Introduction to Logic

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Lecture - 16
Syntax of Propositional Logic

Welcome back. In the last class, we discussed something about Propositional Logic. Minimal feature, what are the important features of propositional logic that is, what we have discussed. We initiated our discussion with Syntax. Today, we will be talking about syntax in greater detail. So, propositional logic is a study of it is a logic of prepositions, where prepositions are considered to be simple sentences in which, you can clearly one can clearly speak them to be either true or false.

So, they are all considered to be declarative sentences. So, these are the declarative sentences are only sentences which are, within our per viewer. So, propositional logic in one sense, it can be viewed as a language, the formal language, which has his own syntax and semantics, you know. Just like English language, ordinary English language has it is own syntax, that is alphabets etcetera and all. An alphabet combines in certain way and form meaning full words.

And these words will combine in certain way and form meaning full sentences and all. And then, the grammar with a help of a grammar, we will be able to know, which sentence is correct. Like a cat is on the mat, seems to be better sentence and all, an appropriate sentence. Whereas, if you one can talk about mat cat all and and all. So, anyone who learns, who knows grammar and all, then we immediately come to conclusion that, it starts an appropriate sentences and all.

Just like that, just like in the case of English language, in the case of formal languages. Like, language of propositional logic, we have our own syntax and semantics and all. So, in this class I will be focusing on the syntax of propositional, logic and all. In the last class, we discussed in some detail about the syntax. How, what you mean by syntax etcetera and all, in the last class.

So, we will going to the details of, what exactly we mean by syntax? And how different kinds of formulas are generated, with a help of some kind of synthetic rules. So, just like ordinary English language has grammar and all, grammar decides which sentence is appropriate, meaningfully grammatically correct and grammatically in correct sentences, etcetera and all. In propositional logic, we have what we called as well formed formulas and all.

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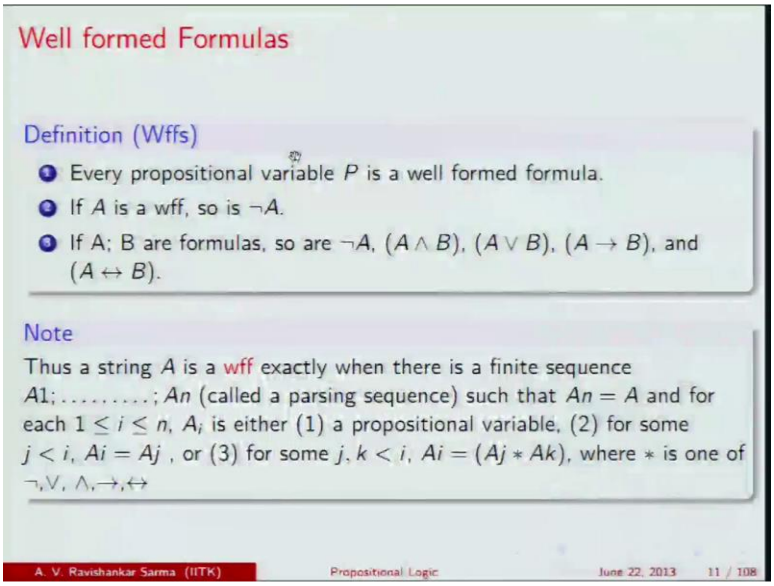

So, to start with the language of propositional logic has some kind of alphabets, which are considered to be propositional variables, which represents some kind of propositions and all. Like for example, if I say it is raining. It is simply represented as the basic units that is, the proposition. So, that is it can be represented as simple letter or. Suppose, if you say it is raining or it is not raining. Then, you simply represent it as or not are and all.

So, we have to begin with, we have propositional variables. We have n number of variables, which are available to us to represent all the sentences, that are generated in our propositional language. And in addition to that, we have five logical connectives. They are AND, OR, implies, negation, double implication. And in addition to that, just like ordinary English language has punctuation marks like full stop, comma, etcetera and

all.

In the same way, in order to read the formulas properly, we need to have some kind of punctuation marks. Punctuation marks in the case of propositional logic are parenthesis, left parenthesis, right parenthesis. And sometimes we use even comma, full stop. It is all over convenience. So, propositional logic has I mean, what we need to talk about when these meaning full strings combine together and form a kind of meaningful formula and all.

Now, at all kinds of strings combine together and form some kind of formula in propositional logic. So, for that we need to define, what we mean by a well formed formula and all. For example, if you say in English language mat, cat all and all, it is not well formed kind of thing. It is at least grammatically incorrect at all. In the same way, an analogy in propositional logic is that. If there are many strings and all p, q and with logical connectives, punctuation marks and all. Suppose, if you have sentence like this, which is generated in this way.

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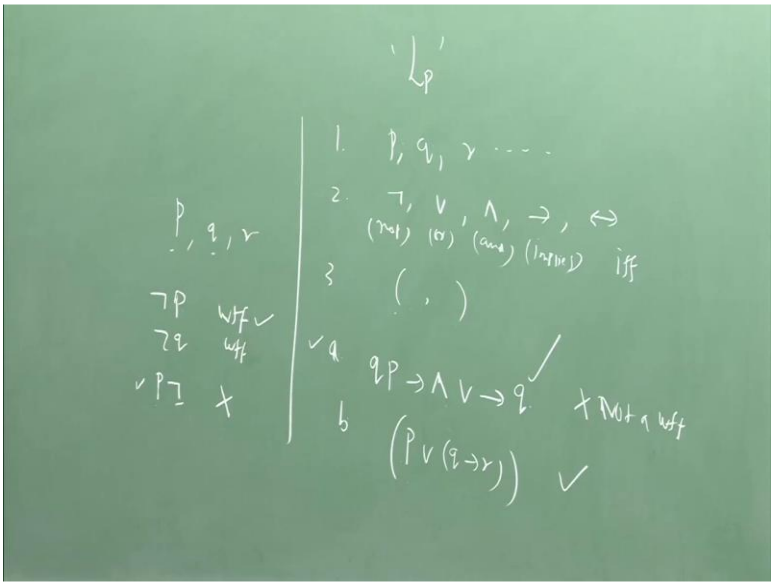

So, we have propositional variables p, q, r, etcetera and all. And then, this is a language of propositional logic. And then, we have logical connectives OR, AND implies end if and only if. So, this transfer AND. This transfer implies. The material implication and then it stand for if and only if, which is used for invoking necessary and sufficient conditions and all. In addition to that, what we have said is simple. That is, this is left parenthesis, in this is right parenthesis. And then, this is a punctuation mark, which sometimes we use it.

So, sometimes suppose if you, there are many formulas which can be generated from these things and all. So, like for example, if you have, if you write like this, OR, implies, q, etcetera and all. So, this is a string which is generated in the language of propositional logic. But, it is not a well formed formula. So, then what we mean by a well form formula. So, we require a definition for a well formed formula. For example, if you write like this p OR q implies r.

So, this is considered to be a well form formula. Whereas, whatever is written about that is a, q, p, implies, AND, OR implies q, etcetera and all, is as good as mat, cat, OR, etcetera and all. So, these are not considered to be a well form formula. Whereas, this is considered to be a well formed formula, in propositional logic. So, what is the definition with which, we can say that. The first one is a well formed formula. And the, is not a well form formula and the second one is a well formed formula and all.

So, here is a definition which we have. So, the definition of well formed formula is like this. Every propositional variable like p's, q's, r's etcetera and all. Suppose, if you write like this. Just p's, q's r, this stand for atomic sentences, there are already well formed formulas. So, that is the first thing. And the second thing is that, something is a well formed formula. Let say, p, q, r anything. And the negation of that one is also, a well formed formula and all.

So, that means suppose if you have this p's, q's, r’s, etcetera. There are all well form formulas and all. Atomic sentences or automatically well formed formulas. If you put one symbol, this is stands for naught and all. If you write naught p, this is a well formed formula. The same way, naught q is a well formed formula. And whereas, if you write p and negation, this is not a well formed formula and all. So, that is a way, we defined it all.

