Introduction to Logic
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Lecture - 17
Logical Connectives: Truth Tables

Welcome back. In the last lecture, we presented syntax of propositional logic, where we have to seen how to construct well form formulas. And not only that, we constructed well form formulas, we also came to know how to read a given well form formula and all with the help of parenthesis etcetera. And we gave some kind of preferential altering for the connectives not end implies are etcetera and all. With that, you know you came to know how to read a given well for formula, especially when the parenthesis are not given enough. But you know usually we find that you know, parenthesis are usually given and all, in case if it is not given we need to follow those particular kinds of rules.

So, in this class we will be talking about semantics of propositional logic. So, what will be doing simply is that, we will talking about what you mean by the well form formula is that we have a formulated area. For example, if you have come up with a formula a p implies plus q implies plus r for example and what you mean by that formula. So, we go with yes lower that, meaning means giving truth conditions true a given formula and all. So, following will be going in to the details of the semantic of propositional logic, with an idea that, meaning of a given formula means, mean it giving truth conditions and all.

So, in this lecture, I will be focusing on 1 simplistic method, which is called as truth table method, which is very simple and very easy to use. So, it works for, it works especially when the number of variables propositional variables are less; that means, 2 or 3, it works better. Of course, it works even more than 3 variables also, but it difficult for as to formulate, difficult for us to construct truth table and all. A computer can easily do it, when the number of propositional variables is more than 5 or 6 etcetera. So, today will be talking about the semantics of propositional logic.

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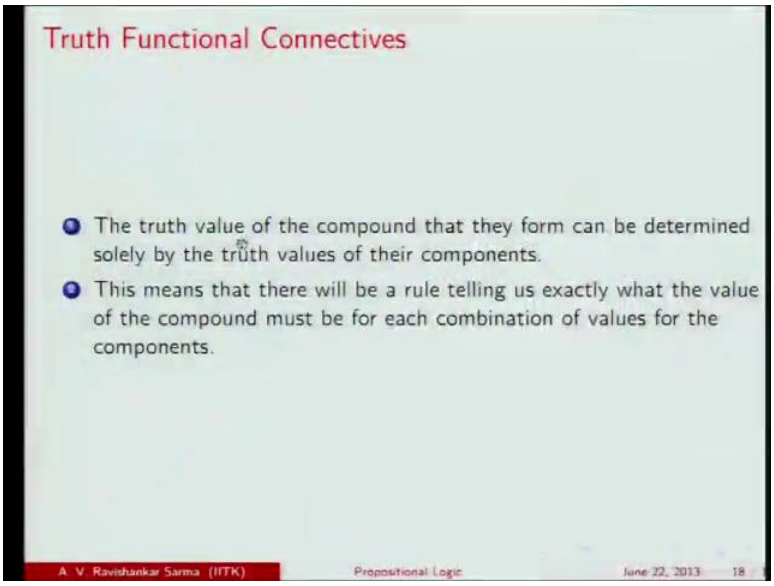

So, there are 5 connectives that we have discuss so far, that is, negation and or implies and double implication and all. So, what you mean by these connectives and all. So, the semantic will take care of is particular kind of thing and all. So, before going any further, we need to talk about these connectives, in the sense that, these connectives are all truth function.

So, why these are truth functional which is because, of this that the truth value of the compound statement that is generated out of this propositional connectives, can slowly be determine by the truth value of its individual components enough. For example, if you have a formula p implies plus q, the truth value of p implies plus q you can be slowly determine by, whatever truth values the propositional variables that occurring the formula that is, p q etcetera takes.

Suppose, if p take value t and q takes value f, the truth value of the compound formula p implies q plus q takes a value f. So, this means that, there will be a rule telling which tells as exactly what the value of the compound must be, for each combination of values for the components enough. So, in this if the components take some values and all, you can determine the truth value of a compound sentence. It is in this sense, these connectives that we have to time discuss are construct to be truth function connectives enough.

So, if the truth value of a compound formula is not slowly determine by the truth value of its constituents, then it is not considered to be a truth function connective enough. So, they are many non truth functional, the connectives which are used in a non truth functional way. But we are not going to talk about those particular kinds of things and all.

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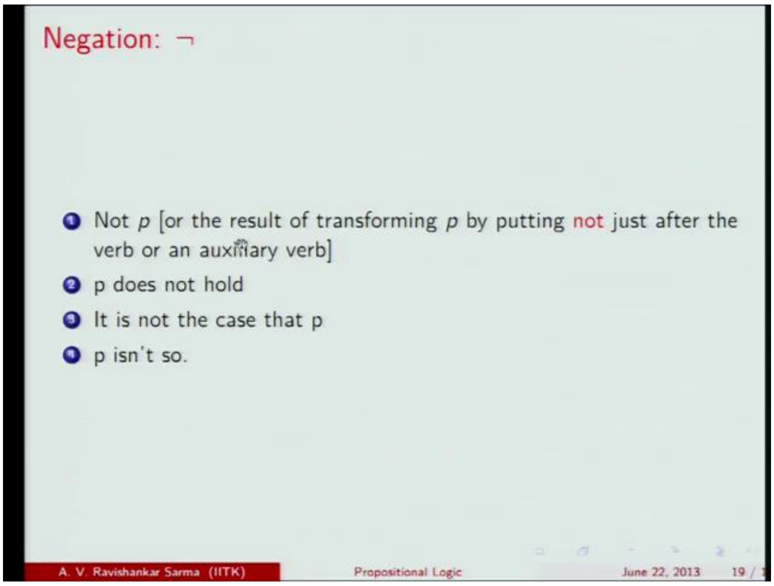

To start with, a simplistic kind of connection that we had logical operator that we are try to talk about, is a negation. So, negation is simply represented as not p, which simply results especially when we transform a given sentence to its negation and all. For example, if you say it is rainy, then the negation of that 1 is; it is not raining and all. You have to put just not before the verb or some kind of auxiliary verb and all. All may are mortal than the negation of that, 1 all manner not more than enough fix.

So, it is good as saying the same thing; p does not hold enough. Suppose something as a property p metal expanse upon eating and if it does not hold any say that, metals does not expand, this metal does not expand upon eating enough. That particular property does not hold for this particular kind of thing and all. Or if you want to say it is not the case that p enough. It is not the case that, this is a choke pieces or something like that.

So, in the same way, if you represent p is it, so you have to represent in terms of negation. So, the negation some kinds, it is used with this particular symbol, sometimes we use this particular kind of symbol.

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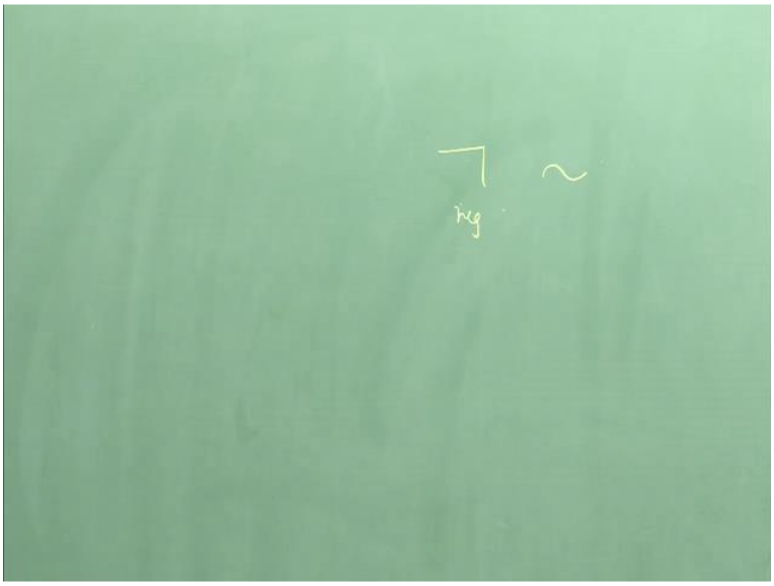

It stands for negation only. In some text books it is written in this sense. So, these 2 are same symbols. So, it is only for our convenience, we are using this particular kind of thing.

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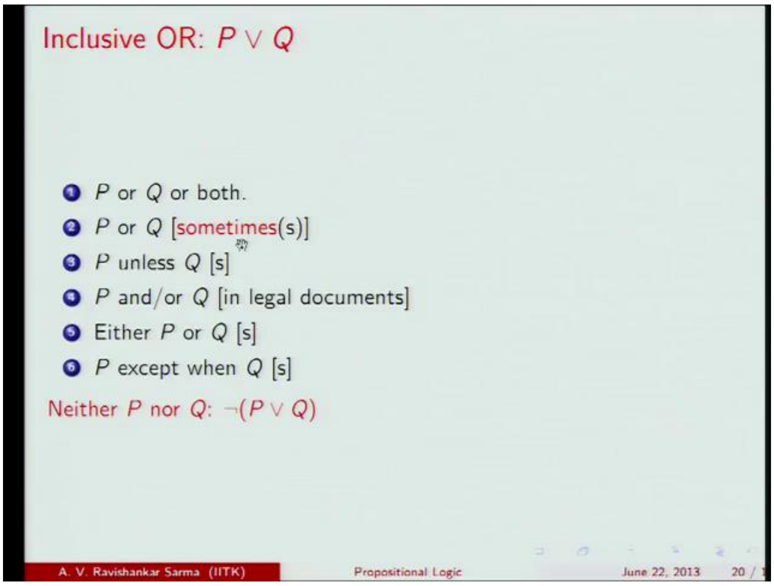

The next connective is or can be used in to different senses, 1 is inclusive sense and other 1 is exclusive senses, understandably it is stated in classical logic, at means the propositional logic, we treat it as inclusive or out. So, what is inclusive or? It is simply P or Q. It is raining or it is not raining. Or it can be both; it is raining and it is not raining, it can be both also like, its pay tea or coffee or both of them. Or it can be simply represented get as; P or Q are P unless Q is also represented as P or Q. And some times in the legal documents, we write this kind of language, where you write like this P and oblique or Q. It is use for some specific kind of purpose in all.

So, we did not going to the details of this legal documents etcetera all. Sometime the use this kind of long language and; that means, it means P is a P or Q or you can be both coffee or tea a coffee and tea. Either you can be satisfied with coffee, you can satisfied with tea, or even if you serve both of them, it will work for us. So, P except when Q; is also represented as P or Q and all.

So, if you want to represent the negation of P or Q, you have to say it in an English language as; neither P nor Q. It is not the case at, the P or Q. So right now, we are just try to talked about what we mean by this connectives, at what exactly this connectives are. Then, will talk about the meaning of these connectives enough; that means, how these connectives behaves, in the formal logic that is the propositional logic, that we are trying to discuss.

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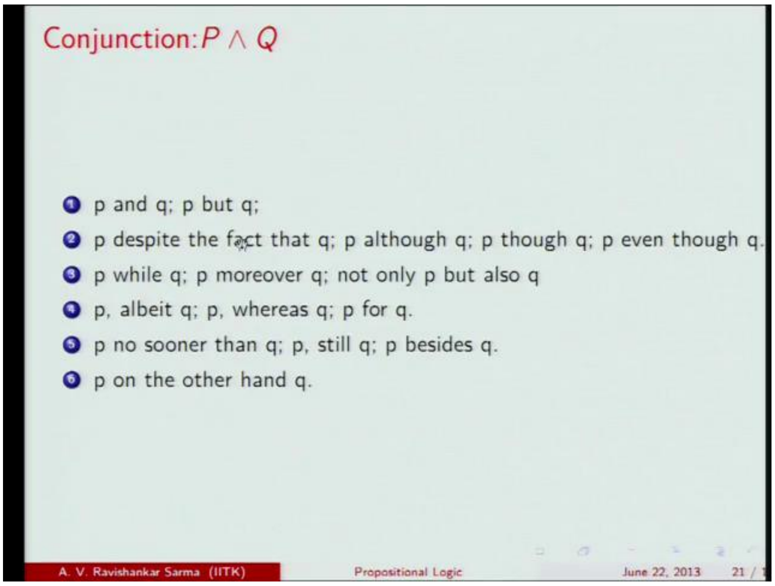

In the conjunction this you come across like this p and q or p, but q is translated in to p and q. And this is the list of things and all, which can be translated in to simply p and q about. p despite the fact that, q a p although q, p though q, p even though it is q, etcetera, all this things are translated into conjunction where all that is p and q, p where as q, p for etcetera, p no sooner than q, p still q, p besides q, all this things which you are come across in date to day.

