Mathematics for Economics – 1
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Lecture – 30
Dynamic stability of time path, Cobweb model

Hello and welcome. So, this is another lecture of this course Mathematics for Economics part 1. The present module that we are going through is on difference equations. So, we have introduced what are difference equations and presently we are trying to understand how to solve a difference equation of the first order and in particular these are linear difference equations.

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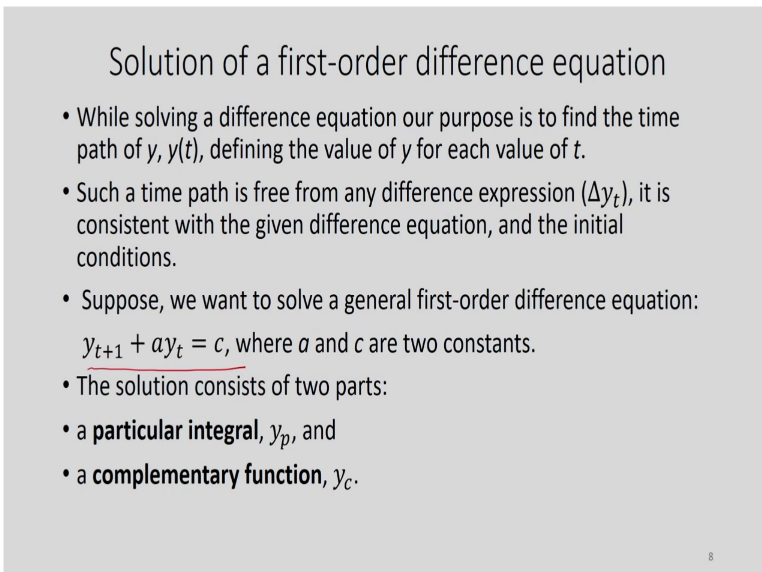

So, you can see on your screen the title slide. We have talked about the general form of a linear difference equation. So, this is the form $y_{t+1} + ay_t = c$ where a and c are two known constants. We have seen that there are two parts in the solution one is the particular integral y_p and the other is the complementary function y_c. The complete solution is the summation of these two.

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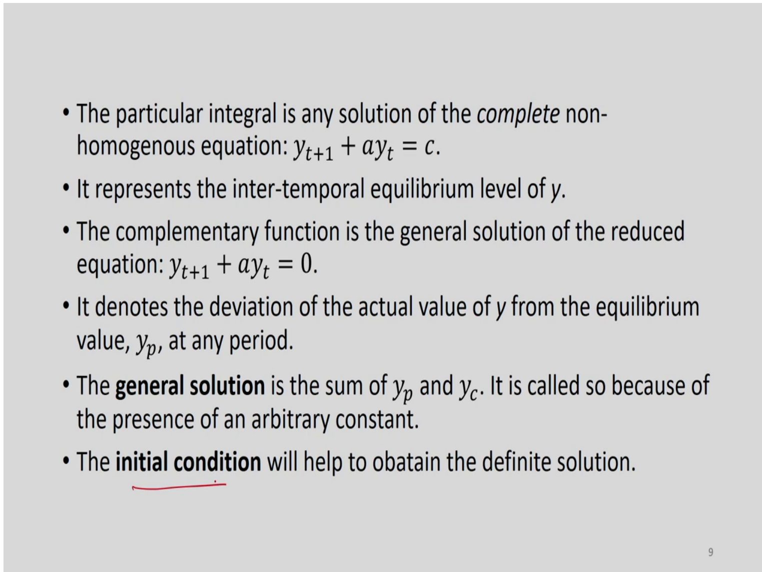

And if we know the initial condition then actually, we can find out the exact solution.

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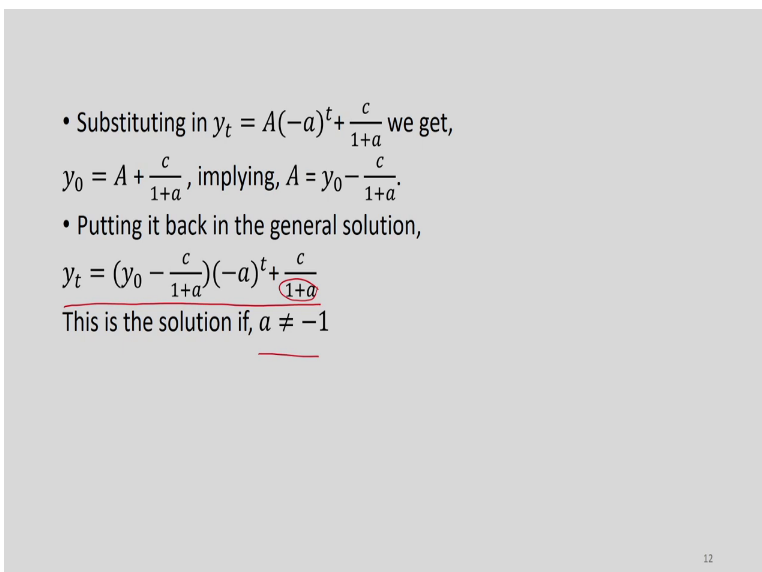

And exact solution will be of this form $y_t = (y_0 - \frac{c}{1+a})(-a)^t + \frac{c}{1+a}$. However, this condition, this solution will be valid if $a \neq -1$. Because you can see if $a = -1$ then the denominator becomes 0 and that is a problematic thing.

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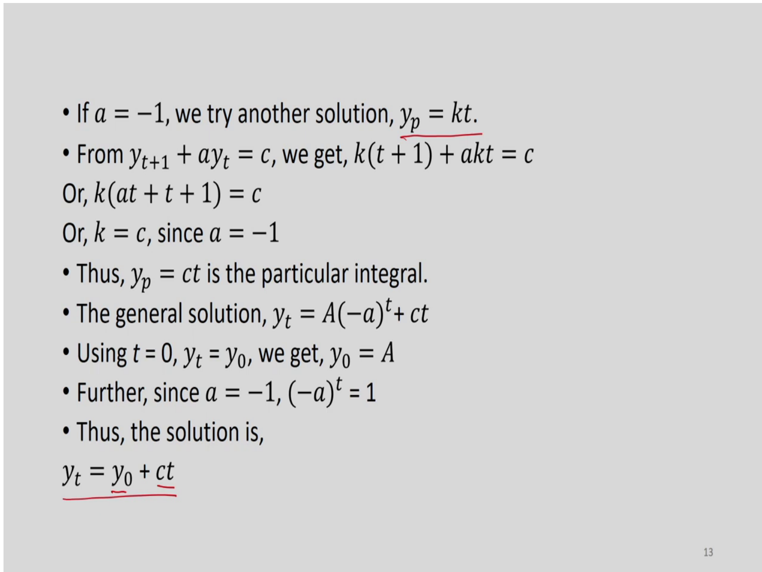

So, what happens if $a = -1$ then the particular integral we take a different solution to that we take $y_p = kt$ and if we do so, then the solution is $y_t = y_0 + ct$, here ct is the particular integral and y_0 is the initial condition that is the value of y at the initial period.

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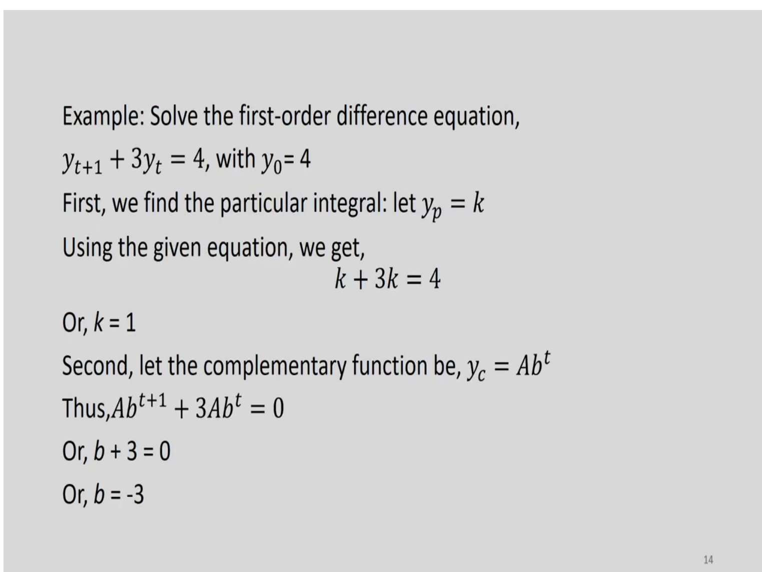

So, this is how we solve difference equations of first order and we have talked about one example.