According to our definition, the only case in which the negation comes in the left hand side of this propositional variable is considered to be well formed formula. Whereas, if I write like this p followed by that, there is a negation. Then, it is not a well formed formula and all. So, this is all our convenience and all. So, first we need to define, what we mean by well form formulas and all. And we stick to the definition. And then, it makes our life simpler.

So, that is the way we follow, this particular kind of definition. So, if something is a well formed formula, immediately the negation followed by that is also considered to be a well formed formula and all. And the third thing is that, they are different clauses and all. With which, we can decide what is considered to be a well formed formula, etcetera. Just like, we have so many grammatical rules to say that, which sentence is grammatically correct, which sentence is grammatically incorrect, etcetera and all.

So, we have very limited rules and all here. Not like grammatical rules or many and contextual, etcetera and all. But, here we have only three rules. Simple rules with which we can say, that a particular formula is a well formed formula and all. So, why we are worried about this well formed formulas. First, we generate the well formed formulas. And then, you talk about what you mean by, this well formed formulas while providing the semantics and all.

So, that is what we are with, which we will be occupying or attention in the next class. So, we will focus our attention on the syntax. So, how these strings are generated? And out of all the strings which are generated, only few strings are considered to be well formed formulas. Whereas, others are considered to be not well form formulas at all. So, out of these well formed formulas, some of them are valid, some of them are invalid, etcetera.

Or sometimes, you can even classified into tautology, contradiction and contingent well formed formulas and all. So, will talk about little bit later. But, you focus our attention on this particular kind of definition, with which we can know. We will be able to know, whether a given formula is a well formed formula or not. So, now the third clause is that. Since, these connectives are binary connectives. AND, OR, implies, if and only if, they are all binary connectives.

That means, at least we require two atomic sentences. And these two atomic sentences are joined by these binary connectives and all. It is in that sense, AND, OR, implies if and only if, they are considered to be binary connectives. So, the third clause says that, if A AND B are well formed formulas then, whatever is written in the brackets A AND B and A OR B, A implies B and A if and only B are also considered to be a well formed formulas and all.

So, in the fourth clause which is not explicitly written here, which says that nothing is considered to be a well formed formula, which is not formed by using this three clauses and all. So, which is already implicit in it but, you want to state it explicitly, you can state the forth rule also. I think, it is a well formed formula, which does not follow these three rules and all. So, this one is considered to be ((Refer Time: 10:58)) not a well formed formula, because it does not follow any one of these three rules and all.

So, whereas this one is considered to be it is a kind of a well formed formula and all. So, these are the four clauses which are considered to be important in judging, whether or not a given formula is a well formed formula or not. So, now once we identify that, this is a well formed formula. Then, we can generate a unique three structure for these well formed formulas. And then, we say that every well formed formula has it is unique three structure and all.

So, before that we define this well formed formulas in a kind of formal sense. So, it can be read as follows. A string means, any alphabets, etcetera and all. That means, a propositional variables are combined with a help of logical connectives and parenthesis. And that will constitute a string around. A string is considered to be a well formed formula, exactly when there is a finite sequence. Let say A 1 to A n, which is considered to be a parsing sequence.

That means, we will be parsing from left to right. Of course, one can parse it from right to left also. But, we usually a convention is in that we follow from left to right, we will go from left to right. So, you have a sequence A 1, A n, etcetera and all. It is like a formula, in which a first letter A 1 may be taking care of p or may be A n is considered to be q or something like that. In between, there are some logical connectives.

So, it is such that, the n'th letter, that is A n is nothing but, A only. So, that is considered to be you know, we said in the beginning. That every propositional variable, p is a well formed formula here. If the n'th variable is itself is A only, that means it is an atomic sentence p only. So, that is considered to be obviously well formed formula and all. We are exactly saying the same thing, with the help of some kind of formal definition and all. So, if A n is equal to A and for each one less than or equal to i less than or equal to n.

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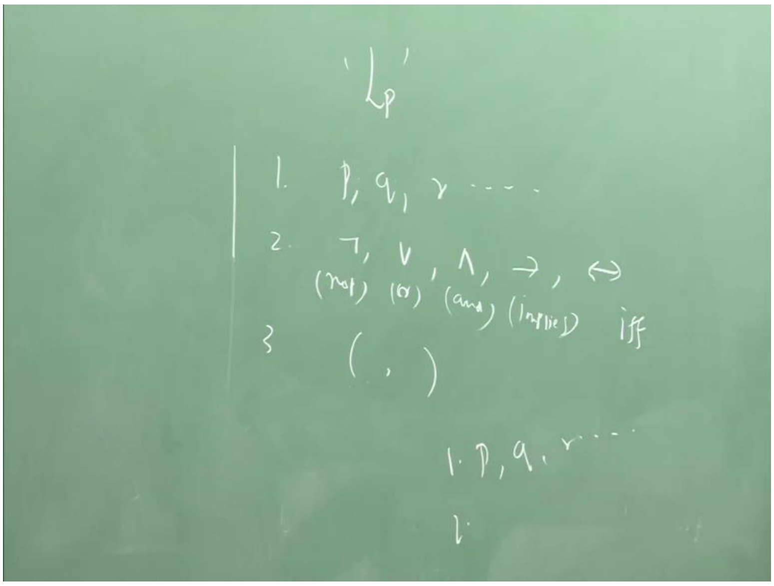

So, this particular kind of A i, which is that has to be a propositional variable, like simple p, q, r, etcetera and all, first one. And the second one is that, for some j less than i. If this A i is equal to A j, that has to be the case r. In the third case, if j and k less than i, then that particular A i has to be a combination of these two things and all. A j and A k and all. And this star indicates any logical connective, apart from the negation and all. It can be OR, it can be AND, it can implies, it can be if and only if or only.

We are just exactly stating the same thing, as we have done earlier and all. So, you know the first condition takes care of the first one, which we have defined in the well formed formula. There is every propositional variable p, is a well formed formula. The second one is taking care by the second condition here. That is j less than i, A i is equal to A j. And the third one is taking care by the third condition and all. So, that is what it essentially says.

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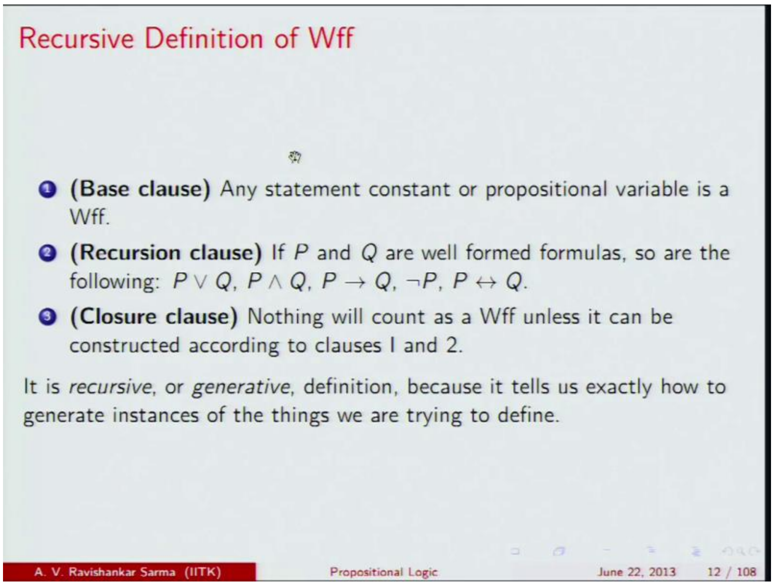

So, this is the you can recursively use this particular kind of definition, of well formed formula. In this, we have three clauses and all. One is the base clause, which says that any statement constant or a preposition variable is a well formed formula, which is the first condition. Second one is recursive clause that is, if P AND Q are well formed formulas, the following things are also considered to be a well formed formulas and all.

