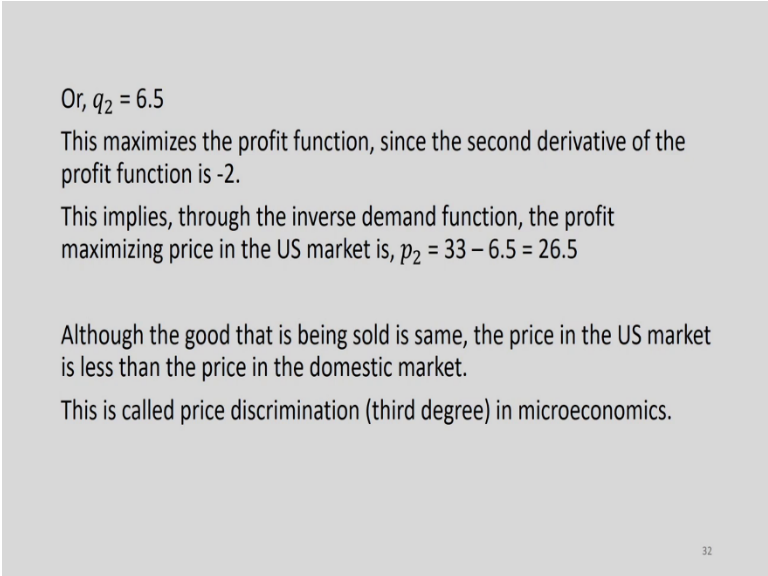
Mathematics for Economics – I
Professor – Debarshi Das
Humanities and Social Sciences Department
Indian Institute of Technology – Guwahati
Lecture – 34
Tutorials – 2b

Hello and welcome to another lecture of Mathematics for Economics part 1. So, we have been doing some tutorials in this series of lectures now. The tutorials actually have two purposes for the students, number one, they help the students to solve the actual problems that the students will encounter in the tests. So, that is the practical part of the tutorial and the second purpose of doing these tutorials is that it also helps the students to review the entire syllabus and the different topics that have been covered in this course.

So, the topics that we have so far covered in our tutorials are the following, we have covered the real number system after that we talked about the mathematical logic and then we talked about the idea of proofs and then we went to a differentiation the idea of limits, differentiability and then we also talked about the idea of series and sequences, idea of convergence and then the last topic that we covered is about optimization.

Optimization is a very important application of mathematics in economics, so we have covered the topic of optimization in our tutorials, we talked about the first order condition and the second order conditions and we also talked about the practical problems of maximizing profit for example, for a producer, for a farmer. So, those are the things that we have done. Now today what I propose to do is to take a further topic which we have covered in this course.

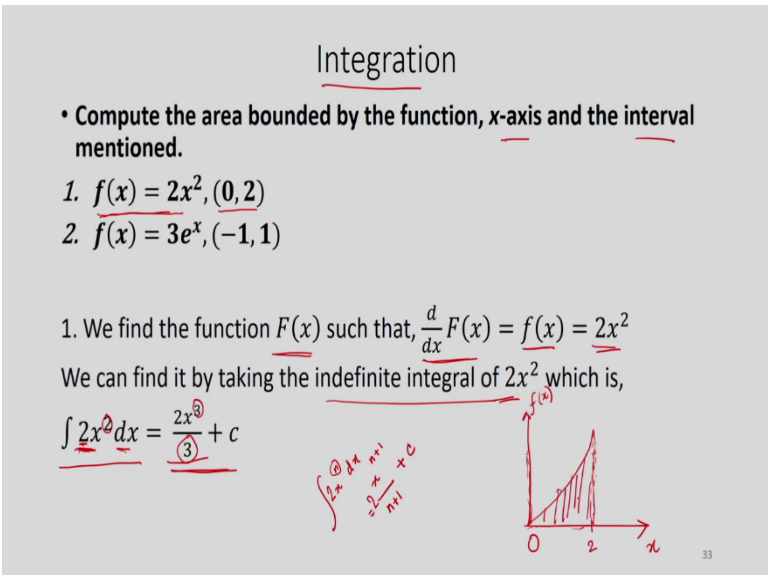
(Refer Slide Time: 2:59)



So, after optimization what we shall do today as you can see on your screen is this is the second series of tutorials and here optimization has been done, we have done this before and let me take you to the last problem that we have done. This was the last problem, this was about maximizing profits in two simultaneous markets, the producer is selling his product.

So, in this case, the producer decides how much good he will sell in these two markets and what will be the prices that he will charge from these two markets. This is called price discrimination that we have seen in the previous lecture.

(Refer Slide Time: 3:47)



So, today we are going to start with this new topic of tutorial which is integration. So, let us straight away start with a problem, so this is the problem, compute the area bounded by the function x-axis and the interval mentioned. So, a function is given here, this is the function in the first problem, $f(x) = 2x^2$.

So, we have to compute the area bounded by this particular function, the x-axis and the interval mentioned. Here in the first problem the interval is (0, 2). Just visualize this in terms of our diagram, so we have this first quadrant here along the horizontal axis you have x and suppose this is (0, 2) that is the interval that we have been given.

Now here is the x-axis here is the interval so just imagine two vertical lines at points $x = 0$ and $x = 2$. So, the three sides are known to us, the x-axis, this vertical line and this vertical line but what is bounding the area from above? So, what is bounding this required area from above is this function $y = f(x) = 2x^2$.

Now how does this function look like, $2x^2$, it is going to be something like this. It is going to be increasing because as x is rising, this $f(x)$ is rising and you can see we have x^2 so it is not a linear function, it is a quadratic function in a particular quadratic form, $2x^2$. So, it is rising and rising at an increasing rate, it is a convex function.

So, we have to find out the area here that is the problem. Now as we know this can be easily found out by the application of integration. So, the main problem that we have to solve is that we have to find a function $F(x)$ such that $\frac{d}{dx}F(x) = f(x)$ and what is $f(x)$?

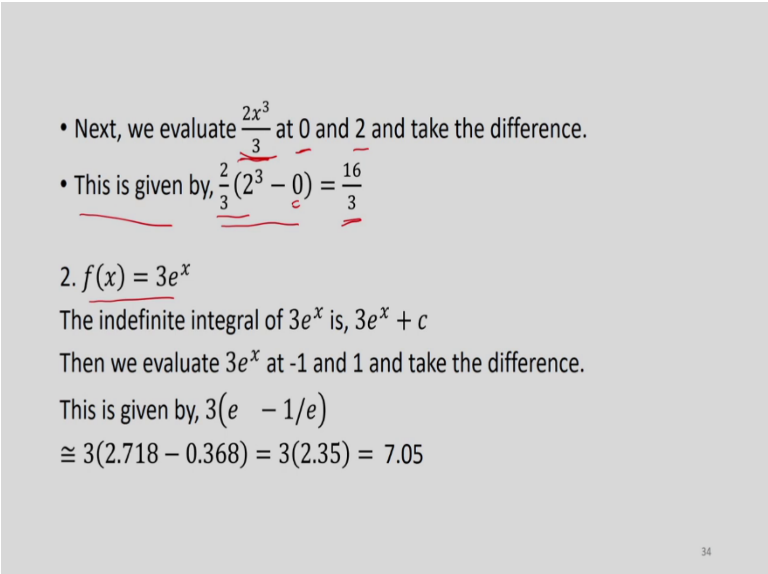
$f(x) = 2x^2$. So, we have to find out the antiderivative of $2x^2$. What is that function? If we take the derivative that gives me $2x^2$, we have to find that function, so we are going in the opposite direction of the derivative here that is why we are calling it antiderivative. So, anti-derivative as we know can be rephrased as an indefinite integral of $2x^2$.

