Introduction to Logic
Prof. A.V.Ravishankar Sarma.
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 15
Nature and Scope of Propositional Logic

Welcome back. In the last few lectures, we have presented basic concepts. And then we also presented traditional logic which has reminded as paradigm in logics, for more than 2000 years, that is, Aristotelian logics. In fact, Aristotelian logics; needs to be presented while studying the predicate logics, in fact, they are closer to predicate logics and the preposition logics. We are going to study right now. So, what will be doing now is; will be studying the basic concepts of propositional logic. So, as a name suggests, propositional logic is a study of is a logic of propositions.

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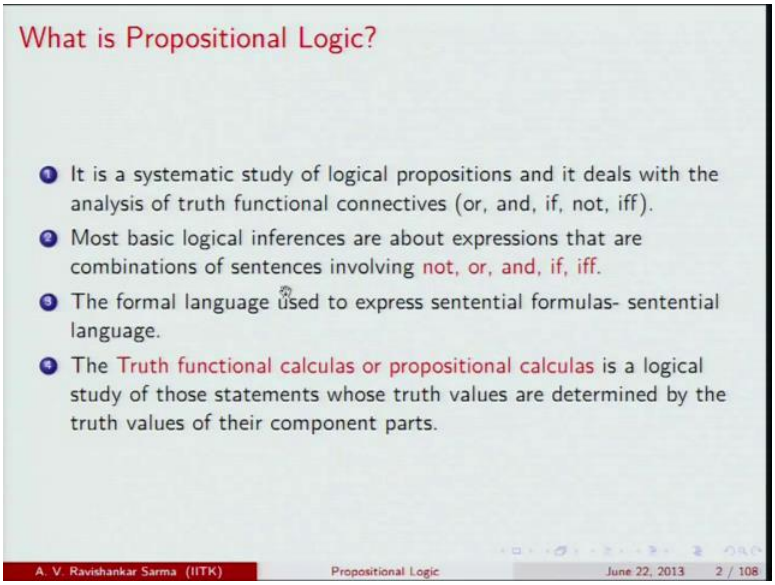

And then in this a lecture; what will be doing is; will be talking about what we mean by propositional logic, basic introduction of propositional logic.

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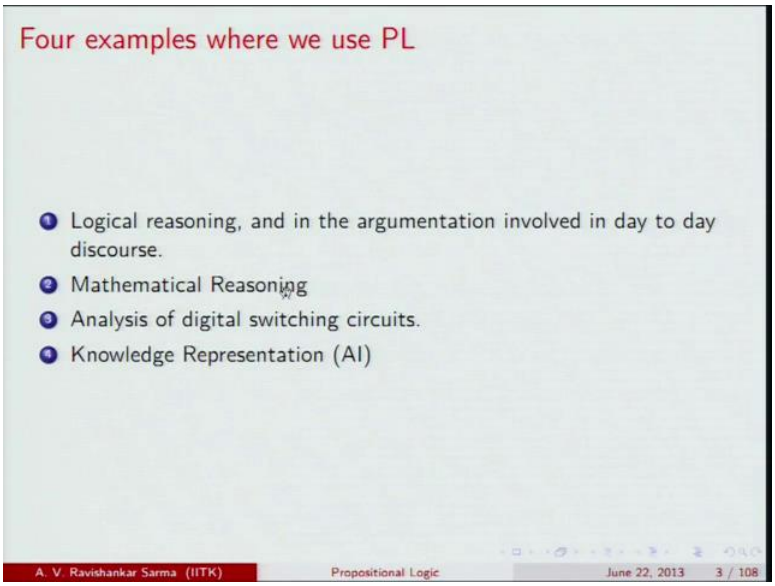

And we try to cover the syntax of a propositional logic.

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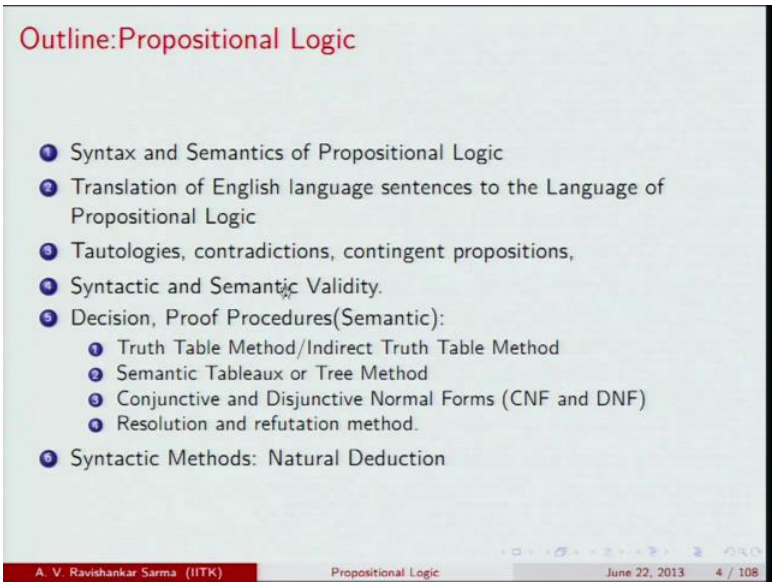

So, in this coming lecture, forth coming lecture, what will be covering is like this. We will start with a syntax and semantics of propositional logic. So, any language has its own syntax and semantics. So, in the same way language proposition logic is also viewed as a language; which as it is own syntax on, just like our English language has its own syntax and semantics. And then we will talk about how this is syntax and semantics are related to each other.

So, just like that, in propositional logic is also a kind of language, which has its own syntax and semantics. And we will be talking about in this lecturer, will we focusing on syntax part alone So, later we will be studying a some of the translations of English language sentences into the language of propositional logic, that is what we have we will be studying from the next few lectures. And then we will be studying 3 important kinds of statement, which occurring propositional logic. They are the statements which are always true, which are consider to be tautologies and the statements which are consider to be always false, they are consider to be contradictions. And the statement which are neither either true or false, are consider to be contingent kind of propositions enough.

So, will classify our group of statements into 3 categories and then we will only be interested in tautologies because, it is so happen that, all the tautologies in our language thus is the language of proposition logic, they are all turned out to be valid formulas. So, ultimately our journey begins with constructing some kind of well form formulas. So, that is what syntax will take care of it. And then after constructing basic formulas etcetera an all, then what you mean by these formulas and all. That means, will start giving some kind of truth conditions to a given formula; that means, meaning of a given formula means, giving truth conditions to a given well form formula.

So, then we will talk about a validity in the context of syntax and in the context of semantics. And then we will be talking about some of the important decision procedure a method, which tells us whether or not a given sentence is valid or when 2 groups of statements are consistent to each other, are when how can we check whether a given formula is tautology or contingency or a contradiction etcetera. All these things are come will come under the category of decision procedure methods.

So, there are few decision procedure methods, to start with will begin with the simplest possible method, that is, truth table method. And truth table method works as long as the numbers of variables are considered to be less in number; that means, 2 or 3 variables are there, then it will be easy for us to handle. And then if number of variables increases 5 to 6, and then there will be 2 to the power of n entries in the truth table. So, things will be difficult for us, but may be a computer can manage it in a better way, but for as it will be difficult to handle.