P OR Q, anything which joins these things, any binary connective which joins these prepositional variables and all, is also considered to be a well formed formula and all. Of course, brackets are not there here. You can insert it, appropriately here. To read the formulas in better way, we need to know parenthesis, which we will come to it little bit later. So, now the closure clause is that, nothing will count as a well formed formula unless it constructed according to clauses 1, 2 and all.

And of course three clauses are there earlier. But, we have only two clauses here, because negation is also incorporated in the second one. So, these are the three clauses which are important to judge, whether a given formula is considered to be a well formed formula or not. So, this definition is considered to be a recursive definition. We can repetitively use these things for n number of variables and all. And it is a generative definition, because it tells us exactly how to generate instances of things that, we are trying to define and all. So, this is considered to be a recursive definition of well formed formula and all.

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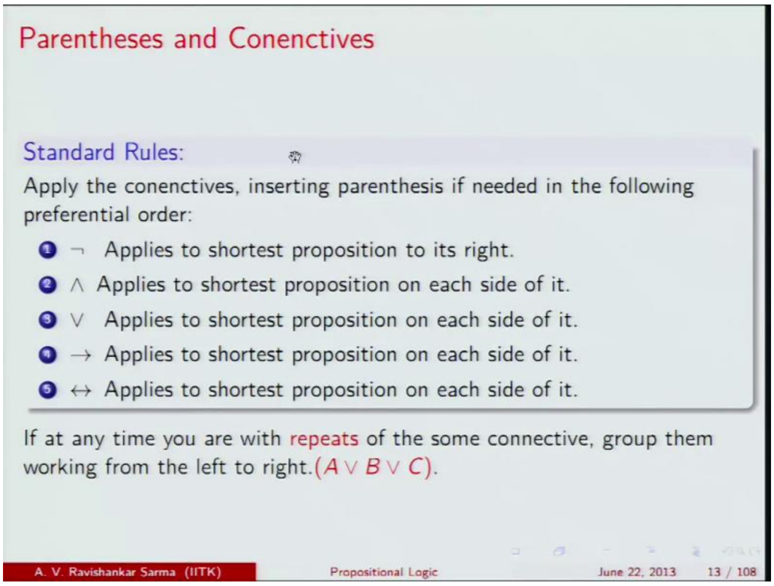

So, we identified what you mean by a well formed formula. Just like, cat is on the mat. It seems to be a meaning grammatically, correct sentence. Whereas, mat cat on, does not seem to be appropriate for us. Anyone, who knows the minimal grammar rules. They will immediately come to the conclusion that, it is not a grammatically correct sentence and all. So, in the same way in the case of propositional logic, it is a formal language.

In this language, it has it is own syntax in which we came. With a definition with, which we will come to know, whether a given formula, we came to know whether it is a well formed formula or not. So, now coming back to this readability question and all. So, there will be some kind of confusion to read these propositional, logical formulas and all.

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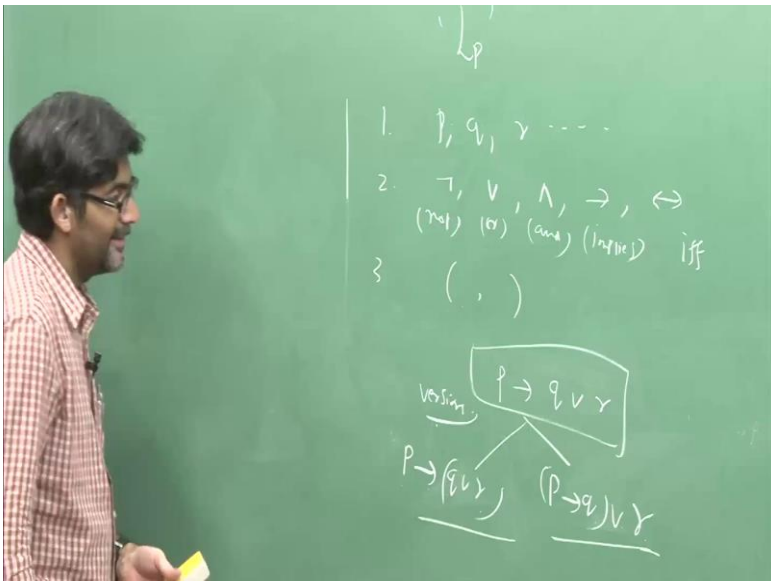

For example, if you have p implies q OR r. So, how to read this particular kind of formula? It can be read in two different ways and all. First p implies q OR r. You can be write in this way or there will be obviously, confusion and all. If there is no parenthesis, which is given here, then it can be written as p AND q. And then, forward by that there is a letter r. So, these are the two versions which are possible for this thing. Two ways, one can read.

The same formula here. So, that adds as to it gives us some kind of confusion. Some kind of confusion arises, in the process of reading this formula and all. So, in order to avoid this particular kind of confusion, so what will be doing is we follow some kind of convention. Again this convention is, comes out of practice and all. Logicians in many text books, follows this particular kind of conventions and all.

The first convention is that, you read formulas from left to right and all. So, we do not write, you do not read formulas from right to left and all. Of course, one can do it. But, your definition of well formed formula and all other things changes it could. But, our convention is like, just like convention that, you will be following left hand side traffic and all.

So, like that you know we will be following meticulously, religiously following. We will following, some kind of conventions and all. Although, logic is considered to be rigorous and all, it is also by need by some kind of conventions and all, like this. It is only makes our lives simpler and all. Otherwise, there is lot of difference between p implies q OR r and p implies q OR r and all. These two are two totally different sentences and all, the main two different things and all.

But, in the case of syntax how do we know that, these two formulas are synthetically different at all. Symmetrically, we know that it is they are different at all. The main different things at all. So, it is p, if q OR r is the case in the first one. The second one, p if then q OR, where there is another conditions like r and all. There are totally two different things at all. How do we know that, these two are different formulas and all. Although, it is generated from the same kind of formula and all.

So, now these conditions have come up with some kind of tree structures for these particular kind of formulas. And then, they use the definition of well formed formulas. That means, the clauses that we have discussed earlier. In making it, in making this formulas distinct and all.

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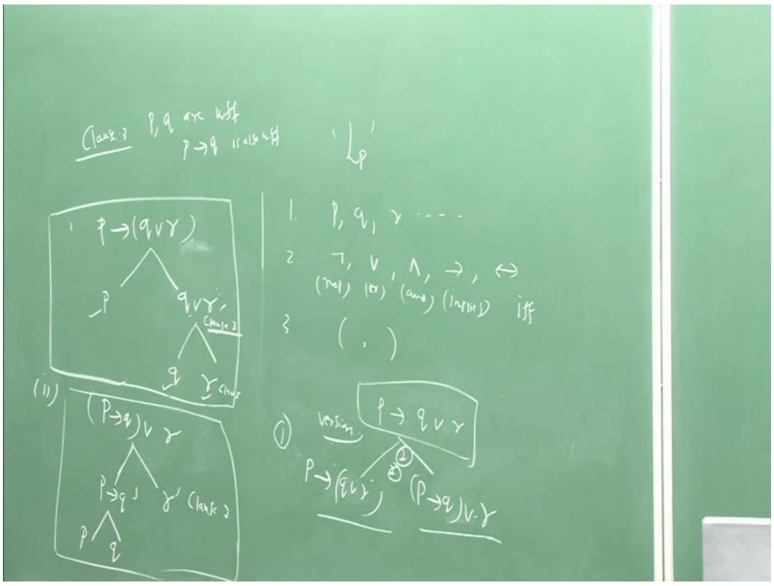

For example, if you have the first formula p implies q OR r. So, in this one the first thing that we will be using is this one, p and q OR r. So, how did we get this one? Suppose, if p q are well formed formulas, p implies q is also well formed formula. That is, what we have said it in. I think, in clause 3. So, these among which we have discussed in the last slide. So, we will be drawing tree structures for these formulas.