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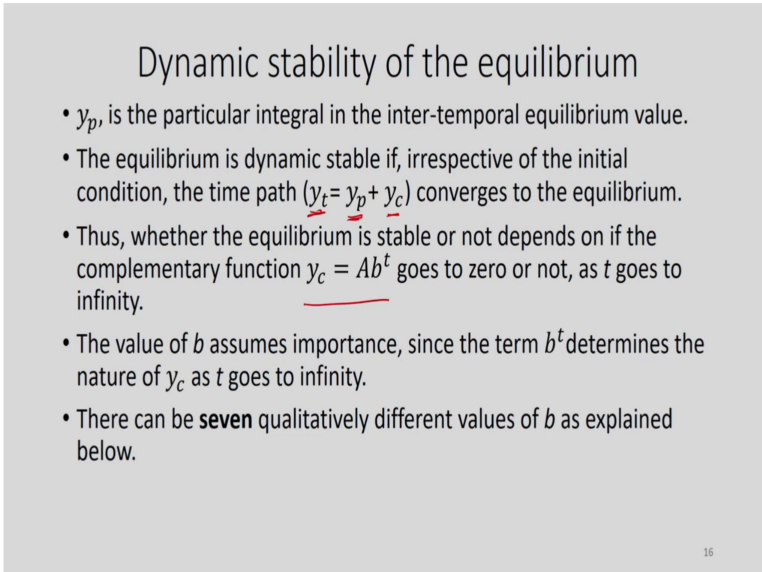

Now, the question is, what about the dynamic stability and that is what we were talking about as the last topic of the lecture. So, when we say dynamic stability what is meant by that is that does y_t which is the complete solution does it approach to y_p the particular integral, y_p as we know is the intertemporal equilibrium value of y. So, does the solution which is the complete solution y_t , does it approach intertemporal equilibrium?

Well, that depends on what happens to y_c, if y_c converges to 0 only then we can say that y_t converges to y_p. This is very simple. Now, what happens to y_c, as t goes to let us say it goes on increasing in general it can go all the way up to infinity. If t goes to infinity then what happens to y_c, well, here there are different cases that can come up.

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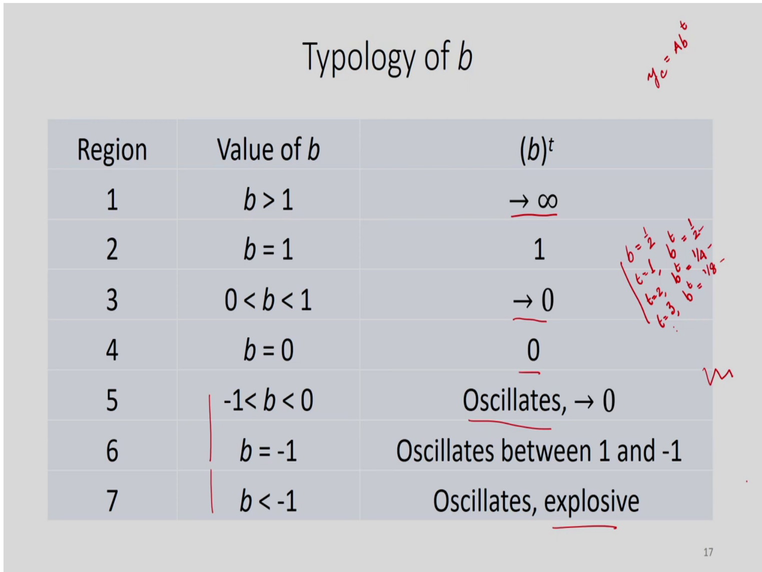

And in general, there are seven possible regions or cases and these are enumerated here. So, the first case is b is, what is b remember $y_c = Ab^t$. So, here there is a b term, b is a constant, but constants can have different values. Now, if $b > 1$ then as t goes to infinity this y_c goes to infinity as well, this is the first case. Suppose $b = 1$ then b^t remains constant at 1.

So, in this case it does not go to 0 that is what we wanted. Thirdly, suppose $0 < b < 1$ in that case as t goes to infinity, what happens to b^t in this case actually it goes to 0. I mean take an example suppose $b = 1/2$ in this case as $t = 1$ $b^t = 1/2$, $t = 2$ $b^t = 1/4$ $t = 3$ $b^t = 1/8$.

So, likewise it will then become 1/16 1/32. What is interesting to note is that the denominator that is the terms which are appearing here, they are going to infinity as t goes to infinity and as we know $\frac{1}{\infty}$ is close to 0. So, that is why I am saying that as t goes to infinity, if $0 < b < 1$ b^t goes to 0. If $b = 0$, then 0 to the power whatever power you put there, it will remain at 0.

If $-1 < b < 0$ then something interesting happens then b^t actually oscillates. So, start with $t = 1$ let us suppose, if you put $t = 1$ then let us suppose $b = \frac{-1}{2}$, $(\frac{-1}{2})^1 = \frac{-1}{2}$, $t = 2$,

$(\frac{-1}{2})^2 = \frac{1}{4}$. So, you can see from $\frac{-1}{2}$ it becomes plus $\frac{1}{4}$ then it becomes $\frac{-1}{8}$ then it becomes $\frac{1}{16}$.

So, in other words the value of b^t oscillates, it moves in a zigzag manner, it oscillates. And in the process however, it approaches 0 because the denominator like in this case the denominator goes to infinity. So, the b^t term approaches 0, this was the fifth case. What happens if $b = -1$. In that case also it oscillates between 1 and -1. So, put $t = 1$ $b^t = -1$ put $t = 2$ $b^t = 1$ $t = 3$ $b^t = -1$ $t = 4$ $b^t = 1$.

So, it oscillates between +1 and -1. This was the sixth case. And the last case, that is the seventh case suppose $b < -1$. Now, like before in all these two cases the b^t was oscillating, here also it will oscillate, because $b < -1$. So, you can think about $b = -2$. In that case also as t goes on rising, b^t will go on oscillating, but in this case it will be explosive.

So, the oscillation will become explosive that is b^t will become far and far away. It will become far and far away from the value 0. So, that is why I am saying that it oscillates and it explodes.

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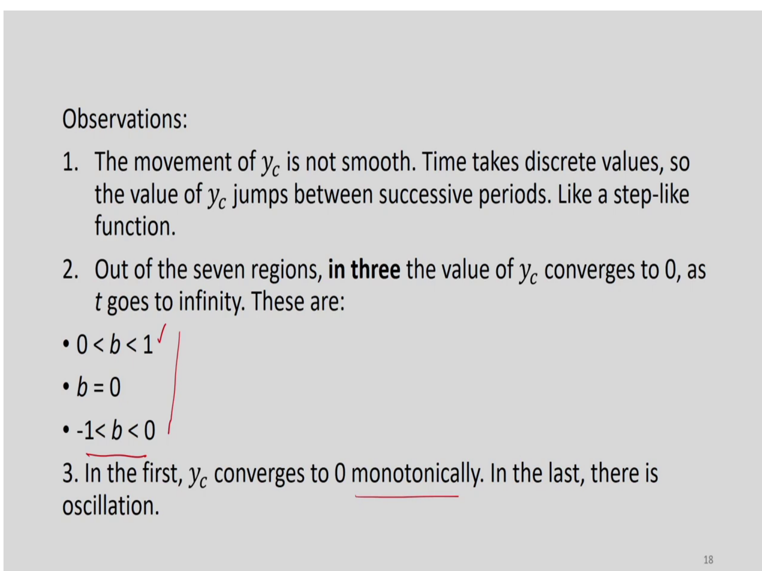

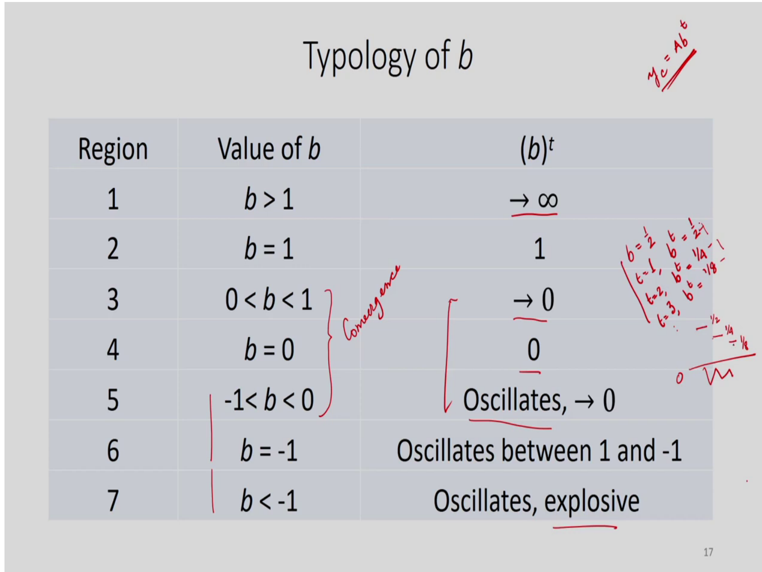

Typology of b

Region	Value of b	$(b)^t$
1	$b > 1$	$\rightarrow \infty$
2	$b = 1$	1
3	$0 < b < 1$	$\rightarrow 0$
4	$b = 0$	0
5	$-1 < b < 0$	Oscillates, $\rightarrow 0$
6	$b = -1$	Oscillates between 1 and -1
7	$b < -1$	Oscillates, explosive

Now, let us come to certain observations from this table, these seven cases. First thing to note is that the movement of y_c, that is the complementary function is not smooth. Time takes discrete values, so, the value of y_c jumps between successive periods, like a step-like function that is obvious, let us take this example that I have just constructed, b^t, it is 1/2 then it is 1/4, then it is 1/8, then it will be 1/16.