So, here you have the integral sign and then the function that we have to integrate that is called the integrand. Integrand is $2x^2$ and you have dx, x is the variable of integration and so, this will give me this expression here we are simply using the formula that if you take the $\int x^n dx$ then what you get is $\frac{x^{n+1}}{n+1} + c$, c is the constant of integration.

So, that is the formula that we have used here and we have also noted that this 2 is constant, so if you have 2 here, it will just get multiplied by 2. Whatever the constant is with x^2 or x^n, the result that we are getting here is going to get multiplied with that 2. So, here $n = 2$ itself, so the power of x is 2 here.

In general, it can be n so it will be x^{n+1} so you are getting 3 and, in the denominator, you are getting again $n + 1 = 2 + 1 = 3$. So, this is the result $\frac{2x^3}{3}$. Now this is the indefinite integral but we have not yet found out the area of this shaded region that we have to find out.

(Refer Slide Time: 9:45)

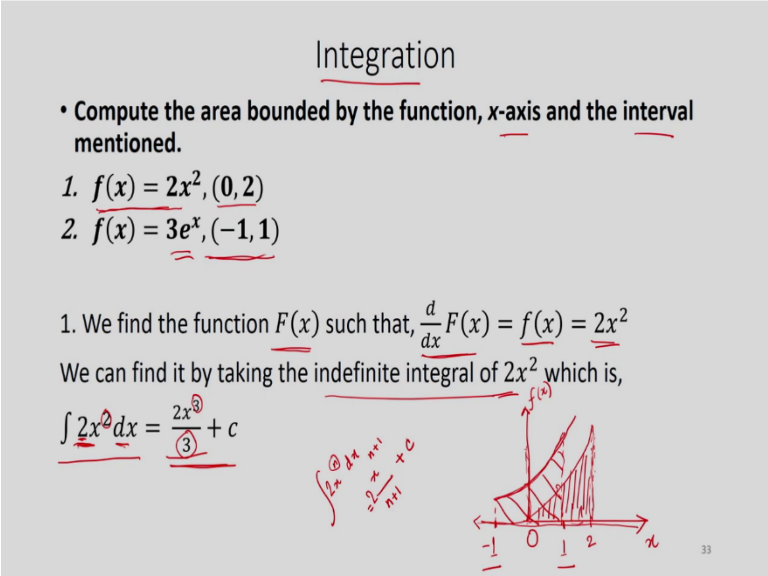


Well, basically the idea is that we have to evaluate this function that is $\frac{2x^3}{3}$, at these two numbers 0 and 2 and we have to take the difference. So, here we are doing that, remember here we are ignoring that constant part that is c, that c actually will again get cancelled if I take the difference, so it is not making any appearance here.

So, if I take the value of this function at 2, I will get $\frac{2}{3}(2^3 - 0) = \frac{16}{3}$. we are going to get $\frac{16}{3}$. So, this is the answer: the area of this region, this shaded region is $\frac{16}{3}$.

And here is another function $f(x) = e^x$, so it is an exponential function here, e^x.

(Refer Slide Time: 11:20)

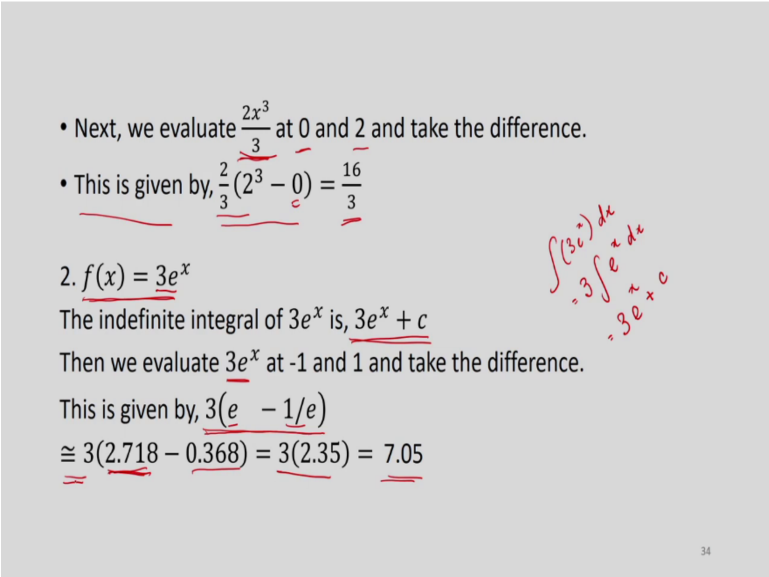


And the question is similar but here the interval is a little bit different, it is starting from -1. So, actually we are going to the left of the origin, so suppose here you have 1 and here you have -1 and how this function looks like? This is a different function $f(x) = 3e^x$, this function is not starting from the origin because if you put $x = 0$ here then $e^0 = 1$. So, at 0 the function is taking the value 3.

So, it has a vertical intercept. So, the function will roughly look like this, so what you have is this area. This is the area that we are required to estimate, the interval is from (-1,1), the downward limit that is the vertical limit in the bottom is the x-axis itself and the upward limit that is the vertical limit on the top is this function. So, this is the area.

The method is going to be just the same as before, first we find that capital $F(x)$ which if differentiated gives me $3e^x$ and after I find $f(x)$, then I will evaluate that capital $F(x)$ at 1 and -1 and take the difference. So, this is the strategy.

(Refer Slide Time: 13:16)



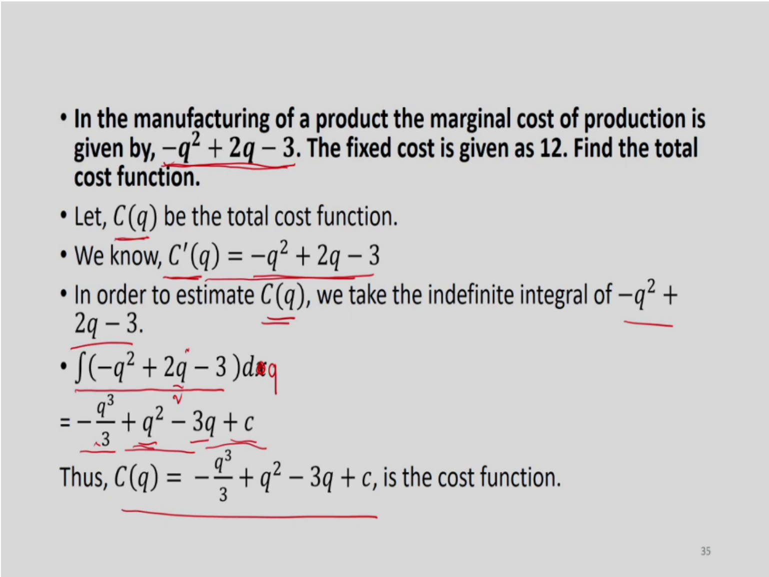
Now what is the indefinite integral of this, as we can see it is written as $3e^x + c$, so this can be easily checked. So, what you have is $\int 3e^x dx$. This is the indefinite integral and again $3\int e^x dx$ and what is the $\int e^x dx$? It is $3e^x + c$.

