**Mathematics and Economics – 1 Sets and Set Operations**
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**Lecture 4: Set operations**

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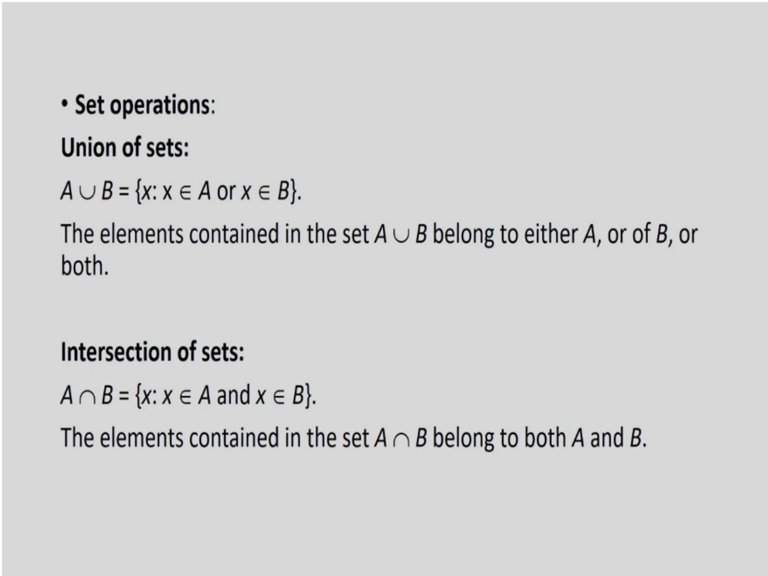
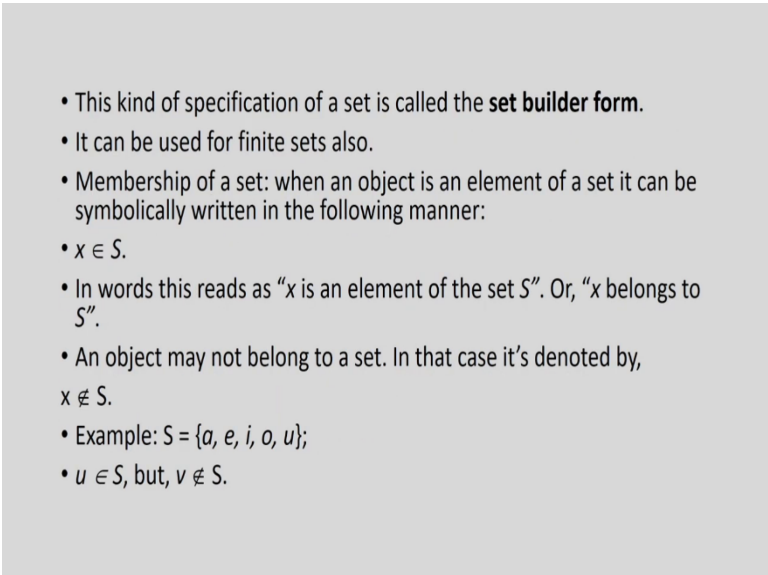


- **Subsets**:
- Suppose there are two sets *A* and *B*, such that every element of *A* is also an element of *B*. Then *A* is called a subset of *B*.
- Example, $A = \{x, y, z\}$, $B = \{w, x, y, z\}$
- $A \subseteq B \leftrightarrow [x \in A \rightarrow x \in B]$
- Is a subset a smaller set? Not always, because the two sets may be equal.
- If they are not equal, then *A* is called a **proper subset** of *B*.
- $A \subset B$ (*A* is a proper subset of *B*)

**Minus of sets:**

Contains the elements of the first set, but not of the second set.

$A \setminus B = \{x: x \in A \text{ and } x \notin B\}$

**Example:**

$A = \{a, b, c, d\}$, $B = \{d, e, f\}$ then,

$A \cup B = \{a, b, c, d, e, f\}$

$A \cap B = \{d\}$

$A \setminus B = \{a, b, c\}$, $B \setminus A = \{e, f\}$

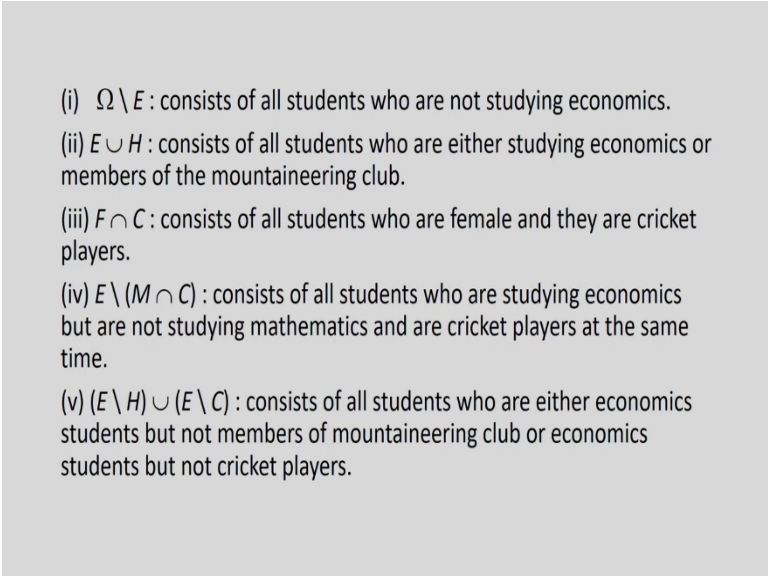
- **Example:**
- Suppose *A* is the set of positive even numbers, *B* is the set of positive prime numbers. What is $A \cap B$ ?
- $A = \{2, 4, 6, ....\}$
- $B = \{2, 3, 5, 7, 11, ...\}$
- $A \cap B = \{2\}$, it has a single element. This is because all other elements of *B* cannot be elements of *A*. By belonging to *B* they must not be divisible by 2, whereas all elements of *A* are divisible by 2.

- **Example:**
- In a village 50 people don't own a bicycle, 25 don't have a pucca house, 10 have neither.
- *A* and *B* denote the sets of people who don't own a bicycle and don't own a pucca house respectively.
- $A \cap B$ is the set of people who own neither a bicycle nor a pucca house.
- $A \cap B$ has 10 elements or people

- $A \cup B$ = set of people who either don't own a bicyle or don't have a pucca house. The set $A \cap B$ is a subset this set.
- $A \cup B$ has 65 elements/people.
- $A \setminus B$ = set of people who don't own a bicycle, and own a pucca house.
- It has 40 people.

- When two sets have no common elements, these sets are called **disjoint sets**.
- We write, $A \cap B = \varnothing$, where $\varnothing$ denotes the empty set, the empty set has no elements.

- In the above example, the set of all people of the village can be thought of as *the source* of all sets.
- Sets of different descriptions such as people without bicycles, without pucca houses, with tube or without tube wells, with or without TV sets, etc. are subsets of this big set.
- This big set is called the **universal set**, often denoted by $\Omega$.




Good afternoon everyone. So, this is the fourth lecture of this course called Mathematics for Economics part 1. So, what we have been doing in the last few lectures, I am giving you the preliminaries of this course. In the last lecture, we started with this topic called Sets and Set Operations. So, I have defined what is a set and I have defined what is meant by an element of a set.

And then we talked about set operations we talked about, for example, intersection of sets, union of sets, these are the things that we have done. We also talked about what are known as subsets, proper subset and general subsets. And we also discussed some examples.