So, then we will be talking about 1 interesting and important method; which serves as

core of this course and all. So, that is the semantic tableaux method or it is also called as tree method etcetera. So, then there is another method with which you can talk about; validity etcetera and all. So, that mean in that method, what we will be doing is; we reduce the complex well form formula into the that well form formula consists of implication, double implication etcetera. And then will reduce it to a formula which consists of only negation disjunction and conjunction.

So, if you can reduce that complex formula into this particular normal form, which are usually we usually call, then these things these formulas are called as conjunctive and disjunctive normal forms. And then once you reduce a formula into conjunctive and disjunctive normal forms, you can clearly you easily come to know that, are a given formula is it tautology or not. All these things you will be studying in the forth coming lectures. And then we will also talk about 1 of the interesting methods, that is, resolution and refutation method.

So, these are all semantic kind of methods. There are some other methods with which we can still talk about validity in the context of syntax. So, that is what we call it as syntactic consequence or syntactic validity etcetera. So there, we study the natural deduction method, with which we can talk about some of the syntactic consequences etcetera, which are there are some formulas which are; obviously, valid an all. Those in we talk about in the context of syntax, we present natural deduction method.

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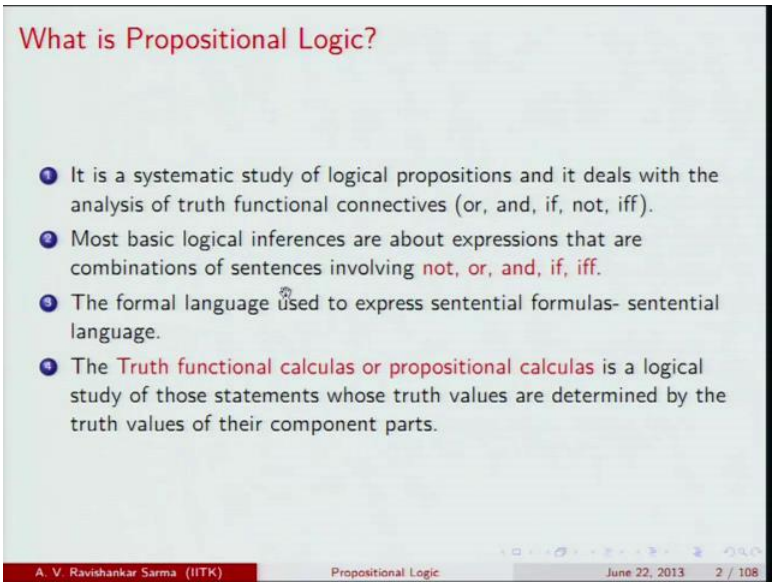

So, now what is propositional logic and why we need to study propositional logics. So, first of all it is a systematic study of logical propositions. Logical propositions in the sense, there a proposition is a sentence, which can be a clearly spoken as either true or false. In the basic concepts, we have said that language can be used in 3 purposes. 1 is logical usage, another 1 is expressive usage and then there is other kind of way in which you can use a language. For example: language can also we use to express some kind of feelings etcetera.

So, we will be considering or we will be interested in only those statements, which can be clearly spoken either true or false. Suppose, if I say that this is a choc piece, if it is differing to the fact etcetera and all, an actually this is a choc piece and that is the statement is construct to be true. So, only those sentence as we will be taking into consideration and then these sentences are combined with the help of what we call it as kind of truth functional connectives, which are construct to be; or and if not if an only if. These are the things which we express it in the English language, but in the formal language we express it in terms of it is own symbols along.

So, will talk about little bit later about this symbols, with which we can represent these phrases or and if not and if and only if. So, proposition logic, is says systematic study of these things. Logic of propositions are logic of study of the connectives etcetera. All consider and all come under the category of propositional logic. And we it only studies about these connectives only and there are no quantifiers etcetera and all in this kind of language, quantifiers in the sense for all x there exist some x etcetera and all. This is a most simplistic kind of language, which usually with which you can represent mathematical kind of reasoning and all.

So, 1 should note that, there are different kinds of reasoning which we have said in the beginning of this course; that you know reasoning can be divided into deductive and inductive kind of reasoning. And the deductive reasoning, proposition language proposition logic is one of the outcomes of this deductive reasoning and all. There are other kinds of deductive reasoning, in which some of the fundamental principles of logic their weekend and all. And then we will enter into some other kinds of logics which are called as non-classical logics.

So, see it is also considered to be formal language, which is used to express sentential

formulas are sentential language and all. It is also called as sentential language etcetera. It is a most basic logical influence are about, usually there expression that are combinations of sentences involving; not or and if and if and only if. It is very difficult for us to talk about connecting 2 sentences without using these logical connectives and all.

Suppose if you say this is a chock piece and this is a duster and all, if what combine this 2 sentences, there should be some kind of connective which you should connect these 2 sentences and all. So, in that case, either you say this is a choc piece or this is a duster in that case it is p or q, if it is choc piece then it is a duster it is called as p, it is expressed as p and q. In order to join this atomic proposition which we call, atomic proposition are the proposition which cannot be further used into any other kind of proposition and all. Just like p'c q's r's etcetera, they are all called as atomic proposition and all.

So, these atomic propositions combine with this logical connectives and form complex formulas and all compound formulas etcetera. So, this is also considered to be truth functional calculus or it is also called as propositional calculus. It is a study of those statements whose truth values are determined by the truth value of their component parts.1 of the important point when needs to note is this that, a truth value of a given formula in the propositional logic, is solely determined by whatever truth values that it is individual constituents takes and all.

Suppose in the formula p r q, the truth value of p r q is only determined by whatever values p and q takes and all and depending upon how the logical connective, that is, in this case are behaves and all. So, we will be talking about semantics under the context, when we talk about semantics of proposition logic, we will see how this these connectives behaves and all or and implies etcetera and all. So, they behave in a certain way in the ordinary day to day language. And as far as possible, we are trying to convert we are trying to translate the English language statements into the appropriate language of propositional logic.

So, in the process of translation, 1 might even be disappointed. For example, a simple example could be like this; usually we say when we are sick we go to the doctor and all and this can be put into this format. So, I became sick and I went to the doctor, that is what we usually do and all and that is represented as a and b. The same sentence can be

represented as b and a also because, a and b and b and a are identical to each other. They are logically equal into each other because, they have 1 same truth values and all.

Suppose if you say that, I went to the doctor and I became sick. So, this 2 are different things and all. Nobody goes to the doctor to become sick and all. So, I went to the doctor and I became sick is totally different from, I became sick and I went to the doctor and all. This is, they seems to be some kind of other operators, which are important to analyze these particular kinds of statements. So, that means, truth value of a and b is not only determined by the truth value of a and b which is, but it depends upon some other extra logic some other factors and all.

So, these kinds of things are called as intentional logics and all. Intentional logics are not part of our course and all, but we will be talking about only extensional logic. That means; the truth value of a given compound formula is only determined by the truth value of its individual constituents.