So, that is what I have written here, $3e^x + c$. The next stage is that we have to take the value of $3e^x$ at -1 and +1 and take the difference. So, that is what we have done here, $3(e - 1/e)$.

Now e, as we know it is an irrational number but it can be approximated by this number 2.718, we are taking up to the third decimal place and I divide 1 by this number 2.718 and that will give me 0.368. So, I simplify this and I get this answer 7.05.

So, that is my answer that the area of this shaded region is a little bit over 7, 7.05. But again, this is an approximate value because at this stage we have taken the approximate value.

(Refer Slide Time: 15:33)



So, this is how we apply the idea of integration to evaluate the area under a curve within a particular interval. Here is another practical instance of application of integration. In the manufacturing of a product the marginal cost of production is given by this, $-q^2 + 2q - 3$ The fixed cost is given as 12. Find the total cost function. So, marginal cost is given, fixed cost is given, we have to find out the total cost function.

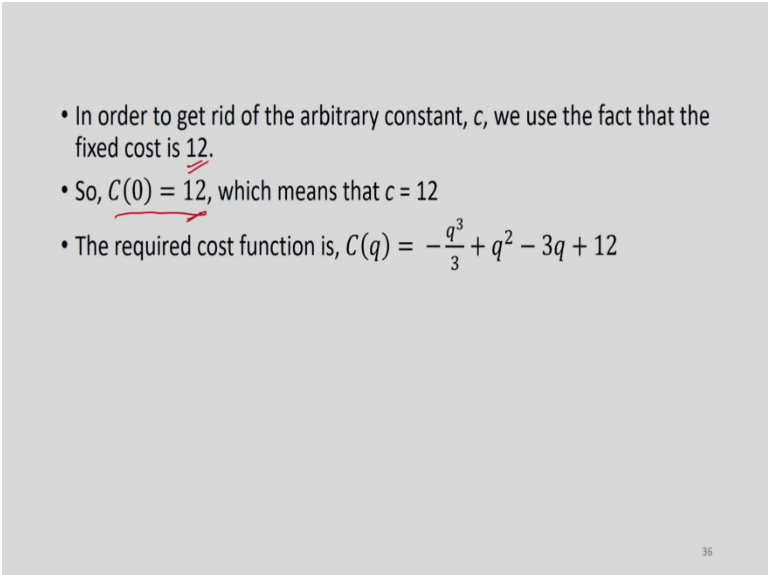
So, we start by assuming that the total cost function is given by this $C(q)$, so this is the total cost function so if the total cost function is given to us we know that the marginal cost can be found out by taking the derivative, first derivative and we are also given the information. In this case $MC(q) =- q^2 + 2q - 3$. So, we combine these two things and I will get $C'(q) = - q^2 + 2q - 3$.

Now our purpose is to find out what is $C(q)$? So, $C(q)$ is the antiderivative of this marginal cost function. So, therefore we take the indefinite integral of this marginal cost and so that is what we are doing here, $\int(- q^2 + 2q - 3)dq$.

So, the next stage is just to find out the indefinite integral, at this stage we are using that formula, that power rule kind of formula that we have encountered in the previous problem. So, that will give me, $-\frac{q^3}{3} + q^2 - 3q + c$.

So, this is the cost function, the cost function $C(q) = -\frac{q^3}{3} + q^2 - 3q + c$. But here there is an arbitrary constant c and we cannot leave it like that.

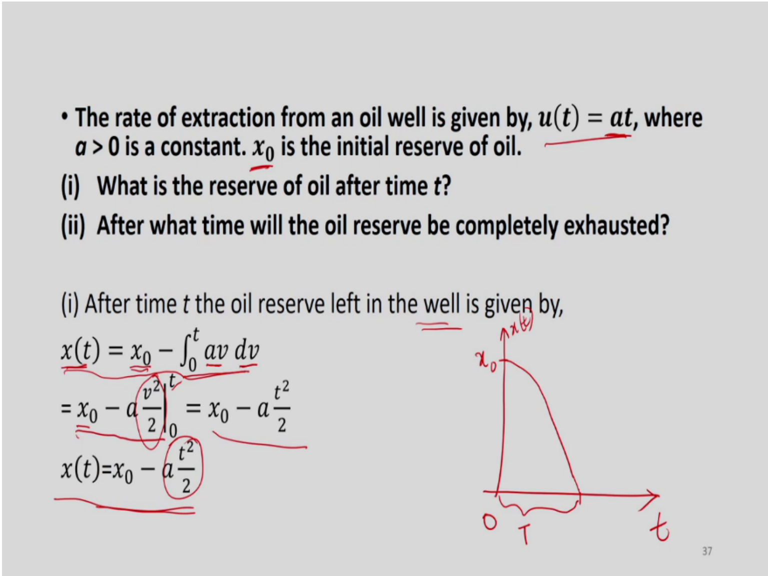
(Refer Slide Time: 18:50)

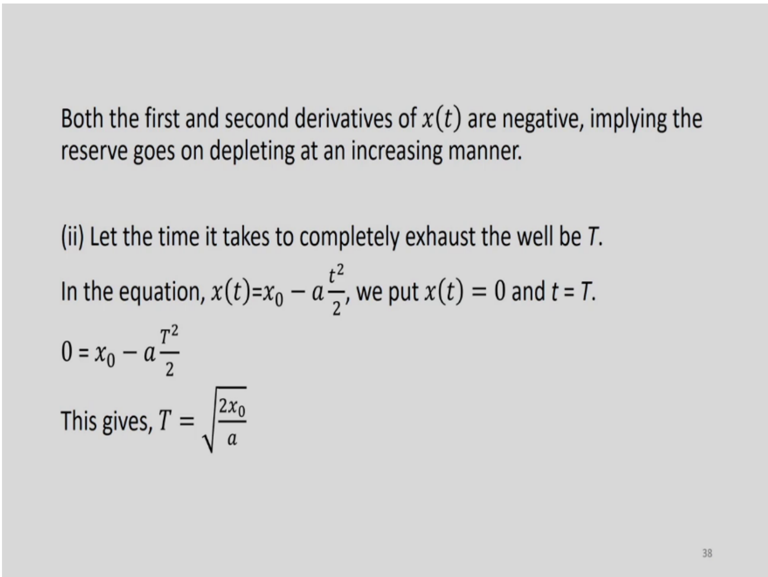


In order to get rid of the arbitrary constant small c we used the fact that the fixed cost is 12. Now the fixed cost is 12 means what? That is if the producer is not producing, he has to bear some cost and that cost is 12. So, basically it means that at output 0 the producer's cost is 12. That is what I have written here in mathematics $C(0) = 12$ and if you put $q = 0$ in this function this part will become 0 and $C(0) = 12$ and therefore $c = 12$.

Therefore, c has been found out and therefore the cost function is $C(q) = -\frac{q^3}{3} + q^2 - 3q + 12$. So, this is another application of integration.

(Refer Slide Time: 20:10)





Here is another sort of application. The rate of extraction from an oil well is given by this $u(t) = at$ where $a > 0$ is a constant, x_0 is the initial reserve of oil. What is the reserve of oil after time t? After what time will the oil reserve be completely exhausted? So, there are two parts to this question.