So, these are all translated into simple translation, that is, p and q, p on the other hand q and all, which commonly use in the scientific discourse, is also translated in to p and q. It is translation kind of list enough, 1 has to go through this thing in greater detail. Once we solve the a examples, then will recruit our self better than with this translation.

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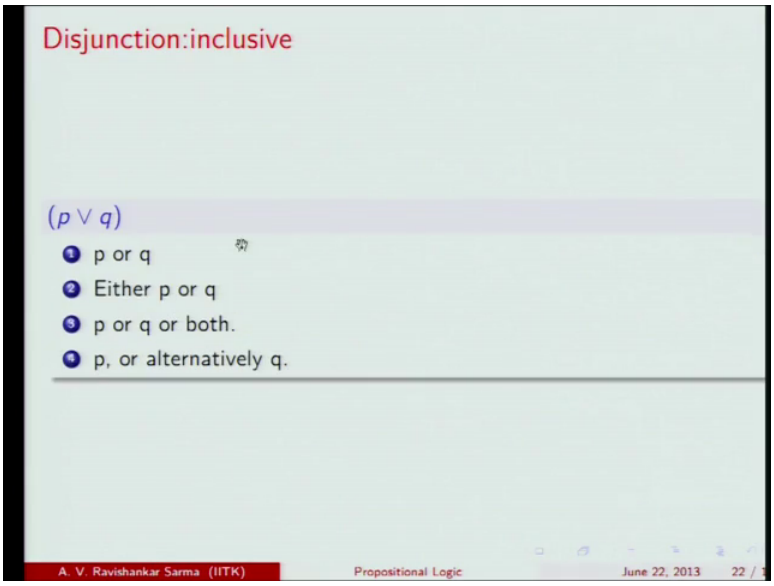

So, this is also called as disjunction and all; either p or q and all. It is simply either p or q are p or q are both p are alternatively q, these are simple things.

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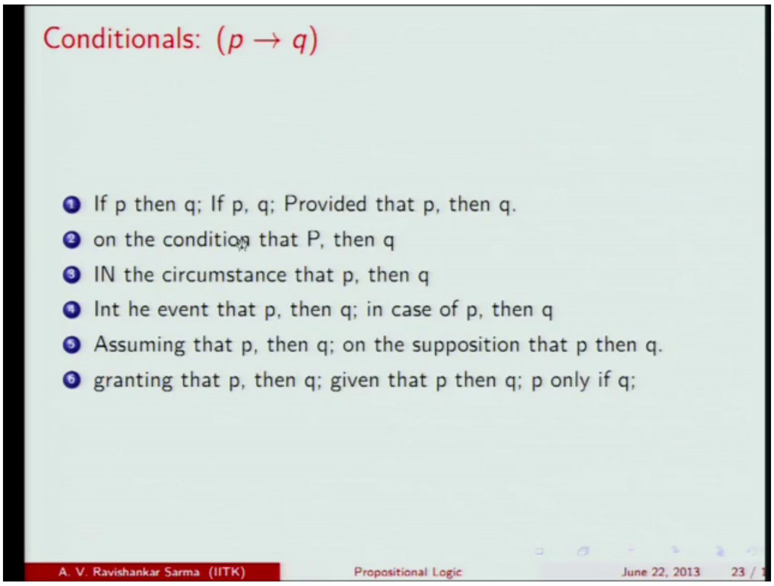

So, now, the conditional which is represented as p implies q.

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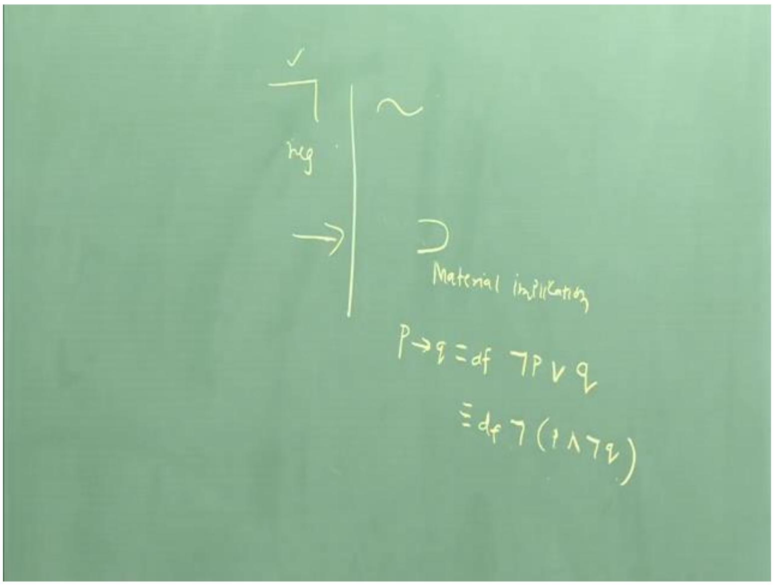

In many text books, we implication is used in this particular kind of sense; this is called as material implication. So, in many text books; so we will find this particular kind of symbol, but it is convenient for us. So, we are using these things, for negation we use this, for implication we use this. So, p implies q by definition means; it is not the case that p or q. Or if you applied demorgan's rules for this, 1 then it becomes it is not possible that p and q enough.

So, this is same as this, this not p negation of this is conjunction will become disjunction, in negation of negation of q will become q n, these are all 1 and the same. So, whenever you come across material implication, this is what we been enough in propositional logic, we take in to consideration this particular kind of definition for the implication. That means, implication can be expressed in terms of disjunction enough or in implication is expressed in terms of or operator.

So, all this things come under the category of p implies q. Example in the circumstance that p, then q in the event that p and q and the case of p then q etcetera. Assuming that p then q, on the supposition that, p then q etcetera and all, all these things are translated in to a simply p implies q enough. In p implies q p is usually construed as an antecedent hypothetical situation, if that said get satisfied and the q follows enough.

So, how did it is a the material implication is distributed to came with this idea.

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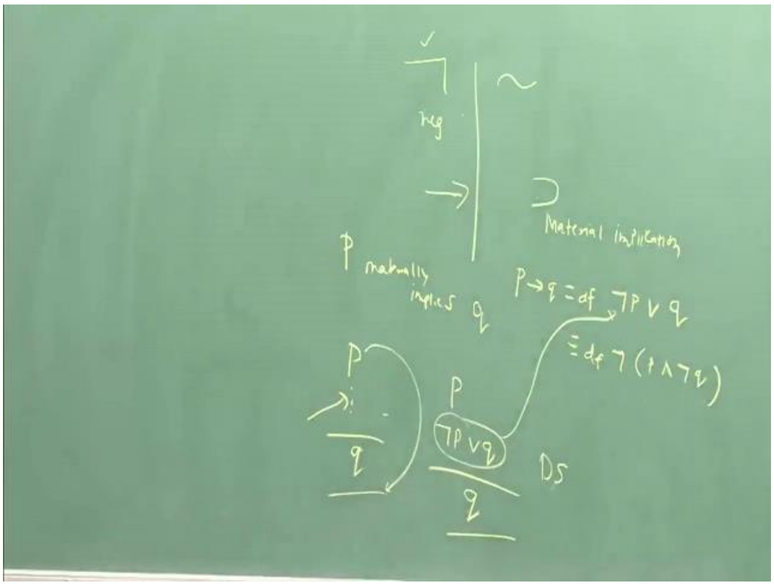

How is p materially implies q. So, he was; that means, some hope p q and or. So, what substitution 1 has to make so that, p materially implies q enough. They were testing we so many a different kinds of formulas here, in ultimately what feasted well here is this particular kind of q an or... If we can substitute in set the missing proposition, compound proposition that is not p or q, then these 2 by disjunctive syllogism its 2 q. So, it is because of this particular kind of thing to they thought, so this is the definition of this. p materially implies q, in no other way, then is particular kind of thing.

If p is the case not p or q is the case, then only p materially implies q. So, way of transform, so way of moving from p 2 q enough, see it is in this sense; is not p or q should as definition of material implication.

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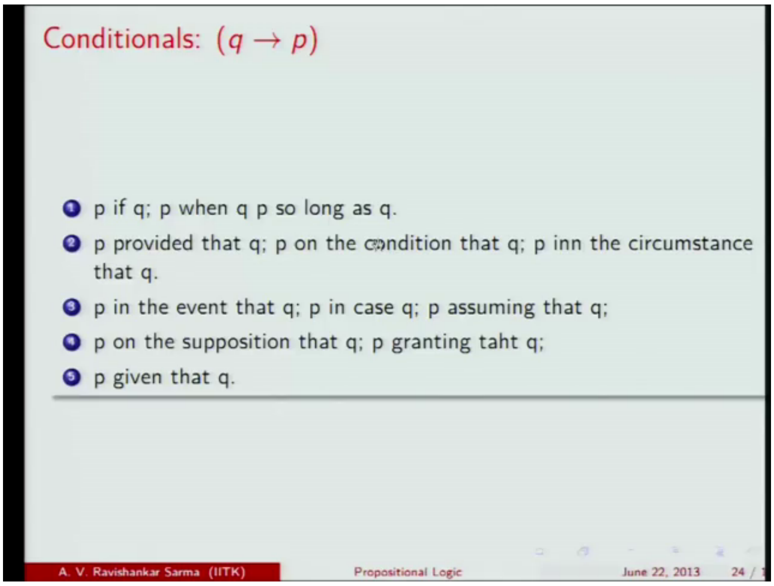

So, this is what we mean by this see p if q. So, these are translated in the reversed, instead of p in plus q we translated as q implies p. So, this is things which we come across in day today discourse, enough p in the event that q, p in case q a assuming that q etcetera, all this things a translated in to q in plus p enough. So, we need to go in to the details of these things a little bit later. It will be boring, if i go in to the details of each and everything, if a read out all this things.

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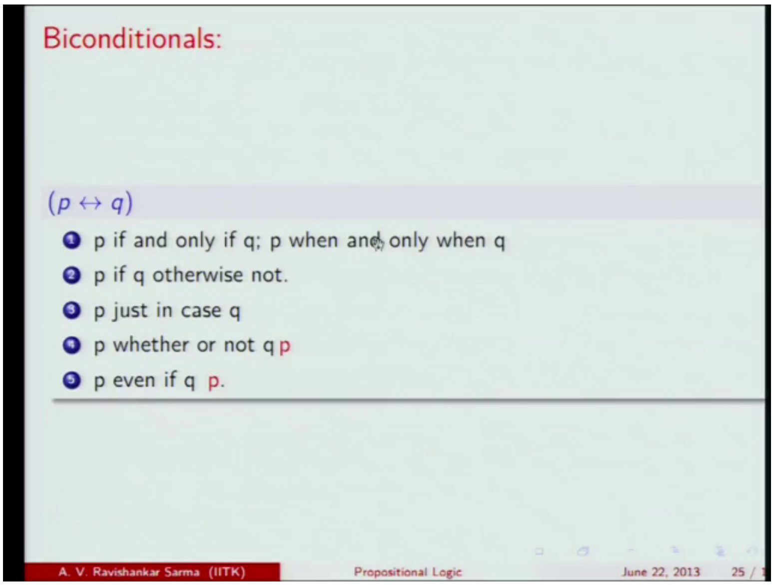

So, now by conditions now, just we are training see what this conditioner; what this connectives are all above. Then, we tried to talk about meaning of these connectives enough, with the help of truth stable. So, p if an only if q, this kind of by conditioners are usually used in natural sciences especially, when you are training invoke necessary and sufficient, can be since or when you try to bring, bringing equal between 2 things, when you want to say that p and q are logically identical to each other. And you will say p if an only if q. So, it is simply represented as p if an all if q. Then, in some text books it is return in this sense.