So, you can see that between these two values between 1/2 and 1/4 for example, there are plenty of other values. For example, there could be 1/3. So, one can think of other values between 1/2 and 1/4, but b^t is not assuming those values. Similarly, between 1/4 and 1/8, there are plenty of values which are left out, which means that the b^t, that is the complementary function is actually jumping from one value to another.

So, you can think of something like this. So, this is 1/2, 1/4, 1/ 8. So, all these values are coming down, and they approach 0, but there are gaps. So, that is what I mean that the movement of y_c this is the y_c depends on b^t, but b^t is not a smooth function it takes the values of t which are discrete, therefore, b^t also misses many values between the successive two values. It is like a step-like function. So, that is what I mean by this, you can imagine this to be steps. So, it is a step-like function.

Out of the seven regions that we have listed in three, the value of y_c converges to 0 as t goes to infinity, these are these three cases, $0 < b < 1$, $b = 0$ and $-1 < b < 0$. So, these are the three cases where you have convergence. It is written here more clearly that it goes to 0 it stays at 0 and it oscillates, but again it goes to 0. So, these are the three cases where there is convergence.

Third observation in the first, that is this one, $0 < b < 1$, y_c monotonically, monotonically means it does not oscillate, non oscillatory that we have seen before. So, here is the case where it does not converge in an oscillatory manner, it goes to 0 but in a monotonic manner. In the last there is oscillation where $-1 < b < 0$. We shall see the diagram in a minute.

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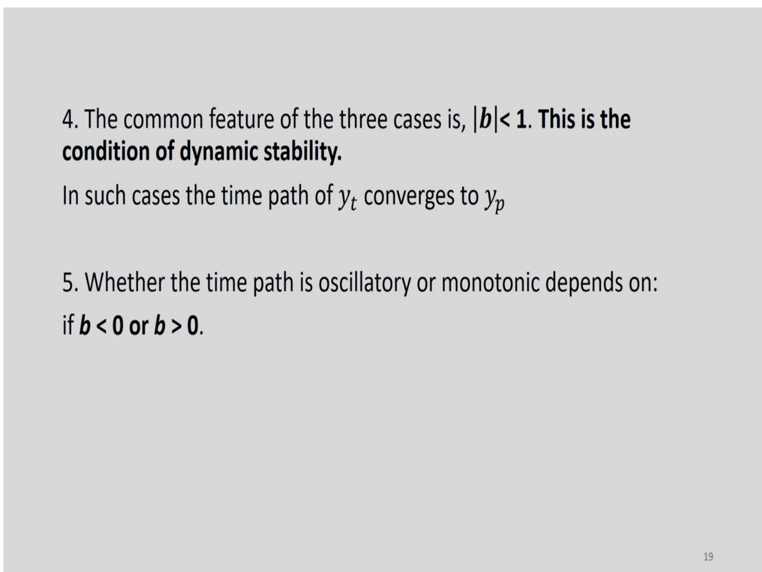

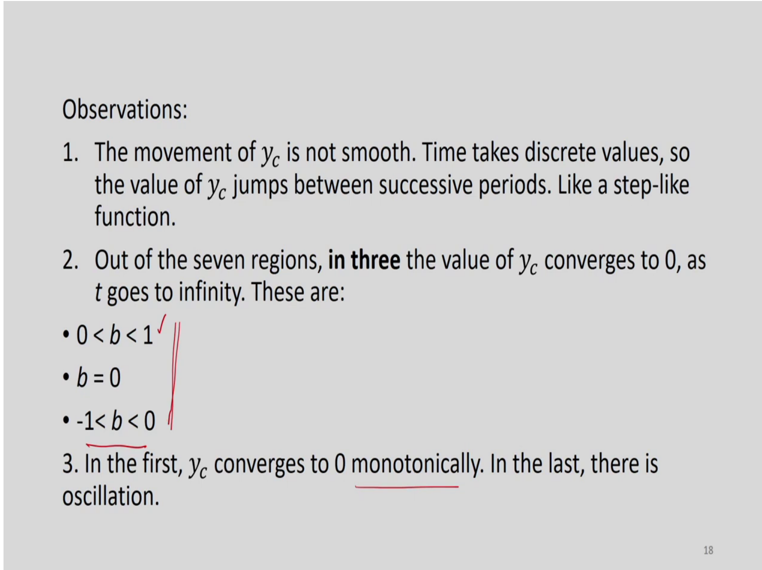

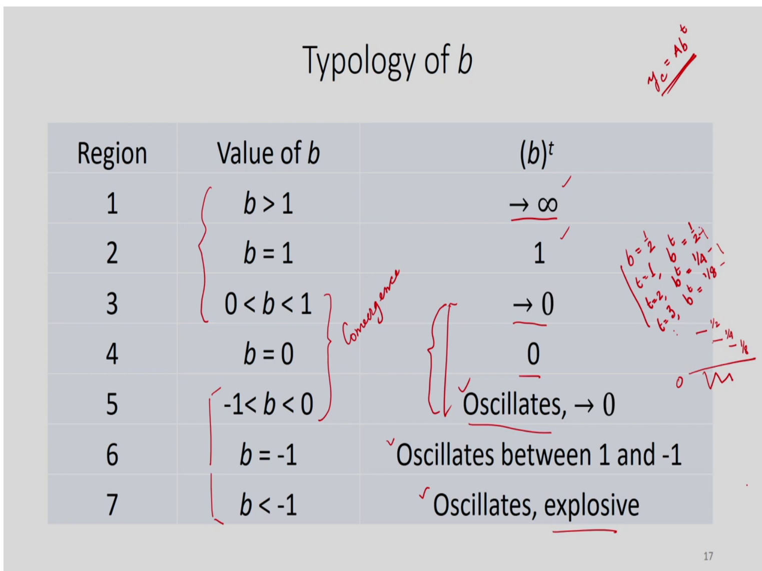

Fourth observation is this, the common features of the three cases is $|b| < 1$. These three cases mean the cases of convergence. In these three cases what is common is that the absolute value of $|b| < 1$. This is the condition of dynamic stability. In such cases, the time path of y_t converges to y_p. So, this is the thing we wanted to figure out, that, when do we have dynamic stability? And now we can answer that question.

The time path of y_t converges to y_p that is we have dynamic stability if $|b| < 1$. So, this is the case, the three cases that we are talking about these are the three cases convergence. And the last observation whether the time path is oscillatory or monotonic depends on if $b < 0$ or $b > 0$. So, if $b < 0$ then it will oscillate the time path, it will oscillate. Let us see the table so, these are the three cases where you have $b < 0$, in all these three cases you see oscillation is there.

Whereas, if $b > 0$, so, these are the three cases where $b > 0$, you do not have oscillation, you have monotonic movement either it is explosive or it does not change at all it stays at 1 or it converges to 0, but in all these cases whatever be the case convergence of divergence the movement of y_c is monotonic. So, this is something to remember that dynamic stability is ensured if the $|b| < 1$, that is number 1 number 2 if we have oscillation or monotonic movement depends on the sign of b whether it is, $b > 0$ or $b < 0$.

