least one of the criteria. So therefore, they belong to $A \cup B$. How many elements are there in $A \cup B$? It has 65 elements or people? How do I know that? That $A \cup B$ is having 65 People? The reason is this that look at A , number of elements in A , let us suppose I am representing by this number A this is 50. And how many people do not own a pucca house?

This is 25 all these are given how many. People belong to A intersection B that is also given. So, the number of people who are there in $A \cup B$, that will be how much? That will be this number plus this number minus this number. So, 50 plus 25 minus 10. And this becomes 65. Why did I do this operation? This plus this minus this, because 10 elements are common in A and B .

So, if I just add this and this, then we will be counting some of the elements twice because they are both in A and B . So, to avoid this double counting, I take out those people who are common in both the sets. So, 10 is taken out, so I am getting 65, so that is there. This has

been done. $A \setminus B$ = set of people who do not own bicycle and own a pucca house. So, there are some people who do not have a bicycle, they belong to A.

At the same time they have a pucca house that means they do not belong to B. If you belong to B, you do not have a pucca house. So, if you do not belong to B, you have a pucca house. So, who are these people who do not have a bicycle, but have a pucca house? So, how do I find that number? So, that will be like 50, 50 is the total number of people who do not have a bicycle and from that I have to take out the intersection elements, the people who have a pucca house, but they do not have a bicycle.

So, if I do that, then I get how am I getting that? This is 50 minus 10 that is 40. So, this is how am I going to get this, this $A - A \cap B$, because within A there are some elements who are in B also, but if you belong to B, then you do not have a pucca house, but I want people with a pucca house, so I do not elements from B, that is why I have deducted this.

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- When two sets have no common elements, these sets are called **disjoint sets**.
- We write, $A \cap B = \emptyset$, where \emptyset denotes the empty set, the empty set has no elements.
- In the above example, the set of all people of the village can be thought of as the source of all sets.
- Sets of different descriptions such as people without bicycles, without pucca houses, with tube or without tube wells, with or without TV sets, etc. are subsets of this big set.
- This big set is called the universal set, often denoted by Ω .

- **Example:**

- In a village 50 people don't own a bicycle, 25 don't have a pucca house, 10 have neither.
- A and B denote the sets of people who don't own a bicycle and don't own a pucca house respectively.
- $A \cap B$ is the set of people who own neither a bicycle nor a pucca house.
- $A \cap B$ has 10 elements or people

$$\begin{aligned} \# A &= 50 \\ \# B &= 25 \\ \# A \cap B &= 10 \\ \# A \cup B &= 50 + 25 - 10 \\ &= 65 \end{aligned}$$

Then we come to another concept related to the sets, which is called disjoint sets, when two elements have no common elements, two sets have no common elements, these sets are called disjoint sets and we write it like this $A \cap B = \emptyset$ is this is called an empty set or phi. This denotes the empty set. The empty set has no element. So, this is a trivial set in the sense that it does not have an element.

So, what can be an example of an empty set? For example, if you think about 2 sets A and B , A is the set of odd numbers or positive numbers and B is the set of even numbers then there is no set, there is no element which belongs to both A and B . So, in that case, $A \cap B = \emptyset$. In the above example, the set of all people of the house can be thought of as the source of all sets, so I am talking about this example.

So, there is a village in which some people meet some criteria, some people do not own a bicycle, some people do not have a pucca house, etcetera, etcetera. Now, this village itself or more precisely the people of the village itself could be thought of as the source of all this small, small sets.

The set of all people of the village can be thought of as a source of all sets. Sets of different descriptions such as people without bicycles, without pucca houses, with tube well, this should be tube well, with tube well or without tube well, with or without TV sets, etcetera. So, these are the different descriptions of sets, these are subsets of this big set. So, for example, in this example, A and B are subsets of this big set of people of the village.

And we can think about other kinds of sets also which can come from this big set for example, people who are literate in this village for example, people who belong to a particular caste in this village. So, those will be different sets that one can think of this big set is called the universal set, often denoted by this sign, Ω , this is called the gamma sign, the capital gamma sign in the Greek alphabet, this is called the universal set.

So, in this example, all this A capital A, capital B and one can think of it capital C, capital D, all of them belong to this universal set gamma. And what is gamma? gamma is the people of the villages of that particular village.

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- If S is a subset of universal set Ω , then we can define another set, $\Omega \setminus S$. This set is called the **complement** of S in Ω .
- $\Omega \setminus S$ is also written as S^c .

Example: Let the universal Ω be the set of all students in a college.
 Let F be the set of female students,
 E the set of all economics students,
 H the set of students in the mountaineering club,
 M the set of mathematics students,
 C the set of all cricket players.
 Describe the members of the following sets.