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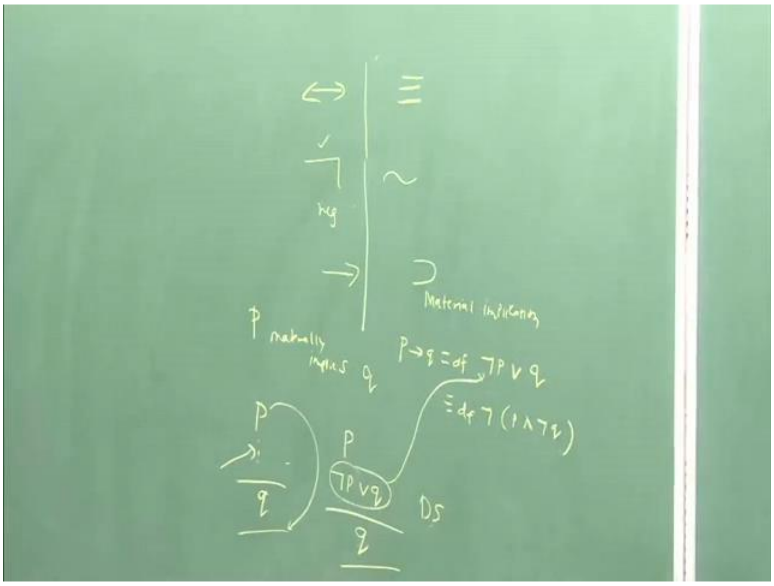

Now, we are using this by conditioner. In some text books, it is simply represented in this these 2 symbols are 1 of the same. So, all this things come under the category of whenever you find p if q otherwise not a p just in case q, p whether or not q, p even if q all this things, will be translated into p if an only if q enough within go in to the retails of this things little bit late.

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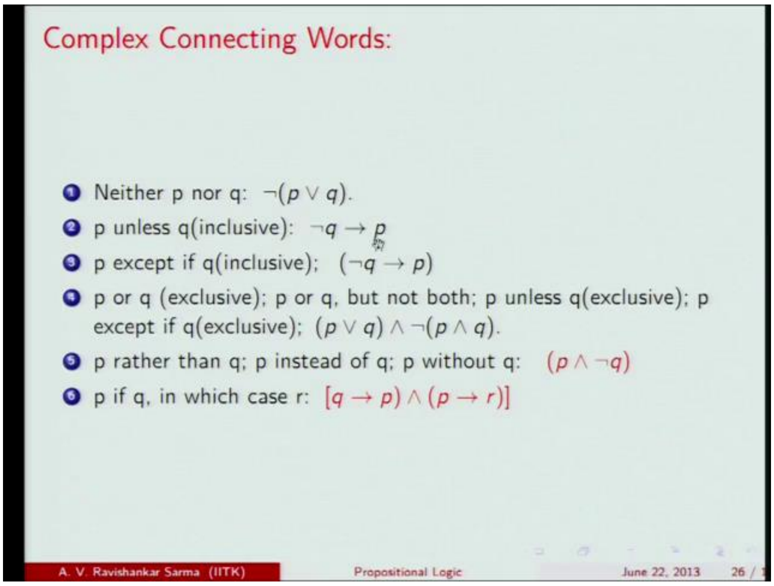

So, let us considered some of the examples, how we can translate that accruing in the English language into language of propositional logic. The first 1 is neither p nor q, it is exactly is the negation of p or q. So, that is we are just put negation before p or q. So, now, p unless q, if you take it in inclusive sense, it is simply not q implies p and all. If you want to say p except if q, again its used in inclusive sense, it is retain the sense not q or, not q implies p enough, p or q if you want to say expression transit of exclusive operator; that means, 1 exclude the other 1 other possibility enough.

For example, in the parties, usually you will say we are organizing some kind of function, the halls for example, you will put only you will put restriction in such that, either you have to take soup or salad in for example. That means, if you take soup you are not suppose it take salad, if you take salad you are not suppose take soup salad. The 1 excludes the other possibility.

In the same way you put ice cream or some kind of sweet an or, if you take as ice cream an or suppose takes sweet put, a sweet even not suppose you take ice cream enough. But if it is usually increases sense and all, if it is little bit flexible enough, you can take either ice cream or even sweet. Suppose is as ice cream is finished, you can even cake sweet also. So, inclusive or has at fix flexibility in; that means, p or q and even both of its there is p and q also. So, p 6 translation is like this; p if q in which ever case or an all. So, it is represented as q in plus p and p implies are ever, p rather than q or p instead of q, p without q simply translated as p and not q enough.

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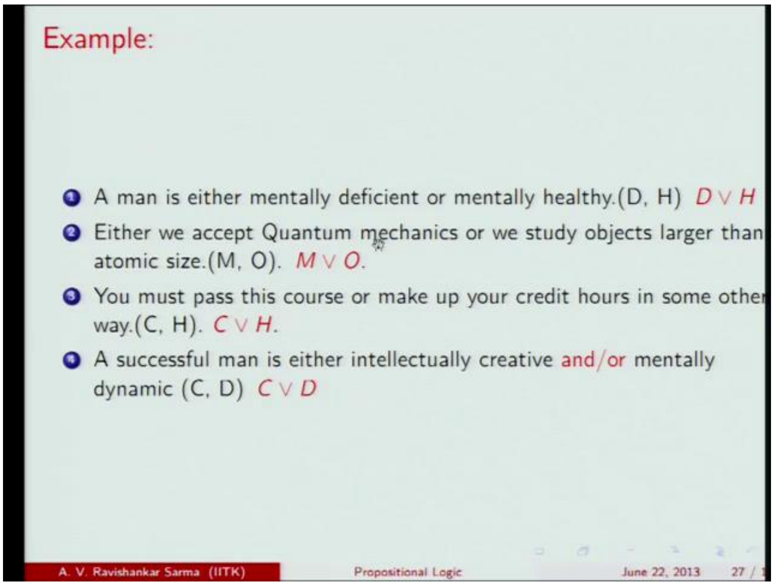

So, this is only for of translation in nor, which we are training to mention all. First the translation parties important, then we can talk about, you have to convert the given English language sentence into the appropriate language of propositional logic enough. For example, if you have given this thing; a man is either mentally deficient or mentally healthy enough. So, the first 1 is the represented as mentally deficient is represents as D and mentally healthy is represented as H. Then it is simply disjunction in all, so is D or H enough.

In the same way, we accept quantum mechanics or we study objects larger than atomic size, this represent as M or O. A successful man is either intellectually creative or mentally dynamic; it is represented as C or D and sometime you can even represent at as C or D or it is not the case, it both of them and all. This is not is not the cases C and D.

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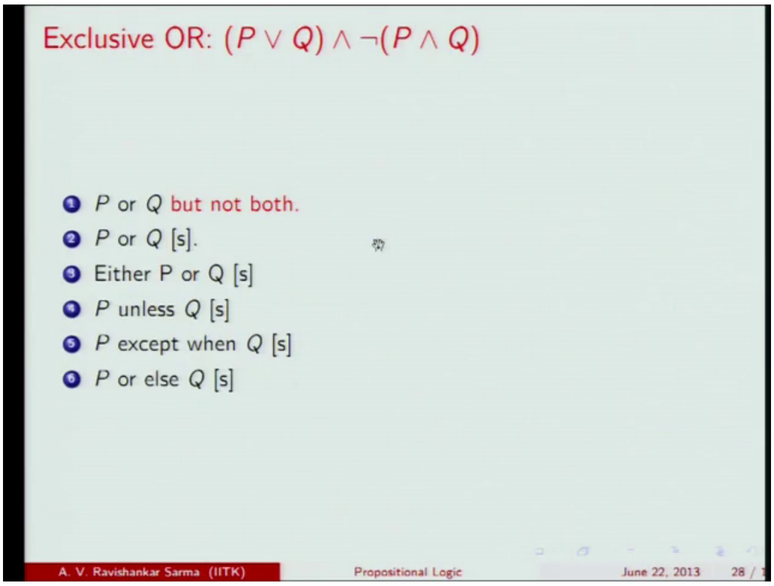

Exclusive or the definition of exclusive r is this; P or Q, but not both of them. So, we sometimes we use it in exclusive or, sometimes we used in the sense of inclusive sense we use, is particular kind of operator or ever. So, P unless Q can be represented in includes sense, it can be represented in exclusive sensible. So, this creates some kind of confusion and all. So, the same kind of connective can be used in both the senses. In 1 sense, it is like this.

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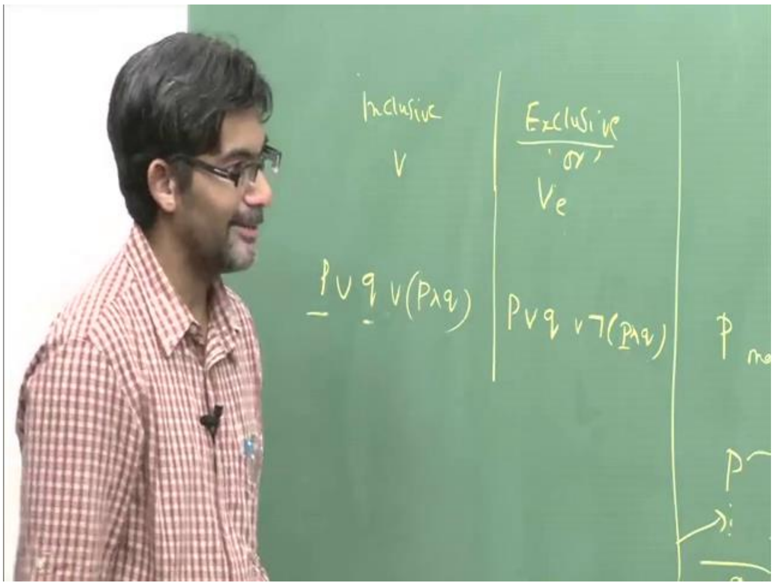

This is inclusive or an all and this is exclusive or. So, both are represented in this way only, usually we represent exclusive or in this way, we do not write in anything here. It is p or q or both that fix enough, this is not we mean by as that p or q or even both of them along. I will here ice cream or sweet a jalebi by a something like that, or it can be both enough; ice cream and jalebi by also, we can have any 1 of this things enough. But, if you put restriction on this particular kind of think, suppose if you take a ice cream you are not suppose to take jalebi, then you are write it in this particular kind of p or q or, but not both of them.

So, definitely you have ruled out 1 possibility enough. So, if you take this 1, you are not suppose take the other 1 and all. It is or the p or q, but definitely it is not both of them, both of them you are not suppose to take in all. It is in this sense; exclusive or and or, either as scheme are jalebi, but you are not suppose take the both the things ice cream and jalebi, forward buy ice cream you are not suppose that jalebi here. So, these what we mean by exclusive or. So, this is what is expressed here.

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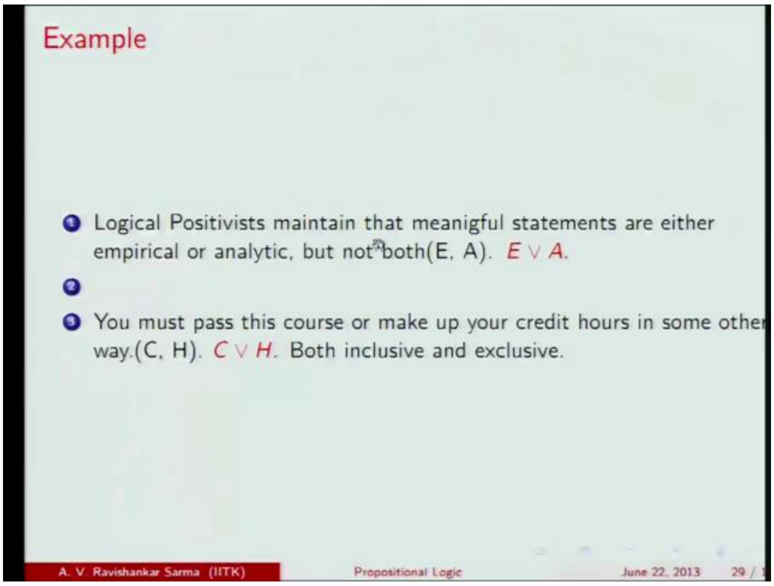

So, these things can be represented in either x in the exclusive sense or a or inclusive sense and all. So, what we try to say here this is that, same thing can be expressed in both the things inclusive or exclusive or, but in classical logic of the propositional logic, if unless until it is striated clearly, it is usually take in by default as inclusive or only. That means, the first definition is the 1, which we will be taking in to consider p or q or p and q or both of them.

So, that is a definition we usually taking to consideration for the operator or and or, that it is exclusive or we do not mean by it as exclusive or at or. For example, these things can be interpreted in even both inclusive and exclusive are as well. It sometime it is difficult to find out, whether it is used in exclusive sense or inclusive sense enough. But by default, it is used as inclusive sense of you must pass this course or make up the credit hours in some other way enough. It seems to if it is the case there is use common sense and all, it seems to be the case that 1 exclusive the other possibility of.