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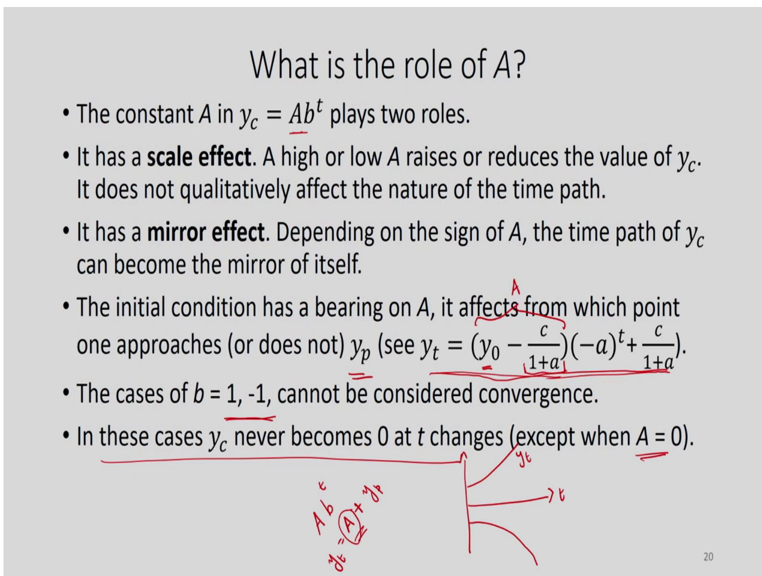

Now, it may seem that we are focusing our attention only on b but let us now talk about A. A remember was the coefficient of the b^t term. Now, this A plays two roles: first it has a scale effect a high or low A raises or reduces the value of y_c it does not qualitatively affect the nature of the time path. So, what do we mean by saying that it does not affect qualitatively the nature of the time path it means that if it is dynamically unstable even if A changes, the path remains dynamically unstable.

If it is dynamically stable then A does not affect that particular quality of it. It basically gives it a scale effect, maybe the path shifts up a bit or shifts down a bit but qualitatively the nature of the equilibrium does not change. The second role is that it has a mirror effect depending on the sign of A. The time path of y_c can become the mirror of itself and this is not difficult to understand. So, suppose you have a time path something like this y_t with respect to a particular value of A.

Now, that A was supposed to be positive. Now, suppose the sign of A changes then the time path can become just a mirror image of itself that means, it becomes something like this. So, in that sense also A can affect the time path it has a mirror effect. The initial condition has a bearing on A it affects from which point one approaches y_p or it does not approach y_p. And this can be verified from this general solution here.

So, this was a general form of the solution y_0 was the initial condition. That is the value of y at period 0. Now remember this is A. A has this term y_0 in it. So, if y_0 changes A will change and depending on the value of y_0 one can have a positive A one can have a negative A. For example, if y_0 is greater than this intertemporal equilibrium $\frac{c}{1+a}$ then A becomes positive otherwise, if $y_0 < \frac{c}{1+a}$, A becomes negative.

So, that is why I have written if effects from which point one approaches y_p. So, the initial condition has a bearing on A. The cases of $b = 1, \ -1$, so, these are the two borderline cases where b is taking certain specific values +1 or -1 now, these two cases cannot be considered to be cases of convergence. If $b = 1$ then what happens to Ab^t, it is equal to A just A because $b = 1$, $1^t = 1$.

So, $A + y_p$, this is my solution. Therefore, since A is constant therefore $y_t = y_p$. In other words, y_c does not become 0 as t changes, because t has no bearing on A unless A is just equal to 0, but that will be a very rare case of coincidence, where you have the first term dropping out. So, in general, if you have $b = 1$ then it cannot be considered to be a case of convergence. Similarly, if

$b = -1$ then also it cannot be considered to be a case of convergence then y_t will go on oscillating.

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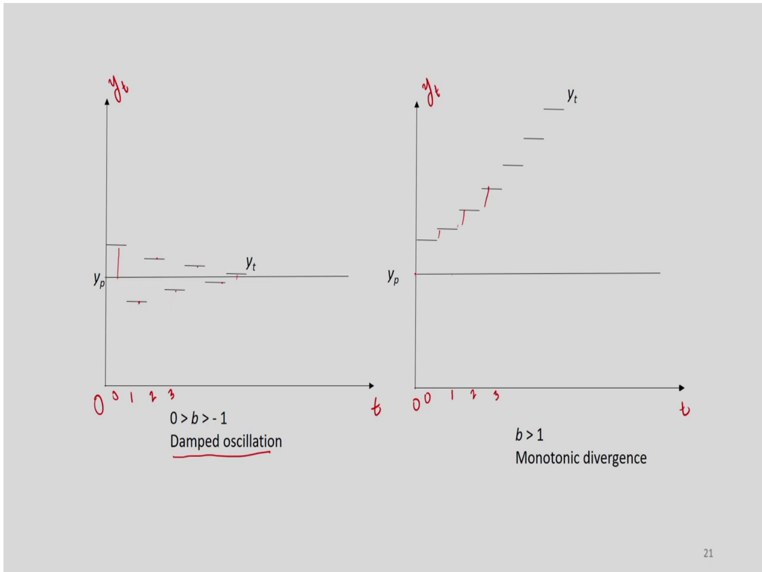

This was the role of A. Now, I promised that I will show some diagrams about this movement of y_t. So, here you have time on the horizontal axis, on the vertical axis the y_t is represented. Now, in both these diagrams on the left hand side panel or in the right hand side panel, I have demarcated something like y_p, y_p is the intertemporal equilibrium value of y. Question is do we approach y_t as t changes.

In the first case, we do, you can see this is like $t = 0$, $t = 1$ $t = 2, 3$. And you can see the values of y_t, it is here to begin with, then jumping down then it is going up above the y_p and again it is coming down below y_p again it will go up y_p below y_p, likewise, it will go on in an oscillatory manner. But the point to note is that the gap between the y_t and the y_p is declining over time. So, initially it was this much, but at this stage you can see the gap has come down a lot.

So, that is why it is called damped oscillation. And this is the case where $-1 < b < 0$. And the second case is a case of monotonic divergence. Here you have explosive movement of y_t. It goes away. So, you are starting from 0. In period 1, the y_t is farther away from y_p. In period 2, it goes even farther away. In period 3, the gap rises. And as you can see the gap between the successive y_t, those gaps are rising.

The gaps are rising which means that the gap between y_t and y_p is also rising. Because y_p was below y_0, to begin with and since the gap between the successive y's are rising and they are rising in increasing manner therefore, as time goes by you have an explosive movement y_t is getting farther and farther away from y_p in an increasing way. So, this is called monotonic divergence, you do not have oscillation here. So, this is the case of $b > 1$.

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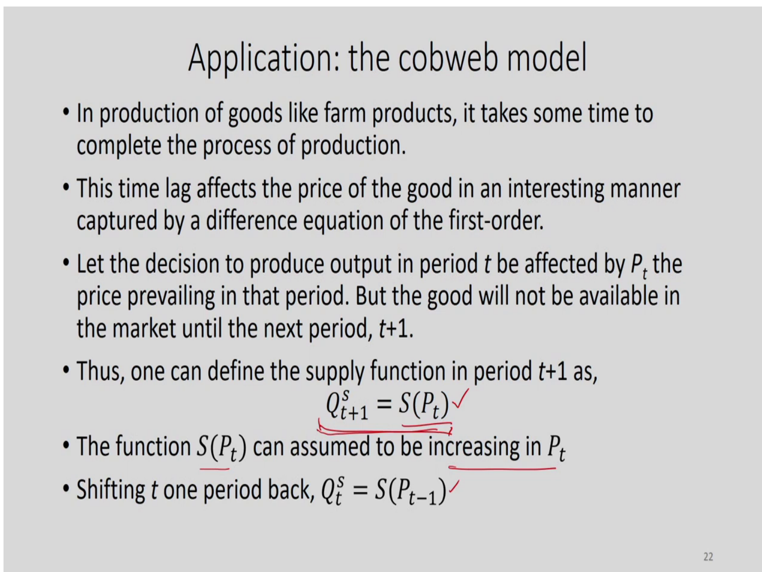

We have talked about the theory. Now we are going to talk about certain applications. The first extremely well known application of the first order difference equation is the Cobweb model. So, what is this about? In production of goods like farm products, it takes some time to complete the process of production. So, this is also known as gestation lag. Let me explain this, this time lag affects the price of the goods in an interesting manner captured by a difference equation of the first order.

So, how does this lag come into being, let the decision to produce output in period t is affected by P_t the price prevailing in that period. But the good will not be available in the market until the next period t+1. Thus, one can define the supply function in period t+1 as $Q_{t+1}^{s} = S(P_t)$. So, this is the function that captures the lag. So, in a particular time period t, let us say the farmers have decided how much they will produce.

Now, when they decide how much they will produce in their farm they look at the current market price, that is what they know off they are sure of based on that price they decide how much they will produce. But, the decision to produce does not automatically translate into production, it takes some time for the production to actually materialize. So, the production will come into the market in the next period whereas, the decision was taken in the previous period.