Till to such an extent, where you will end up with only atomic sentence and the end point and all. So, since this is a complex sentence and all, it further reduces to q and r. So, this is the key structure which is generated for this formula one. Now, that tree structure for the second formula is, like this. Now, this formula says that first we have p. And then, this is joined, just combined with q OR r and all. So, must be q OR r. Then, q OR r again we need to state these rules and all.

So, this is clause 3. Clause 3 is that, p and q. If p is a well formed formula, q OR r is a well formed formula, then p implies q OR r is also a well formed formula. So, q OR r again we apply clause 3. And then, we can say that q has to be a well formed formula and r has to be a well formed formula. This is the tree structure of this one. So, now at the point we have only atomic variables and all. So, now coming back to this particular kind of thing, p implies q OR r. This is second formula.

So, what I am trying to say simply is that, every well formed formula has it is unique tree structure end. No two formulas have the same kind of tree structure and all. So, unless until they are logically identical and all. Suppose, you write the same thing, I will state about this things little bit later. So, now this is another kind of well formed formula. This can be read in this way. First one, we have drawn tree structure for this one.

And the second one is like this. So, now you apply the same definition of well formed formula. Then, it reduces to this one. This is again clause 3. So, now this can be further reduce to p and q. So, now if we see the structure of tree diagrams for this particular kind of thing, then these two are totally different and all. And the left hand side, you have only one atomic variable here. You have two atomic variables here. And you have two atomic variables here, in the right hand side.

But, you have only one here. These two are totally different. These two have different structures and all. So, that makes this formulas p implies q OR r. It is totally different from p implies q OR r. So, you have to note that, we did not invoke any meaning or anything for these formulas and all. You have just drawn tree structures, according to the definition of well formed formula. And then, we have stated that, the first one is having different tree structure compared to the second one and all.

That makes these two formulas, distinct to each other. So, in this way we can distinguish these formulas to be different and all syntactically. Because, they have we come up with the propositions, pre propositions that every tree, every formula every well formed formula has the unique tree structure and all. So, this formula has a unique tree structure like this. And this formula has a unique tree structure, like this. So, with this you can even know how to read these formulas.

First you read this one, p plus q then r then, this p and q are read and all. So, there is another way of with this tree structure. You can also come to know, how to read this particular kind of formula and all. So, this is the confusion that arises and all. Suppose, if you write a formula p implies q OR r then, it leads to two different versions and all. So, how to avoid this particular kind of confusion and all? So, here is the convention that we follow.

And what comes to a rescue is, how to use this parenthesis and all. So, these are some of the standard rules, which you will find it in the standard text book and all ((Refer Time: 24:49)) in logic. Of course, one can come up with once own kind of rules and all. But, you have to be, it has to be uniform throughout the thing and all. So, the standard rules are like this, when it comes to identifying. Suppose, if nothing is given in all, suppose a formula is there like this.

So, in the text book or somebody gives you a formula, like this. So, how to put parenthesis and all? So, that you avoid this particular kind of confusion and all. So, now here are some of the rules, that we followed. So, apply the connectives and inserting the parenthesis, if needed in the following preferential order. So, we have said that there are five logical connectives and all. AND, OR, implies, if and only if, negation, etcetera and

all.

And now, among these five logical connectives, preference should be given in the following order and all. The first preference should be given to negation and the second one, it applies to the shortest preposition to its right. And whatever preposition, which is immediately following the negation within which that, it has scope and all. And connective applies to the shortest preposition at each side of it, because it is a binary connective.

Because it connects both the things and all or applies to the shortest preposition again and the each side of it. In the same way, implies double implies etcetera. There are all binary connectives, because of in the binary connective. So, it is because is binary connective. So, it connects the immediate sentences on the left hand and the right hand side it. So, if at any time you are with repeats of the same connective and all, group them and you work them from left to right and all.

We follow some particular kind of convention and we group them in certain way and all. Like in this example, A OR B OR C. The repetition of or connective here and you move from left to right and all. And you put brackets appropriately and all. So, we will try to see some examples and all. And we will apply, this particular kind of thing. This is the most important thing.

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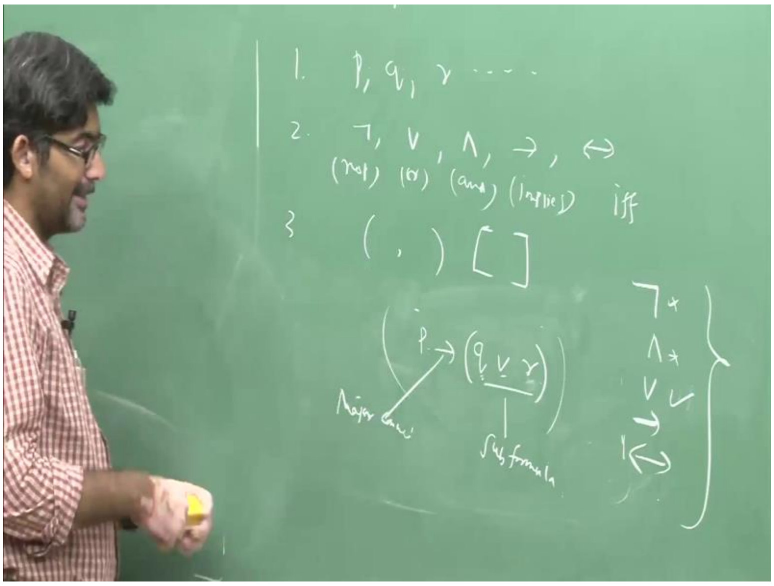

Otherwise, there will be confusions like this one. You should be, it would be read as p implies q OR r. Or it can be even read as p implies q OR r. So, we need to follow some particular kind of convention and then. First, we will talk about this simple example. And then, we will move on to the complex example and all. So, what is that we are trying to say? First preference should be given to negation, and then conjunction disjunction implication and if and only if and all.

So, now we are to apply this particular kind of things. Since, negation does not appear here. So, we need not have to worry much about it. So, now the next connective, next preference should be given to AND. That is also not there. So, you not have to worry about it. And we will take some examples which involve, these things also. Now, the next immediate preference should be given to the OR connective and all. So, now what we need to do is...

What we have said here? This is the OR connective combines the shortest proposition on the left hand side of it. So, that is q is the shortest kind of proposition, which is in the left hand side. And r is the preposition, which is the shortest kind of preposition which is on the right hand side of it. So, we will put bracket like this. So, this is taking care off. So, now the next in the order we have implies. So, now we need to put another bracket and

all.

So, p implies q OR r. So, now we need to put brackets for p and the whole formula here. So, in order to distinguish it, we use even you can use square brackets also, to separate it. Otherwise, you know it is becomes impossible for us to read the things and all. So, one can use even square brackets or may be some other things as follow. So, now the whole thing. Now, implication should be given priority and all. So, now this is going to be the correct parenthesis, of this one.

So, one need not have to... Sometimes, one can omit this parenthesis as well. So, even if you omit this particular kind of thing and all, it does not make a big difference and all, because you can read the same thing. It means p implies q OR r. So, that is what we mean by the formula, which we have written before. So, there is another way in which it tells and all. So, in a given well formed formula, the last preference that we have given is, if and only if or not.

So, whenever you have this particular kind of connective, that particular kind of connective which comes at the end, is going to be the main logical connective. So, it will be definitely useful, especially when you are evaluating a given well formed formula to be true or false, tautology or contradiction etcetera, using truth table and all. One needs to know, what we mean by a main... What do you mean by the main logical connective and all?

Usually, the main logical connective here is, this one. So, this connects the whole formula and all. Main logical connective means, it is the connective which combines the maximum number of propositional, variables and all. So, now here is a formula, which connects only two variables and all, q and r. So, this is a sub formula. So, this is considered to be major connective and all, because it connects one p q and r also.