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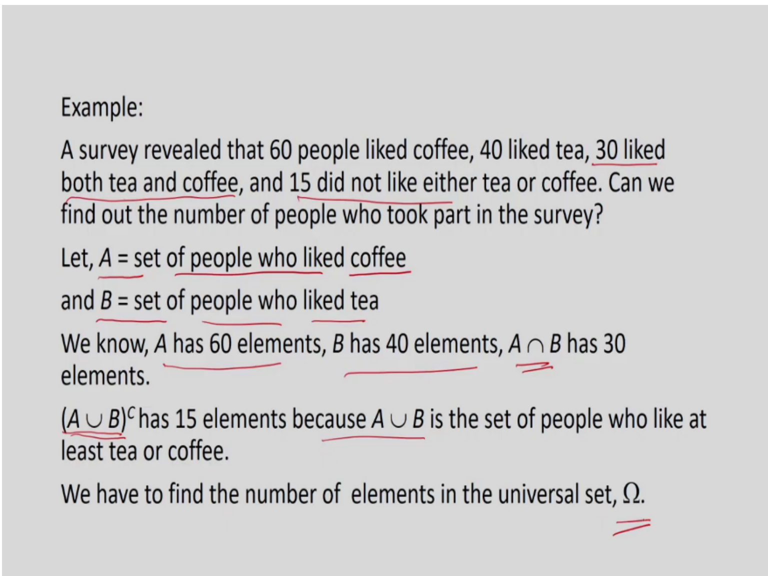

Now, I am going to talk about another example today. And then maybe we can talk about some other topic in today's lecture. So, this is an example, rather an exercise and we are going to solve this. A survey revealed that 60 people liked coffee, 40 liked tea, 30 liked both tea and coffee and 15 did not like either tea or coffee. Can we find out the number of people who took part in the survey? So, this is the question and let us see how we can try to solve this. So, first we define certain terms suppose A denotes the set of people who liked coffee.

So, here you have A: set of people who liked coffee in that survey. And B: the set of people who liked tea. And we further know from the information available in the question itself, that how many people liked coffee? 60 people liked coffee. So, A, which is the set of people who liked coffee has 60 elements. We further know that 40 people liked tea that means that this set B has 40 elements.

We also know that $A \cap B = 30$elements, why? Because it is given in the question that 30 liked both tea and coffee. So, when a person likes both tea and coffee, he belongs to in the set A because he likes tea, but he also belongs to the other set of B. So, he belongs to both the sets at the same time. And we say that he belongs to A intersection B. So, $A \cap B$ has 30 elements.

And we also know the last information that we have, is 15 did not like either tea or coffee. Now, what is this set we are talking about people who do not like either tea or coffee. So how do we represent this in terms of these notations. And we represent this by this $(A \cup B)^c$. So, we first construct this set A union B, and if we take the complement of that, then that set is the set of people who do not like either tea or coffee and so that set has 15 elements as given in the question.

So, $(A \cup B)^c$ has 15 elements because $A \cup B$ is the set of people who like at least tea or coffee? That is by definition, $A \cup B$ means the people who belong to that set at least likes tea or coffee. So, in the complement of that set will mean those people who do not like either. So that means that $(A \cup B)^c$ has 15 elements, we have to find out the number of elements in the universal set, Ω.

Remember, we defined what is an universal set in the last lecture, universal set is the set from which all these sets are coming, this is the kind of source. So, universal set in this case is the number of people who were surveyed. So, in that survey, some people said I like tea, some people say that I like coffee, some people said I like both and some people said, I do not like either. So, all of these put together, there all of them are coming from the set Ω. We know the number of elements in Ω is what?

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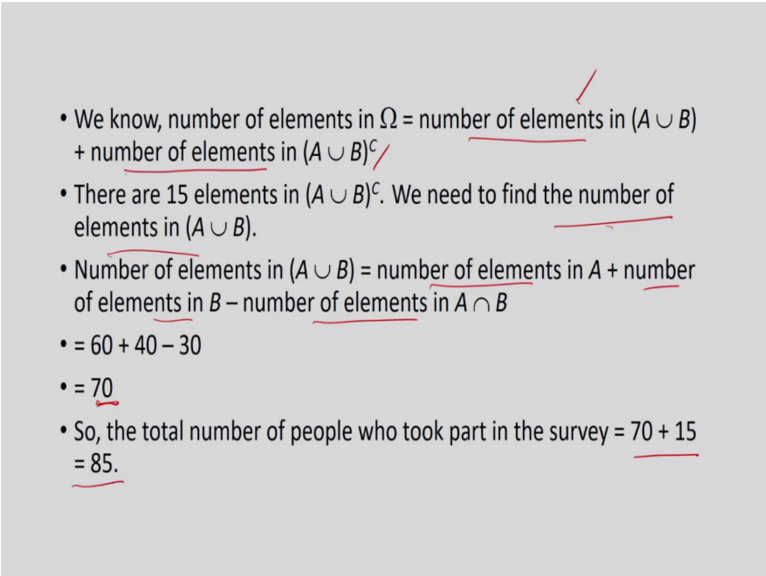


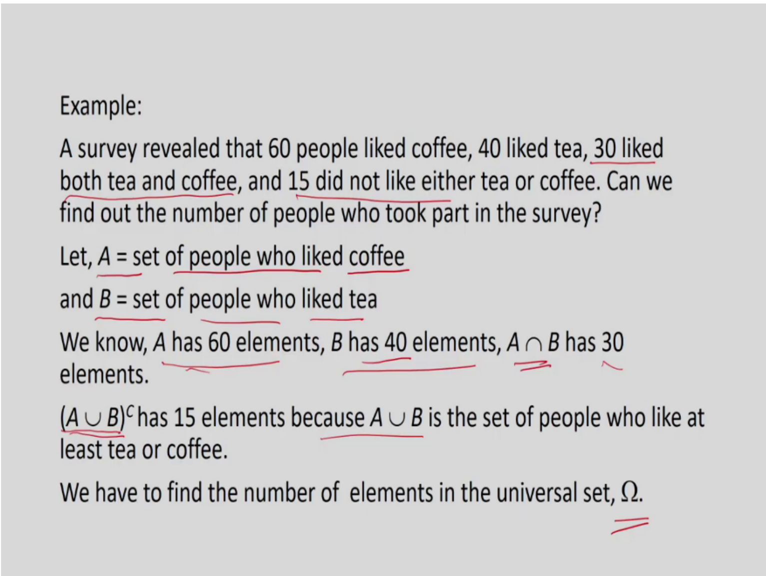


The number of elements in $A \cup B$ + the number of elements in $(A \cup B)^c$. So, if you take any set, which belongs to gamma, that is the universal set, and you take the complement of that set. And if you put them together, you get the universal set. So, that is the idea we are using here. What we know is that there are 15 elements in $(A \cup B)^c$, people who do not like either.

So, we know this number, the later one, if we can find out the number of elements in $A \cup B$ , then we are through. Therefore, we can find the number of elements in Ω. So, therefore, we need to find the number of elements in $A \cup B$. So, now, let us use again the old definition that we talked about in the last lecture, that how many elements are there in $A \cup B$?

So, number of elements in $A \cup B$ will be the number of elements in A plus the number of elements in B, then I have to subtract the number of elements who are in the intersection set that is $A \cap B$. So, I have to use this sort of identity and actually on the right-hand side, I know all these numbers, the number of elements in A is 60, it is written here, the number of elements in B is 40 and $A \cap B$ has 30 elements.

So, it will be 60 + 40 - 30. So, that is 70. So, this is the size of the $A \cup B$ set, so, we have got this also, we have got this also, now, we just have to add them up. So, 70 + 15. And so, the total number of people who took part in the survey is 70 + 15 is equal to 85. So, this is our answer. I do not want to spend more time on this discussion of sets and set operations.