So, therefore, I can write this as Q_{t+1}^{s}, that is the quantity supplied in period t + 1 is a function of price that prevails in period t. Now, what kind of function is this this supply function now, we just assumed that it follows the law of supply which is saying that $S(P_t)$ is an increasing function of P_t and this function can be written as this function $Q_t^{s} = S(P_{t-1})$, that is we have taken t to be one period back but the lag obviously remains.

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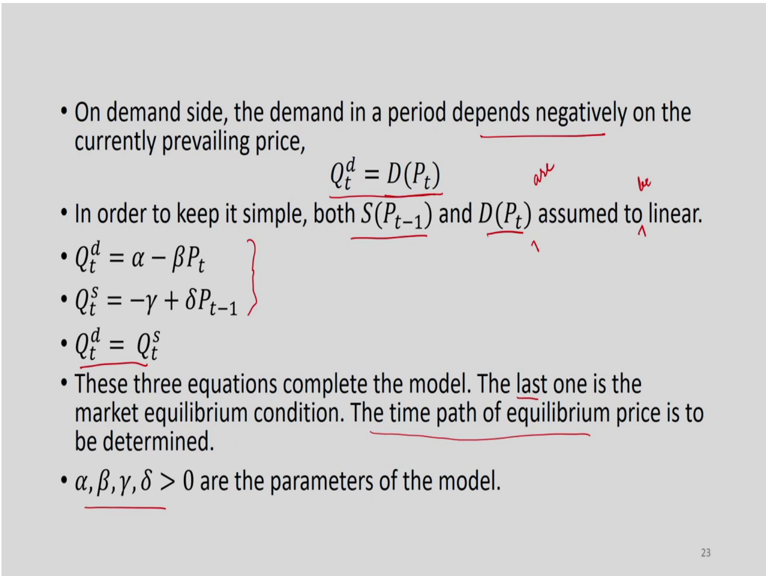

Now, on the demand side the demand in a period depends negatively on the current prevailing market price. So, there are no surprises here: quantity demanded $Q_t^d = D(P_t)$. And since we have assumed that negatively depends on the price, so, $D(P_t)$ is a declining function of P_t. In order to keep the story simple, both $S(P_{t-1})$ and $D(P_t)$ are assumed to be linear. So, we take this particular form $Q_t^d = \alpha - \beta P_t$ and $Q_t^s = -\gamma + \delta P_{t-1}$.

So, these are the two functions, demand function and the supply function. And thirdly, we assume that $Q_t^d = Q_t^s$. These three equations complete the model, the last one is the market equilibrium conditions, this one. As the quantity demanded is equal to quantity supplied that is the market equilibrium condition. We are assuming that in each period the demand should be equal to supply that is in each period the equilibrium should prevail.

What we want to find out is the time path of the equilibrium price. So, although the market is in equilibrium in each period, how does this equilibrium price in the market behave over time? So, that is what we are after. What about these parameters α, β, γ *and* δ, these are all assumed to be positive. So, that is what makes the demand function a declining function of the price of that period and the supply function to be an increasing function of the price of the previous period.

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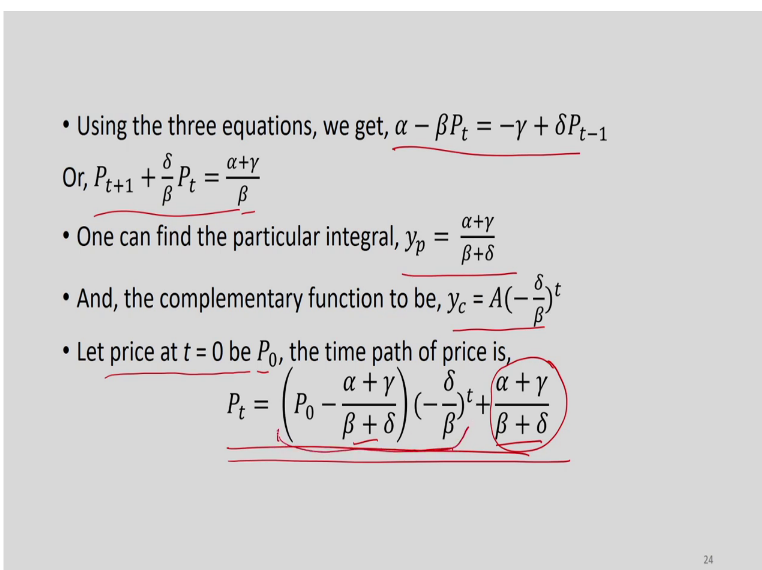

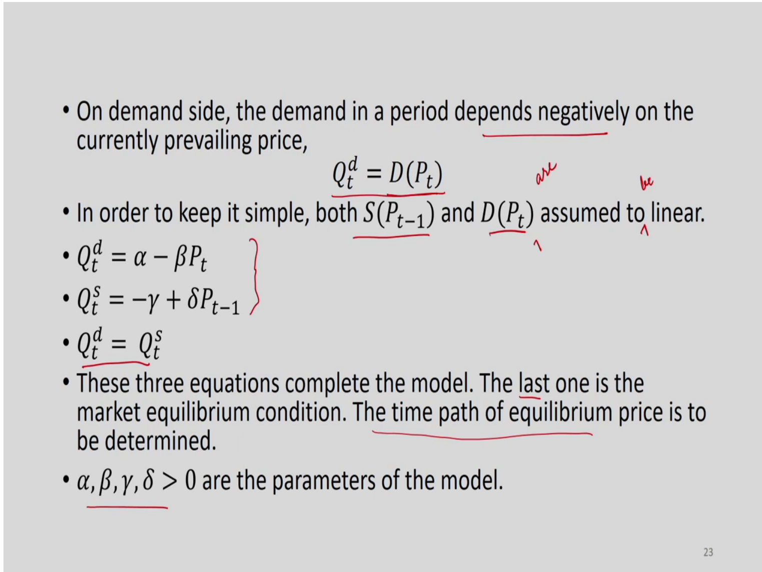

Now, using these three equations these three conditions we can actually write demand is equal to supply and from this I get a first order difference equation, linear difference equation, non homogeneous difference equation in price. And we know how to solve such difference equation of the first order linear difference equation. We can find what is the particular integral y_p and this is I am not going into the detail of that it will be $y_p = \frac{\alpha+\gamma}{\beta+\delta}$.

Similarly, we can jump certain steps and find out the complementary function it will be, $y_c = A(\frac{-\delta}{\beta})^t$. And suppose we have a certain initial condition at $t = 0$, the price prevailing in the market is given by P_0 let us suppose that is known to us in that case the price will have this time path. So, it looks like a cumbersome expression.

But I mean this is quite logical we are just replicating what we have seen before as the solution $P_t = (P_0 - \frac{\alpha+\gamma}{\beta+\delta})(\frac{-\delta}{\beta})^t + \frac{\alpha+\gamma}{\beta+\delta}$. So, this is the time path in this case as we have seen that this is the particular integral and this thing is the complementary function.

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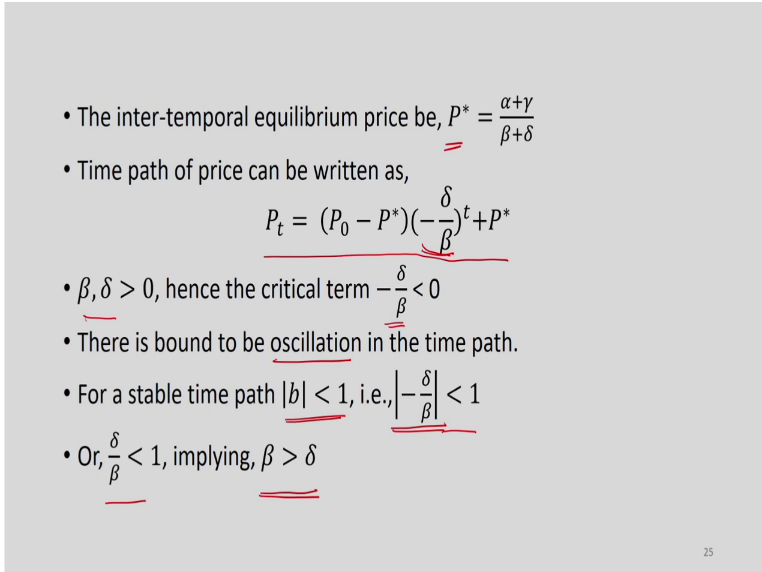

And as we know the particular integral itself is that intertemporal equilibrium price and let us suppose that is denoted by P^* to reduce the clutter, in that case the time path can be written in this form $P_t = (P_0 - P^*)(\frac{-\delta}{\beta})^t + P^*$. So, this is the time path as we know all these parameters are positive, in particular β and δ are positive which means that this term this is the b term.