Now after time t the oil that is left in the well is given by this formula, if you recall. So, let us assume that the oil that is left in the well after time t is given by $x(t)$. So, $x(t)$ is what? $x(t)$ is the initial reserve, initial reserve is $x_0 -$ the oil that has been extracted already.

How much time that has elapsed, it is t. So from 0, 0 is the initial time, the point of origin to the time t how much oil has been extracted. If I subtract that from the initial reserve, I will get the oil that is left. Now how much oil has been extracted that is given by this. So, here comes the application of integration and in this case, it is a definite integral.

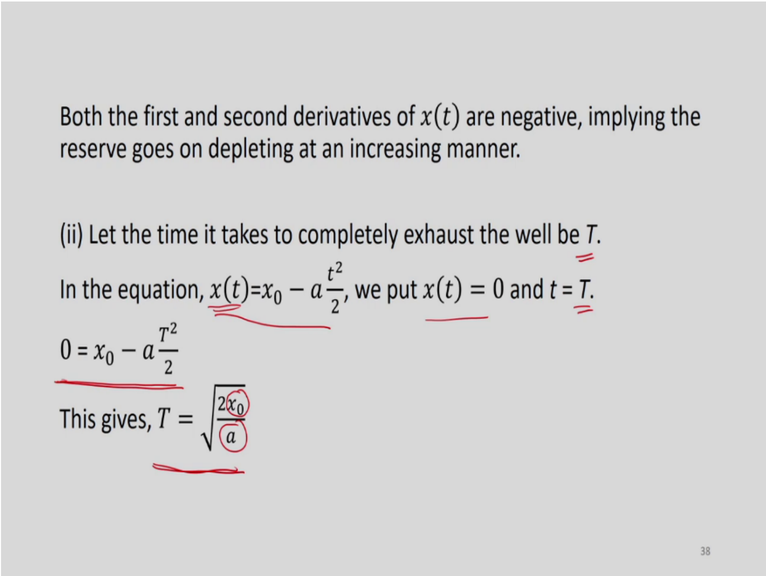
So, from 0 to t I am taking the integral and what is the integrand? It is av and I am integrating with v so dv. Why av? Because this is the rate at which the extraction is taking place at. $x(t) = x_0 - \int_0^t av\, dv$So, then the problem now becomes a simple problem of integration, the first term is $x_0 - a\frac{v^2}{2}$ and there are these limits, the upper limits and the lower limits t and 0.

I am writing that and then I am evaluating this $\frac{v^2}{2}$ at t and 0 and taking the difference so I will get $x(t) = x_0 - a\frac{t^2}{2}$. So, that is what the answer is $x(t)$ that is the oil that is left after time t is equal to $x(t) = x_0 - a\frac{t^2}{2}$. As you can see the first and the second derivatives of $x(t)$ are negative implying that the result goes on depleting at an increasing rate, at an increasing manner.

So, as you can see as t rises, this part is rising. So, $x(t)$ is declining and not only it is declining it is declining in an increasing manner. So, just again imagine this in terms of a diagram at $t = 0$, $x(t) = x_0$. So, you are starting from this point vertical intercept and it is declining like this. It is a concave function but a declining concave function. The second derivative is negative.

The second part was this, let the time it takes to completely exhaust the well be T. So, this was the question: how much time does it take to completely exhaust the oil well? So, that time we have to find out. In terms of this diagram actually we have to find out this time. So, this is the T that we are trying to find through our mathematics.

(Refer Slide Time: 25:00)



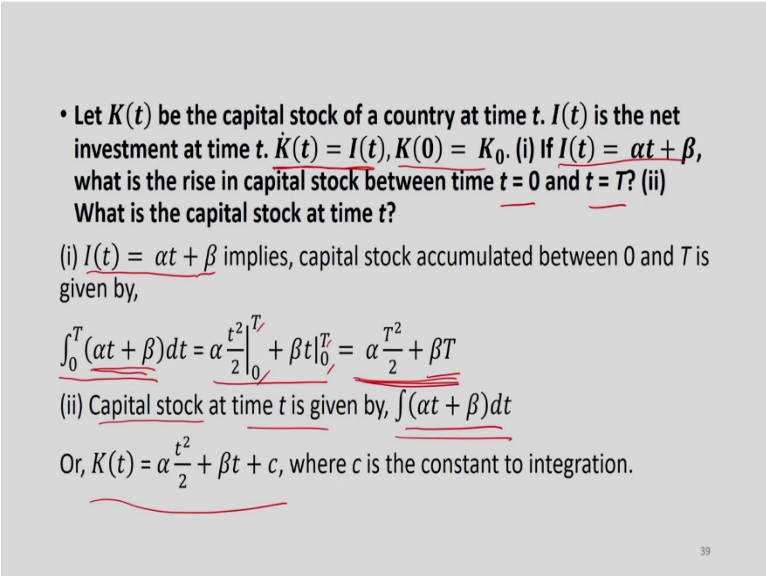
So, what do we do? So, in this equation, this is the equation of the oil that is left, we put the oil that is left equals to 0 that is left hand side suppose is equal to 0 and at what time it is getting equal to 0 that time is T. That we have assumed, so I put small $t = T$ and I put the LHS to be equal to 0. So, that will give me this equation, $0 = x_0 - a\frac{T^2}{2}$.

And then I just have to solve this for T, I take the positive root and this gives me $T = \sqrt{\frac{2x_0}{a}}$. So, as you can see what is the intuition of this T, T rises if x_0 rises. And that is intuitive if you have a large reserve to begin with it will take more time to extract all the oil.

So, that is why as x_0 rises the time it takes to exhaust the well also rises. As a rises, which is coming in the denominator, T actually declines. What is the intuition for that? The a is appearing here, that is where a is appearing. So, a is an indicator of how fast the extraction is taking place per unit of time.

If a is high that means the rate of extraction is high and that will also mean that the exhaustion of the well will take place at a shorter point of time, that is the intuition. So, if you are extracting at a faster rate per unit of time then it will take a shorter time to completely take out the oil from the oil well. So, in that case T will decline.

(Refer Slide Time: 27:30)



Here is a problem from macro-economics. Let $K(t)$ be the capital stock of a country at point t. $I(t)$ is the net investment at time t, so K dot t = $I(t)$, K dot t means the derivative of the capital stock with respect to time and this is also given to us $K(0) = K_0$. That means at point 0 the stock of capital is given as K_0. So, there are two parts of this question: one if $I(t)$ is given, $I(t) = \alpha t + \beta$ What is the capital stock? What is the rise in capital stock between time 0 and time T?

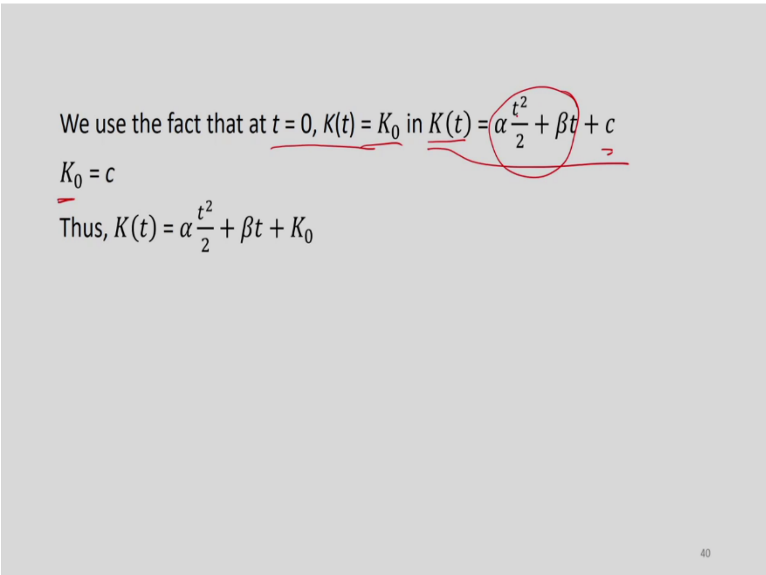
And secondly what is the capital stock at time t. Let us start with the first question, we have been given this particular function of the net investment, $I(t) = \alpha t + \beta$. Now how much will be the rise of capital stock, accumulation of capital stock between 2 points as $t = 0$ and $t = T$. That is what we have to find out and that is easily found out by applying the idea of definite integral.