So, three variables it is connecting at all. So, that is why it is considered to be the major connective at all. So, this is the simple example, in which you put parenthesis and all. So, one can talk about some kind of complex example, in which it has all these connectives and all.

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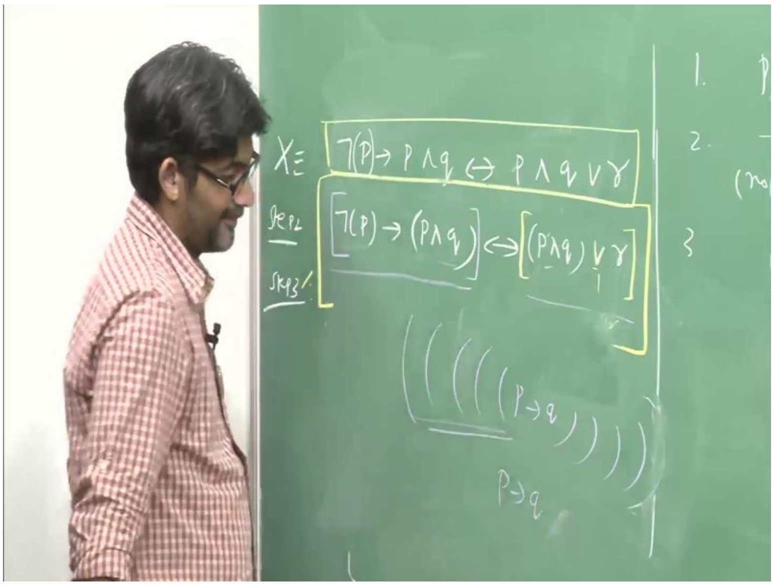

So, just I am writing, whatever comes to my mind and all. Then, we will see what we mean by p AND q OR r etcetera. So, here is a formula, well formed formula like this. Suppose, if the parenthesis is not there at all, it can be read in thousand may be four and five different ways etcetera and all. Ultimately, you may not reach any consistence to, what we mean by this particular formula and all. So, this conversion is very important to judge, whether how to read this particular kind of formula and all.

So, now we apply this kind of, preferential kind of ordering and in, the first preference given to negation and all. So, in the first step what we do is, this one. So, there is no negation here. So, this is the one which we have. So, negation of p AND. So, the first one is taking care of. This is the first step. So, now in the second step the preference should be given to AND correctly. So, now in the case of this one, what we need to do is...

You have to follow some kind of connection and all. Either, you move from left to right or you move from right to left and all. Usual conversion is that, you move from this to that. Then, you come across conjunction here and here also. Repetitively, we are using this particular kind of thing and all. So, two times you will find this conjunction and all. That means, you need to move from left to right. So, now in this case, we are applying this preferential order for conjunction and all.

So, it will be like this. So, conjunction rule is that, I mean the shortest preposition on the left hand side and the shortest preposition variable, on the right hand side. We connect this one. So, you need to put parenthesis like this. And then, this is as it is. Now, again you need to put this particular kind of thing and all. So, the shortest preposition which connects this conjunction is p and the right hand side, it is given OR it is r. So, this is step number 2.

So, now step number 3. So, now we have given preference to and all. We are taking care of AND. So, now coming back to OR and all. The next preference should be given to OR. So, I will use different color chock piece and all. So, that you came to know, how to write this particular kind of thing. So, now here you do not have any OR and r. So, now the only OR connective. This is the OR connective. So, now this has to be bracketed and all.

The immediate preposition, which connects these two things. On the right hand side we have OR. The shortest kind of preposition formula, which is on the left hand side of this particular kind of connective is p and q. It is not take all the propositions into consideration. But, this considered to be shortest kind of propositions and all. So, now you put in brackets like this. So, now there is no OR here. So, now you have take in care of this one.

So, now coming back to implication and all. So, now again you apply the same rule and all. Then, you say that whatever connects this one, on the left hand side the shortest proposition is this only, negation of p. On the right hand side, you have p AND q. So, now you put a connective like this. So, now implication is taking care off. So, now till it is not at over and all. So, now there is one more connective, that is implication double implication and all, which stands for if and only if.

So, now the shortest proposition on the left hand side is, the whole thing and all. And the shortest proposition on the right hand side is, the whole thing p AND q OR r. It is already a kind of complex proposition. So, now the whole the parenthesis now becomes, like this. So, now this is the final kind of thing and all. So, it should be read as NOT p implies p AND q if and only if p AND q OR r is the case and all. So, in this way we can decide.

So, what we how to read this particular kind of formula and all? If you do not have this convention and all, there are many ways which you can read this formula and all. For example, NOT p plus p AND q implies p and then q OR r. That is one way of reading it or you can read it in this way, NOT p implies p AND q if and only if p AND q OR r. That is one way of reading. There are several ways, you can read it and all. In order to avoid the confusion and all, we follow parenthesis and all, like this.

So, now once you have done this particular kind of thing. So, we can safely omit some of the parenthesis and all, because excessive information also causes some kind of unnecessary occupation of information and all. In computer science language, there are all characters and all. So, if unnecessarily put, so many brackets and all. Like for example, p implies q. You put it in several brackets, like this. So, waste of energy and all.

Although, it is consider to be same and all. Three brackets here, of course four. one two three four etcetera and all. So, left and right brackets, parenthesis matches and all. We can write p implies q, like this and all. We can omit all these brackets and simply say that, p implies q itself. So, it is saves our space and all.

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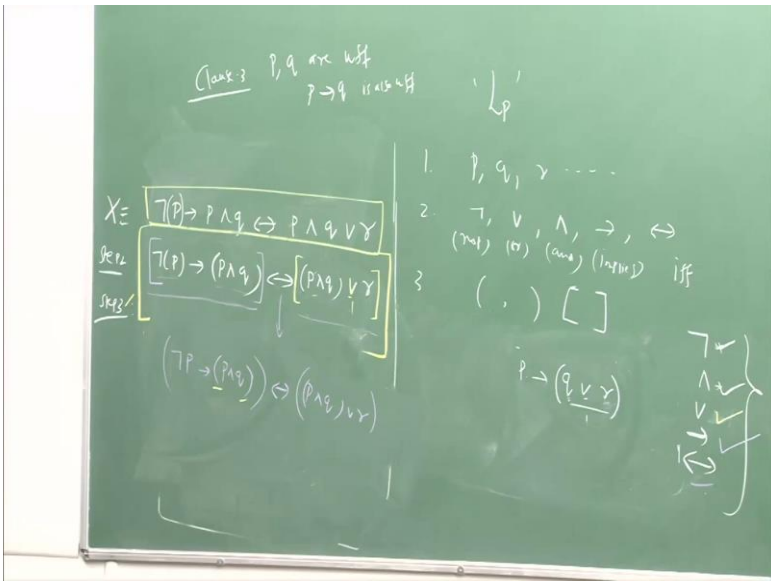

So, in this way the main logical connective here is this one, if and only if and. So, although we give least preference to this particular kind of thing but, this is going to decide whether this formula is going to be true or not. We will talk about this, little bit later. This is also considered to be the major logical connective. In the sense that, it connects maximum number of prepositional formulas and all. So, now in this case you can omit parenthesis in this way, naught p.

You do not require any kind of brackets here. And now, you can even omit this particular kind of thing also. So, now you can write p AND q. And put one bracket here, implies p AND q OR r. And this whole bracket need not have to be put at all. You can remove this particular bracket. And you can simply read in this particular kind of thing and all. So, this brackets should match and all. Otherwise, computer show some kind of synthetically error and all.

So, now what we discussed so far is simply, like this that. We can generate n number of meaningful, n number of strings and all. But, not all strings are considered to be well formed formulas, that is the first thing ((Refer Time: 38:22)). And the second thing, that we have discussed in detail, with some examples is that. When the parenthesis are not given, how to decide how to read a particular kind of well formed formula and all?