There are plenty of examples in the book, the book that I have referred to is Sydsaeter and Hammond, Mathematics for Economic Analysis. So, there if you look up the appropriate chapter, you will find plenty of examples. So, it will be a good idea to try to solve those exercises. Some of the solutions are given in the book itself, the even number of questions. So those questions which have even, sorry, odd serial numbers, their solutions are given in the book itself, so it is very helpful for people who want to try out.

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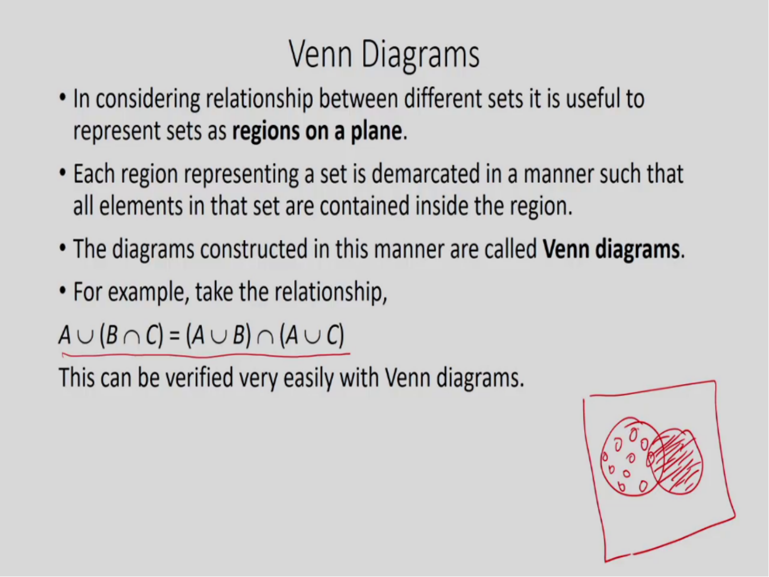

I now go to a new topic, which is called Venn diagrams. Now, actually, this topic of Venn diagrams, we talked about Venn diagrams in some previous discussion already, when we talked about for example, this Mathematical Logic. We talked about what is a necessary condition and what is a sufficient condition. So, there I use the idea of a Venn diagram. So, a set and a subset of that.

So, that we have seen but it was a more kind of intuitive kind of idea that we use now, we are going to introduce Venn diagram in a more rigorous manner. In considering relationship between different sets, it is useful to represent sets as regions on a plane. So, suppose this is the plane that I am talking about, this is itself a plane, two-dimensional plane. So, some regions could be thought of as those elements, consisting of those elements which belong to a set.

So, this circle that I have drawn here, this basically contains all the elements which belong to a set. So, I can draw another circle and elements within that circle will be contained in this set, the new set, etcetera. Each region representing a set is demarcated in a manner such that all elements in that set are contained inside that region. So, you have this region here all the elements of this set are inside this region this shaded region and you have this other set and all elements are inside this region.

So, this is just a visual way to represent sets. The diagrams constructed in this manner are called Venn diagrams. For example, take the relationship $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. So, this is a relationship between set operations, now it can be verified very easily with the help of Venn diagrams let us see how we can do that.

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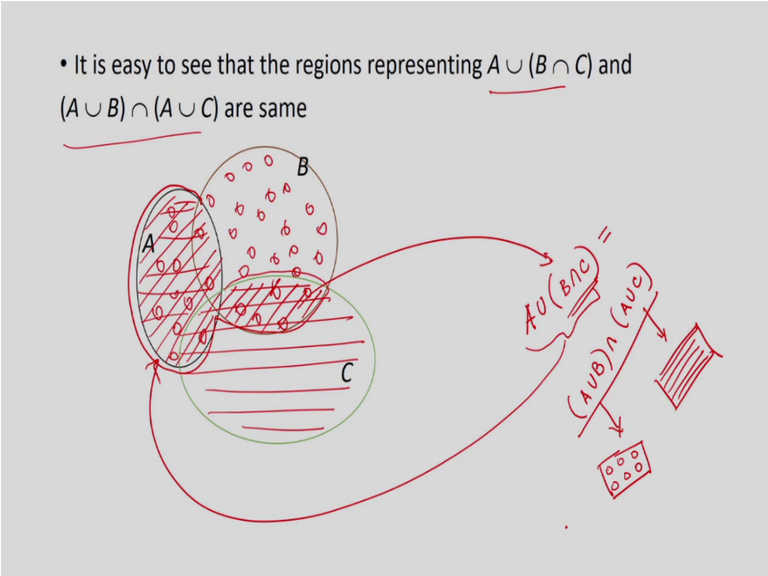


So, here I have drawn three circles one is representing A on the left and then you have another one B on the right and top and on right in bottom we have C. So, there are these three circles they represent three sets, any arbitrary three sets and I have for the benefit of our viewers, I have color coded them. So, A is black circle B is red and C is green and I have tried to draw the circle in such a way that there are intersections, you can see that within A and B there is an intersection.

So, they are overlapping the circles are overlapping. When the circles are overlapping, so, intuitively it means there is an intersection set A and B are not disjoint. So, I have intentionally done that the intersections such that the intersection set is non empty. Similarly, I have also tried to make sure that $A \cap B \cap C$is non empty. So, there are some elements which belong to all three sets, why did I do that?

I could have talked about non intersecting sets also, but that would not have been very interesting. So, now, we try to see whether this is equal to this let us try to see that. Here on the left-hand side you have $A \cup (B \cap C)$. So, since you have $(B \cap C)$, so let us first talk about that what is $B \cap C$, $B \cap C$ is the intersection set, that means it is the overlapping region.

So, we are talking about this one, this is $B \cap C$. So, here you have $B \cap C$ and what about $A \cup (B \cap C)$, so, that will be the set which will include this $B \cap C$ region, additionally it will also include the complete set of A. So, this one and then you have the entire A added to that. So, this is this red shaded region is what is B intersection C.

So, this is the left-hand side, mind you, now I have to prove that this shaded region that I have discovered on the left-hand side is actually the shaded region that will be represented by the right-hand side. So, on the right-hand side what do I have? I have $(A \cup B) \cap (A \cup C)$. So, let me see what is $A \cup B$. $A \cup B$ is this entire set. So, let me draw, let me use another shading strategy.

So, $A \cup B$ is this sign. So, this entire two circles put together A and B together including the intersection of A and B and what about $A \cup C$? So, let me use another shade which is kind of horizontal strokes. So, this will be what? This will be A and C put together. This is C and this is A, this and this together. Now, it is becoming clear what is then this entire thing, $(A \cup B) \cap (A \cup C)$, it is this horizontal shades intersection of these circles.

So, you actually are getting this part and this part that is the common because in these two parts only you have both the circles as well as the horizontal parts. But remember this is the also the portion which is on the left-hand side also. So, that we have found out before. So, therefore, we have proved that the LHS is equal to RHS by the help of the Venn diagrams.

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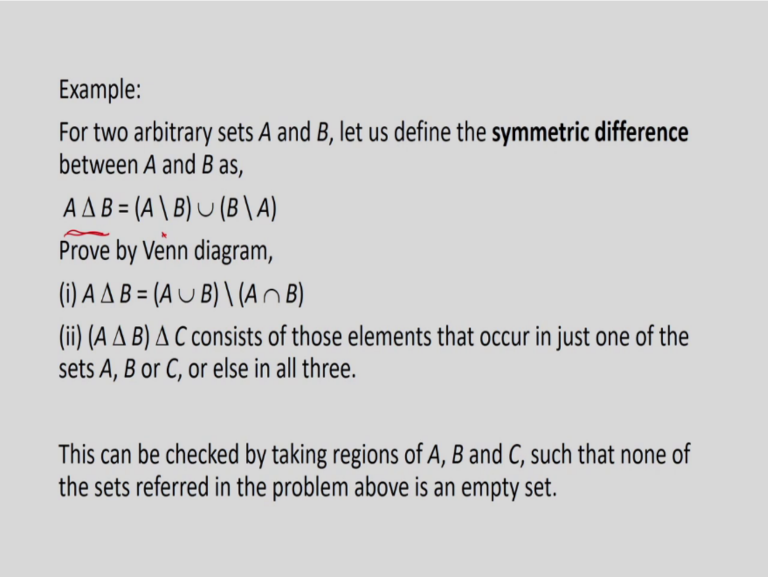


This is another example of use of Venn diagram. For two arbitrary sets A and B, let us define the symmetric difference between A and B as $A \Delta B$, a new symbol is being introduced which is delta, $A \Delta B$ is defined as what it is, $(A \setminus B) \cup (B \setminus A)$.