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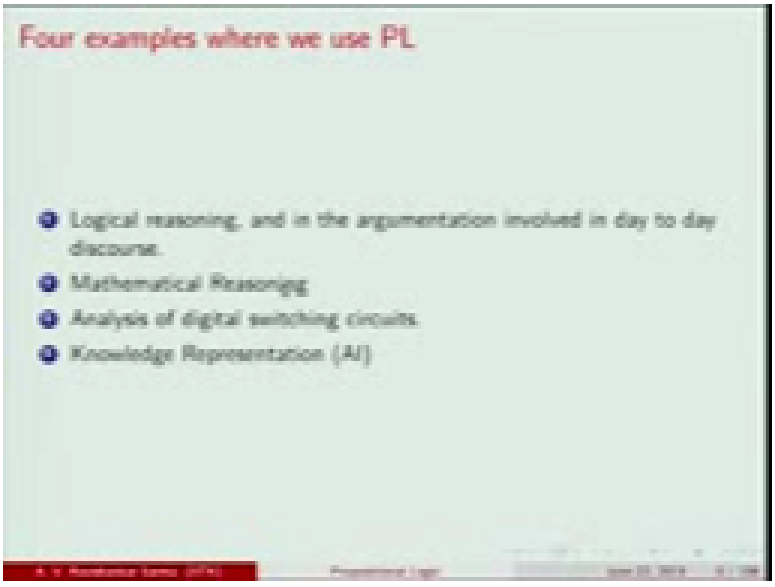

So, to motivate our self, you will ask this fundamental question; where we use this propositional logic and all? So, usually in general, you know we present propositional logic as a formal language, which tries to capture basically mostly mathematical reasoning and all. Or it can be used to capture logical reasoning and argumentation that involved in day to day discourse, but not all the all kinds of propositions can be captured in this proportional logic and all. For example: if you say all birds flies, tweety is a bird

and tweety flies and you got an extra information that tweety is a penguin and penguins does not fly. So, in that case, you need to give up some of the conclusion that you have derived earlier, that is, all birds fly is a may be tweety is a bird that flies. There are some some other things which you have derived earlier; we need to withdraw those conclusions, which are not permitted in the language of propositional logic.

So, these are the cases which come under the category of defaults. And default reasoning is not the 1 which we are trying to consider here. There many other kinds of reasoning like; counter factual reasoning. For example: if you say, if I become the Prime Minister, I would wipe of all the corruption etcetera and all. So; that means, you had not become the Prime Minister; that means, the anticorruption of that particular conditioner is fault. And the semantics of proportional logic tells us that, if the anti cirent is false, a irrespective of whatever the consequent, irrespective of whatever truth value the consequent takes, your conditional the truth value of the conditional is of always true and all.

In that case, there is no distinction between; if I had become the Prime Minister of India, let say pigs would fly and if you say if I had become the Prime Minister of India, pigs would not fly. This does not makes any big difference and all because, in both the cases, the anti falls and irrespective of whether pigs flies and pigs wouldn't fly does not matter. It makes a whole conditional sentence, if I had become the Prime Minister of India, pigs would fly is true. In the same way equally, if I had become the Prime Minister of India, pigs would not fly will also be true.

So, these are things which we do not want in day to day because, this goes against our intuition. So, in that case, we need to maybe we need to extend this propositional logics and we should talk about some other things and all. So, that is not what is of concern to us, but there are some cases in which, the propositional logics works better; mostly the mathematical reasoning and all. And the another interesting point is that, proposition logics are directly used in analyzing digital switching circuits and all. We have a simple complex digital switching circuits; which has on and off switches and all.

There are various kinds of gates that we use and all; and or any nand gate etcetera and nor gate etcetera gate etcetera. All this things are the under lying logics are nothing, but logic I mean logics are usually is also called as a propositional logics you know because, proposition logics is; they are talking in the same way as bull, represented these kinds of

reasoning with the help of algebraic operation plus and multiplication. And this plus in proposition logic stands for or a connective or and multiplications stands for the connective and.

So, they are more or less the principle of geology, helps us in translating the algebra into the language of propositional logic and all, which we will talk about little bit later. So, you are given a complex circuit and all. So, you will consider, you will translate that complex switching circuit into an appropriate formula of a propositional logic again a complex formula and all. You will forget about the switching circuits for a while. And then you simplify that complex formula, maybe it is been or something like that p r q or something like that big compound formula. And then you will reduce or simplify it using all kinds of rules, laugh absorption laugh distribution and etcetera and all, when you simplify the logical formula and all.

So, then again you will go back to the switching circuit. And then you will directly study the simplified formula and all a complex formula, let us say it is reduces into just p r q and all. That means, if you have a complex formula which involves a 4 or 5 switches and all which somehow reduces to just p r q and all; that means, you just require only 2 switches. That means, the complex digital circuit is exactly same as the simplified digital circuit there is a simplified formula that we called, we arrived at. So, in that sense, proposition logics are directly used in analyzing ,only simple digital switching circuits where you have on and off switches.

So, we can also how this logical connectives behaves by using sand and or gates etcetera exclusive or gates etcetera. That is a practical application of is propositional logic. And 1 of the most important thing is that, suppose you want to talk to the machines etcetera and all, if a talking in English and all do not help us a in any way, it want answer you anything. So, the first thing 1 is do is to represent our knowledge, that knowledge claims etcetera an all into some kind of formal language an all. At the most simplistic kind of formal language that we use, is the propositional language of proposition logic, which is also consider to be a formal language. Once you represent it in terms of some kind of formal language, then next thing is; 1 can write 1 can prove some theorems in all, 1 can write programs, 1 can write algorithms etcetera an all. All these things follow.

So, basically, proposition logics are used as a knowledge representation tool in artificial

intelligence and even in the computer science. So, our concern is to capture from mathematical reasoning or reasoning that is that we use it in as for as possible in the day to day discoes as per.

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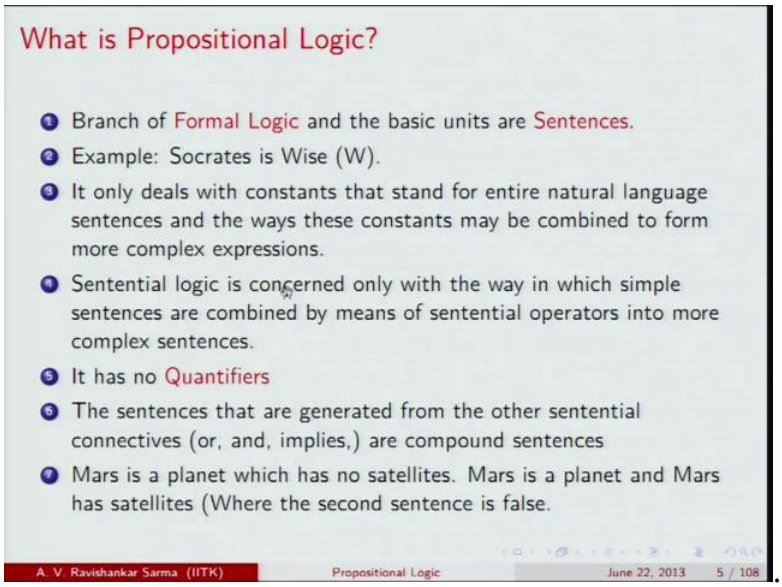

So, somewhat is propositional logic. So, when we a study the history of logics into consideration in greater detail, I will talk about how this proposition logics supinated originated and how Aristotle logics were gradually logistician have gradually have to ignore this Aristotelian logic because, it could not account for many things. Although, it can still be used as 1 of the pyridine cases of logics in all because, it dominated from more than 2000 years.