So, we are integrating this function, the $I(t)$ function because we know this, that if I take the derivative of the capital stock I get the net investment, $I(t)$ so here we are going the opposite direction and the limits of integration are 0 and capital T. So, these are the upper and the lower limits, well after I do the integration, what I get?

I get, $\int_0^T (\alpha t + \beta) dt = \alpha\left[\frac{t^2}{2}\right]_0^T + [\beta t]_0^T$ but I have to take the upper and the lower limits so those are written here and this is coming out to be $\alpha\frac{T^2}{2} + \beta T$. So, that is the answer. So, this is the rise in the capital stock between 0 and T. The second part is, what is the capital stock at time t?

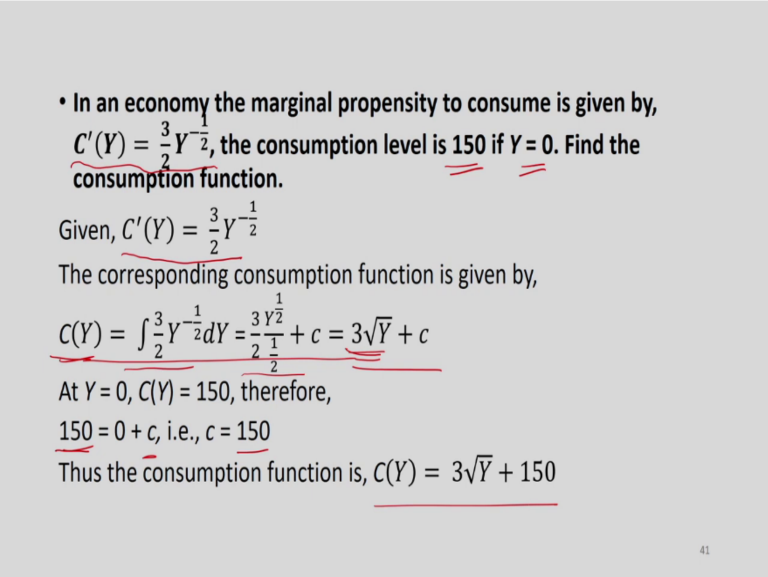
Well, that is found out by taking the definite indefinite integral of the investment function. So, what are we doing? We are taking the indefinite integral of $\int(\alpha t + \beta)\, dt$. Here we are not interested in the rise of capital stock. We are just finding out, what is the capital stock at a particular point of time? So, that is why we are doing the indefinite integral and we get this result: $\int(\alpha t + \beta)\, dt = \alpha\frac{t^2}{2} + \beta t + c$. But here there is an arbitrariness which is c is there and we have to get rid of that.

(Refer Slide Time: 31:20)



That can be got rid of by using this information, that at $t = 0$, the capital stock is K_0. So, that information we use here we put $t = 0$ here. So, here I will get K_0. K_0 is this and that is $K_0 = c$ because this part will just become 0. So, c which was sort of arbitrary is now given a value it is equal to K_0, so therefore, the stock of capital at any point of time t is given by this function $K(t) = \alpha\frac{t^2}{2} + \beta t + K_0$.

(Refer Slide Time: 32:10)

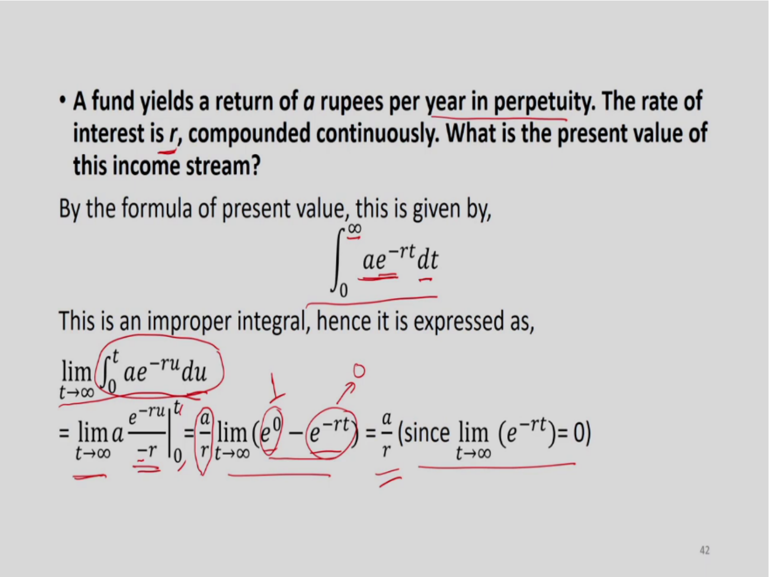


In an economy the marginal propensity to consume is given by this, MPC, marginal propensity to consume is given by $C'(Y) = \frac{3}{2}Y^{-1/2}$. The consumption level is 150 if $Y = 0$. Find the consumption function? So, this is going to be a similar application of the idea that we have been using, the $C'(Y)$ is that is marginal propensity to consume is $\frac{3}{2}Y^{-1/2}$.

Now as we know the marginal propensity to consume is found out by taking the derivative of the consumption function, first derivative. So the consumption function therefore is given by the integration of this MPC. We are taking the indefinite integral and we just use the power rule and I will get this, $C(Y) = \int \frac{3}{2}Y^{-1/2}dY = 3\sqrt{Y} + c$, c is the constant of integration.

We know that the consumption level is 150, if $Y = 0$. So, I put $Y = 0$ here in this form. So, on the left-hand side I will get $C(0)$. $C(0) = 150$. On the right-hand side, this part will drop out and so I will get only c. Therefore $c = 150$. Thus, the consumption function is this $C(Y) = 3\sqrt{Y} + 150$.

(Refer Slide Time: 34:05)



Here is a little bit of a difficult question compared to what we have been doing. A fund yields a return of a rupees per year in perpetuity. The rate of interest is r, compounded continuously. What is the present value of this income stream? Now the formula of present value is if you have compounded continuously, then what is the formula it is given by this. So, integration of 0 to infinity, why infinity?