So, that is the definition of delta. Now, we have to prove with the help of Venn diagrams, two propositions, one is $A \Delta B = (A \cup B) \setminus (A \cap B)$, and second, $(A \Delta B) \Delta C$consists of those elements that occur in just one of the sets A, B or C or else in all three. So, let us try to check this. This can be checked by taking regions A, B and C such that none of the sets referred in the problem above is an empty set.

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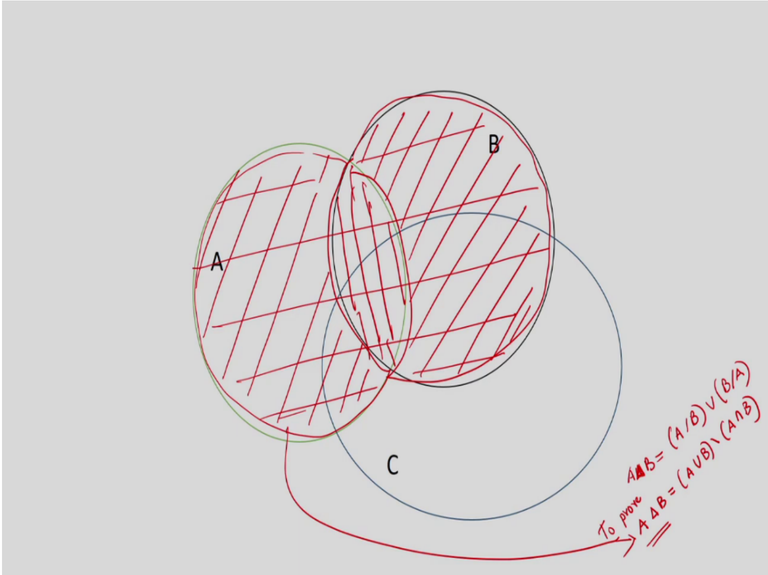


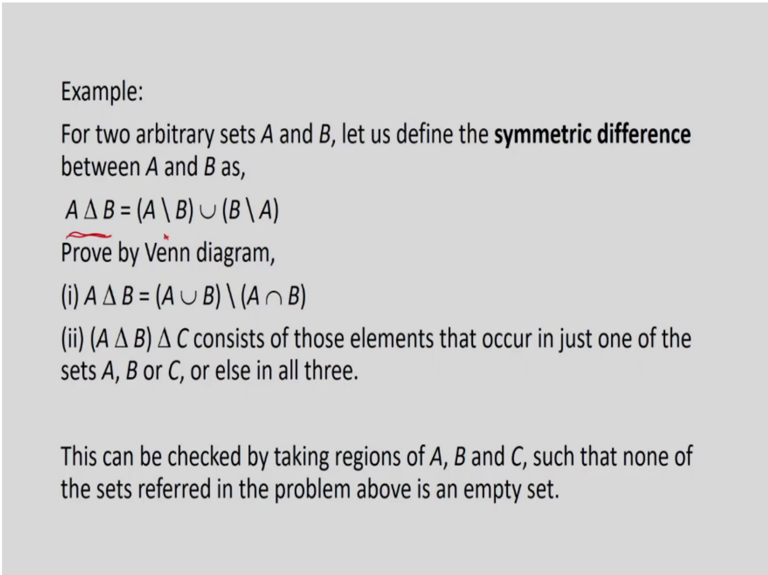


So, I am just going to make sure that this intersection sets are not empty sets. So, just let me write down the thing that we are supposed to prove. We are supposed to prove this $A \Delta B = (A \setminus B) \cup (B \setminus A)$, this we have to prove. But what is the definition of delta? Oh no sorry, I have just defined it $A \Delta B = (A \setminus B) \cup (B \setminus A)$, what do I have to prove? I have to prove something else, I have to prove that $A \Delta B = (A \cup B) \setminus (A \cap B)$.

So, let us try to prove this one and then we will go to the second question. What is A intersection sorry, $A \Delta B$on the left-hand side? If we go by the definition, we are using this definition it is A minus B. Now, what is $A \setminus B$? Let us suppose $A \setminus B$ is this shade that I am using here, this is $A \setminus B$, so, I am taking all the elements of A but I am not taking the elements of B.

So, this is $A \setminus B$ and what is $B \setminus A$? $B \setminus A$ means I am taking all the elements of B but I am not taking the elements of A, so, that will be this set. So, this shaded region that I have just demarcated on this plane is $A \, \Delta \, B$. So, this is the left-hand side, this is the left-hand side. Now, if I can show that this is exactly equal to the right-hand side then I am done. On the right-hand side what do I have?

On the right-hand side I have $(A \cup B) \setminus (A \cap B)$, what is $(A \cup B)$? $(A \cup B)$is the whole thing. So, this whole thing is A union B. And from this I have to take out this one, suppose I demarcate by this horizontal portion. So, from the horizontal portion, I am deducting the vertical portion. So, I will be left with only this one, this part and this part and that exactly what we have found on the left-hand side also. So, basically, we have shown that the left-hand side is equal to the right-hand side.

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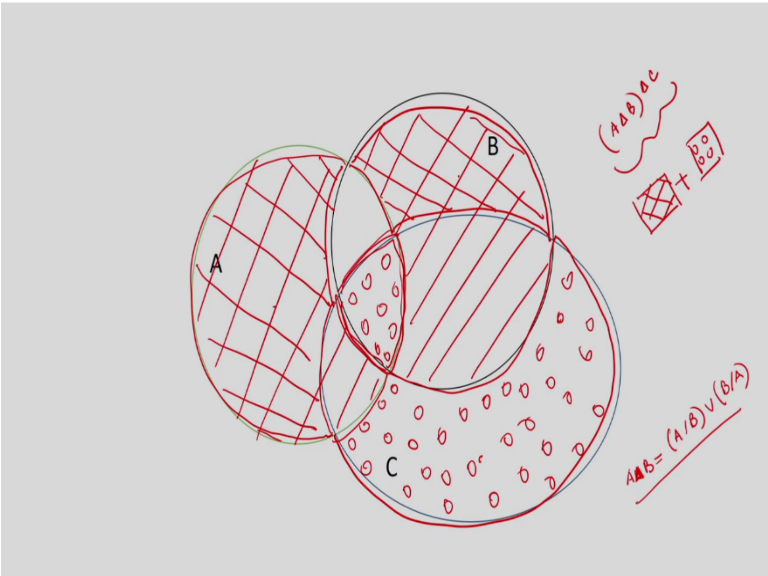