So, that means, the C or H are it cannot be both of them enough, there is in 1 sense you can use this things or you can use it in inclusive sense in such a way that, C or H are even both C and H enough. In both the sense we can use, is particular kind of thing, but I am training say simply there is a un unless amount till it is stated explicitly, we mean the connective that we have used that is a means it is in inclusive, it is used in inclusive sense.

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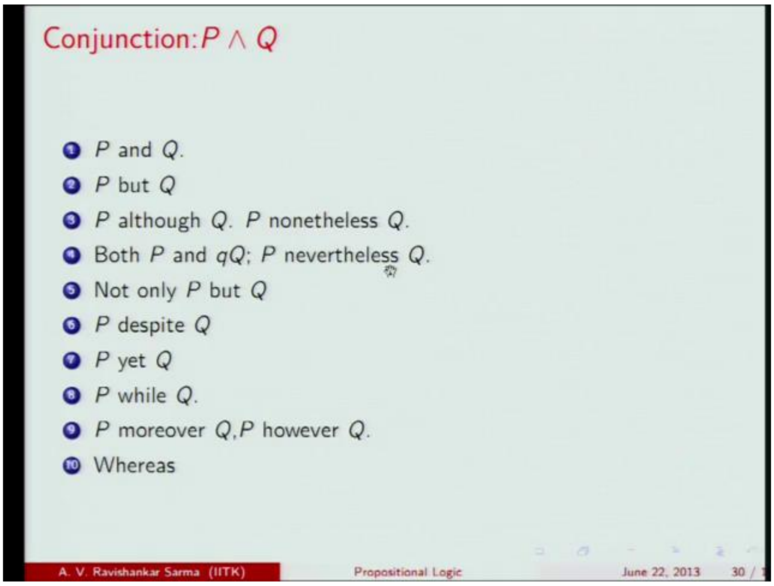

So, the conjunction has a list of these things; P and Q, P but Q, P although Q, P nonetheless Q, whenever you come across that kind of thing, is simply substitute as P and Q, not only P, but also Q is also written as P and Q, then is P and Q have to be there and all. So, that is what you mean by not only P, but also Q, P yet Q we also translated as P and Q etcetera, P more over Q, P however Q all in this things translated into simple conjunction an all.

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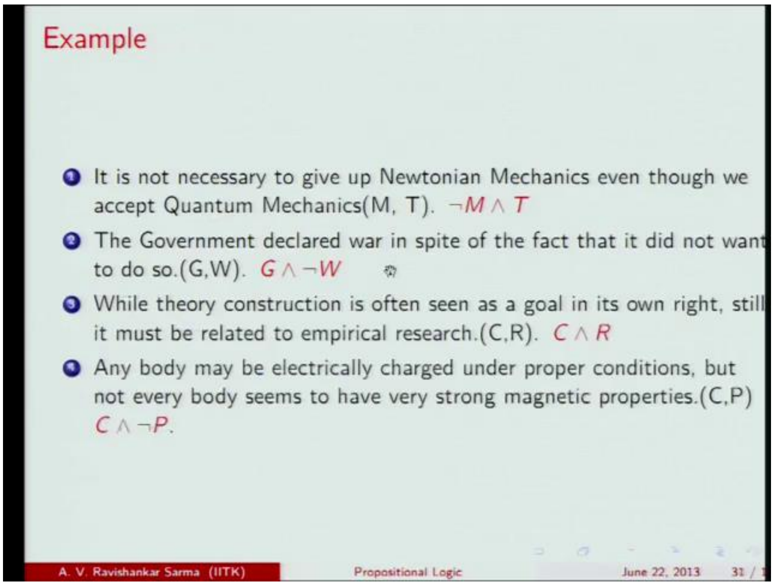

P and Q. If, you examples of translation, we considered then we will move on to what we mean be this correct is now. So, this class is all about what we mean by this connectives, how these connectives behaves enough, in what sense they had considered to be truth functional connectives enough, that is what we are interested in. But before that, we are training to translate some of the English language sentences in appropriately into the language of propositional logic.

Suppose, if you want to express this particular kind of thing, it is not necessary to give up Newtonian mechanics, even though we accept quantum mechanics. So, this is in this 1 P although Q etcetera, it appears to be like P although Q an all, third 1. So, it is simply translated as not M because, it not necessary to give up, it is necessary to give up means; it is written as M, but here it is saying that, it is not necessary means not M and all.

The next 1 is accepting the quantum mechanics that is T as presented as letter T and all. These are simple translation and all. 1 is to make lot of translation so that once give the idea here is that, once you translate English language sentences into the language of propagation logic, then we can talk about many beautiful logic properties and all, when you can say that 2 groups of statements are consistent of each other or when consolation follow from the premises ,or when we can say that the give sentence is totally that always true and all or when given formula is always false, that is conditions etcetera all. All these things we can talk about, only when only once you translate English language sentence into language of propagation logic.

So, last 1 we last example we take into consideration; anybody is electrically charge under proper conditions, but everybody seems to be seems to have a strong magnetic properties and all. If you represent first sentence as C and second sentence as P, then the second sentence not everybody seems to have very strong magnetic fields, is represented not P. So, it is connection of 2 statements that is, C and not P, it represents the whole sentence we have translated as C and not P.

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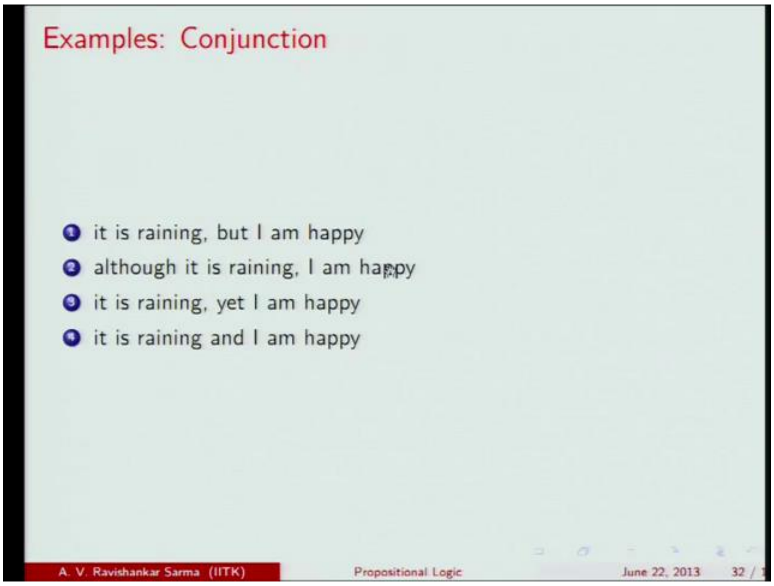

We solve the examples which we take into consideration. So, it is raining, but I am happy, again it is translated as r and h. Although it is raining, I am happy, again same as r and h. It is raining, yet I am happy is also translated to r and h. It is raining and I am happy; so r and h.

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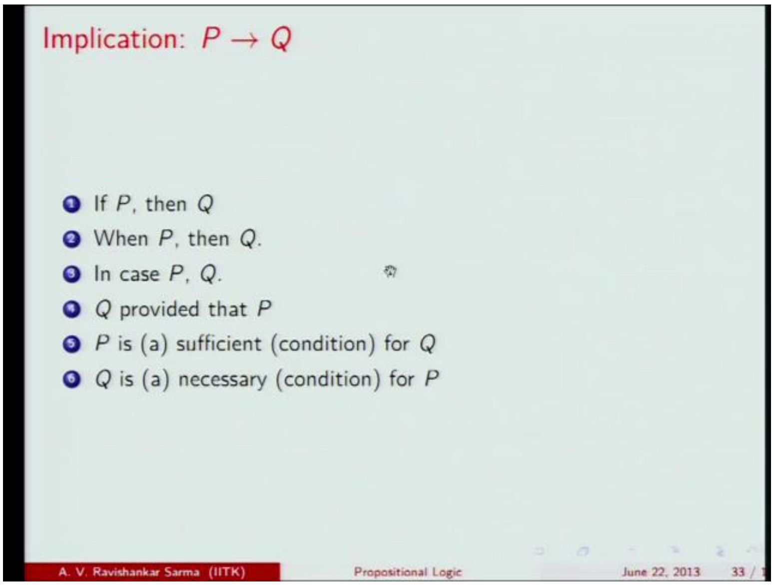

So, you have solve the simple examples, come to know how to translate given English language sentences into language of propagation logic and all. This, whatever I presented here, is all some kind translations guide an all. 1 needs to go into details of each and every thing, whenever some kind of examples an all, you can look like this translation guide and see and how to translate given English language sentence into the language of propagation logic.

So, if you want to express this thing, it should then P than Q simply translated as; 1 which is mention in red color, that is, P implies Q an all. In case of P Q is also translated as P implies Q, Q provided that P, P implies Q only. Here is little bit difference and all, if you want to express sufficient condition, P is sufficient condition for Q, is translated as P implies Q, whereas if you want to see P is necessary condition for Q, it is represented as Q implies P and all

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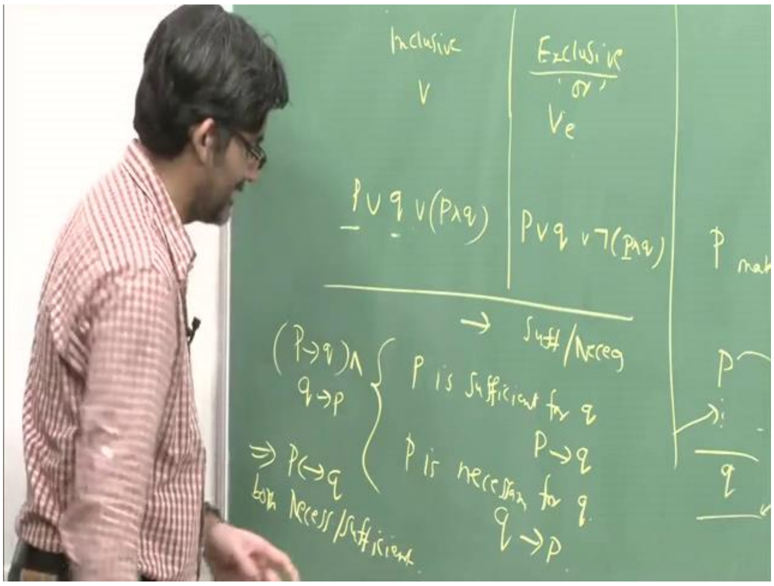

So, it is simply like this. So, usually condition are used for sufficient and necessary condition and all. So, P is sufficient for q which is translated as which P implies q and all. So, P is necessary. For example, if you want to say that oxygen necessary for life and all, but oxygen it is not sufficient for survival for an all, we need fat, carbohydrates and other thing an all. So, P is necessary condition for q means; it is represented as q implies P an all. So, this is the only difference.

If, both the cases happens and all; that means, P implies q and q implies P, P is sufficient and necessary condition and all. Then we write it as this 1; both it is both necessary and sufficient an all. So, this is what important in natural sciences, especially when you want you say that, some is sufficient necessary condition for all something and all. Here in mathematics also use this necessary in sufficient kind of condition.

So, something is sufficient for Q mean P implies Q , something is necessary for Q, P is necessary for Q means Q is P Q implies P an all. So, that the reason for why 6 1 Q is in condition for P means; it is give 1 P implies Q. Suppose, you want to say P is necessary for Q and all, it as to be Q implies P an all. This is the thing which we will be using it in the example which follows.

So, all these thing are simply represented as simple P implies Q an all. Q if P, that is a kind of necessary condition an all and it is represented as simple P implies Q an all.

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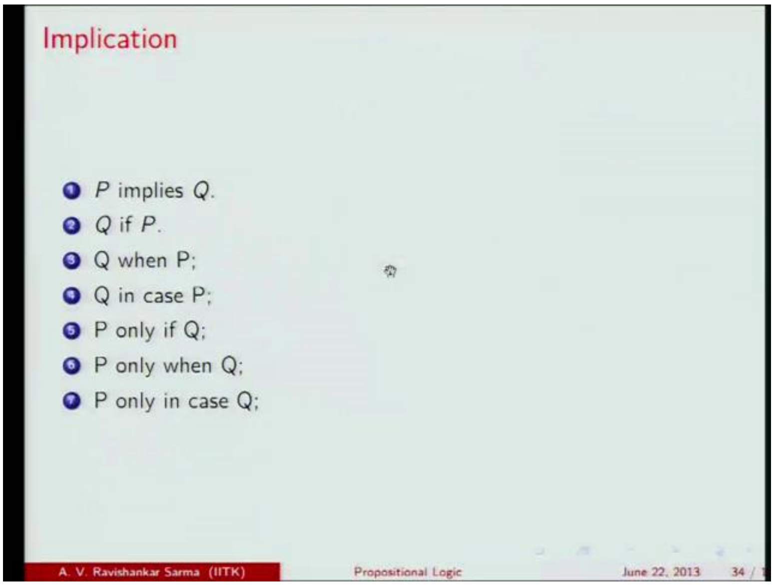

P only if Q is also represented P implies Q, P only when Q, or P only when case of Q etcetera, all this thing represented P implies Q an all. So, what do we need to substitute this P's Q's with some kind of simple sentences, atomic sentences that, we commonly seen today discussed. Then, we see the importance of this translation of an all.