So, 1 can again fall back in Aristotelian logics and you can talk about some of the important points, in important things at you can borrow from Aristotelian logics to the modern logic. What is propositional logic? It is a branch of formal logic; formal logic in the sense, there is a distinction between content and formal. Suppose if you are argument involves the analysis just forms like all x or y all y s are z an all, in that particular kind of argument are pattern. So, what is important is a form, all x implies y y implies z x implies z. The only form matters in all, does not matter what you substitute for x y z, it can donkeys, it can be cats, it can be anything in all.

So, whatever you substitute into x and y and z, suppose if you it is in this particular kind of form; x implies y y implies z an x has to be z. As per as you believe in the true value

logic, I means is your sentences are only true or false, I can be neither true and or false all these things are ruled out. So, then the formal logic means; your studying the you are interested in the forms of argument logic. But there are some other argument, which you have presented in the basic concepts, while studying a what the basic concepts they are like this. For example, if is a all items are invisible. So, all items all this room is made for items, for example, let say this room is made up of atom's atoms are invisible. So, this room is invisible.

So, you will not be able to find out any flow in this argument and this until you analyze the terms that you have used in your arguments. So, in that case, there is a shift in the meaning of usage of the term atoms. So, in the first sentence, it is used in a different sense, the second sentence is used in a totally different sense. But in propositional logics are the formal logics which we take into consideration, it is pre suppose that is no shift in the meaning of the words are phrases, that you have used in your logics.

So, it is branch of formal logics, were the basic units are sentences. So, all the sentences will take this simple letters and all, which are called as sentential variables. Suppose if you want to represents, so critics why, it can be represent as you a simple letter s or it can be represented as a simple letter w and all. It up to us to take our own letters and an all because, we have any number of letters which are available, so you can take any other letter in all. You can be represented even all false. So, it only deals with the constants, that is, stands for a entire natural language sentences. And the ways these constants may be combined to form more complex expressions. What are these Constance and are implies if an only if etcetera.

So, we have some sentences which had out there and then we have some logical connectives. And these sentences combined with this logical connectives and form complex compound formula in all. So, now, sentential logic is also concerned with, the way in which simple sentences are combined by means of sentential operators; again negation, conjunction, dis junction etcetera. And they form the most complex kinds of sentences enough. And you have to note that, sentential logics or proposition logics are propositional calculus, all these things are 1 and the same in all.

So, they have no quantifies and quantifies in the sense, for all x there exist some x etcetera and all. For example: if you want to represent all men or mortal, it is simply

propositional language you represents it as m later m or something like that. You represented in terms of quantifies in all, you say for all x; x is a implies x is mortal in all. So, you do not have quantifies in all that sets some kind of limitation to the propositional logic.

If you want to make your language little bit richer in all, then you need to add is quantifies enough. But again, proposition logic on the 1 hand, there elegant, very simple and it has wonderful features in all like a there are wonderful features, everyone every logician would be starving for that is, consistency completeness etcetera in all. A system is called as consistent, especially when you are not able to derive, you cannot derive both x and not x is the same time or a same set of assumptions etcetera. If you can derive both x and not x and your system is consist to be inconsistency.

Suppose if you talk about it is raining and it is not raining, given some kind of assumptions then this something wrong with your system, not wrong is a system is called as inconsistent. And logicians will hate this kind of inconsistency. In fact, even mathematicians has well, in consistency is treated as a some kind of hell why because, if you are given an inconsistency statement, you can derive anything any kind of strange kind of proposition in all, without violating the truth value of your calm your proposition formulas. So, that is 1 problem which arises.

So, we look for only consistent kind of, look for us system to be consistent. And the second wonderful feature is completeness. So, there are in our language, that we are going to construct will forms, some kind of well form formulas, some well form formulas are consist to be valid some are in valid. So, all the well form formulas which are considered to be valid, so that all can be provable an all; so you have to be find proof for these all the well form formulas. And in the same way, if all the proof all the formulas that you are proved, that has to be a end of the day it has to be truer. So, it proves lots of thinks in all, but at the same times false in all. This is no you are not of purpose; it is not of purpose in all. At the end of the day, just treatment has to be prove just has to be true in all; that means, it has to be tautology in all.

So, whatever is provable is true and whatever is true is provable, then they then your system is called as complete. And 1 other wonderful features of this proposition logic; it is construct to be the minimal kind of representation of your knowledge, these are

consistent, complete, compatible. All these wonderful features are there a present in these proposition logic. Once you at quantifies etcetera in all, then the complexities arises and will talk about incompleteness etcetera in the context of, at the end of this thing. While talking about predicates logics, will talk about incomplete and theorem, which tells us that, there are some formulas within the language of first order logic, in the sense first order logic means; it is a propositional logic plus predicate logic. In those language is in particular in the predicate logic, there are some formulas which are; obviously, true, but cannot find proofs in all.

So, that makes a system in completed. So, it has no quantifies, propositional logic has no quantifies. And the other thing is that, the sentences that are generated from the other sentential connectives are considered to be compound sentences in all. So, you have some sentences like; it is raining etcetera in all and you have other side statement. If you combine these 2 sentences, with it is a raining I mean implies grass is that will become a compound kind of kind of sentence.

So, these out of these connectives and are implies if and only if there are binary connectives because, it connects at least 2 sentences in all, were as the negation is consider to be a connective an all. So, suppose if you say is example, mars is a planet which has no satellites'. Suppose if you say mars is a planet and mars has satellites in all. So, it is a conjunction of 2 statements in all. If at least 1 of the these statement is false and it makes a whole conjunct false in all. That is the way that conjunction behaves an all.

Suppose if we say, mars is a planet which has no satellite, that makes may be making the sentence true and all. The second sentence in particular, at least 1 of the conjunctives falls there; obviously, makes a whole conjunct falls in all. Even the mars is a planet is true, but the second 1 mars has satellite etcetera, if that has to be that is false in all, it makes a whole sentence false in all.