That $b = \frac{-\delta}{\beta}$, δ *and* β are both positive which means that this $b < 0$. And as we know if b is negative if its sign is negative then there is bound to be oscillation in the time path. So, monotonic movement whether you are talking about convergence or divergence is not going to be there. So, that is number one, number two stability. When do we have stability?

We have stability if $|b| < 1$ which means this has to be satisfied $\left|\frac{-\delta}{\beta}\right| < 1$ and we know this can be written as $\frac{\delta}{\beta} < 1$. So, this is the condition of stability which is implying that $\beta > \delta$. So, this is the condition of stability. So, two conclusions we are getting from here, one there is bound to be oscillation in the time path of the price and number two if we have to have stability then $\beta > \delta$.

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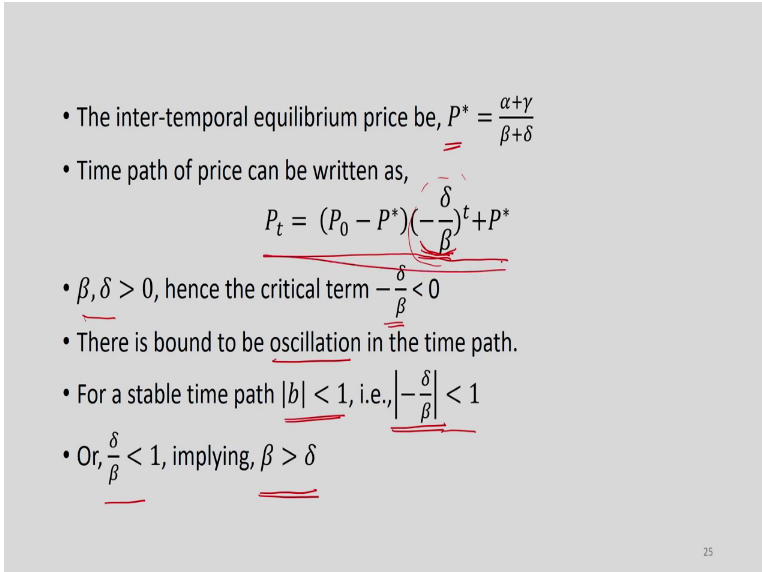
- β, δ are the absolute value of the slopes of the demand and supply functions respectively.
- The stability condition implies that the slope of the demand curve (line) is higher than the slope of the supply curve (line).
- If this condition is satisfied there will be **damped oscillation** toward the equilibrium.
- On the other hand, if $\beta = \delta$, there will be uniform oscillation. The price will alternate between P_0 and $-P_0 + 2P^*$, maintaining equal distance from P^*.
- If $\beta < \delta$, the demand line is flatter than the supply line. There is explosive oscillation. P_t will diverge from P^* by fluctuating around it.

- On demand side, the demand in a period depends negatively on the currently prevailing price,

$$Q_t^d = D(P_t)$$

- In order to keep it simple, both $S(P_{t-1})$ and $D(P_t)$ assumed to linear.
- $Q_t^d = \alpha - \beta P_t$
- $Q_t^s = -\gamma + \delta P_{t-1}$
- $Q_t^d = Q_t^s$
- These three equations complete the model. The last one is the market equilibrium condition. The time path of equilibrium price is to be determined.
- $\alpha, \beta, \gamma, \delta > 0$ are the parameters of the model.



Now, what are β and δ? Just recall β and δ are the absolute value of the slopes of the demand and supply functions respectively. These are the two functions. So, β and δ are the absolute values of the slopes of these two functions, demand and supply. The stability condition implies that the slope of the demand curve or line in this case, because we are assuming a linear function, the slope of the demand line is higher than the slope of the supply line.

I mean the absolute value of the slope if this condition is satisfied there will be damped oscillation towards the equilibrium. Because you know in that case the $|b| < 1$, on the other hand, if $\beta = \delta$ there will be a uniform oscillation the price will alternate between P_0 and $-P_0 + 2P^*$ maintaining equal distance from P^*. Let us try to see how this is happening if $\beta = \delta$ this becomes -1.

So, suppose $t = 0$ if $t = 0$ then this part is just 1 in that case $P_t = P_0$. So, that is one possible value of P_t if $t = 0$ then you have this term becoming equal to just P_0. So, that is mentioned here P_0. On the other hand suppose this is equal to 1, $t = 1$ in that case this becomes -1 in which case this entire expression becomes $2P^* - P_0$.

So, in this case where $\beta = \delta$ P_t will alternate between these two values P_0 and $-P_0 + 2P^*$ in every two successive periods. And these two values are actually equidistant from P^*. And the third case is where you do not have a $|b| \leq 1$ but $|b| > 1$. So, this is the case where $\beta < \delta$, the demand line is flatter than the supply line. There is explosive oscillation P_t will diverge from P^* by fluctuating around it.

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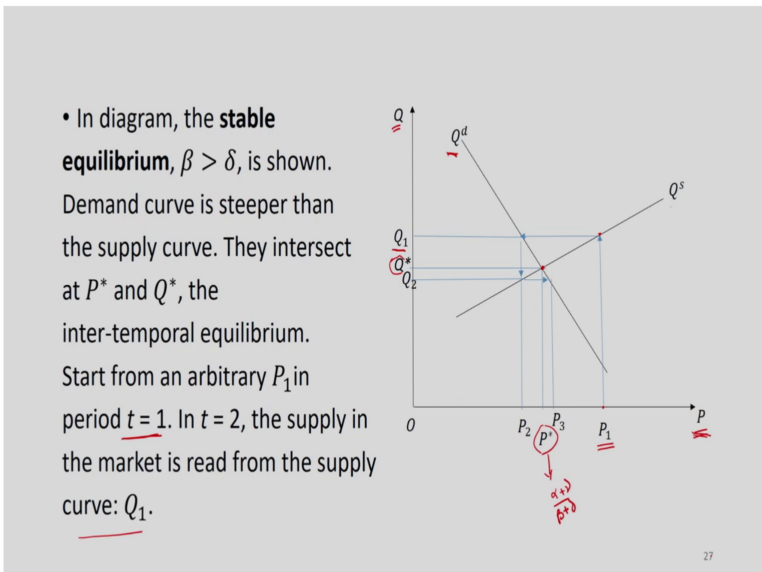

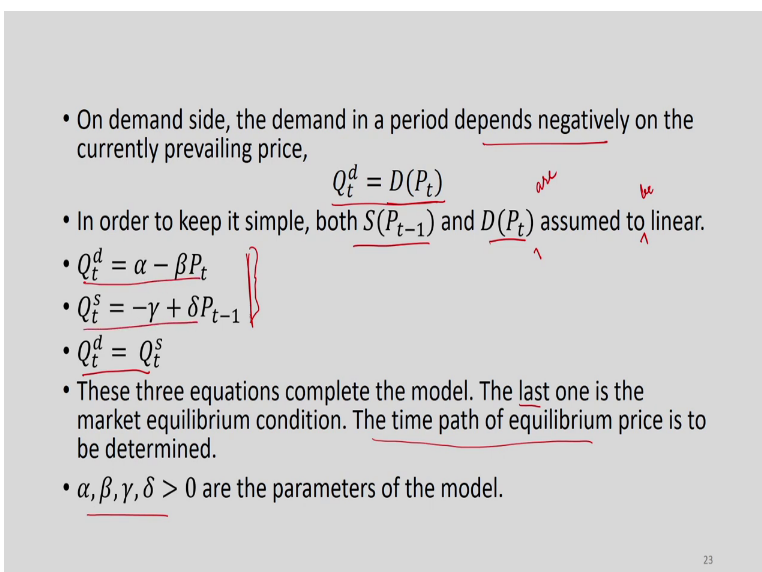

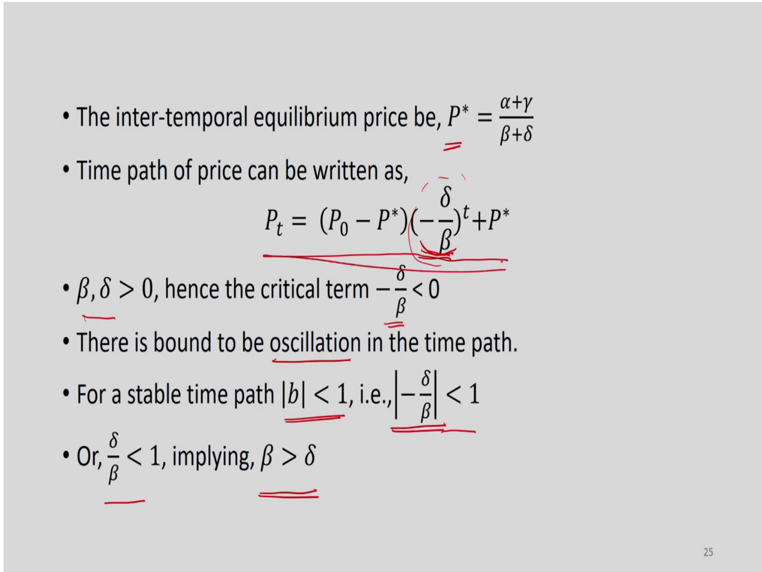

I have drawn the diagram for this is the case of stable equilibrium where $\beta > \delta$. So, as you can see the line Q_d, this is quite steep, whereas the line Q_s is not that steep, it is relatively flat. In this case, if you have noticed I have represented price in the horizontal axis. And quantity along the vertical axis and this is just to make sure that we are getting the slopes correct that $\beta > \delta$.