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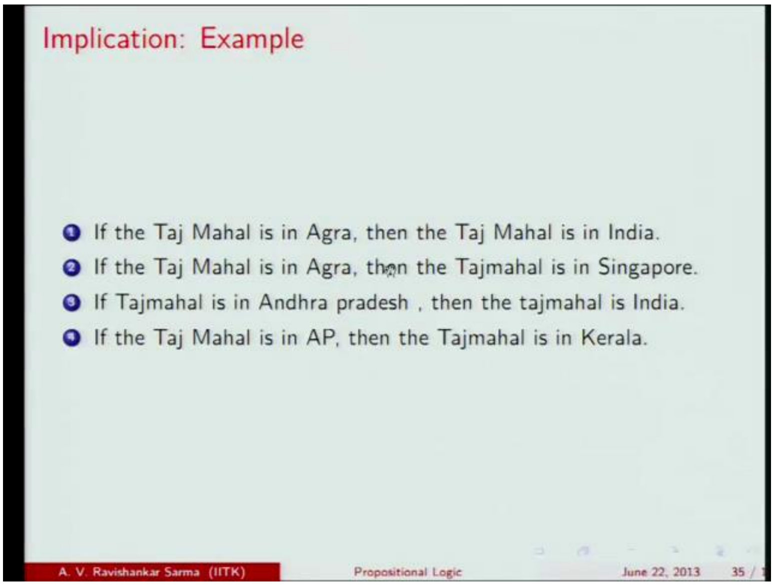

One simple example of condition sentence is like this; Taj Mahal is in Agra, then Taj Mahal is in India. It seems to be making some sense and all. If Taj Mahal is in Agra, then the Taj Mahal is in Singapore, the first sentence is true and second sentence is false. And what happens, if you have this particular kind of thing and all, p is true and q is false an all and what will happen to the whole condition, that is, p implies q that what we are going to talk about in a right from now; that means, semantic of material implication.

If Taj Mahal is in Andhra Pradesh, that is, false then the Taj Mahal is India true. Even then, this antecedent is false consequent is true and condition is going to be true only the whole condition is going to be true. If the Taj Mahal is in AP, then the Taj Mahal is in Kerala, both are false and again the condition is going to be true. So, how do we know that the first conditional east true and the last conditional is also true, whereas the other 1 second 1 is the Taj Mahal is Agra, the Taj Mahal is Singapore that is false. So, how do you know that the given condition is false, given condition is true, we need to have some kind of semantics this material.

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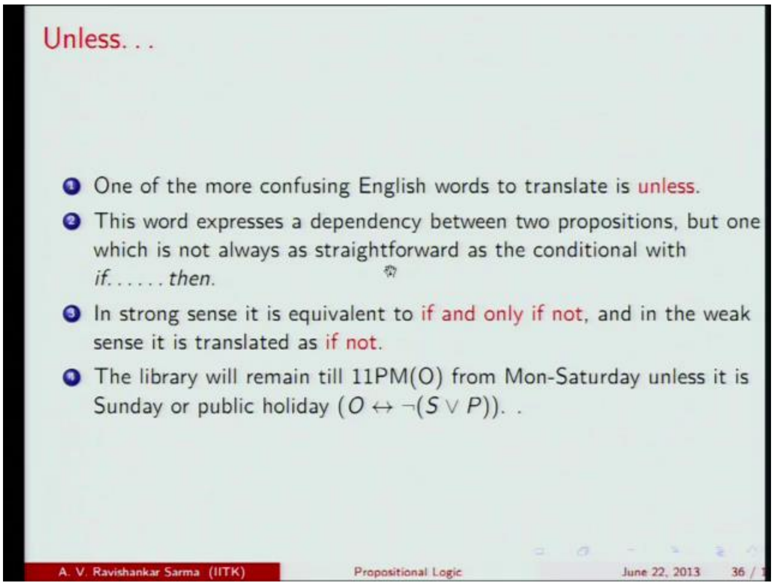

Talk about little bit later, sometimes this kind of connect operator unless, all this things which have train to talk about, there all truth function connectives. So, I did not entering to the truth function kind of truth, but we are just training to see, how truth translating given in English language sentence of into the language of preposition, especially we come across unless in kinds all those things, is 1 of the most is kind of English words to be translated in English is unless.

So, this word expresses at dependence between to prepositions; is binary kind of operator sometimes, it came difference of sometimes it came be implies etcetera. But 1 which is not always what is not straightforward is that, there are not to representative as if then are sometimes it representative are etcetera. In strong sense it is equivalent to if an only not. And in the week sense, it is translated as if not if not p then q or all.

So, suppose if you want represent this particular different kind of sentence, it depends upon either you are try to make it strong kind of statement or tiring to make a week statement in all. Suppose if you want say this particular kind of thing, librarian remain till 11 o clock from Monday to Saturday, unless it is some kind of Sunday or some kind of public holiday as some kind of strike happens or students close it some other reason etcetera.

So, it is simply represent as a conditional is P then Q; that means, O. O stand for the librarian remain of till 11 am, if an only not at all. This is used in a strong sense, if only if it is not a case that is P an all. That is it is not a case that either it opens from Monday, either it is Sunday, that is S are it is public holiday it is means P. It is used in this strong sense or it is simply representative as low impress not is all P. Only 1 kind of condition will apply and all, by condition will not applying, that is not p then q all.

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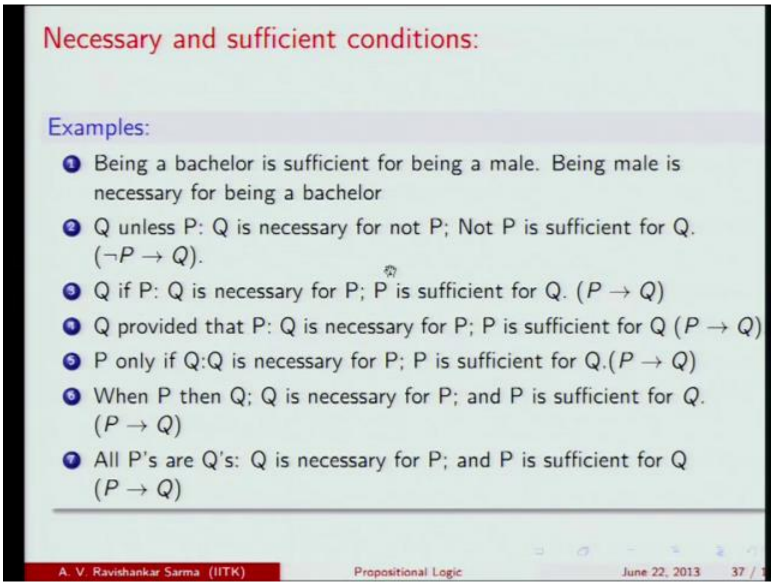

So, that is way we translated into particular kind of think and all, unless is always create sometimes problem to us. So, now, this is what we have already discussed in all the. Whenever you want to invoke some kind of necessary submission condition, you used condition all either P plus Q or Q plus P. Some simply examples are like this; being a bachelor sufficient for being a male and being a male is necessary and necessary condition being a bachelor and all. So, being a male is constant to be necessary condition a being a bachelor.

So, this and all saw the translations which are which needs to be a need to going to the details of this thing. So, Q provide the P Q is necessary for P, P is sufficient for all these things are translated into P implies Q. So, all these things passed 20 minutes which I am trying to talk about, will come under the list of a kind of we are trying to provide a kind of translation guide it with the help of which we can safely translate its all the sentences in English into the language of propositional logic.

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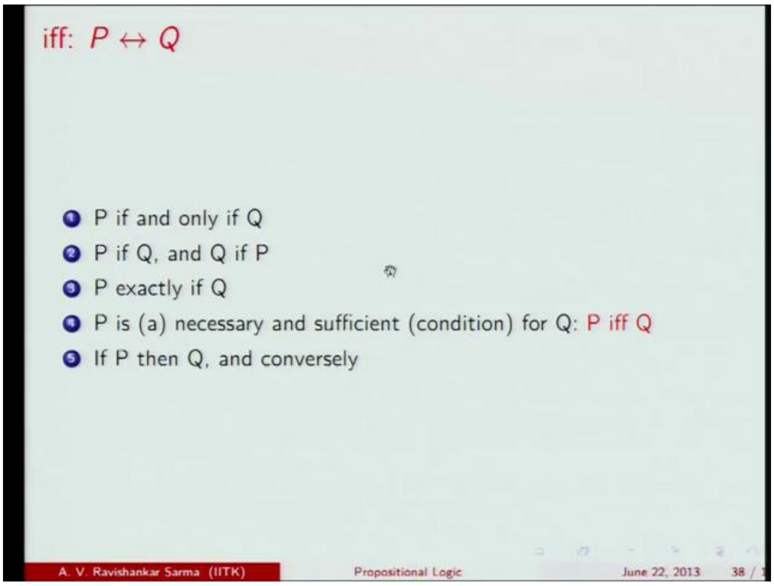

So, these things also we have covered and all.

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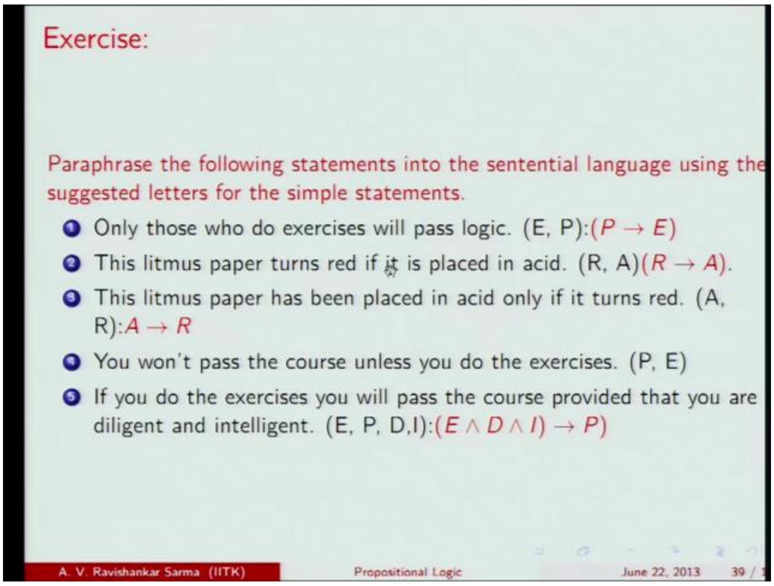

So, let us considers some simple examples. Then, will move on to the semantics of propositional logic. Only those who do exercises will pass logic and all; that means; you must do exercises to pass the examination and all. It is a kind of suppose, if you represent it like, only those who do exercises is E and who will pass as the examination is P. Usually if it is a sufficient condition, you will represent it as; E implies P and all. But, it is a necessary condition; that means, it is clearly stating that if you do not solve the exercises passing the exam.

So, usually we simply write it as E implies P and all, but it only satisfies sufficient condition. Here it expresses the necessary condition and all. 1 is necessary for the other; it is like oxygen is necessary or existence and all. So, instead of translating this; the first sentence into E implies P, we translated as the reverse 1 that is, P implies E and all. So, this is the necessary condition; that is why P implies E and all. That means, in the last case, we have seen if P is a sufficient condition of Q, then you translated it as P implies Q, P is a necessary condition for Q means, it is Q implies P and all. So, instead of Q, we are here P and instead of P here E and all. That is why which is translated as P plus E.

So, in the same way, you would not pass the course, unless you do the exercises, you can also be translated as a you would not pass the course, unless until you do the exercise and E implies it is also translated the P implies E and all. So, last 1 if you do the exercises, you will pass the course, provided that diligent and intelligent. So, it is like E and T and I, the conjunction of all this statements implies P and all.