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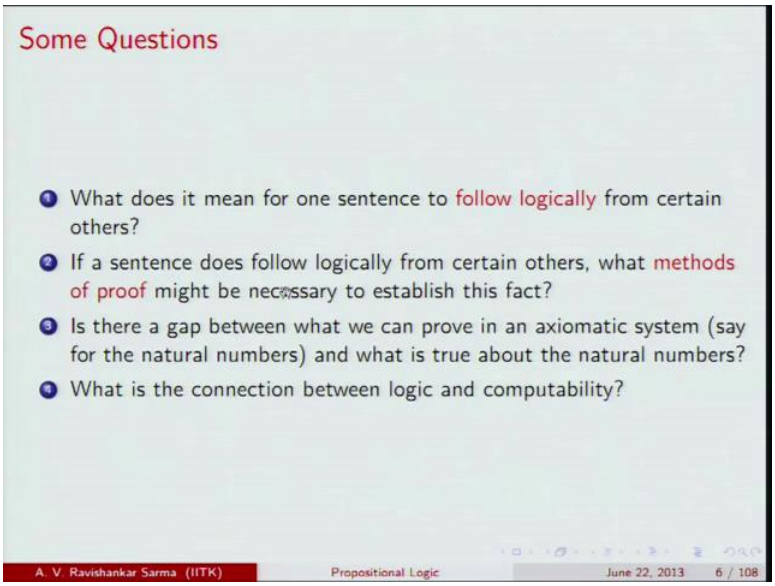

So, will talk about truth and false it is a little bit later. So, these are some of the pertinent or fundamental question that we will asking our self, which motivates as to considered are steady; the proposition logics which are consider to be the minimal kind of logics, with which you can represent basically mathematical reason. So, these are some of questions. The first question is; what does it mean to say that 1 sentence logically follow from others certainly from others. That is what is validity will take care of this particular kind of question in all.

So, logic is after all, all ever all about what follows from what. You have you sentences and after all why need to use tools of logic in all. You have you few sentences and then you want move from these 2 sentences to another sentence in all. So, there are some kinds of techniques which we use. Either it the way you move from premises to the conclusion, if it requires that require some kind of necessity, some kind of certainty are, if you do not want any new information to be there in the premises, do not want any con new information to be there in the conclusion, which is not there in the premises etcetera, then use deductive reasoning an all.

In the same way, if you want we have to few premises to begin with any, you want to concludes something then concludes something and then your argument to be stronger weak an all. We cannot be 100 percent certain etcetera an all. In that context you use in reasoning an all. Natural sciences usually follow this particular kind of reasoning an all.

So, this is what we have studied in the basic concepts in all.

So, 1 of the fundamental questions, many logician would be interested to ask is; what follows from what? So, that validity, the concept of validity will take care of this particular kind of thing in all. We know that particular argument is valid. For example: it is raining and grass is waited on, from that you can derive it is raining in all, a and b you can derived a. So, how to know that this a and b, a can be derived. Is there any logical procedure with which we can tell, as you can tell whether it is raining is derived from it is raining and grass is wet.

So, if a sentence does follow logically from the others, what are the methods of proof etcetera, which are necessary to establish is particular kind of fact you know. So, in that context, we use truth able method, we use semantic tableaux method; we use some other interesting and important methods. And then after finding out all these things, we will ask our self is any gap between what is what we have proving using some axiomatic system. That means, you start with fundamental axioms and then use some kind of transmission tools, some kind of natural principles at use in the logic mode respondents etcetera. And then in proves something; that means, a last step of your proof is considered to be a theorem.

So, is at any gap between what we can prove and what is true about, let say natural numbers etcetera and all. That means, whatever is provable is not true a whatever is true is not provable and there is a gap in all. If that is a case in this system is called as incomplete, but in the case of proposition logics, is problem will not arise, there are no gaps. So, propositions logic is consider to be complete consistent etcetera. And the other question that a computer scientist must be interested in is; what is the connection between logic and computability knows. So, you will not talk much about this particular kind of thing, but will try to cover the first 3 things.

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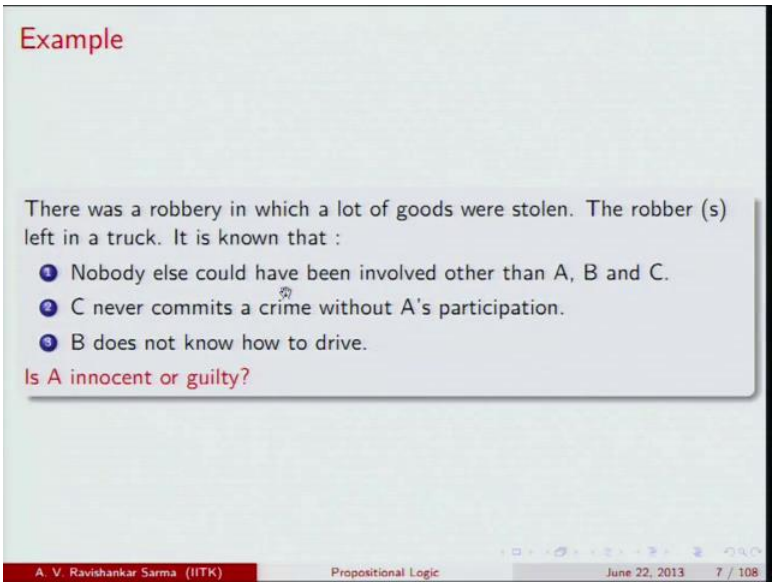

So, let us start this thing with a which some kind of motivating example, the 1 which we use it in the day to day discourse. We have said in the beginning of this lecture that, proposition logic also covers some of the important things, that will tried to arguments related to day to day discourse. So, it is an interesting example. There was some robbery in which lots of goods were stolen and we are trying to find out who is considered to be guilty here.

So, these are the conditions which are given. The robbers left in a truck after the steeling's everything in all. And a bank they have might a steels stolen something; goods etcetera, the left in a truck. And our information has this particular kind of things enough. So, we have seen only 3 robbers running away from steeling everything etcetera an all. So, nobody else could have been involved other than naming them, it is a b c in all. So, that is the first thing which we know, at least some information that we know. And we also know that, somehow the history of this an all, guilty people are robbers, on some information we have may be.

So, c never our commits a crime without, a's participation an all. That means, whenever c does some kind of thing, a will always be there in all; that is what we assure of. Let me c will not come it any robbery etcetera an all, unless a's participation is there. So, there will be coordinating with each other nicely and all. So, c and a will always with there, in that particular kind of wherever the robbery as taken place, that is 1 thing which we

know. And the third thing which we know is this thing; b does not know how to drive; that means, after let say yes stolen lot of money in all, but he himself cannot drive an all; that means, only may be a and c knows driving in all. But in this case, may in might have to depend upon a and c. He himself cannot drive. So; that means, suppose if c alone and all then he cannot will be caught an all.

So, now, using this information an all, how to be known that whether or not he is innocent are he is guilty know. So, in this case, let us that you are seeing here a b c etcetera an all. So, they all stands for; a means a is guilty, b means b is guilty, c means c is guilty an all. Of course you can say might say that, you know I will take I will represent guilty as a not a and all. I can do it in that way as well, but in general we treat whose over is involved in a robbery is called is found to be guilty.

So, a b c is means; say a is guilty, b is guilty, c is guilty etcetera and all. Suppose if you say not a, not guilty means; is innocent in all, there is opposite of that. So, we are already use some kind of connectives in all. So, now, you can say that, this puzzle or problem can be solved, you can use some kind of mental reasoning etcetera an all. You do some kind of exercise and then you can come up with your answer and all. That is seems to be very difficult, you know see it will be very difficult if you do not have some kind of symbolic form and all.

So, this is what is taken place in the history of mathematics in particular ancient in the ancient days in the period of Egyptians. So, they do not have any knowledge of the unknown number's in all. They were all struggling about this particular important thing.