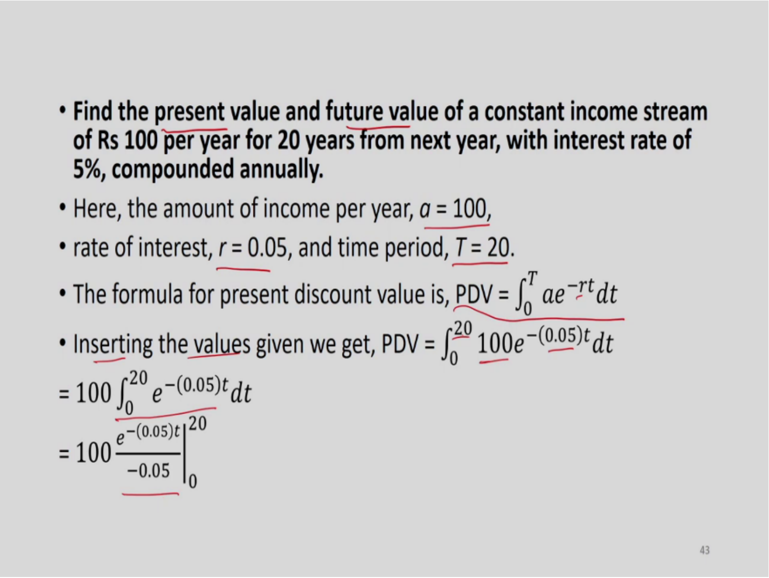
Because this return is coming in perpetuity forever and what is the return in each year? This is given by a, that is a, but this is in future. So, therefore I have to discount that. So, we are getting e^{-rt} and dt. So, here r is the rate of interest which is given to me. So, this is the present value but this is an improper integral because the upper limit is infinity.

So, I write this as this, $\lim_{t\to\infty}\int_0^t ae^{-ru}du$. I have taken u to be the variable of integration. So, as to not confuse it with t. So, this is easy, so I have to integrate $ae^{-ru}du$. a can be taken out, a is a constant and so integration of e^{-ru}.

So that is $\lim_{t\to\infty} a\left[\frac{e^{-ru}}{-r}\right]_0^t$. $\frac{a}{r}$ can be taken out because this is a constant so $\frac{a}{r}\lim_{t\to\infty}(e^0 - e^{-rt}) = \frac{a}{r}$ $(since \lim_{t\to\infty}(e^{-rt}) = 0)$. Remember there is a minus sign here, that is why it is coming out as the lower limit minus the upper limit. And this part is just 1 and this part goes to 0.

So, the second part is 0, the first part is 1, therefore, the answer is $\frac{a}{r}$. So, this is the present value of a fund which gives you an amount of money per year in perpetuity and the interest rate is compounded continuously.

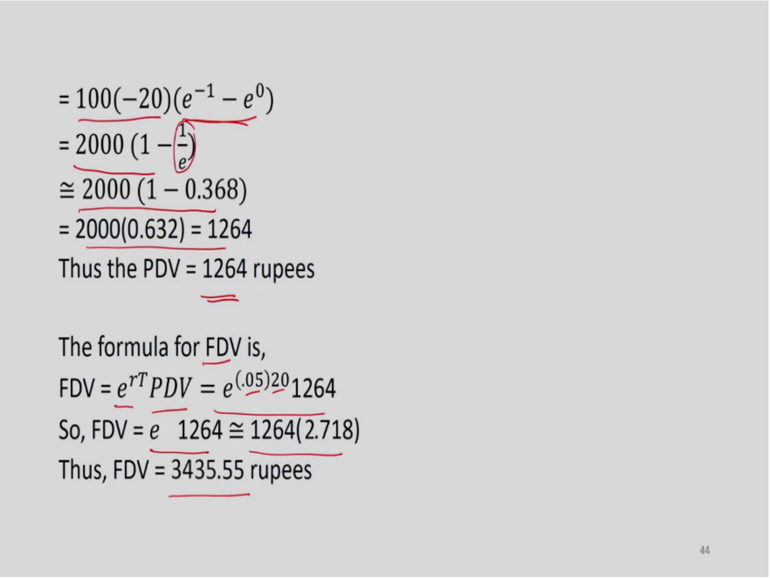
(Refer Slide Time: 37:15)



Find the present value and future value of a constant income stream of 100 rupees per year for 20 years from next year with the interest rate of 5 % continuous annually. We have to find out two things: the present value and the future value. So, here the amount of income that we are getting annually is 100 rupees. The rate of interest is 5 %, so it is 0.05.

The time period is 20 years. It is compounded continuously, so the PDV will be this. This is the standard formula that we have just used in the previous problem also. So, here I just have to insert the values that are given to us $a = 100, r = 0.05, T = 20$. So, we have put those values here. So, this is the integration that we have to do and this again the same formula $\frac{e^{-rt}}{-r}$. And the limits are 20 and 0.

(Refer Slide Time: 38:37)

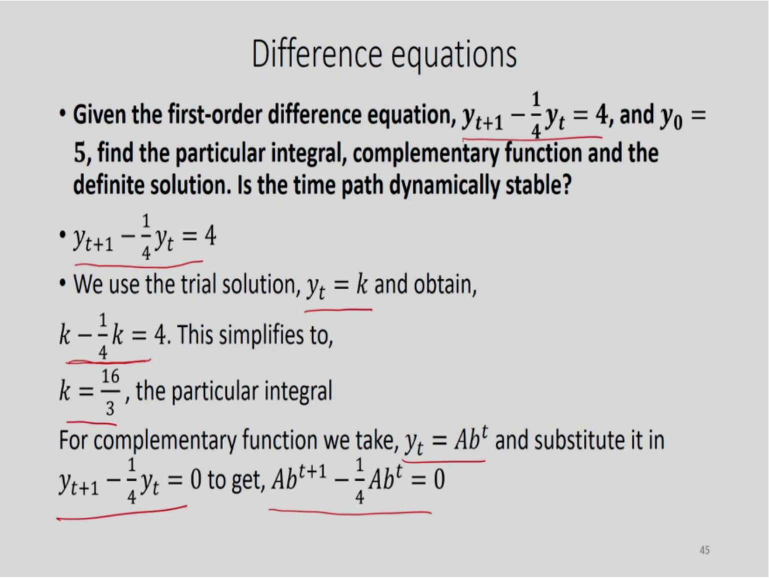


And this simplifies to, here I have - 0.05. So, that will come out to be -20 and within the brackets these are the values if I take the limits and take the difference. So, I can just multiply this thing with the minus sign so it becomes $(1 - \frac{1}{e})$ and this is approximately equal to $2000(1 - 0.368)$. I have taken the approximate value here and this is simplifying to be 1264 rupees, so this is the present discounted value.

Now we also have to find out the future value, not only the present value but also the future value, what do we do to get the future value? I multiply the PDV that is the Present Discounted Value with e^{rT}. r is known to me 0.05, T=20, so I just have to put those values here and then I will simply get $e.1264$.

And that is approximately equal to this, FDV=3435.55. So, this is the future discounted value. That actually completes our discussion of integration. Now this was the last topic that we covered in our course; this is different equations. So, let me take some problems from this last topic and then I think we can call it a day.