The fourth 1 is the 1 which is used, it can be used in strong sense or it can be used in week sense and all. It is again unless creates some kind of problem in the translation and all.

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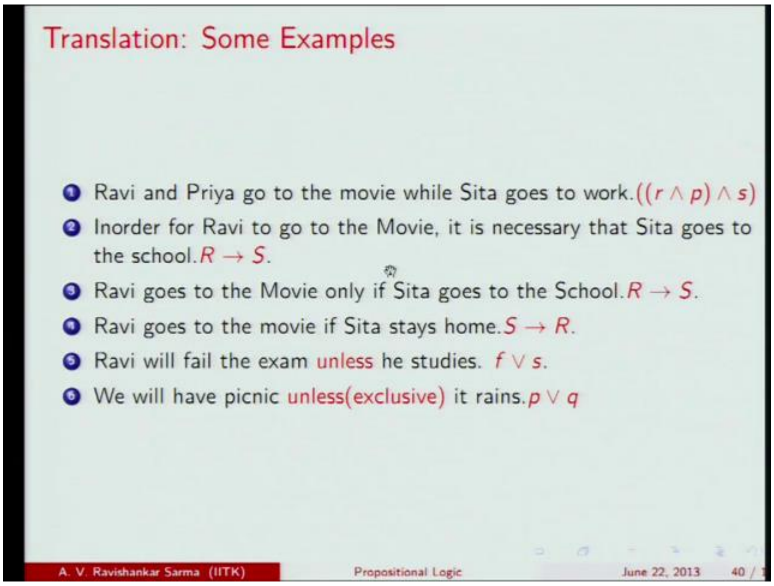

So, this is the way to translate the things and all. Some more examples if you like this Ravi and Priya go to the movie while Sita goes to the work and all. So, Ravi and Priya, it represented as R and P and while Sita goes is also represented as conjunction operator that is why it R and P and S. Now, second 1 in order for Ravi to go to the movie, it is necessary that Sita goes to the school and all. So, this is R implies S and all. Ravi goes to the movie, if Sita stays at home and all is some kind of sufficient conditions or that is why a it is a necessary condition and all. Sita must stay in the house so that Ravi can go to the house.

So, it is simply the first 1 is represented as R, second 1 is represented as S. Since it is a necessary condition, that 1 must be there in the house. So, it is S implies R and all. Ravi fail in the exam, unless he studies and all. So, either he also study the F is a case or S is a case and all. It is used again in inclusive or exclusive sense and all. So, here you will see clearly here; sometimes unless is translated as if P then Q. And some other cases it is translated as F or S and all. That is why unless the phrase unless presence lot of problems and all in the process of translation.

The last example you will have picnic, unless it rains and all. If it does not rain, we will have we will go for a picnic and all. So, it is simply we can be used in both the senses and all. It can be used as include the sense P or Q and both and all or it can be in this case at least it is used in a exclusive sense and all. So, it is 1 excluding the other possibility and all. So; that means, if it rains you will not go to the picnic and all. If you go to the picnic does not rain and all. So, 1 excludes the other possibility and all. So, it is simply represented as P R Q.

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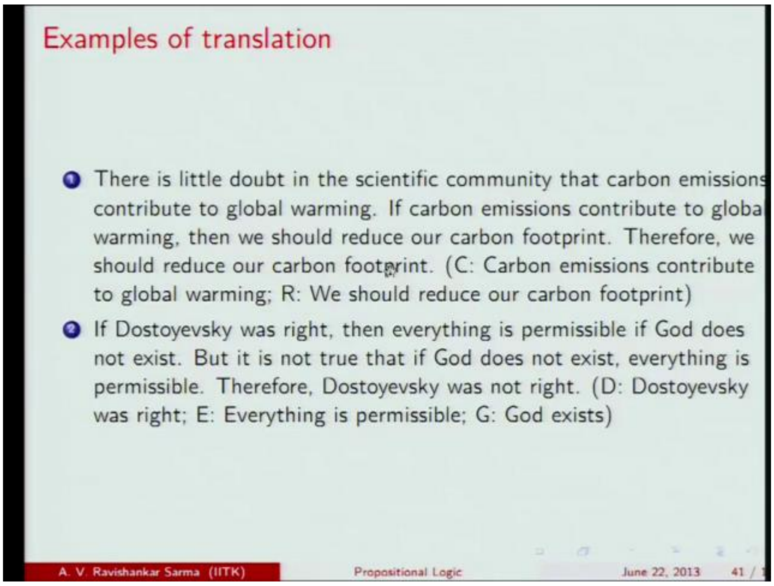

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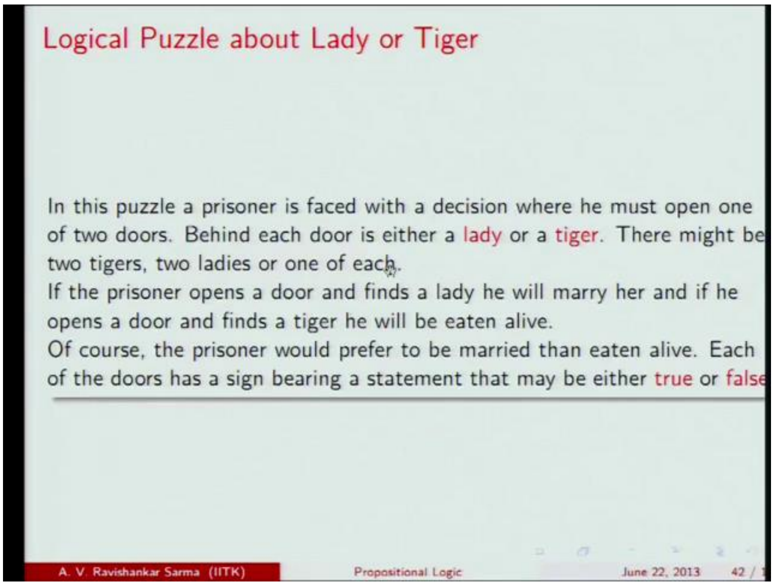

Some more examples, which will training to consideration. So, now, before a going any further and all. So, we need to see how these connectives actually behave and all. So, let us a try to talk about the semantics of a propositional logic and all, so how these connectives behave and all. So, to start with we have negation then, we trying to expressing how this connectives behaves.

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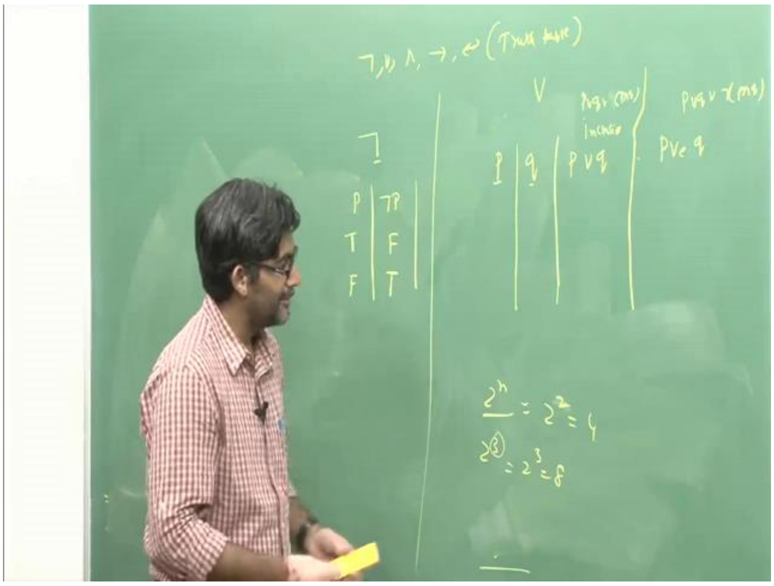

So, to start with we have simple negation and all. Suppose, if you have a sentence p and the, the negation of this 1 for example, if this becomes T and all and the negation of p will become false. Suppose if you say this is the duster, the negation of that 1 is it is not a duster and all. So, that is represented by not p and all. So, this is the way this connective behaves and all. If the sentence is false and all, then the negation of that 1 will automatically be true and all. This is the most simplistic kind of thing and all, then coming back to this 1.

So, this is the truth table for this. So, all this connectives that I am try to mention on the board there all truth functional connectives and all. That means; truth value of a compound sentence is only determined by the truth value of its individual constituents and all. So, here you have p and you have q and then p r q. And this p r q can be used in 2 different senses and all. So, this is inclusive r which is defined as p r q, but not both the case. And all and this 1 p r q r, but not it can be both and 1, but not both the case. So, this is the definition of this 1.

So, now there are 2 variables in a propositional variables p r q; that means; there are 2 to the power of n entries in the truth table and all. So, what we are trying to do simply is that, we are trying to see how this 5 operators like negation r and implies and if and only if behaves, in logic and all. So, for that we are trying to discuss truth table, we are trying to construct truth tables with which we can say something about this behavior of this connective.

So, suppose if you have only 2 variables and all, then you have 2 to the power of n entries possible in a given truth table and all. That means; 2 square, 4 entries are possible in the truth table and all. Suppose there are 3 entries and all, we have 3 variables and all then you have 8 entries possible in the truth table and all. So, 8 rows we need to inspector. So, as the number of proposition variables increases and all, truth table will become bigger and bigger and all.

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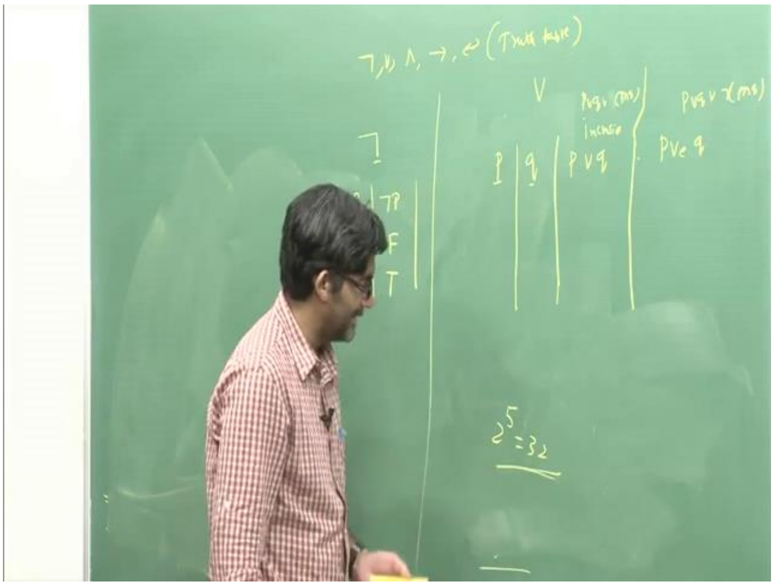

And the number becomes 5 4 and all, there will be 32 entries in the truth table and all, because 2 to the power of 5 is 32. So, let us talk about a simple thing and all.

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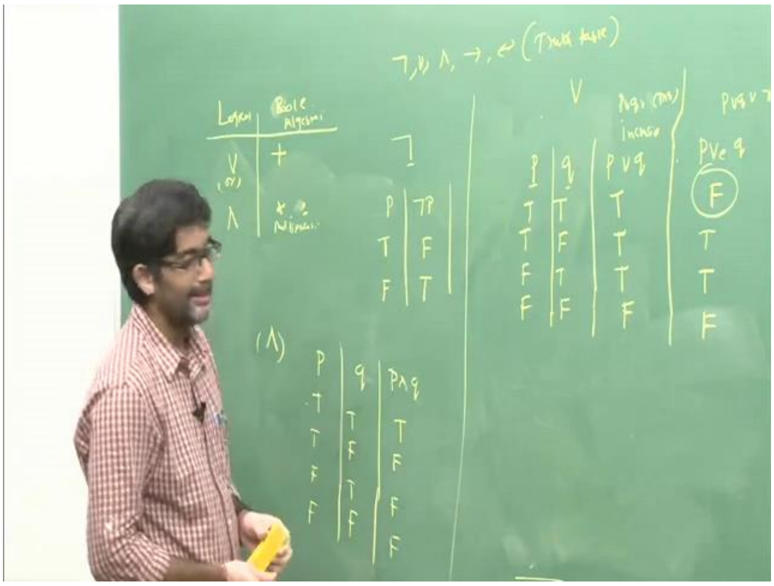

So, first you will write alternatives and F, and then so 2 T's and 2 F's and then you write alternative F's and all like this. And then, this connective is going to be false only in this case; that means, both constituency individual constituency p and q are false, then only it will become false. In all other cases it becomes true and all. So, this is the way this behaves and all this connective behaves and all, p or q is going to be false only in this case in both a both constituency are false, constituency means p r q. In all other cases, if p becomes F, q becomes T, then also it becomes T or whenever p is T, q is F then also it becomes T etcetera.