At least, often leads to confusion and all. But, if you follow this particular kind of convention, then till simply our thing and all. This is one way of reading this particular kind of thing and all. So, usually in good standard text books and all, this convention is already given and all. Usually, parenthesis are already given and all. If you do not give it and all, then if I follow some convention I read this formula, in this way ((Refer Time: 39:00)).

So, what happens if someone comes, someone follows a different convention and all, like they reverse the same thing ((Refer Time: 39:07)) For example, first preference they will given to if and only then implies OR, AND, then double negation and all. It might lead to some other kind of formula and all. So, that is a reason why, it is always good to state. It is important to state this parenthesis appropriately and all. Otherwise, this will be a confusion of reading this particular kind of formula and all.

So, when there are groups of connective like this, which you find it. Then, there is a way again convention is that, you move from left to right and all. So, there are all conventions only. But, one can follow one's own convention, to find out what we mean by a given formula and all. But, what you mean by, what exactly a how this formula is well formed formula and all.

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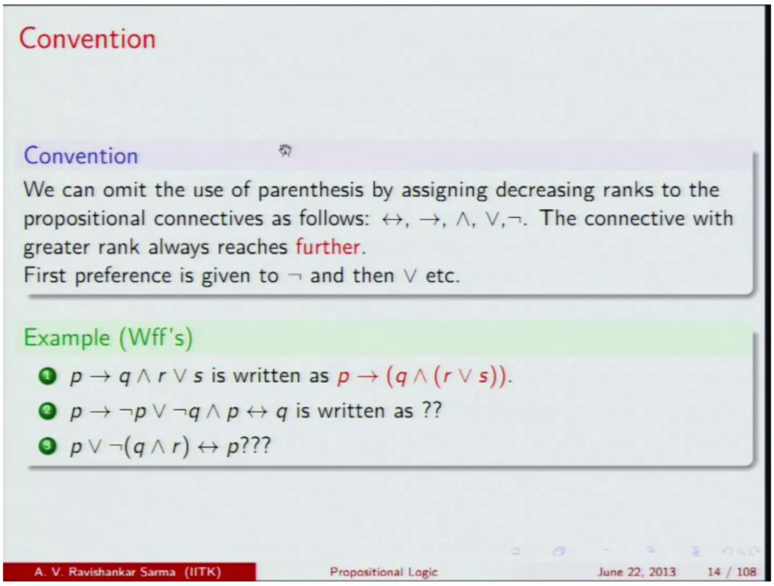

So, these are some of the things which we have discussed and greater detail already. The convention here is that. We can omit the use of parenthesis by assigning, some kind of decreasing ranks to the prepositional connectives, as follows. So, if you find if and only if, you can safely omit the parenthesis and all, like the one which we have done there. And the next immediate thing is, this particular kind of thing and all ((Refer Time: 40:24)).

In this case also, p implies q OR r. You can put one bracket here. But, you can omit this particular kind of parenthesis and all. You can just simply state, p implies q OR r. You can be simply read in this sense. And the next in the decreasing order of rank, which is p AND q. And then, next one is r and negation and all. So, the connectivity with greater rank always reaches further and all. Greater rank in a sense that, if and only if.

It set reaches further and all. It corrects as many sentences as possible and all. Prepositional formulas are as possible. So, in the examples that are there here, in the slide. So, example if you have a sentence like p implies q AND r OR s. This can be written in this particular kind of thing, I will just write it and then come to know.

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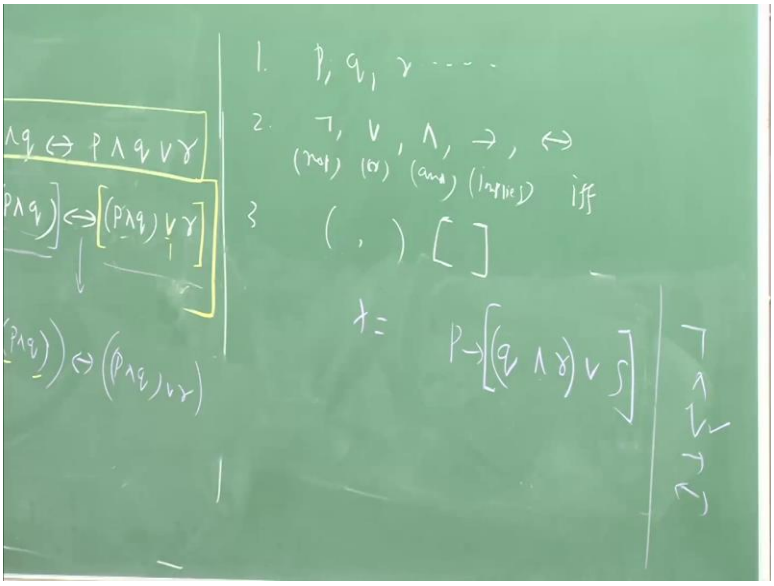

And then, talked about p implies q AND r OR s. Suppose, if you have a formula like this.

So, now again what we have done is, we have to follow this particular kind of order, implies AND if and only. So, the first thing which we need to take care of is connective AND and all. So, you put bracket like this. And then, followed by that, you need to give importance to this particular kind of connective. And then, you put bracket like this in the second step. So, this is taking care of OR.

And now, what need what is left is? P implication and all. So, now this should be taking care of. So, this is considered to be the formula in this. p implies q AND r OR s and all. So, now even if you omit this parenthesis and all, this one, it is not going to make it different and all. It is same as this one, p implies q AND r OR s, this one. So, in this way you can omit the paragraph parenthesis and all, in order to see some kind of space and all.

So, likewise you can draw tree structures and they can say that these two formulas are different and all.

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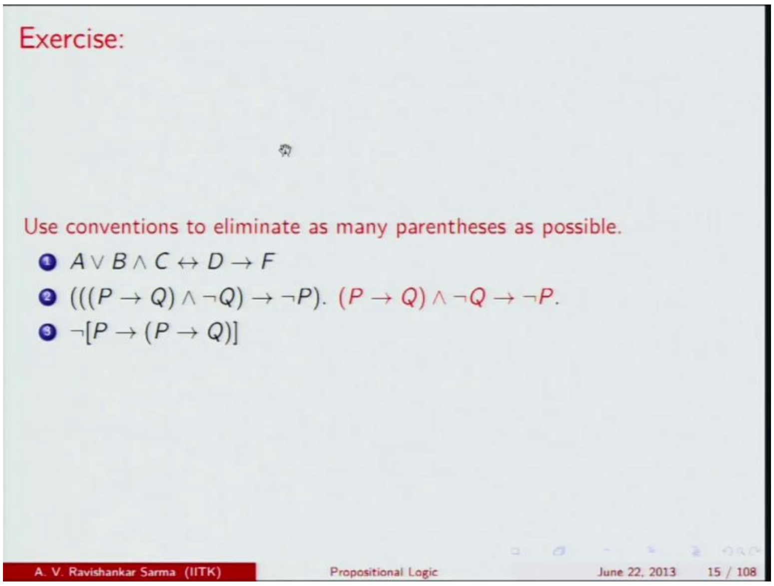

So, you need to use some kind of conventions to illuminate, as many parenthesis as possible and all. The conventions, that we have mentioned earlier. Unnecessary, if you put too many brackets and all, it will occupy unnecessary space and all. So, better to omit this parenthesis by using, that decreasing rank kind of thing and all.