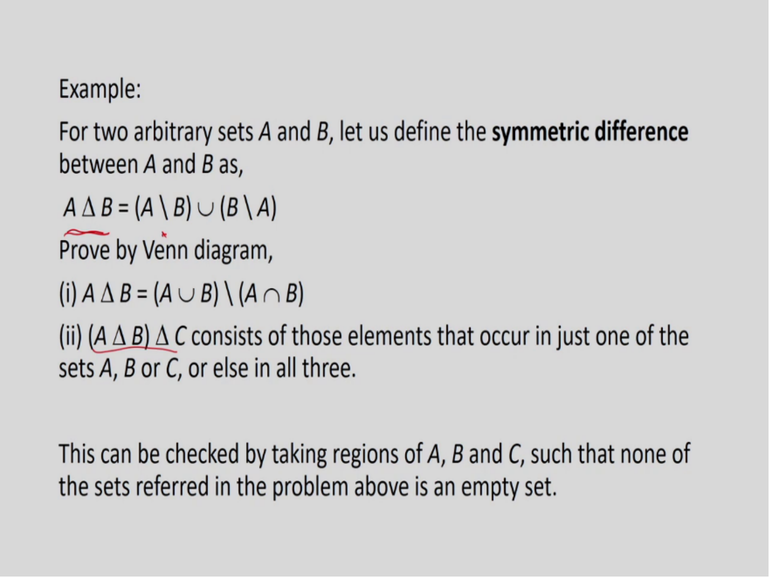


Now, I am going to take up the next one. This was the first question. The next thing we have to prove here is $(A \Delta B) \Delta C$consists of those elements that occur in just one of the sets A, B or C or else in all 3. So, $(A \Delta B) \Delta C$, this consists of elements in either A exclusively or either be exclusively or either C exclusively or they occur in all of them simultaneously.

So, that is what we have to show. First let us try to find out this region $(A \Delta B) \Delta C$. $(A \Delta B)$ actually we have already found out it was this region this was $(A \Delta B)$. Now, delta C, now, how do I understand $(A \Delta B) \Delta C$ this region this shaded region with that I have to do the delta C. So, if I use the definition here, so it will be this red shaded region minus C.

So, if I take out the elements which are there in C from this red region then what am I left with? I am left with this region. So, this is my $(A \Delta B) \setminus C$. So, this region is $(A \Delta B) \setminus C$ and then I have to do $C \setminus (A \Delta B)$ . So, from C, I have to take out the $(A \Delta B)$ region. So, if I do that what am I going to get? This belongs to C, this circle things, they belong to C, but they do not belong to $(A \Delta B)$.

Similarly this region also they belong to C but they do not belong to $(A \Delta B)$. So, this one this notation $(A \Delta B) \Delta C$ is what region? This is this region plus, this plus this. So, what is the region that we are talking about? So, if I just highlight that it will be this portion, this portion, and this portion and this portion, and this portion. So, it is actually a collection of four different regions.

And you can see the common property of these four regions is the following. They either belong to A, exclusively, or they belong to B exclusively, or they belong to C exclusively, or fourth possibility is that they belong to all of them simultaneously, A and B, and C, this is the grand intersection. So, we have proved what we were supposed to prove, because that is, that is written here, that $(A \Delta B) \Delta C$ consists of those elements that occur in just one of the sets A, B, or C, or else in all three.

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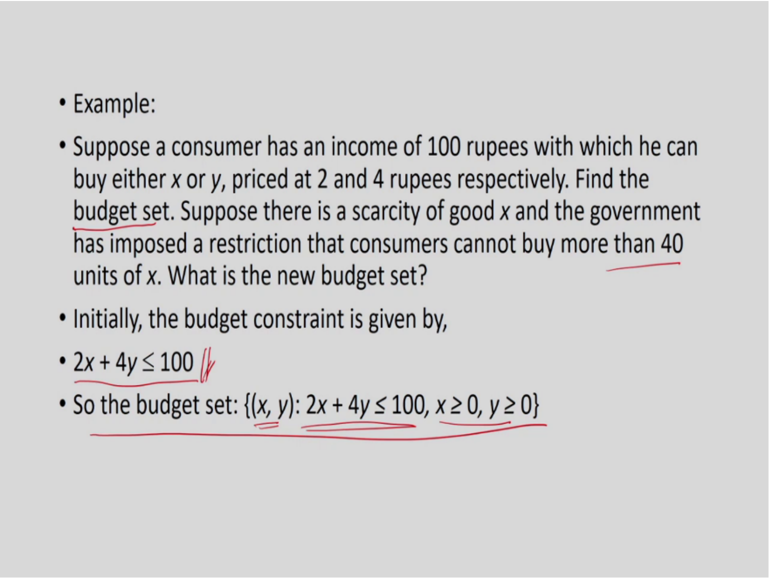


So, we have shown that in terms of our Venn diagram. This is the final thing that I am going to do this is not about Venn diagrams, but something which is related to sets, an intersection of sets. Suppose a consumer, this is an example from economics. Suppose the consumer has an income of 100 rupees with which he can buy either x or y. So, x and y are the two goods that he can buy with the 100 rupees that he has, and the prices of x and y are 2 and 4 rupees respectively.

So, price of x is 2 rupees, price y is 4 rupees, find the budget set. So, that is the question we have to find the budget set. In the last lecture, I have talked about what is a budget set, so I am going to use that definition. Suppose there is a scarcity of good x and the government has imposed a restriction that consumers cannot buy more than 40 units of x, then what is the new budget set?

So, there are two questions here. In the first part, the total income is given the prices are given, we have to mention the budget set. So, that is the first part and the second part is imposing a new thing, which is that the government has imposed a restriction that because of scarcity of good x, the consumers cannot buy more than 40 units of x, in that case, what is the new budget set?

Let us do the first one first. Initially the budget constraint, remember there was something called a budget constraint, what is a budget constraint? It basically tells you how much of x and y a person can buy with the money that he has. So, the maximum amount of x and y that he can buy. So, this is given by this budget constraint is given by $2x + 4y \leq 100$.

Why? Because on the left-hand side, you have the $2x + 4y$, $2x + 4y$ is the total amount of money the consumer is going to spend, if he buys x amount of the first good and y amount of the second good. So, in that case, the total expenditure will be $2x + 4y$, because prices are 2 and 4. Now, this expenditure must be less than or equal to 100 rupees the amount of money that he has, so this is the budget constraint.

Therefore, the budget set is this one, this set: $\{(x, y): 2x + 4y \leq 100,\ x \geq 0,\ y \geq 0\}$. And so, this set is basically a set where each element has two elements, a pair x and y such that this budget constant has to be satisfied $2x + 4y \leq 100$. At the same time, we need to satisfy this one that x and y cannot be negative quantities. They have to be non-negative. This was the first part.

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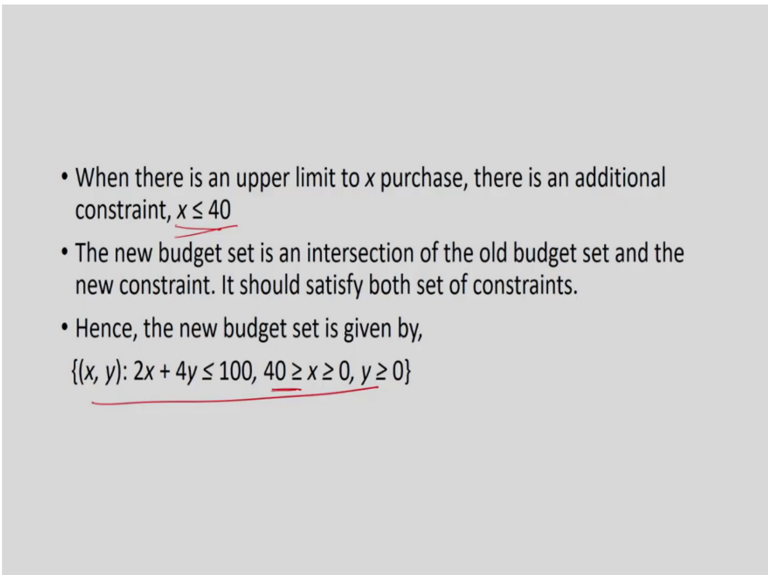