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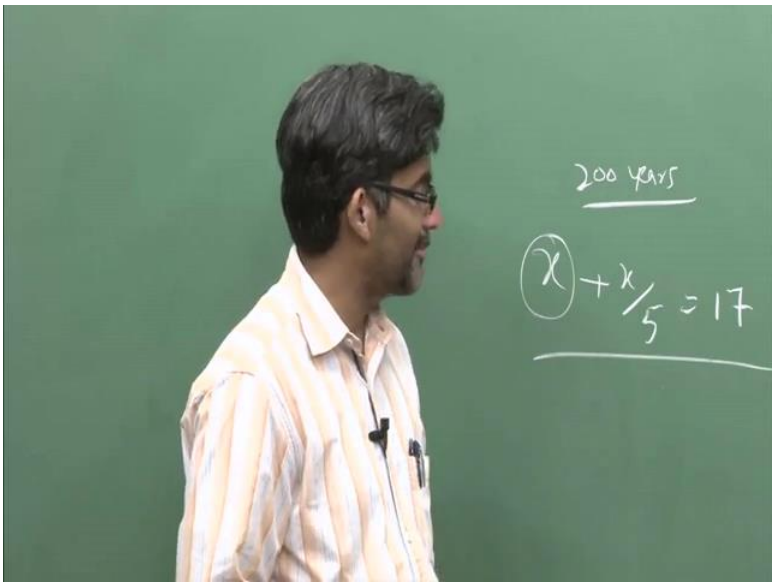

What adds to one-fifth of its number is to 17 in all. So, they struggle from more than 200 years because, they do not have this concept of unknown number in all. So, these days if you ask even 6 class student, easily able to answer this particular kind of thing in all. The problem there was these that, they were no symbols at are available to them, did not did not think in that direction in all. So, only after 200 300 years, after their contribution to the mathematics, people could solve this particular kind of problems in all.

So, what I am, why I am saying these particular kind of thing, is this because, this fact that if you do not have some kind of symbolic formula, things will become very ... You might say that they are only 3 sentences I can do it, I can mentally calculus, mentally know what is going to be the answer and all; if, the number of proposition increases in all, or information is big enough, then the best way to representation in terms of some kind of symbolic formula.

So, this motivates us to study proposition logics in all. If you want to know whether a is innocent a is guilty etcetera an all, we you we have to representation terms of appropriate language.

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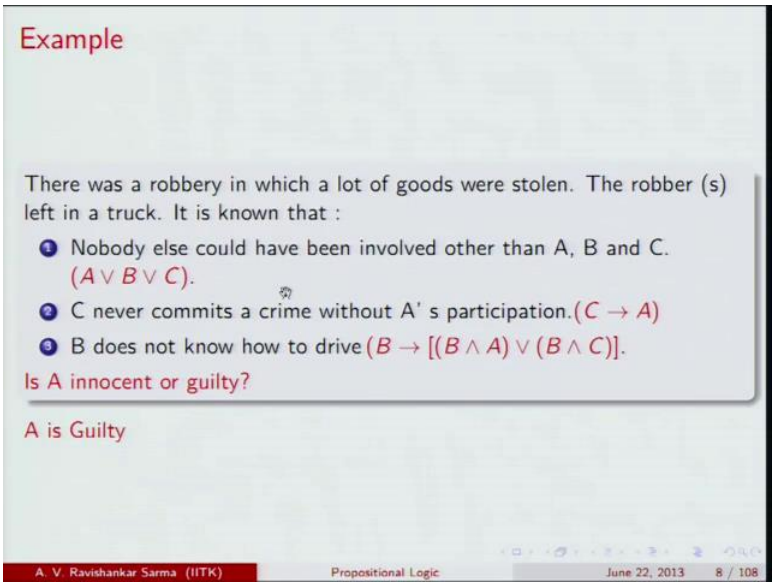

That language could be like this. So, depending upon the sentences, we translate the sentences that occur in the puzzle into the appropriate language of proposition logic. And then we will see when these 3 sentences are simultaneously happen to be true in all, when these 3 sentences satisfies. That means, when these 3 sentences taken together is going to be true in all. So, that gives us some kind of answer for this particular kind of thing and all.

So, this no 1 else could have been involved other than a and b and c, means; either a has to be there, b has to be there or c has to be there where involve. There is kind of thing d is not there. So, a or b or c, that is the 1 with which we can represent the first sentence. And the second sentence is represented as; c never come it is a crime without a's participation; that means, c is important for a and all, presence c is important for a's presence is important for can all. So, these can be translated into c implies e an all. So, it is like a sufficient kind of condition and all.

So, the third 1 is this that b does not know how to drive. That means, b requires of either a or can all. So, in the first case it is like b implies either b has to be with a an all so that he can run away or b has to be with can all as to coordinate with c. So, that he can run away from that site after still here within. So, now once you a represent this thing into this particular kind of formulas in all, then we will come to know, when these 3 formulas are jointly true and all.

So, when under what context, there all consistently each other etcetera an all, we will solve this problem little bit later, but I will post pone this problem to for a while. Then once we talk about either truth tables or some kind of semantic method or some other kind of method, will come back to this particular kind of problem. But our basic thing basic idea that I am trying to give is that, when you have a some problems like this and once you represents in terms of some kind of symbolic formula, and use principles of propositional logics, you can solve this particular kind of problem in all.

So, if you had little bit impatient etcetera an all, answer for this thing is that a is consider to be guilty an all. So, how to be known that is guilty, a some methods which you can show that, a is guilty follows from these particular kind of information. So, 1 way of doing it is; these are the 3 propositional that we have, p 1 p 2 p 3 etcetera. For example, for time being we consider in that p 1 and p 2 and p 3 at all needs to some kind of tautology in all statement which is; obviously, true in all.

If you can show that is a tautology, then instead of showing that this is a t and all and what we have to show is this particular kind of thing in all. Let say the first sentence is represented as p 1 and second sentence is represented as p 2 and p 3. This is a information that we have. And this all should lead to suppose, if a is guilty means guilty means, it should be like this. And a is innocent, innocent means; it is not a in all. Innocent and guilty are opposite to each other know.

So, if you can show that, is whole thing p 1 and p 2 and p 3 all these sentences implies that a a means guilty in all. If you can show that this is a tautology, at means under all interpretations of p 1 p 2 p 3 whatever values that you assign to the variables that occurs here, which is always true, then we can say that a is considered to guilty an all. So, if you want to show that, is guilty what show that at particular formula, which is there in the brackets, is should be a tautology an all. If it is not at all tautology, then a is not said to be guilty an all. So, that is 1 way. There are there are varieties of ways, which we which you can show that a is guilty follows from this particular kind of information.

So, 1 important method that you will be using a little bit later is; the semantic tabular method. So, this is the motivating example, which leads us to study the propositional logics. There are lots of puzzles, you will be talking about some other puzzles little bit later, which are cooked up by a famous logician Raymond’s mullion. He has constructed

wonderful puzzles, which are which come under a category of puzzles. So, these puzzles goes like this.