Because when we are writing the demand function and the supply function in this manner, in this manner and this manner, we are taking P as the independent variable and usually independent variables are represented along the horizontal axis. Although in economics the prices are generally represented along the vertical axis. So, in this case to get the slopes correct, I have followed the convention of representing the independent variable in the demand and supply functions along the horizontal axis.

They intersect each other at this point $P^* Q^*$. Here is P^* and here is Q^* and this $P^* Q^*$ is the intertemporal equilibrium in particular this $P^* = \frac{\alpha+\gamma}{\beta+\delta}$. Now, we can see how this model works. We start with some arbitrary value P_1 and suppose this is period $t = 1$. Now, this is the price that is prevailing in period $t = 1$ as we know the quantity supplied in this next period will be affected by this price. So, in period $t + 2$, the supply in the market is read from the supply curve and this is given by this so, this is Q_1. This is the supply in period 2.

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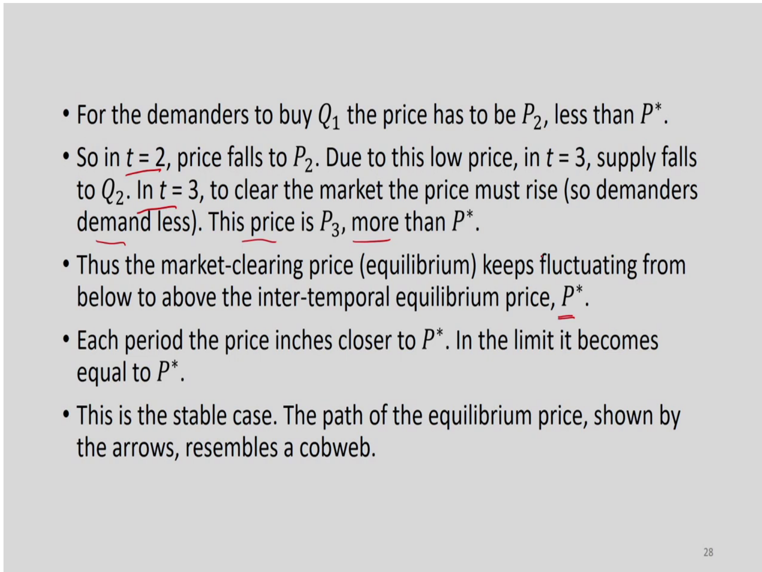

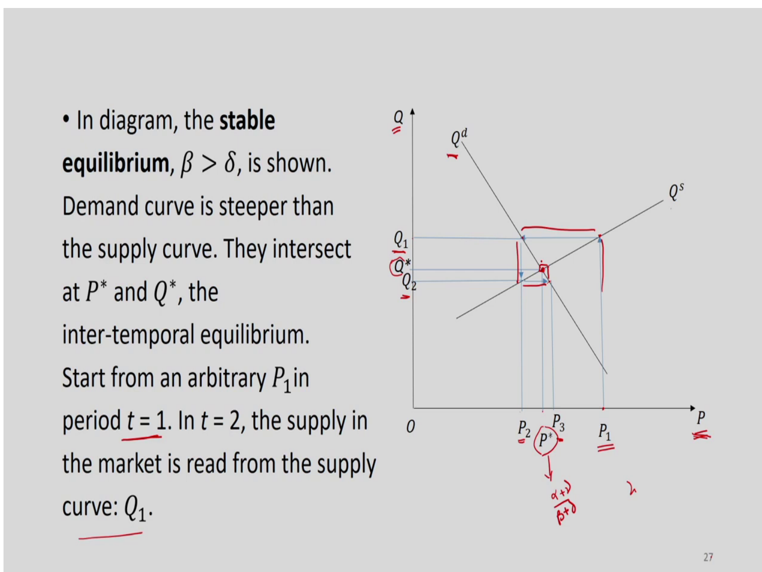

For the demanders to buy this amount that is Q_1 the price has to be $P_2 < P^*$. Let us suppose where is P_2. So, P_2 is here if the demand function has to intersect this line that is $Q = Q_1$ then this is the intersection point from this we read the price which is P_2 that means the price has to be equal to P_2 in period 2. So that the demanders that is the buyers, are willing to buy this much amount of goods, which is Q_1 that has been produced by the producers in period 2. So, the price you can see from P_1 it drops to P_2 and in particular $P_2 < P^*$.

So, in $t = 2$ the price falls to P_2 due to this low price in $t = 3$, the supply also falls to Q_2. If the price is P_2 what is the supply that can be read from the supply line. So, this is the supply and this is given by Q_2 mind you this Q_2, supply will come to the market in the next period, that is period 3. So, in period 3 to clear the market the price must rise because the quantity supplied is very low.

The price must rise, so the demanders demand less and this price is P_3 and in particular it is more than P^*. So, I read the P_3 from the demand function and this is the P_3 price. In particular you can see that the $P_3 > P^*$. So, we started with P_1 which is greater than P^* then we came to P_2 which is less than P^* and then again we go to P_3 which is greater than P^*. In other words, we are oscillating around the P^* price which is the intertemporal equilibrium price.

Thus the market clearing price which is the equilibrium in each period keeps fluctuating from below to above the intertemporal equilibrium price P^*. Each period the price inches to P^* in the limit it becomes equal to P^* and that you can see in the first period the P_1 price is quite far away from P^* in the next period P_2 is quite close to P^*. In the third period it is even closer to P^* and in the next period will become even closer and likewise we will go on.

The fluctuation or the amplitude of the fluctuation is coming down in each successive period. So, in the limit it becomes equal to P^*. This is the stable case: the path of the equilibrium price shown by the arrows resembles a Cobweb. So, this is the path you are going there and here you are going and then you are coming down and in the next we will go up and like this will go on so in the limit it will approach the intersection point but it looks like a spider Cobweb. So, it resembles Cobweb that is why it is called the Cobweb model.

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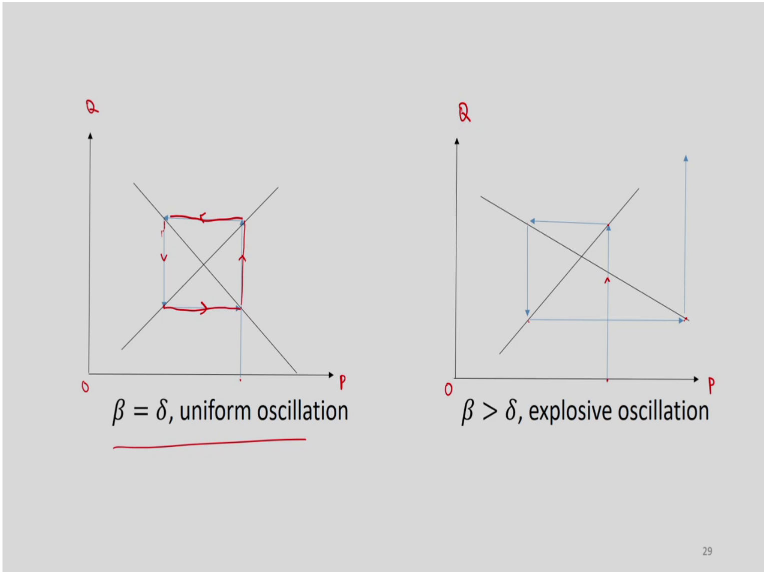

We can also have the other cases in the diagram, here you have the uniform oscillation case where β = δ. As you can see, I can start from any arbitrary price suppose this is the price I am starting from going up this is the supply in the next period to clear the market the price has to come to this level and if the price is this level, then the supply comes down and to clear the market again the price has to go up, but you can see the price in the third period is same as the price in the first period.