So, now, this is what is used in the inclusive sense. In the exclusive sense, 1 excludes the other possibility and all. That means, it cannot be both true and all. So; that means, it becomes false. And the same way it cannot be both false also. So, if; that means, 1 is false, the other 1 has to be true and all. Or if this is q is tree T has to be false and all. So, now this automatically any how this becomes false and all. And this may become T and all. So, this is the only a difference that you will find it in exclusive r n. It is like ice cream or jalebi and is that possibility needs to be ruled out and all cannot be both and all. So, that is why it becomes false and all other cases which remains the same and all.

So, now coming back to the connective end and all, again let us assume that, there are only 2 variables are p and q. So, why we are talking about this truth tables and all because, well form formula should get its meaning and all. So, the meaning means providing meaning means providing truth conditions for a given formula and all. So, this is the compound formula and all, we are trying to provide meaning of this formula and all.

So, meaning how it gets meaning and all, whenever you provide truth conditions for the individual constituents that exist. Once you provide, once you assign some kind of truth values to this individual constituency and all then, the behavior of the connective is such that whenever you have this thing T F T F. So, whenever both the conjunctions are true and all is going to become T. And in all other cases it becomes false and all.

So, this is exactly in correspondence with what we have here 1 enology that we java seen here r stands for this is boole representation on, usually algebraic interpretation. So, this is the logical interpretation that we are given, the same thing which we are trying to talk about. This is the correct r which means plus in boole in algebra and n stands for multiplication and all. So, usually represented in this way, but you can represented it is star or it sometimes it can be represented algebra and dot also.

So, this stands for multiplication. So, now, these p's T stands for 1, for example, value 1 additional number or F stands for 0, then usually you the see 1 plus 0 is equal to 1 1 into 0 automatically 0 and all, so this 0 into 0 that is 0. So, it is in this sense we can understand is particular kind of think and all. So, now this is how the connective and behaves and all. You have to note that, this is the way it behaves in the propositional logic and all. But in actually day to day practice, you might be surprised with this particular kind of thing.

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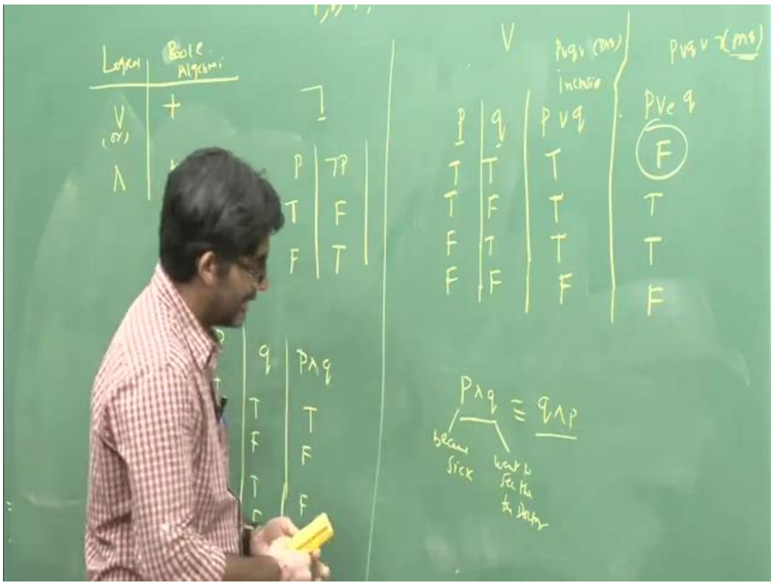

Suppose, if p and q is there. This is same as q and p and all. Suppose if we represent p as; I became sick, let us say I became sick and all and. Then, usually when you become sick and all you went to see the doctor and all went to see the doctor. So, now, p and q is same as q and p. So, now, the sentence will become compound sentence will become like this.

I went to see the doctor and I became sick, there is 1 thing. Nobody goes to the doctor to become sick and all. That means, q and p and p and q is are became sick and I went to the doctor that make sense to us, but the second 1 does not seem to be make a sense and all. These 2 word totally 2 different things and all. I went to the doctor and I became sick and I became sick and I went to the doctor.

So, p and q and q q and p are totally different, especially when you use it in the day to day disclose and all. But in classical logic, especially p and q is same as q and p because, the truth table of p and q matches with the truth table of q and p; that means, these 2 are logically identical to each other. It will by saying that computability low holds and all. So, that why p and q is q and p, but in actual day to day it disclosed, it is not just the meaning of a meaning of a compound sentence. It is only determined by the meaning of its constituency and all, but there is something more and all.

So, those kinds of connectives are called as non truth functional connectives, which we do not talk about it at this movement.

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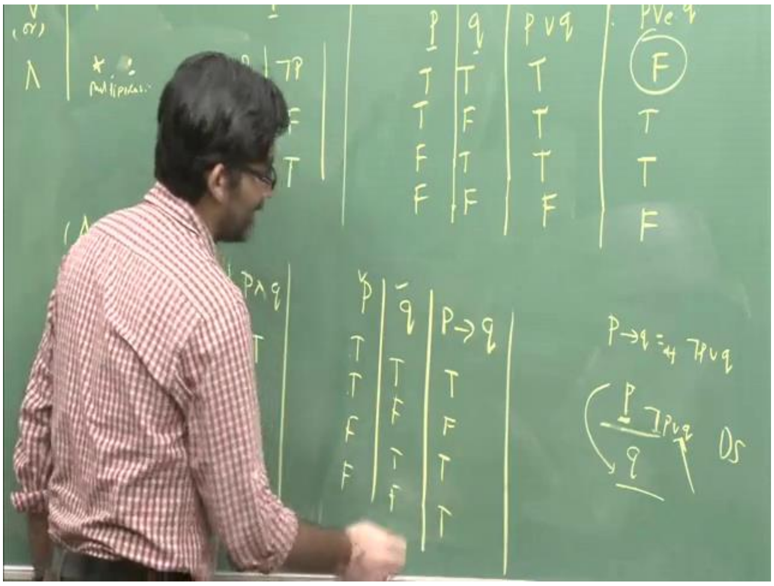

So, how this the other 1 p in plus q. So, we have said that, p in plus q by definition is not p. So, by can by training to understand how p materially implies q an all. What kind of substitution you make it here. So, that p materially implies q and or. So, the substituted this 1 and these 2 p not p or q does not civil the p is to q and all. So, that is why this, the substitution world the formula that is the instead should as the definition of p implies q and all, so not p or q is a definition of this 1. We follow this particular kind of definition then, again if you have 2 propositional variable existing your truth table then you have 4 entries possible enough; that means, 4 rows which are possible. First you write it 2 T's and 2 F and then alternative F an all 1 T 1 F 1 T 1 F and all. And then, this becomes false only in this case. In all other cases, it becomes T an all. So, this is away is connective behaves.

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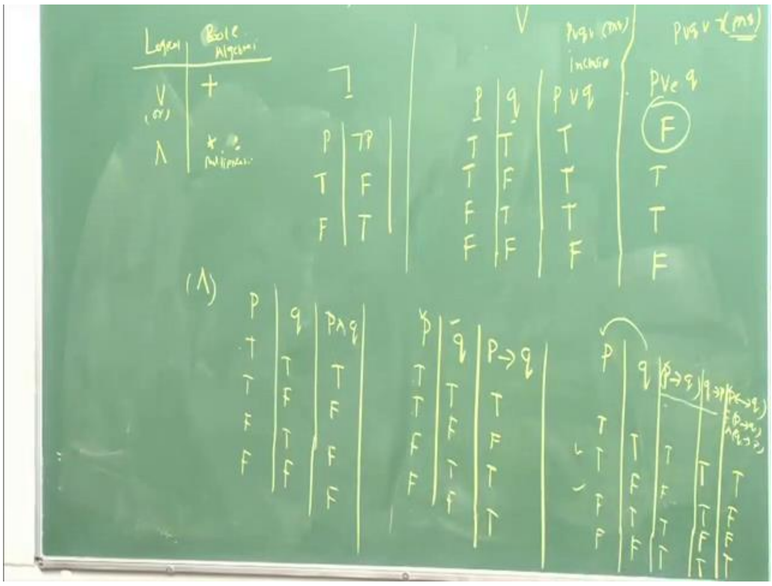

And the only connective that is left p is p implies q p plus q q plus p. So, now, this is what you mean by p in plus q. So, that is both p in plus q, it is sufficient condition and q in plus p is the case. So, now, again you write alternative T's and alternative F's T F T F and all. So, now, first you like it for p in plus q is the same as this 1. You write the same team p F T T. So, now, q implies p, now you have to move from positives. So, it becomes false only in this case; when q is T and p is F it becomes false. In all other cases, it becomes T F.

So, this is the way the connective behaves enough. So, because of that are by that by using the definition, we are training to write the truth value of a given compound formula. So, now, is a conjunction of these 2 things an all, it is both the p implies q and q implies p. So, now, these an end connectives. So, whenever both are true is this T and now this becomes a F and T is F that is 0 into 1 is 0.

So, now, this is also becomes F and it becomes T an all. So; that means, p if an only if q becomes T, when both are true are when both are false enough, in that case, it becomes T F. So, in all other cases, it becomes false an all, in this cases it becomes false. So, this is the way this connectives behaves an all. Based on this thing, we can talk about based on truth cable which is formulated by at least 2 kinds of people, with get stain 2 philosopher of formulated it, the mathematician and logicians. 1 we can attributed to we give stain and other 1 is a attributed to emily post enough.

So, these are 2 logicians which are responsible for this truth table method and all. It is the very intrigued kind of method, is a kind of constructive method, with which we know you talk about whether a given formula is totality when ever given a formula is a contradiction, are in a kind of contingent kind of statement enough.

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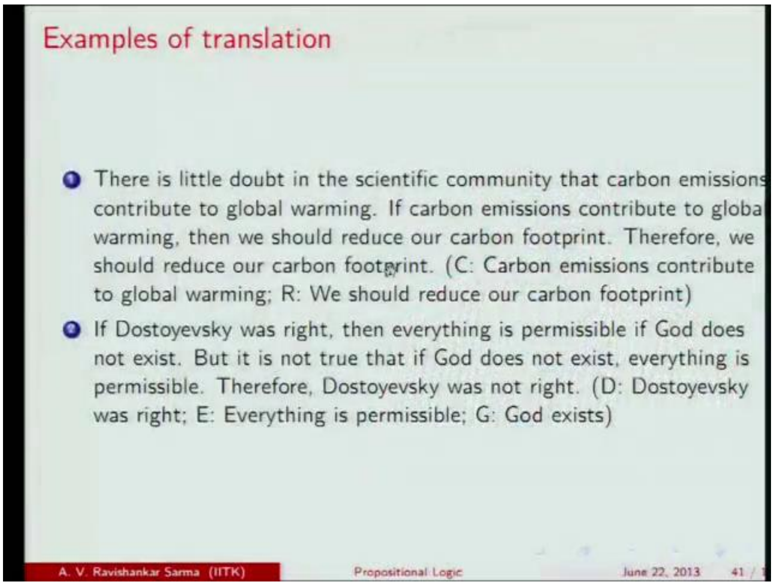

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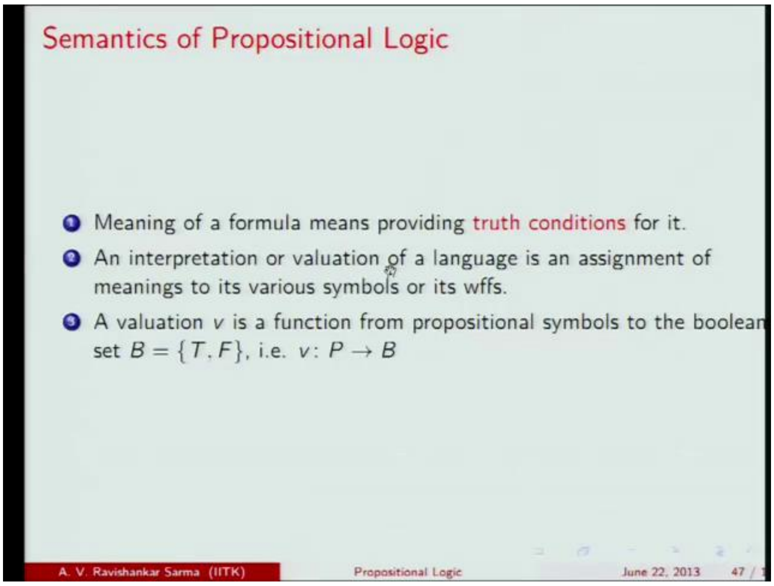

We will go to details of this little bit later. So, this is what we mean by semantics of prepositional logic enough. Meaning of a formula means, providing truth conditions for it, we are used 1 particular kind of method, that is, truth table method and with which we form, we have given meaning to a given formula that providing into truth conditions for a given formula enough.