So, you have to imagine an I land, in which there only 2 kinds of in. So, let a say type a and type b. So, type a always talks truths, type b always tells lies in all. For example, if you ask type a who always tells to is 2 plus 2 is equal to 4, they will say true yes it is true. And if you ask the same question to the news, which are of different types, which are in different type they will answer is 2 plus 2 is equal to 4 they will say no. Suppose, in the same way if you ask them; is 2 plus 2 is equal to 5, news will answer yes. So, news always lies and nites always tells truths and all. It is not the case that news all news tells truths and all. So, a liar cannot tell truths and all, that is a condition that is there you know.

So, now, Riemann sullen Smulian constructed lots of puzzles, in which your stranger goes to this particular kind of island and he asks some kind of questions and all. He wants to find out what type he is, whether he is an a, weather he is a nave or he is a night and all. For example: if say if stranger goes to an island and the stranger says the inebriant says; let us say a says I am not a nave and all and b says I am nite and all. So, based on that, particular kind of information, what is that you are going to infer and all.

So, these are some of the puzzle again, which are represented by the langue presented in the language of proposition logic. And then once you represent it in term of propositional logic and things will become simpler and all. We can solve it with using basically semantic tabluks method, specially these nites and naves puzzle and all. So, basically what is that I am trying to say is; simply this that there are some kind of puzzles which we can solve with a help of the propositional logic, and then there are some instances like the 1 which we have seen earlier. And there are some other things like, how the complex digital circuits can be simplified into simplified digital circuits by using the principles of proposition logic etcetera.

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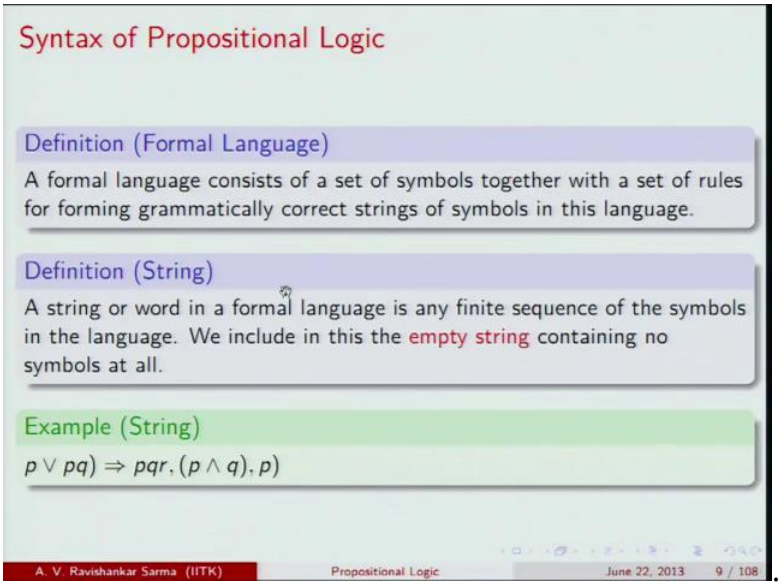

So, far I have discussed only the background of the propositional logic, I have not discussed the origin of propositional logics. In different context I will talk about in the history of logic, I will cover how this propositional logic have originated and all. Basically it began with bool, the mathematical analysis of logic. And then a frag Russell white head advanced it further and then there are some developments of godet etcetera. All this things which we will be trying to cover, basically from 1890s to 1930s and all. So, the developments that we will be covering under propositional logics.

So, now, every language has its own syntax and its own semantics and all. The language that we are trying to talk about, they are consider to be formal languages and all. So, what do we mean by a formal language why and what sense it is distinct from naturals languages like English Hindi Urdu whatever it is. A formal language consist of a set of symbols, those symbols can be; they are the ones which who usually choose. Whatever symbols you can take into consider, first you fix those symbols in the beginning of constructing your language and all.

So, you have some symbols in this case p's q's r's etcetera and all. And then you have some rules, which tells us how this symbols are constructed. And then you can form some kind of grammatically, correct things of symbols, in that particular kind of language. For example, in English language, we have some alphabets to begin with A to Z. And then this A to Z some different alphabets combine together and form some kind

of words, let us say cat, mat etcetera and all.

So, these words combine together a certain way and form some kind of grammatically meaningful sentence and all. For example: if you say cat is on the mat, it makes sense for us and all, to talk about these kinds of things and all, which is constituted out of alphabets A to Z, and then cat is another word which we know may be concept, mat is another concept, cat is on the mat is considered to be grammatically correct sentences. Or you can even say mat is on the cat, this is this is also, usually we do not say that, but you will say we can say that also.

But if you say mat, cat, on, is and all. And any child can easily recognize that it is not a grammatically well formed kind of sentence and all. So, how do we know that it is a grammatically correct sentence, grammatically incorrect sentence we had to learn some kind of gram grammatical rules, with which you can judge, whether it is grammatically correct or not. In the same way in the language of in the formal language in particular, we have some symbols and we know how this symbols are combined together and form some kind of a string.

So, this string is defined in this particular kind of sense. A string are a word just like you know cat is string which consist of c a t and all in the English language. We have string called p q r implies x etcetera and all. A string or a word in a formal language is any finite sequence of the symbols in the language especially. So, it include an empty string also, as a consisting which consist of no symbols at all, that is also considered to be string and all, which will not have any symbol and all.

Examples of strings are; there are may be thousands of things which we have, what we have are simply like this. We have some propositional variables p q are etcetera and all. And we have logical connectives or implies negation, if and only if etcetera and all. And out of this logical connectives and this things, and then we have etcetera, which tells us which are punctuation marks, just like in the case of English language, we have comma full stop etcetera and all. We have and all which tells us that, you know how to read a given well form formula, when a formula is considered to be a well form formula.

So, p r p q and then closed by in the first case implies p q r, they are all strings and all,. But not all strings are considered under the category of well form formula and all. For example, if you say cat, c a t cat makes some kind of sense to, we know that constant of

cat we know that, it is seems to be a correct string and all. It can be t a c also or a c t or something like that. So, you can even talk about this particular kinds of things and all,. But that is not a meaning full kind of word, which we know.

In the same way, in the formal language, if you say p r p q and closed by some kind of parenthesis, then that is not a meaning full kind of well form string and all. So, there are thousands of strings which we can form and all, using symbols and logical connectives etcetera. Then the question arises is; what is considered to be a meaningful kind of string and all or in this case it is well form formula and all.