So, the example that I just gave to you the of odd numbers and even numbers, there you can think of that the set of real numbers is the universal set, from which we picked up one set, which is the set of odd integers or set of even integers. Those are the sets which come from that universal set. If S is a subset of the universal set gamma, then we can define another set which is $\Omega \setminus S$. This set is called the complement of S in gamma, S^c .

So, you have S which belongs to gamma. So, this is the subset and this is the universal set. Then we talk about that set which is within gamma but which is not an element of S . So, this is that set $\Omega \setminus S$. So, this is how we should write gamma minus S , that this is a set which consists of elements that belong to gamma and do not belong to S ,

$\Omega \setminus S = \{x: x \in \Omega \text{ and } x \notin S\}$. Now, that set is called compliment of S but with respect to what? With respect to this gamma.

It is sometimes written as S, you have a small c as the superscript, S^c . So, S^c is the compliment set of S in gamma. So, I have given an example: let the universal gamma, Ω , be the set of all students in a college. Let F be the set of female students, E the set of all economic students, H the set of students in the mountaineering club, M the set of mathematics students, C the set of all cricket players and describe the members of the following sets. So, I have first defined Ω in this problem. Ω is the set of all students in a college and then I have defined 5 sets F, E, H, M, C.

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- (i) $\Omega \setminus E$
- (ii) $E \cup H$
- (iii) $F \cap C$
- (iv) $E \setminus (M \cap C)$
- (v) $(E \setminus H) \cup (E \setminus C)$

Now, we have to describe what these sets mean. So, $\Omega \setminus E$, $E \cup F$, $F \cap C$, $E \setminus (M \cap C)$, $(E \setminus H) \cup (E \setminus C)$.

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(i) $\Omega \setminus E$: consists of all students who are not studying economics.

(ii) $E \cup H$: consists of all students who are either studying economics or members of the mountaineering club.

(iii) $F \cap C$: consists of all students who are female and they are cricket players.

(iv) $E \setminus (M \cap C)$: consists of all students who are studying economics but are not studying mathematics and are cricket players at the same time.

(v) $(E \setminus H) \cup (E \setminus C)$: consists of all students who either economics students but not studying mathematics or economics students but are not cricket players.

Handwritten notes:
 - Red arrow pointing to $(E \setminus H)$ with text "not studying mathematics"
 - Red arrow pointing to $(E \setminus C)$ with text "not studying mathematics"
 - Red arrow pointing to $(E \setminus C)$ with text "not studying mathematics"
 - Red arrow pointing to $(E \setminus C)$ with text "not studying mathematics"

- If S is a subset of universal set Ω , then we can define another set, $\Omega \setminus S$. This set is called the **complement** of S in Ω .
- $\Omega \setminus S$ is also written as S^c .

Example: Let the universal Ω be the set of all students in a college.
 Let F be the set of female students,
 E the set of all economics students,
 H the set of students in the mountaineering club,
 M the set of mathematics students,
 C the set of all cricket players.

Describe the members of the following sets.

Handwritten notes:
 - $S \subset \Omega$
 $\Omega \setminus S = \{x \in \Omega \text{ and } x \notin S\}$
 Complement of S in Ω

So, let us see, $\Omega \setminus E$. Ω is the set of all students, E is the students who have economics as a subject. So, therefore, $\Omega \setminus E$: the set of students who have not economics as a subject. So, consists of all students who are not studying economics. Second, $E \cup H$, H remember students who are members of the mountaineering club and so, $E \cup H$: consists of all students who are either studying economics or members of the mountaineering club because this is a union.

$F \cap C$, what was F ? F : the set of female students, C : the students who are member of the Cricket Club. $F \cap C$: consists of all students who are female students at the same time they are cricket players there, that is the definition I think C . Set of all cricket places. So, $F \cap C$

are female students who are also cricket players. Fourth, $E \setminus (M \cap C)$: consists of all students who are studying economics because, E is coming first but are not studying mathematics and are cricket players at the same time.

So, we have to take out those students from the set of economics students who are studying mathematics and cricket players at the same time. If I do that, then I get this set. And finally, number 5, $(E \setminus H) \cup (E \setminus C)$: consists of all students who are either economics students but not members of mountaineering club, or economics students, but not cricket players.

This should be. This is not member of the mountaineering club. This is what should be the correct answer consists of all students who are either economics students, but not members of the mountaineering club or economics students but are not cricket players. I think I will stop here. I will pick up this thread in the next lecture and finish with this discussion on sets in the next lecture. Thank you.