(Refer Slide Time: 40:35)

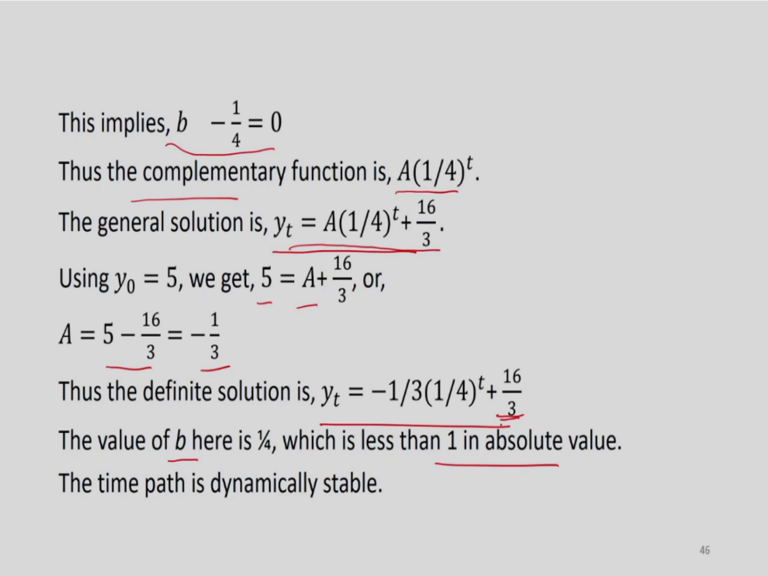


So, difference equations, here is the problem. Given the first order difference equation this, $y_{t+1} - \frac{1}{4}y_t = 4$ and $y_0 = 5$. Find the particular integral complementary function and the definite solution. Is the time path dynamically stable? So, we start with this difference equation, this is a first order difference equation, it is a non- homogeneous difference equation and the degree is 1.

So, first let us find out the particular integral, I will take the trial solution $y_t = k$, k is a constant and that will give me this, $k - 1/4k = 4$ because $y_t = k$ which is a constant, so $y_{t+1} = y_t = k$. So, that fact has been used here and if I simplify this $k = \frac{16}{3}$.

Second part is the complementary function, we again take this as the trial solution $y_t = Ab^t$ and put that into this equation only the homogeneous part. And therefore, I will get $Ab^{t+1} - \frac{1}{4}Ab^t = 0$.

(Refer Slide Time: 42:19)

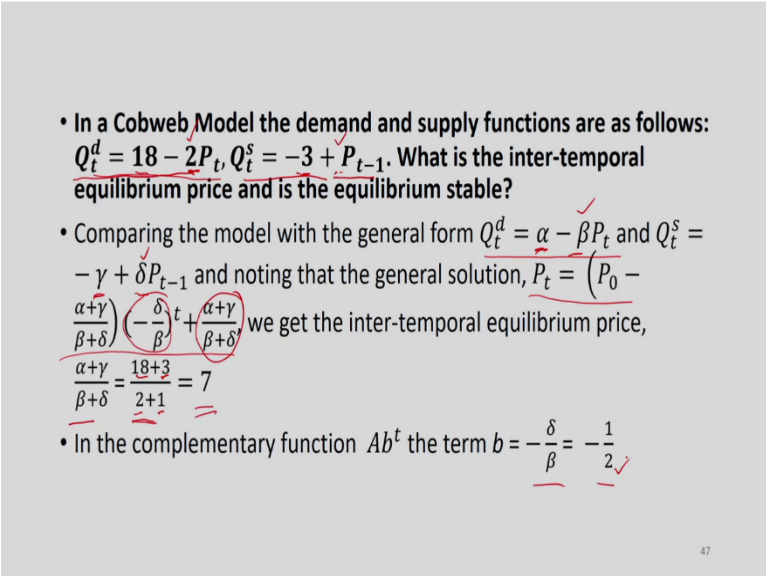


And this we have to solve for b, because a will get cancelled and actually, we are going to get $b - \frac{1}{4} = 0$. That means $b = \frac{1}{4}$. Therefore, the complementary function we had assumed this to be Ab^t, it is actually $A(\frac{1}{4})^t$ because $b = \frac{1}{4}$. Therefore, the general solution is this, $y_t = A(\frac{1}{4})^t + \frac{16}{3}$.

Now we use the fact that at $t = 0, y_0 = 5, y_0 = 5$ means if you put $t = 0$ $y = 5$. So, that is what we have done here. We have put $t = 0$, so that will give me $5 = A + \frac{16}{3}$ or $A = 5 - \frac{16}{3} = -\frac{1}{3}$. So, I put this in this form and I will get this, $y_t = -1/3(\frac{1}{4})^t + \frac{16}{3}$.

Last part was whether the time path is dynamically stable. Now in this case $b = 1/4$ and $1/4 < 1$, in absolute terms and that basically means that the path is dynamically stable, it will converge to the particular integral $\frac{16}{3}$ over time. So, that is the answer.

(Refer Slide Time: 44:09)



Here is another application of difference equations in Cobweb Model. In a Cobweb Model the demand and supply functions are as follows, this is the demand function $Q_t^d = 18 - 2P_t$, p is the price q is the quantity, d stands for demand. And the second is the supply function $Q_t^s = -3 + P_{t-1}$. As you can see that here I have P_{t-1} there is a lag of one period.

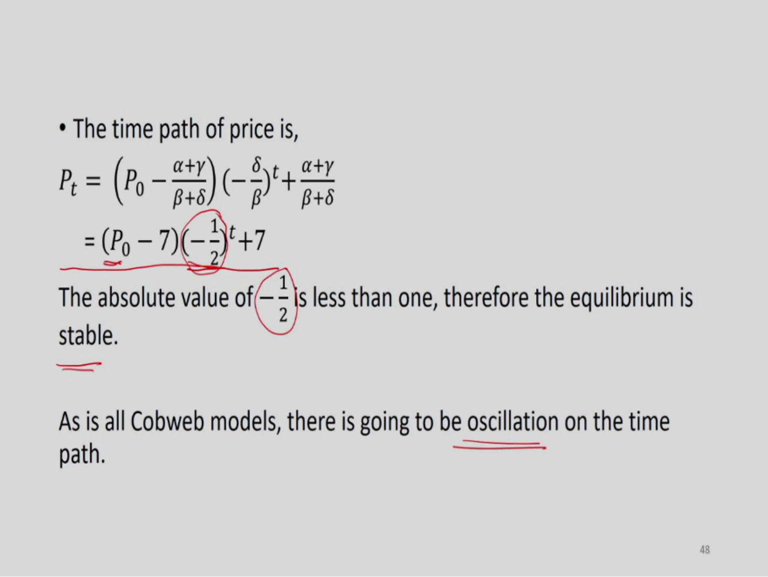
What is the intertemporal equilibrium price and is the equilibrium stable? So, these are the two questions. Now we compared these functions demand and supply functions with the standard model. The standard model was this: the demand function $Q_t^d = \alpha - \beta P_t$ and the supply function was $Q_t^s = -\gamma + \delta P_{t-1}$.

So, we can immediately see what are the α, β, γ, δ in this particular set of equations. Now we know that for the standard model, the general solution or the time path of price is given by this, $P_t = (P_0 - \frac{\alpha+\gamma}{\beta+\delta})(-\frac{\delta}{\beta})^t + \frac{\alpha+\gamma}{\beta+\delta}$ where this is the particular integral or the intertemporal equilibrium price.

So that we can find out immediately by putting the α and γ β and δ here. α here is what? $\alpha = 18$, $\gamma = 3$, $\beta = 2$, and $\delta = 1$. So, this will simplify as 21/3 which is 7. So, the particular integral has been found out.

What about the complementary function? In this case this b is very important, this is the b term, this b here is $-\frac{\delta}{\beta}$ as we have just seen $-\frac{\delta}{\beta} =- 1/2$ because $\delta = 1$, and $\beta = 2$. So, 2 is coming here so it is $- 1/2$, the b term here is $- 1/2$.