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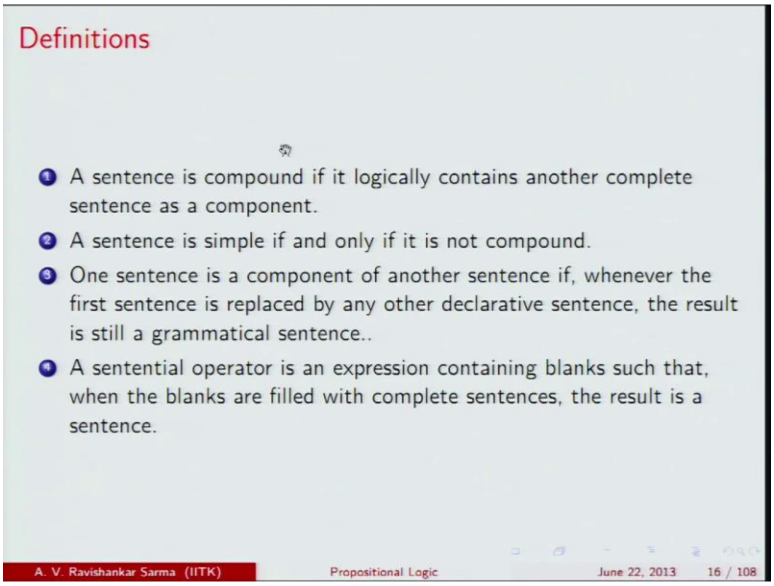

So, now there are some definitions which we need to note. So, what we have discussed so far is, what we mean by well formed formula? How to form a well formed formula? There are some rules for it. And then, how to read a well formed formula and all. So, now there are some kinds of definitions, which we need to note. Usually, p's, q's, r's etcetera are considered to be atomic prepositions, when they combine with some kind of logical connectives. They form compounds sentences and all.

That means, a sentence is consider to be compound, if it logically contains another complete sentence as a component and all, like p q r and all. It consists of another sentence q and all. So, in these two sentences combined together, you will form some kind of compound sentence and all. It is like atoms combine together will form molecules. Molecules combine together, will form compounds etcetera and all. So, like this in analogy you can have it here, in the case of propositional logic.

We have started with basic units. That is atomic sentences, is stands for p's, q's, r's etcetera. And then, we combined with some kind of logical connectives and then form compound sentences and all. A sentence is which, is considered to be simple if it is not compound and all, like you know p's, q's, r's, etcetera and all. You know simple sentences and all. So, one sentence is component of another sentence, if and only if. Whenever the first sentence is replaced by another declarative sentence, the result is still considered to be a grammatical sentence and all.

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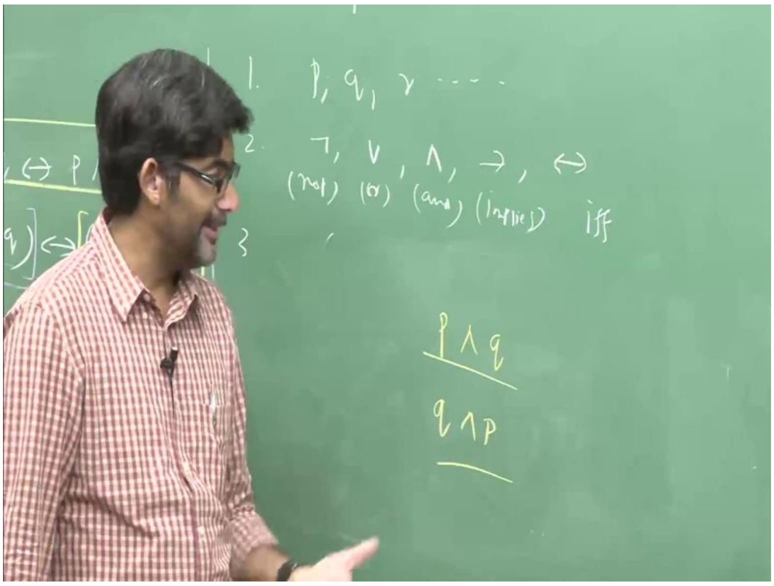

For example, what it essentially says is, this particular kind of thing and all. Suppose, if you have a compound sentence, like this p AND q. So, now you replace q with p and p with q and all. So, this will become q AND p. So, these two are logically identical to each other, which will talk about little bit later. Well, the structures of these two sentences are same and all. So, it has the same tree structure and all, like this p and q and all.

So, we will talk about little bit later. But, what I am trying to say is, this that. So, if you replace p by q and q by p and all, and it will become q AND p. We use commutative property p AND q is same as q AND p. This may not be same. This may not be, may not apply to day to day discuss and all. One simple example could be like this, that ((Refer Time: 45:31)). Usually you know, when you become sick you will go to the doctor and all.

So, this sentence can be put in this way, in a complex compound sentence like this. I became sick and I went to the doctor, that is p AND q. If is the same thing q AND p, I went to the doctor and I became sick and all. Nobody goes to the doctor to become sick and all. So, these two sentences, we mean to different things and all. In day to day discuss but, in the case of propositional logic, they mean the same. Because p AND q is same as q AND p.

They are same similar tree structure and all. They are identical too each other. So, it is in that sense. They are logically identical to each other. So, a sentential operator is an expression, containing some kind of blanks such that when these blanks are filled with complete sentences, the result is also considered to be a kind of sentence, complete sentence and all.

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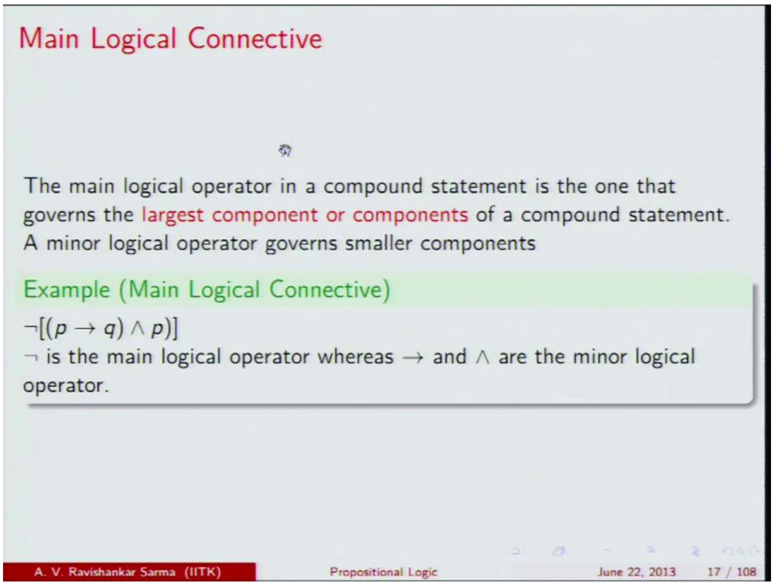

So, now what you mean by a main logical connective and all. So, we have defined already in one of the examples. The main logical operator in the compound statement Compound statement in sense that, it can be p implies q or p OR q or it can be a mixture of all these things and all. A big compound kind of sentence, which is generated by OR AND implies etcetera and all.

It is the one that, governs a largest component or components of a compound statement and all. In minor logical operator, governs only smaller components and all. For example, if you have in this particular kind of formula, which is already there here, in this one.

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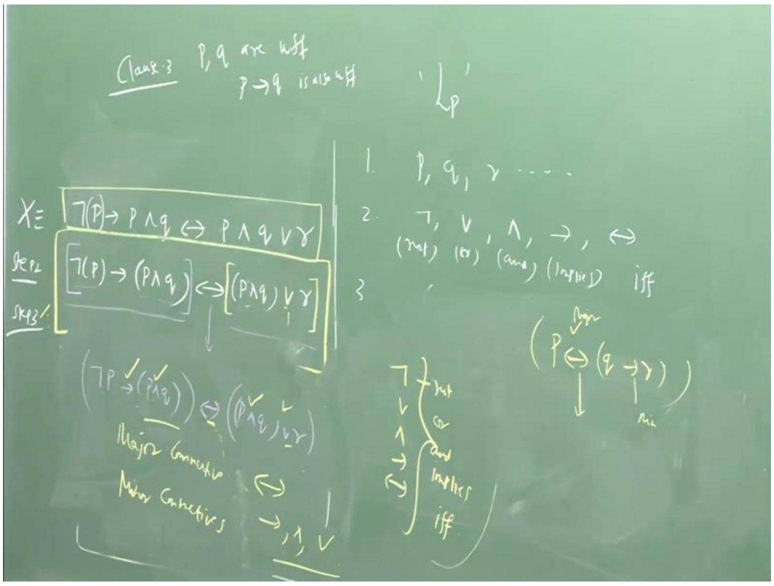

So, the major logical connective now we are trying to find out, what is the major connective here and minor connectives? First of all, what are the connectives? Connectives are these things negation OR AND implies. So, this stands for NOT. Of course, I will talk about this when we talk about semantics in greater detail. But, in this moment even if you know this particular kind of things, it is enough and all.