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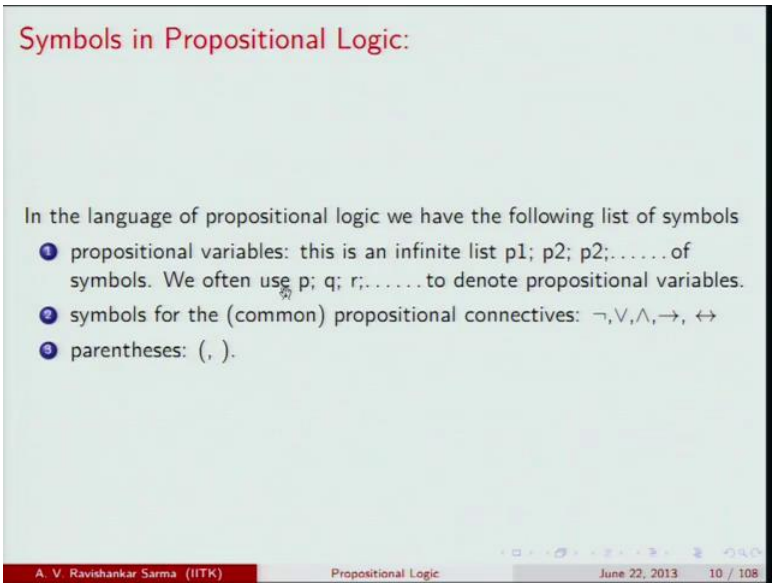

So, these are the things that we are trying to begin with, we have propositional variables which can be infinite and all because, they are so many sentences which we can express it in terms of formal language. So, we our symbols are also and all tough it is finite, but you can even take it as a infinite also; p 1 p 2 p n and all this things. And if it is you can use q 1 q 2 something. We have n number of natural numbers you can use that. So, we often use usually p q r etcetera and all, to represent that propositional variable. Suppose, if you say it is raining and it is not raining, it is represented as r and not r.

So, symbols for the propositional, we have propositional connectives, negation or and implies and if and only if. We need to talk about how these, what we mean by this connectives and all and we talk about semantics we talk about this. So, we do not we are not worried about the meaning of this connectives at this moments, the syntax what will

be interested in is; we just know how this formula generated and all. It is just like the start trying to build a building and all. Suppose if you are trying to build a building, then what you need is a raw material, that is, bricks, concrete and cement and etcetera and all. We just mix it and all, and then later you know find kind of restructuring etcetera that you that it involves. It involves some kind of with that, you can say that it is analogues to semantics and all.

So, in addition to this symbols and logical connectives we have ... the left hand, left prentices, comma, full stop are your right prentices. Prentices tells us how to read a given formula and all.

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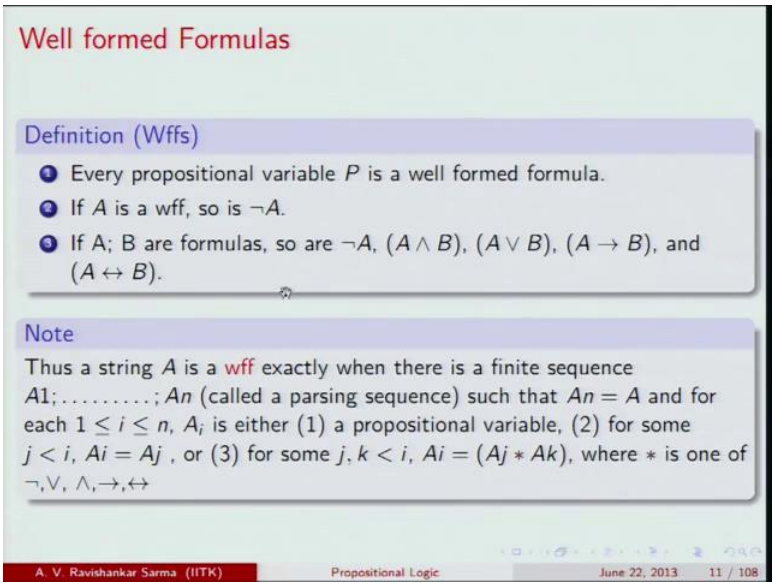

So, now with this thing, I will close this lecture and all. We have, we need to now we have generated lots of strings etcetera and all. So, now, we need to talk about what we mean by a well formed formula and all. So, how do we know that, a particular string is consider to be the well form formula and all. There should be some kind of definition with which, you can talk about a decide about, when you can say that a given formula is a well form formula and all.

So, this is a definition, definition goes like this; every propositional variables p q r etcetera and all this things, are considered to be well form formula. Whatever variable that occurs, is considered to be a well form formula. Empty string is also considered to be a well form formula, which is nothing is there, that is also considered to be a well

form formula. And if a is a well form formula, if assuming that, there is a atomic sentence is; obviously, well form formula. If I add negation the left hand side, that is also considered to be a well form formula ion.

The same way if a and b are 2 atomic propositions and which are considered already considered to be well form formula; that means, they are propositional variables, as is the case of first rules tells us. And then not is well form formula, a and b is a well form formula which is in put in left and right prentices, a r b is also considered to be well form formula, a implies b is also considered to be a well form formula and all. So, the fourth rule tells us that, nothing is a well form formula which does not follow these 3 rules and all. So, for instance, finally, we will see that for example.

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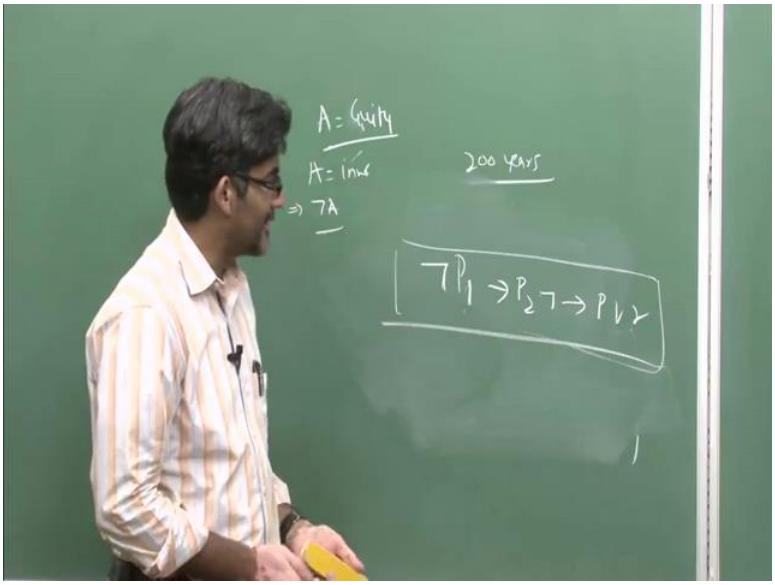

If we have particular kind of sentence like P 1, P 1 implies not something like P 2 not implies P r r and all for example. If you have this whole complex sentence like this. So, this is extinct and all, definitely this is called as extinct because, it is generated out of your symbols and logical characters. But this is not a guard consider to be evil form formula because, of this thing this, is not constructed out of the 4 rules that we have said. Only 3 rules appears here, but the fourth rule is implicit in this 1, that is, nothing is an well form formula which it does not follow this things and all. So, it is none of, it is not in this particular kind of formats. So, it is not considered to be well form formula and all.

So, in this class what we discussed is simply like this; we presented the basic idea of

proportion logic; that means what we are trying to do in the propositional logics. And then we started with the syntax. And then we talked about what we mean by a meaning full string and all, mean not meaningful string, but when you say that a given well form formula, when a when do we say that a given formula is considered to be a well form formula and all. That means, you have generated 1000 of strings and all, not all strings are all of importance to us. Just like cat, c a t is 1 word which we use, we do not use t a c etcetera and all in English.

So, in the same way we have some kind of meaningful. So, we have some kind of strings which are important to us. So, these rules are the definition gives us some kind of idea of whether or not a given formula is a well form formula. And once we have well form formulas, then you can talk about whether or not this given well form formula is a valid or it is a tautology, all this things you can talk about it little bit later. So, with this we will end this lecture.