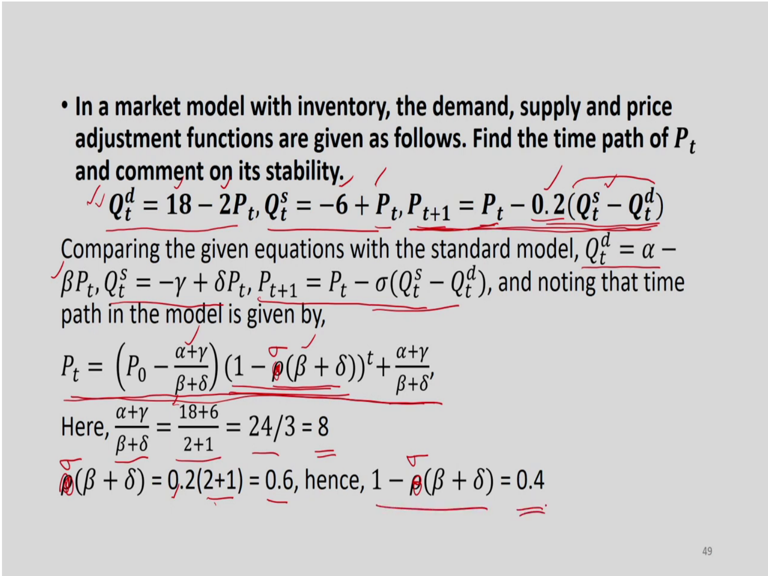
(Refer Slide Time: 47:39)



So, the time path is in short given by this, $P_t = (P_0 - 7)(- 1/2)^t + 7$. P_0 remember is not known to us, it is a constant it is the initial price so unless we have some information of what the initial price was at time is equal to $t = 0$ we cannot say anything about P_0. So, but that does not hinder us from saying something about the stability, because the stability depends on this.

The absolute value of $- 1/2$ is actually $1/2 < 1$, therefore, the equilibrium is stable. So, the stability is there and as we know in all Cobweb Models, this b term is always negative as long as the demand function is downward sloping, supply function is upward rising, the p will always be negative that means there will be a price oscillation over time. So, oscillation is going to be there but as we can see the value of b is less than 1 in absolute terms so therefore, the equilibrium is stable.

(Refer Slide Time: 49:09)



In a market model with inventory the demand supply and price adjustment functions are given as follows, find the time path of P_t and comment on its stability. So, this is the other application of the first order difference equation that we discussed in this course, this is called the market model with inventory.

Demand function is given $Q_t^d = 18 - 2P_t$, the supply function is $Q_t^s = -6 + P_t$, mind you unlike the Cobweb Model there is no lag here as far as the supply function is concerned. And finally there is a price adjustment function $P_{t+1} = P_t - 0.2(Q_t^s - Q_t^d)$.

So, just to recall, to understand the logic of this model, let's refresh our memory. In the inventory model there is no automatic clearance in the market. So, that was there in the Cobweb Model, that what happens to the price depends on the market demand and market supply and the price adjusts automatically according to whether there is excess demand or excess supply in the market.

That was determined by the market mechanism, here the market is not competitive in this model, the inventory model. Here the producers actually have some discretion over the price, they control the price and the price adjustment they do, is given by this. So, the price in the next period that is P_{t+1} will depend on the price of this period but it also depends on the access supply in this period.

So, this is the excess supply $Q_t^s - Q_t^d$ and this is getting multiplied with -0.2. So, if you have excess supply in this period then in the next period the price is going to be adjusted downwards compared to this period's price and vice versa, if you have excess demand in this period this will be negative and negative minus negative is positive, so in the next period the price will be higher than this periods price.

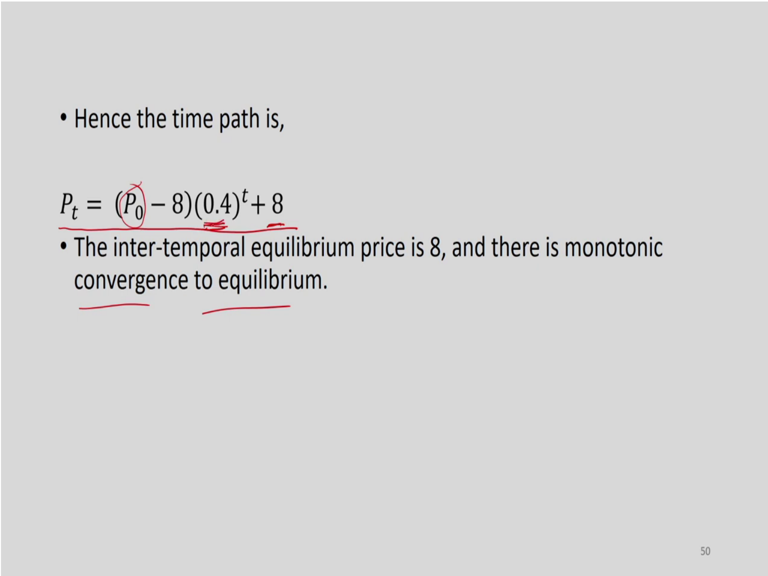
So, there is an element of time involved here that is why we have difference equations to take care of that. Again, what we are going to do in this problem is to compare this set of equations with the standard model. Well what was the standard model? Standard model was $Q_t^d = \alpha - \beta P_t$ and $Q_t^s = -\gamma + \delta P_t$ and $P_{t+1} = P_t - \sigma(Q_t^s - Q_t^d)$.

These were the notations and we can just compare these equations or these functions with these equations and functions and we can figure out what are the α, β, γ, δ, σ in this particular given set of functions. The time path in the standard model was given by this equation $P_t = (P_0 - \frac{\alpha+\gamma}{\beta+\delta})[1 - \sigma(\beta + \delta)]^t + \frac{\alpha+\gamma}{\beta+\delta}$. So, this was the time path.

So, I just have to figure out this value here and this value here and I can substitute those things in this function on this time path. That is going to be our strategy. So, in this case $\frac{\alpha+\gamma}{\beta+\delta}$, $\alpha = 18$ $\gamma = 6$, $\beta = 2$ and $\delta = 1$. So, I put everything here, $\frac{24}{3} = 8$ and think about this term $\sigma = 0.2$ and $\beta + \delta = 2 + 1 = 3$.

So, $(0.2).3 = 0.6$and remember we have to figure out what is $1 - \sigma$. So, this is sigma, so we have to figure out what is $1 - \sigma(\beta + \delta)$ that is $1 - 0.6$, so, that is 0.4. So, I will have to substitute 0.4 here. And this also we have found out this is 8.

(Refer Slide Time: 55:20)



So, therefore the time path is given by this $P_t = (P_0 - 8)(0.4)^t + 8$. 8 here is the particular integral or the intertemporal equilibrium price. And this 0.4 is very critical; this is that b term in the complementary function. Once again, I do not know the P_0 here that is the price at point $t = 0$.

So, that is the arbitrariness but as far as stability is concerned, I can say something about that. Here it is going to be a stable convergence to equilibrium. There is going to be stability and there is convergence to the equilibrium and the convergence is not kind of oscillating convergence because $0.4 > 0$, there is going to be a monotony convergence to the equilibrium price, the equilibrium prices point is 8.

So, that is the answer. So, I think with that I will conclude this series of tutorials that we have been discussing in the last few lectures. And I thank you once again for joining me and all the best. Thank you.