AND implies OR if and only if, which is used to invoke necessary and sufficient condition or equivalence relation and all. So, the major logical connective is a one, which connects as many propositional variables as possible and all. So, now in this one... So, this is considered to be the major logical connective, because it connects NOT p p AND p q and on the other hand p q AND r. At least, three connectives on the left hand side and the right hand side and two connectives on the left hand side and all.

So, it connects as many propositional variables, as possible and all. So, that is a reason why? So, this is considered to be the major connective and all. So, later it will be very useful. This concept is very useful in the sense that. So, what will be doing is, in order to judge whether a given formula is... There are three types of sentences, which are occur in proposition logic, tautology, contradiction and contingent sets, statements and all.

Under this major logical connective, if you get all T’s etcetera and all. That is considered to be tautology, under this major logical connective. Whatever connective that is here, it is if and only if. Under this, if you get only falls and all, if you evaluate the truth value of this particular sentence and under this major logical connective, if you get all, it is considered to be contradiction. If you get T's F's etcetera, it is considered to be contingent sentence and all.

It is for this reason, we need to find out what we mean by what exactly is a major logical connective and all. So, now the minor logical connectives are the once, which connects has many few proposition variables as possible and all. So, now here this is a major minor logical connective. Here, this is also. There are sub formulas and all. Whatever connective that occurs or figures out in the sub formulas, is considered to be the minor logical connective and all.

Here is the one and here is another one and all. All the sub formulas whatever connective is there, that is considered to be the minor connective and all. So, these are all minor connectives. So, in this particular kind of formula, it is not you know. This is a major minor and all. In general, but with respect to this particular kind of formula, this form this symbol if and only if is considered to be the major logical connective and all other things are minor logical connectives and all.

So, exactly the reverse order follows in all here. So, that is whenever you have this particular kind of thing that, will serve as the major logical connective. Next, possible thing is this one. And then, AND OR of same ground and all and this is a negation and all. Example, if you have a formula like this, p implies q implies r and all. So, you have something like this. So, now in this one, the major logical connective is this one.

Major one and this is the minor kind of connective. So, now the reverse order follows in this particular kind of thing and all ((Refer Time: 51:05)). So, now we have identified what we mean by. What we mean? We are seeing, what is the major logical connective and minor logical connective? This we use it in the semantics in particular. That is evaluating the truth value of some particular kind of sentences and all.

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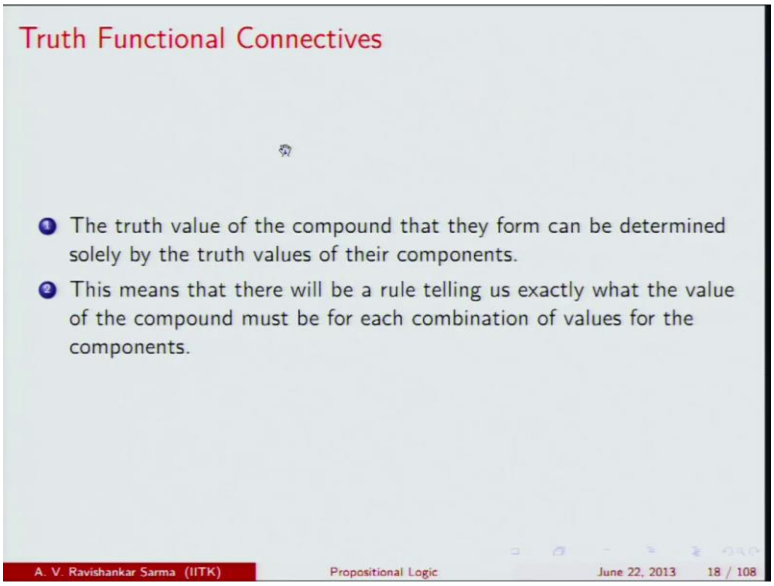

So, now coming back to we have defined, what you mean by well formed formulas and how this well formed formula are generated. And we came to know, how to read this well formed formulas etcetera and all. And then, we also came up with some kind of convention with, which in case you are not given parenthesis and all. How to insert this parenthesis etcetera or how to omit this parenthesis? Unnecessary parenthesis etcetera, needs to be omitted to save some kind of information etcetera.

All these things with which we have discuss so far. And we also identify, what we mean by what is considered to be the major connective and the minor connective and all. So, now we will move on to different kind of thing, which is called as semantics of the propositional logic. That means, we are trying to define what we mean by this formulas and all. So, it is like the analogies like this. Syntax is like producing things.

In the case of, in the language of marketing language, we have just... You start producing the things and all, without knowing the complication etcetera and all. Just production is considered to be syntax, just like that is what we have done here. We have some kind of alphabets. We combine them in certain way. And then, you are saying that, it is a well formed formula. And some other strings are not well formed formula, etcetera and all.

And in the market language, we have distribution. So, distribution corresponds to semantics and all. So, we need to know how to distribute, what you produced and all. So, for that we need to know what we mean by what you produced and all. So, there is what this taking care by semantics and all. So, now the connectives that we are talking about, there are only five connectives, naught, OR, AND, implies, double implication, etcetera and all.

So, these connectives are considered to be truth functional connectives and all. So, why they are truth functional connectives? It is because of this thing. The truth value of a compound statement that the form that means, p plus q, p OR q, p AND q, etcetera. It can be slowly determined by truth value of it is individual components and all. So, this is the main idea proposed by ((Refer Time: 53:40)) particular in which, principle of compositionality.

According to which a compound formula gets it is meaning, only if you can evaluate the truth value of it is individual constants and all. If the truth value of a compound sentence is slowly determined by the truth value of it is constituent. Then, those truth functional connectives are also called as extensional law. So, this means that there will be a rule telling. Rule which tells us exactly, what the value of the compound, must be for each combination value for components have.

So, what you all need is, the truth values of the individual constituents and all. With which you can judge, whether compound formula which is generated by this atomic proposition, with a help of this logical connectives is true or false and all. So, we will postpone this discussion to the next class because, we will be dealing with semantics in greater detail. So, what we discussed in this class is simply like this that. You know, we represented a kind of minimal language for the propositional logic.

So, that is the formal kind of language in which it has its own syntax. So, how we generated this syntax, because it is we generated syntax in this way. We started with a language, which consists of propositional variables and then logical connectives and parenthesis. And these logical connectives and propositional variables combined in a certain way. It will form some kind of strings. But, we said that, we have said that, not all strings are well formed formulas and all.

Only strings which are generated by means of definition of well formed formula is considered to be, what we have calling it as a well formed formula and all. Once we generated the well formed formula, we have seemed that how to read this well formed formulas and all. So, there is a convention which we followed. And then, we gave some kind of preference to this connectives negations etc and all. So, that it makes life simpler and all.

Otherwise, there will be a confusion of reading the same formula in different way and all. And other thing, which we noted in this class is that, every well formed formula has its own unique structure and all. Suppose if the tree structure of two formulas are same and all, that means you are talking about the same formula and all, like p AND q and q AND p. It has the same tree structure and all, in no way different at all.

But, in day to day disclosure, we mean totally different things and all p and q. For example, I went to the doctor and I became sick. Certainly different from, I became sick and I went to the doctor and all. We mean totally different things. In the next class, we will going to the details, what we mean by this well formed formulas and all. That is taken care by semantics how prepositional logic.