So, therefore, actually you are going round and round in the same circle, there is no damped oscillation in this case at least. And here on the right hand side of the panel you have the case of explosive oscillation. I am not going to explain it in detail, but you can see how it is happening if you take any arbitrary price, this is the supply.

So, the market has to clear so this must be the price in the next period and this is the price in the next period this is the quantity then this is the price in the next one at that period. And you can see the price will go on fluctuating but it is getting far and further away from the intertemporal equilibrium. This was the case of explosive oscillation.

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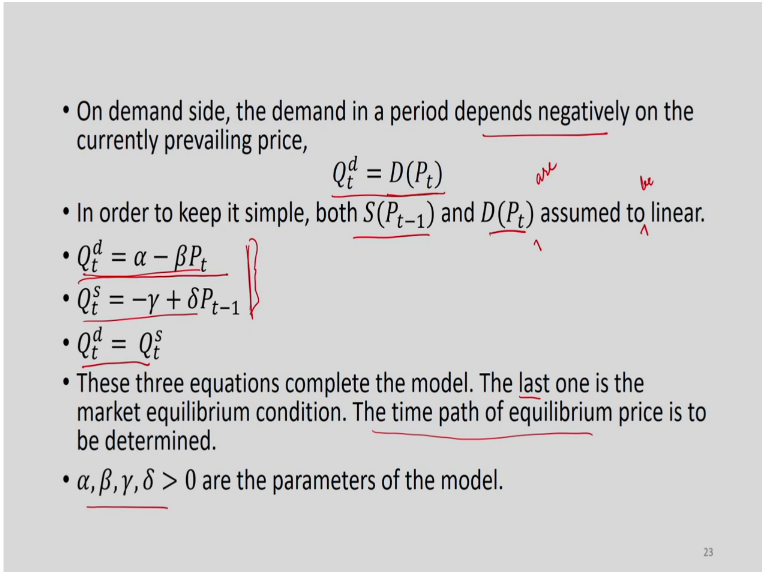
- We have only focused on the time path of P_t, the market clearing price, as defined by market equilibrium condition.
- The time path of the equilibrium quantity can be found by plugging the time path of price in the demand function, because demand depends on the price of that period.
- The stability (or the lack of it) of price will be reflected in the quantity as well.

- Using the three equations, we get, $\alpha - \beta P_t = -\gamma + \delta P_{t-1}$

Or, $P_{t+1} + \frac{\delta}{\beta} P_t = \frac{\alpha+\gamma}{\beta}$

- One can find the particular integral, $y_p = \frac{\alpha+\gamma}{\beta+\delta}$
- And, the complementary function to be, $y_c = A(-\frac{\delta}{\beta})^t$
- Let price at $t = 0$ be P_0, the time path of price is,

$$P_t = \left(P_0 - \frac{\alpha+\gamma}{\beta+\delta}\right)(-\frac{\delta}{\beta})^t + \frac{\alpha+\gamma}{\beta+\delta}$$



Now, so far, we have only focused on the time path of P_t the market clearing price as defined by market equilibrium condition the time part of equilibrium quantity. So, that we have not talked about. But as we know in any sort of equilibrium market equilibrium, there are two sides to it one is the quantity the other is the price. What about the time path of the equilibrium quantity that can be found out by plugging the time path of the price in the demand function because demand depends on the price of that period.

So, let us look at how actually, we can do that. So, this is the time path of price P_t. Now if we take this P_t and we plug it in here. Then we will get the time path of the quantity this will be a function of t only and it will have some P_0 term but as we know P_0 is not a variable it is a constant. So, in short, we are going to get Q_t^d. So, quantity in each period, the equilibrium quantity in each period will be a function of t alone and there will be some parameters.

And that is what is meant by the time path, in this case the equilibrium time path. So, that is how it is done. Now, I said that it has to be plugged into the demand function and the reason being that the demand in Cobweb model in a particular period depends on the price of that period. So, there is no lag there. That is why the time path of the price has to be plugged into the demand function not the supply function and the stability of price will be reflected in the quantity as well.

So, if the price path is stable or it is unstable, then that will be replicated in the quantity, stability or instability as well. So, this was the Cobweb model.

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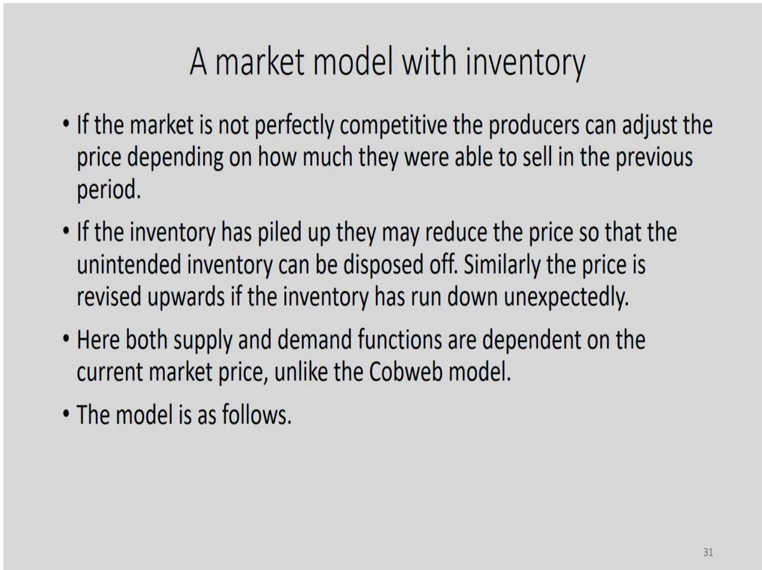

So, we can just talk a few words. You can say a few words about another model, this model is a market model with the inventory and then maybe you can close this lecture. If the market is not perfectly competitive, the producers can adjust prices depending on how much they were able to sell in the previous period. So, here price is not determined automatically by equality of demand and supply. Here the price depends on how much the producers were able to sell in the previous period. In other words, it is the producers who determine the price.

So, this is the market model with the inventory, if the inventory is piled up, they may reduce the price so that the unintended inventory can be disposed of. Similarly, a price is revised upwards, if the inventory has run down unexpectedly, inventory in short means stock. So, if the stock is running down unexpectedly, then the producers might jack up the price or if the inventory is piling up, then they might think that it is a good idea to give some incentive to the buyers to buy in which case they might reduce the price. Here both demand and supply functions are dependent on the current market price unlike the Cobweb model, so there is no lag in the supply function as well as the demand function.

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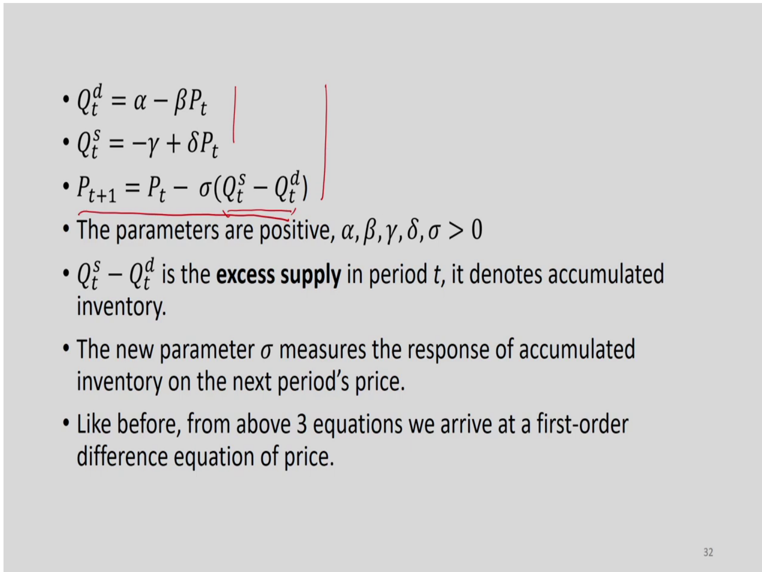

The model is as follows. So, these are the three equations $Q_t^d = \alpha - \beta P_t$, $Q_t^s = -\gamma + \delta P_t$, you can see there are no lags there. And finally, $P_{t+1} = P_t - \sigma(Q_t^s - Q_t^d)$. So, this is the new thing that has come now, the price in the next period depends on the price of the previous period, but it also depends on whether the inventory is piling up or it is running down which is captured by this term. We shall talk more about it in the next lecture. Thank you for joining me in this lecture and see you in the next lecture. Thank you.