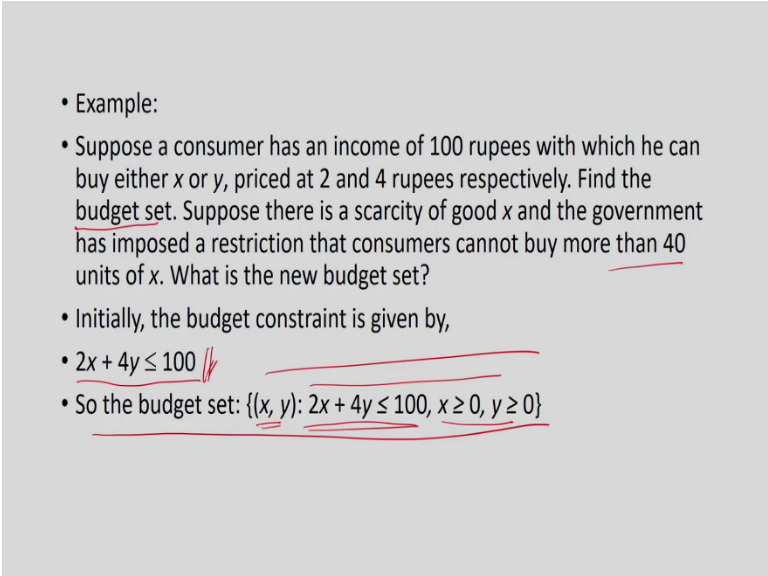


The second part, when there is an upper limit to x purchase, there is an additional constraint of $x \leq 40$. The total amount of x, a person can buy cannot exceed 40. So, it is bounded above by a certain number, which is 40. So, the new budget set is an intersection of the old budget set and the new constraint, and it should satisfy both sets of constraints.

So, this was the old budget set, this was the old budget set. But now, in addition to these constraints, which are written inside the budget set, you have an additional constraint, which is that x cannot exceed 40. So, we just include that inside here, $x \leq 40$. And we have got the new budget set. I guess I will stop here on the discussion of sets and sets operations.

And since we have some time in this lecture, I think I will just start the third module in this lecture itself, and maybe introduce the topics in the third module. So, from the next lecture, we will continue with this topic of the third module.

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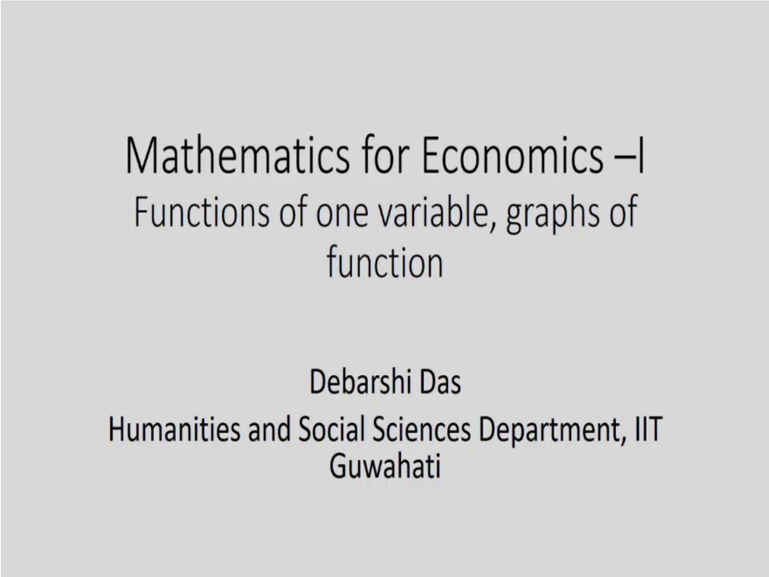


This is the third module and the topic of the third module is functions of one variable and graphs of function. So, this is going to be the topic of the third module, we are going to talk about functions, but with the important qualification that these functions that we are talking about are a single variable function. Now, what are functions?

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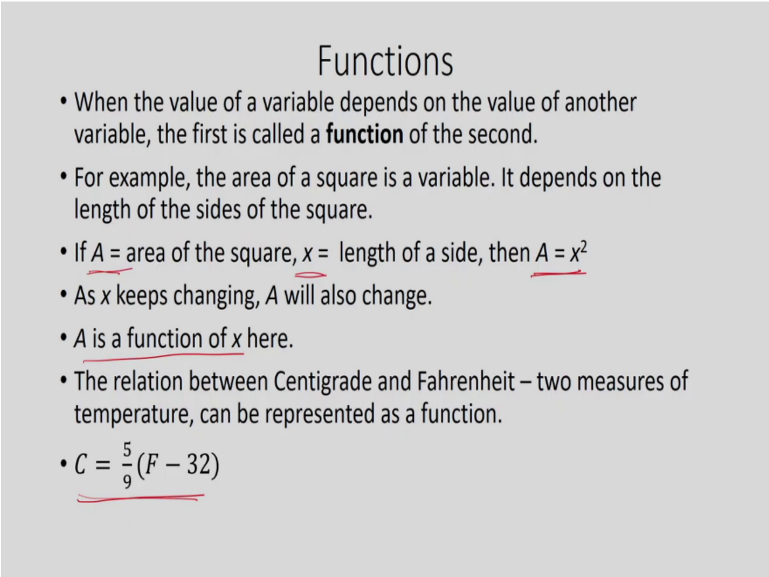


When the value of a variable depends on the value of another variable, the first is called a function of the second. So, here we are talking about real valued functions that I think I should mention in the beginning itself. There could be functions which are not real valued function, but set valued function, but here we are talking about real valued function. So, in the function what is happening is that there are two variables which are linked to each other and the value of the first variable depends on the value of another variable, the second variable and then we say the first variable is a function of the second.

For example, think about the area of a square. Now, area of a square is a variable because it can go on changing depending on what? Depending on the length of the sides of the square. So, suppose A = the area of any square and x = the length of a side of that square then what do we know is that $A = x^2$.

So, as x goes on changing, as the side of a square goes on, let us say rising then A will go on rising. And if x declines then A will also decline. So, in this case we have a function where A is a function of x. The relationship between centigrade and Fahrenheit, two measures of temperature, can be represented as a function. So, here is the relationship that suppose C denotes temperature in the unit of centigrades then centigrades we know, $C = \frac{5}{9}(F - 32)$.

So, here both C and F are measuring the temperature of any place let us suppose, and as F goes on changing, then C will also go on changing by obeying this rule. So, in that sense we can say that C is a function of F.

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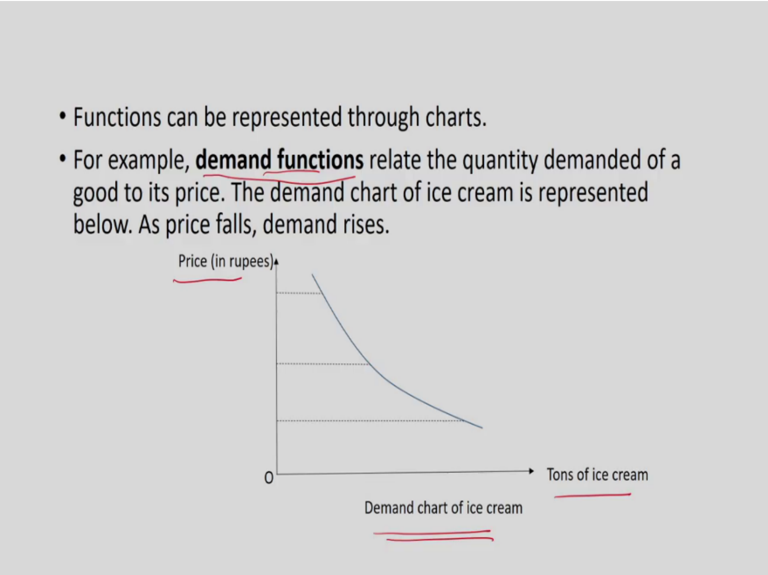


Functions can also be represented through charts. So, previously we represented the functions through some mathematical formula. This is a mathematical formula. This is another mathematical formula, but it is not necessary that the function should be represented only by formula. They can be represented through charts as well. So, if I take an example from economics, here is an example which is called a demand function.