So, what will we doing here is; with we have done it already, we need to know something about interpretation an all, interpretation are valuation of a language it is a assignment of meanings to its various symbols or its well form formulas enough. That means, if you have a formula like any formula take in to consideration.

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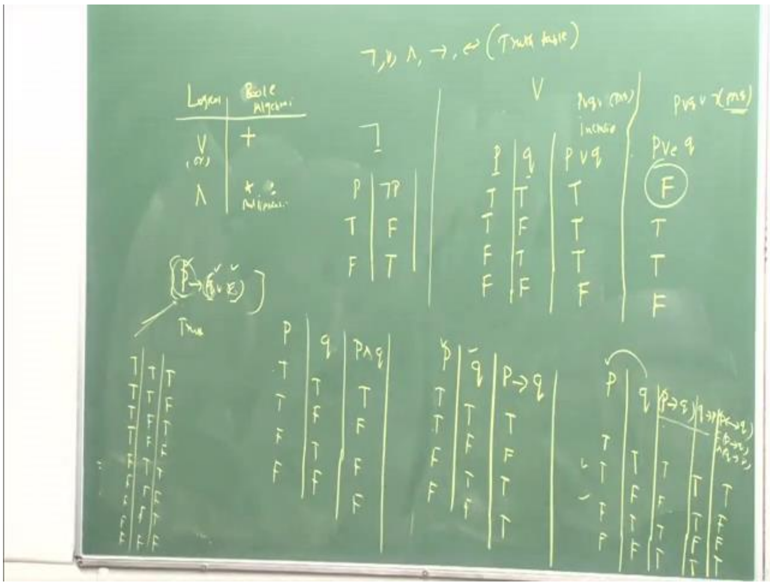

Let to say p plus q r are. So, how it gets meaning an all, assigning some kind of truth values to this 1 this formula enough. So, how can we assign truth values to this formula an all. So, there are some individual components; p q r an all. So, is takes some kind of values an all, either it can take p can take value T, q can take value T or r can take F there many possibilities are like this.

So, whenever you have 3 variables enough, in the case of truth table, first you write all 4 T’s and 4 F’s an all. And then you write 2 T’s 2 F’s 2 T’s and 2 F’s an all. And then, you write alternative F’s an all T F T F T F an also. These are the values that p q r you can only take enough. So, we will assigning some kind of values in the individual kind of components an all. And then with help of this, till be aviating the truth value of the hole compound formula an all. That means, interpretation or valuation of a language is an is a kind of assignment of meanings giving truth conditions to a given symbols or given well form formula an all.

So, in formal terms it is a valuation v is a function from propositional symbol; that means, p q r etcetera, an all that acquires in the formula. And then these p q r etcetera can only take values only 2 values an all, either it can be true it can be false an all. So, there means evaluation function takes a propositional variable, an assigned some kind of value, and that values going to be the kind of binary value, there is either 0 or 1 now. We cannot take any other value an all. If you assign some kind of truth values and if you evaluate the truth value of a given formula that will always p turn over to be either 1 or 0, 1 stands for and 0 stand for false an all.

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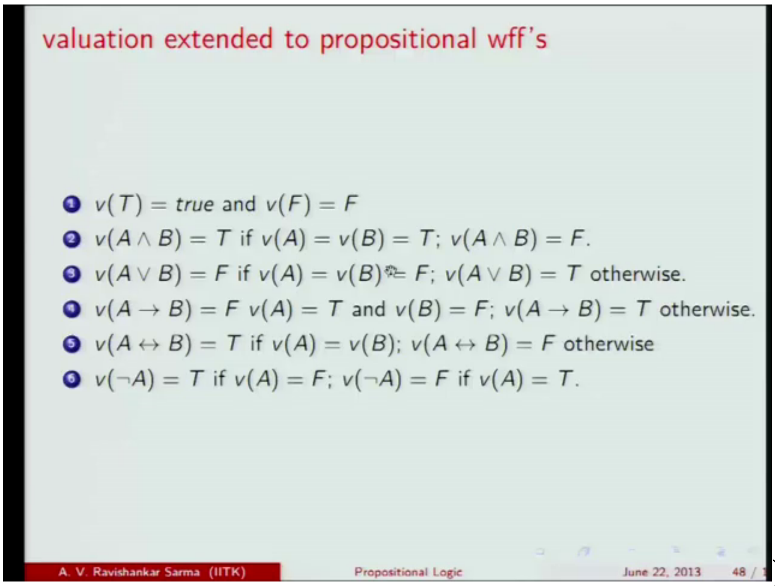

So, this is the way which we a try to talk about the same thing, that we have listed out the bold, in a in a totally little bit different way on. So, v stands the valuation function and there are 2 symbols which we will be using it here. So, that is also very important an all.

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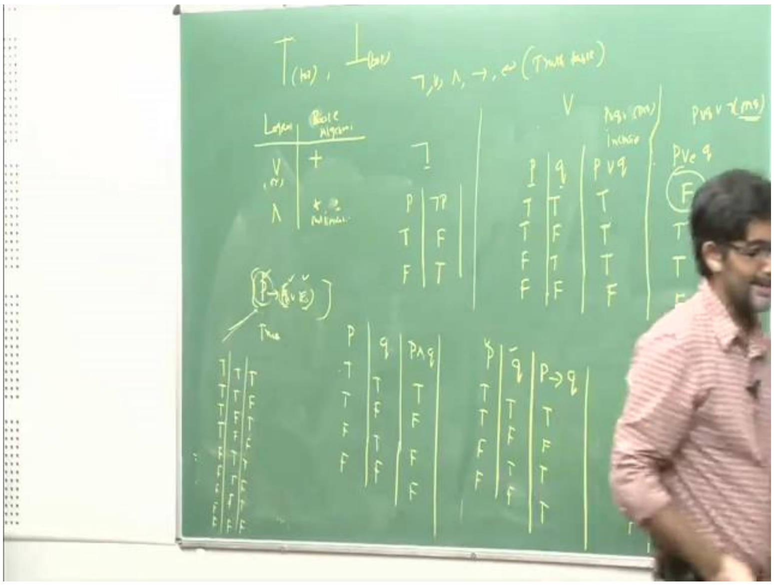

The symbol T means the formula which is always true an all, which is written as top there means, all data like 2 plus 2 residual to 4 etcetera an all. On the 1 held, use another symbol the reverse of that 1, which is represent at the ball; that means, sentence which always false. So, they are conviction etcetera. So, if you give valuation to those formulas which always true, there is; obviously, true and all. True formulas will valuation of truth formula is always be truer. A bachelor unmarried; the truth value of that 1 is always true and all.

So, in the same way, if a truth value of a contradiction is; obviously, false enough, that is what is the case enough. Now the second 1, the valuation function of A and B. So, that is going to B true and all. That means, if you assign various values to A and B, the valuation of and B becomes T only when the valuation of A and the valuation of B is going to be true enough. Otherwise it is going to be false an all. Suppose, if you observe this truth table method.

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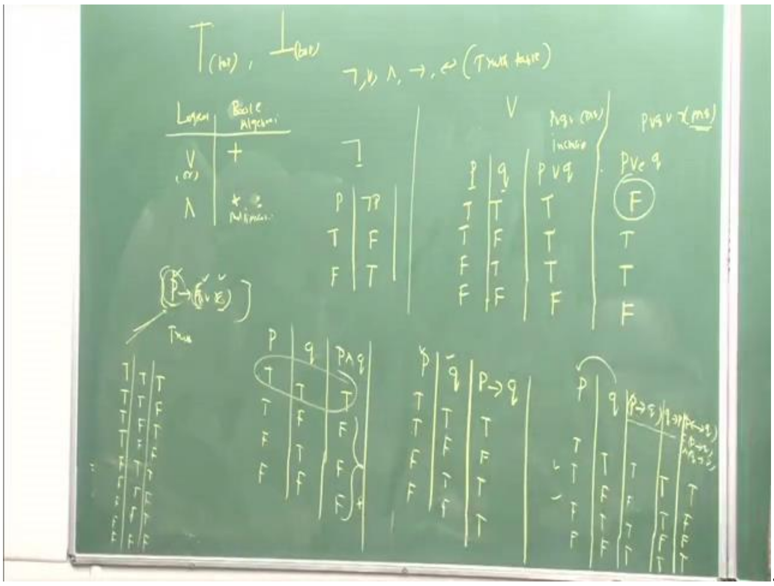

So, the valuation of p and q; so that is going to be true only in this case, in all other cases it is false enough. That is what essentially says an out. In the third case, valuation of a are b there is going to be false. So, now, we to observe is particular kind of thing.

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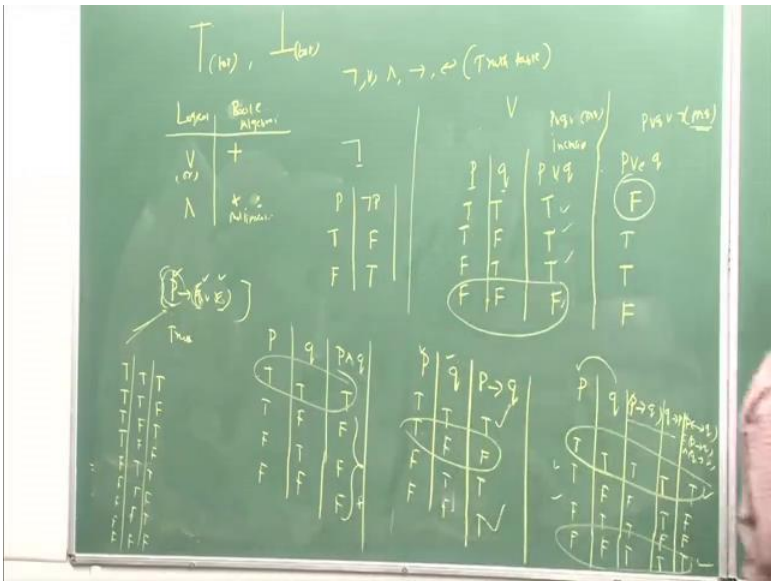

So, the valuation of p or q, there is going to be false when both are false enough. In all other cases, it is going to be true and all. So, there is going decide the truth value of A or B. In the same way well fourth 1; valuation of A implies B, that is going to be false, only when in this case, when p takes value T, q takes value F and this p implies q becomes false. And in the fourth fifth case, valuation of A if an only B, that is going to be true, especially when the valuation of A and the valuation of B is 1 and the same and other wise is going to be false enough.

So, in this case, we need to take this same thing, the valuation of A; that means, F F and valuation of B that is also same and all. In that case, it becomes T and all. So, in this case so in this case and this 1, these are the cases we decides, whether these are 2 are true or false. In all other cases, it becomes false, that is what essentially it says. And the last 1 is straight forward, when very simple, there is evaluation of not A it becomes T, especially even the value of A becomes false and the valuation not A becomes false only when the valuation of A. That means, giving value to this particular formula A, it is always true and becomes T an all.

So, what we have studied in this class is simply this pattern. We started with translation of given English language sentence appropriately in to the propositional logic. Then, after studying in detail about the translation, we moved on to how this connectives behaves in all, how the true truth functional connectives such as negation and r etcetera behaves. And we expressed in terms of truth table and then we expressed same thing in terms some kind of formal language with the help of using that, how when we assigned some kind of values to nothing, interpreting the formulas and all. So, when the formulas is going to be true, when the formula is going to be false etcetera, the things which we have provided, especially in the case of valuation extended to prepositional logical formulas and all.

In the next class, we will be talking about some of the important logical properties such as, when do you say that a given a well form formula is a tautology, then we always true, can you say that a given well formula is always false, when it is contingent. Or when 2 groups of statement are consistence each other, or when some kind of conclusion follows from the given premises an all. All this things, you will be studied in the next class.