So, what are demand functions? Demand functions relate the quantity demanded of a good to its price. So, quantity demanded is a function of the price of the same good. By price what I mean is the price per unit. For example, if you want to buy cold drinks for example, so, if you want to buy let us suppose Thums Up or Coca Cola then buy per bottle you have to pay let us say suppose 20 rupees.

So, that 20 rupees is the price and demand depends on price, it depends on many other things, but it also depends on price. So, there is this thing called demand function which represents how the quantity demanded depends on the price. So, this can be represented in terms of a chart. So, here is the demand chart of ice cream let us suppose. So, demand chart of ice cream is represented below and generally what happens is that as price falls of a good then people buy more of that good, people as a whole.

Let us say in the entire market for ice creams, people buy more of that good if the price declines. So, that is what it is shown here. One thing to note here in this diagram that I have drawn is that I have represented price along the vertical axis, y axis and I have represented quantity, quantity of ice creams of ice cream along the horizontal axis.

And this might seem a little bit awkward, because generally when I say that quantity demanded of ice cream is a function of price, I mean that quantity demanded is a dependent variable it depends on something else, which is price and the thing that is dependent is represented generally along the y axis, but here it is just the reverse. Here price is represented along the vertical axis instead of the quantity.

But this is the convention generally, which is generally followed in economics, that when one draws the demand function or what is known as a supply function, which maybe we shall talk about in the course of this series of lectures. So, demand function and supply functions when they are represented through charts, the price is taken along the vertical axis not along the horizontal axis. So, the quantity is represented along the horizontal axis. So, this is an example of a function demand function. So, here is the more precise definition.

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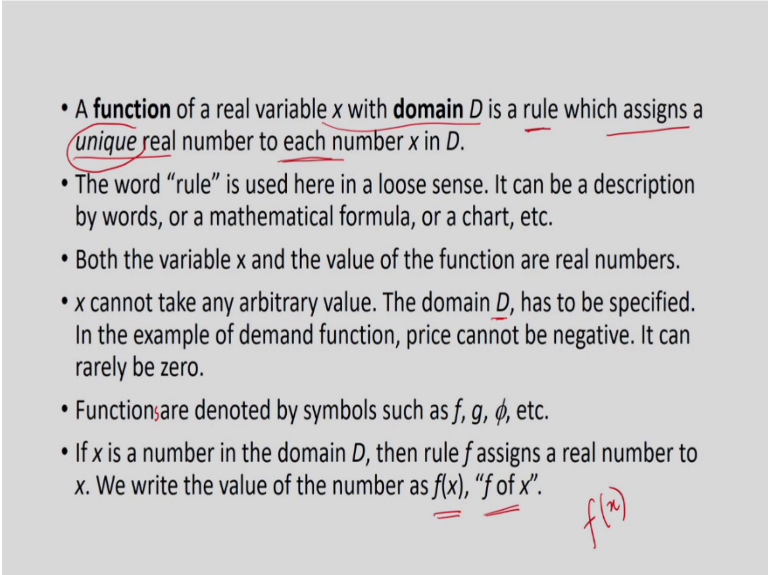


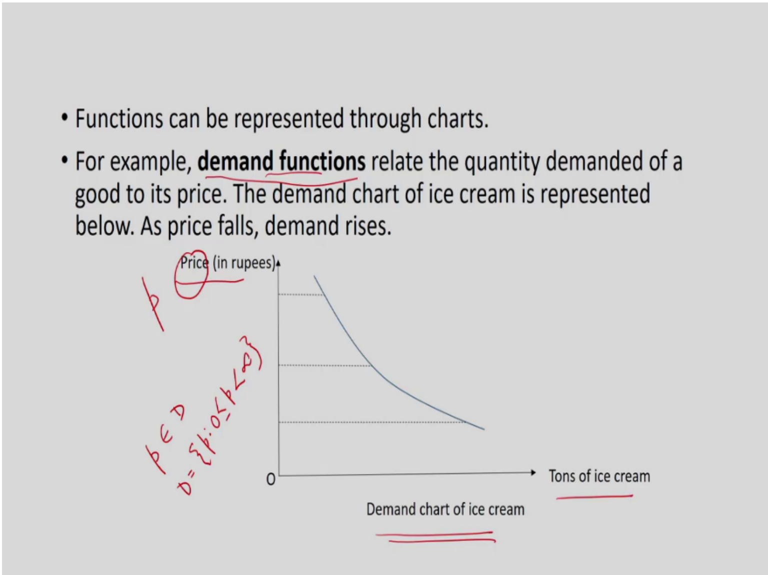


A function of a real variable x with domain D is a rule which assigns a unique real number to each number x in D. So, this is the precise definition. And there are some important terms that have been used here. First is this word rule. I am going to come to this word, domain later on. But let us first look at this word, rule. The word rule used here is in a loose sense, it can be a description by words or a mathematical formula or a chart, etcetera.

So, we have seen two examples of how the rule is specified. So, in the first slide, I told you about the mathematical formula $A = x^2$. So that is the mathematical formula. So that is representing a function. But then, we also talked about the fact that the rule can be also represented by a chart. And it is also mentioned here that it can be through words simple verbally, you can just describe how one variable is dependent on another variable, that is how the rule is specified.

Both the variable x and the value of the function are real numbers. x cannot take any arbitrary value the domain D has to be specified. So, here is the relevance of the domain. So, I said that x is something which is changing, which may change and that affects the value of the function, but x cannot take any arbitrary value. Firstly, obviously, it has to take real values, but more than that it has to belong to a domain and this domain is denoted by this, generally denoted by this letter D.

So, this is basically the set from which the X can belong. In the example of demand function the price cannot be negative it can rarely be 0. So, the p if I represent this price by p. So, suppose this is small p, this p presents the price, then price has to belong to a particular domain. What is this domain? In this case the domain is the set of positive numbers maybe it includes 0 also$p \in D,\ D = \{p\colon\ 0 \leq p < \infty\}$.

So, it. So, it cannot be negative that is the important thing because prices are generally not negative and it is even quite rare to have a price which is equal to 0. So, here we have an example of a domain. Functions are denoted by symbols such as small f, small g, phi (ϕ), etcetera. But in some cases capital letters are also used, but small letters are generally used to denote functions.

If x is a number in the domain D, then the rule f assigns a real number to x. So, this is mentioned here itself, it assigns a unique real number that is also important I have used the italics here to emphasize the fact that f cannot give you two values with respect to a particular value of x, two or three values, it has to give a unique real value for any real value of x.

And another thing to note in the definition is each that for every element of D, the function should give you or this rule should give you some value some unique value, we write the value of the number as f(x) or f of x. So, f of x is basically that value, which this rule will generate and it has to be unique for any given x.

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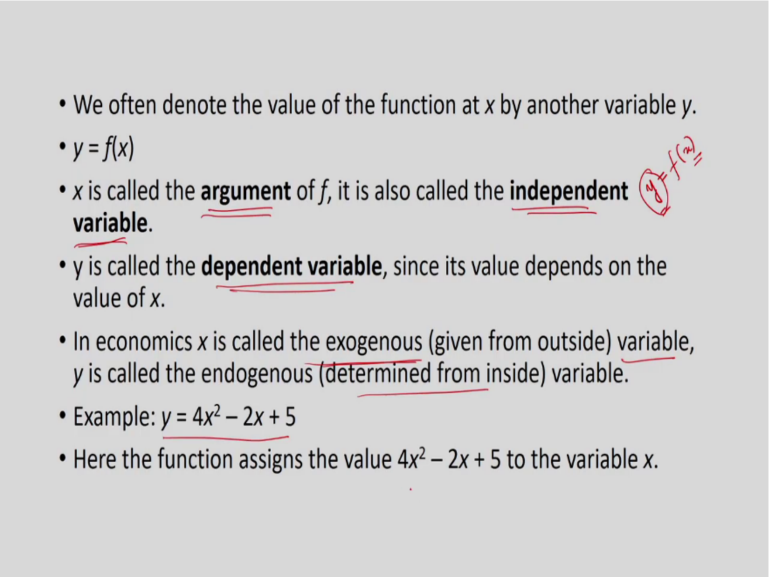


We often denote the value of the function at x by another variable y. So, sometimes since f could be quite clumsy, so, sometimes y is used as a symbol of the value of the function. So, y is equal to f of x. So, f of x is the rule, this rule is applied on this domain, D for each element x in D, you can apply that rule and after applying the rule, you get a set of numbers and that set of numbers, any element of that set is represented by this small y.

x is called the argument of f, it is also called the independent variable. I have already used this phrase independent variable. So, the thing that is moving freely, x is moving freely, obviously, it has to be within the domain, but it has some mobility within the domain it can take different values within the domain. So, that is why it is called the independent variable and sometimes it is called the argument of this function of this particular function f.

On the other hand, there is this other variable that we have generated to the function this y, y is called the dependent variable. Why because, why is it called the dependent variable? Because it depends on the value of x. So, it does not have any freedom of its own, it has to follow the value of x given the rule of f. In economics x is sometimes called the exogenous variable.

Exogenous means it is coming from outside. So, it is given from outside and it can change according to the environment outside and y is called the endogenous variable which is determined from inside, determined by the function. So, it is determined inside. So, y is called the endogenous variable x is called the exogenous variable, they are also called independent and dependent even in economics. So, here is an example $y = 4x^2 - 2x + 5$. Here the function assigns the value $4x^2 - 2x + 5$ to the variable x. So, this is the rule.

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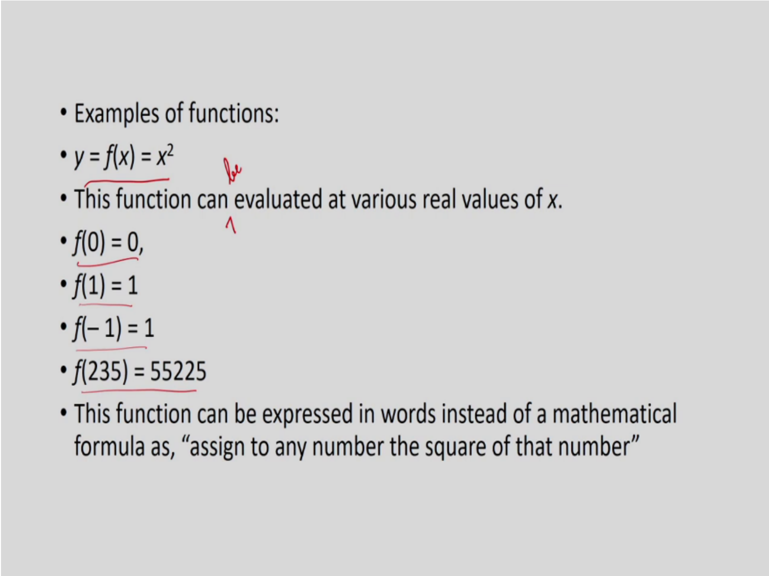


And you have another example here: $y = f(x) = x^2$. This function can be evaluated at various real values of x, you can evaluate at $x = 0$, that is what we have done here. So, f of 0 is equal to what? It will be 0 square, 0 square is 0 and it can be evaluated at 1, 1 square is 1. If you put $x =- 1$ it becomes plus 1. If we put any arbitrary value of x, let us say 235 it becomes a big number 55225.

This function can be expressed in words, I mean, this is mathematical formula, but it can be expressed alternatively in words as assigned to any number the square of that number. So, you take any number, x is that typical number, to that number you assign the square of that same number. So, that is how the rule can be specified by words. And let us take an example from economics.

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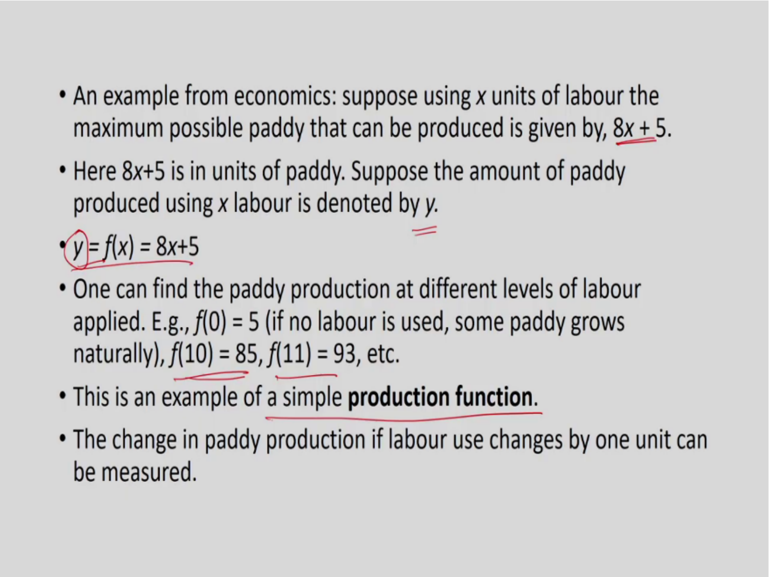


Suppose using x units of labour, the maximum possible paddy that can be produced is given by $8x + 5$. Here $8x + 5$is in units of paddy. So, this is the amount of paddy so it must be in the units of paddy. Suppose the amount of paddy produced using x labour is denoted by y. So, we are writing this $8x + 5$is equal to f of x is a function of x and suppose that is equal to y, y is the value of that amount of paddy.

So, we have introduced a new variable which is y, y is a function of x. One can find the paddy production at different levels of labour applied. So, for example, if you take f of 0 is equal, it becomes what? So, it will be 0 + 5 is equal to 5. So, how do I interpret this f(0)=5?

The interpretation is that if no labour is used, some paddy grows naturally no labour is used because x is the amount of labour if $x = 0$, so, no labour is being used, but even then what the function is telling me is that the value of $y = 5$, that still some paddy is grown naturally.

But if you include some amount of labour, positive amount of labour let us say $x = 10$. So, we just put that value here. So, it will be $5 + 8 * 10 = 85$. So, that basically means that if 10 units of labour are used, then 85 units of paddy can be produced. Similarly, f of 11 is equal to 93. So, as you are increasing the total amount of labour in the production, maybe you are employing more labourers in the field.

Then the output of paddy that you can grow it is rising. This is an example of a simple production function. So, I have introduced a new term called a production function. What happens in a production function? The value of the function gives you the output. So, y is the output, here we are talking about paddy output, but we could have talked about some other kind of output for example, industrial production.

So, that will be like how many cars are being produced in a factory or how many fans, electrical fans are being produced in a factory. So, that will be your y variable. And inside the function that is the argument, argument could be labour or something else could be there. The change in y production if labour use changes by one unit can be measured. So, we are going to end today's lecture here because time is over. So, we are going to pick up this thread in the next lecture. Thank you.