

Introduction to Logic
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Lecture - 18
Truth Table Method: Validity, Consistency, Logical Equivalence

Welcome back. In the last lecture, you have seen semantic of preposition logic; where we introduced every simplistic method that is called as truth table method. So, today what we are going to see is going to study this truth functional connective that is: NOT, AND, OR and implies double implies in greater detail. And then once, we construct the truth table.

We can come to know many things such as: when 2 groups and preposition are logic this are to be consistent, when a conclusions follows from the premises; that means, validity takes **place there** of that particular thing and when a particular formula given a **well form** formula is a valid formula are when a given preposition formula is a tautology, contrition and contingency things, all this is 1 can come to know with this particular decision procedure method; which is called as truth table method.

So, this is a decision procedure method, because given well form formula, we can check whether particular given well form formula is a valid formula, contingency formula, tautology etcetera. So, this is the most simplistic method, with which we begin with and then we will move on to some other kind of decision procedure method that such as: semantic tabular method, resolution, reputation method; time permits go to the details of it in the fourth coming lectures. And then there are other method such as given a well form formula 1 can reduce the formula to conjunctive and disjunctive normal forms

Then 1 can talk about at the particular given formula is a tautology are not. So, once if I find the particular formula is tautology, then we can say that the valid formula.

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The slide is titled "Truth Functional Connectives" in red text at the top. Below the title, there is a blue-bordered box containing the following text: "Truth Functional Connectives" followed by "A sentential connective is used truth functionally if and only if it is used to generate a compound sentence from one or more sentences in such a way that the truth-value of the generated compound sentence is wholly determined by the truth values of those one or more sentences from which the compound sentence is generated, no matter what those truth values may be." At the bottom of the slide, there is a footer with the text "A. V. Ravishankar Sarma (IITK)", "Propositional Logic", "June 22, 2013", and "62 / 108".

So, legations task is to identify, what are the tautology, because all tautology and preposition logic are; obviously, valid formulas and all. So, to begin with, in this lecture what will be doing is we will be talking of some other basic definitions such as: consistency, validity, logical consequence, when 2 formula are logically equivalent each other etcetera using truth table method. So, before that we have define a truth functional connective, in this sense a truth function connectives is a 1, in which the truth value view of a compound is slowly determined in by the truth value of its individual constantine.

Suppose, if you want you determined the truth value of $p \wedge r \vee q$. So, the truth value of $p \wedge r \vee q$ the compound sentence is slowly determined in by whatever, value is $p \vee q$ etcetera preposition variables takes in that given well formula. So, there is a no other thing which comes in to picture in determining the truth value of the particular formula $p \wedge r \vee q$, but, in day today practice is we need to go beyond this truth functional connectives and all. And we need to talk about some other things such as for example, 1 example we already studied in some detail that is I went to I became sick and I went to the doctor.

The second 1 is I went to the doctor and I became sick. These 2 statement are totally different 1 is there is a temporal order present, in this particular kind of thing this example. So, went to the doctor and then followed by that, you became sick there is a

what is following in the second statement in the first statement, you became sick and that is why you went to the doctor. This 2 statement mean to different things and all. So, if the truth values a compound statement are sentence is not slowly determined, by the truth value of its individual constituent then the usage of the truth functional connective and we call it as non truth function usage of that particular kind of connectivity connective.

So, there are many non truth functional connectives, which we come across not in this course, but, in advance courses in logic, especially in model logic etcetera. And all there are some operative, which are consider to be intentional. If the truth value of a compound statement slowly determined by the truth value of its constituent, then the truth function connective is concerned to be extensional. If it is not determined slowly by its individual concerned constituent is called as intentional.

So, there are many operators such as I believe that: p I for example, it is possible that p it is necessary Y that p etcetera all this things are some of model operators, which are concerned to be intentions the truth value of I believe that p is not slowly determined whatever, value p takes and all determined. There are many things which, believe them to be true, but, which may be actually true and all I believe that God exists to be true, but, actually without whether God exists or not it may be the case God does not exists or not.

So, I believe that so many things goes exist that may be true to someone may be others it may be falls. So, the truth line of that particular kind of statement is not slowly determined is whatever, value that the individual components takes and all in that sense it is intentional; which is beyond the scope of our study. So, you will be studying about only extensional operation operators such as NOT implies AND, OR etcetera and all. These are called as truth functional connectives.

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Truth Tables:

Negation

p	$\neg p$
T	F
F	T

Other Connectives

p	q	$p \wedge q$	p	q	$p \vee q$	p	q	$p \rightarrow q$	p	q	$p \leftrightarrow q$
T	T	T	T	T	T/F	T	T	T	T	T	T
T	F	F	T	F	T	T	F	F	T	F	F
F	F	F	F	F	T	F	T	T	F	F	F
F	F	F	F	F	F	F	F	T	F	F	T

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So, this is the 1 which we came off with the negation is the simplistic 1 when the proposition is true, then when against of these; obviously, falls and the p is falls; obviously, not T is true and all it is raining; that means, opposite of that it is raining when it opposite of it is not raining. So, in the same way, we can defined other connectives in this way; I do not want going the details of a all these things, which we studied earlier.

The first to begin with conjunction becomes true only; when the both the constituents are true, where as in all other cases it is going to be false and all. This junction on the other hand it can be used in different senses and all; the 1 which is shown in the red is used for exclusive r the 1 which is used in a black color is meant for inclusive r. In the case of inclusive r p or q that is a dis junction, you can began also called as a disjunction are you can called as a either p r q etcetera and all etcetera; that is going to be falls only when both are both constituents falls in all other case it is going to be true it.

In the case of condition little bit little bit tricky. So, condition will becomes false only when, the antecedent that is p is true in the consequent true here, is false that means, p is q T is false then p plus q is false, in all other cases it is going to be true So, this is quiet difficult for us etcetera, but, it works for mathematical reasoning specially, even the

antisubmarine the fall is the constituent false then also conditionally is going to be true and all for example, if I say if Tajmahal is in Andhra Pradesh then; Uttar Pradesh is in Pakistan; example, if you say that thing both statements are false.

So, even then that case the conditional total conditional the truth value of the conditional is going to be true all, because both p and q are false. So, that is what is a Semantics of material implication So, it false perfectly alright for a mathematical reasoning. For example, simple examples can be 2 plus 4 and then 3 plus 2 is equal to 5 So, in this case condition is going to be true if 2 plus 2 is equal to 5 and 3 plus 2 is equal to 6 is say that thing both p and q are false, but, at the conditions is going to be true.

So, this is what is mean; by a material implication is defined simply as p and q is nothing but, not p are q and the by implication, which is especially used for invoking necessary and sufficient conditions or it can invoke and some kind of equivalence relation; so that is simply like this. So, when both when p is T q is false like conditional p if an all q is false are p is false q is false like condition p if an all if q is false are p is false q is true then p implies if an all if q is false in all other cases it is true. So, this is what how we defined the connectives and all this is the way, the connectives behaves and all behaves.

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Use of Truth Tables:

Truth tables are used to determine:

- 1 to determine whether a proposition is a logical truth or a logical falsehood;
- 2 to determine whether a set of sentences is satisfiable (i.e. whether the sentences can be simultaneously true);
- 3 to determine whether two propositions are logically equivalent;
- 4 to determine whether one proposition follows from another; and to determine the validity of an argument

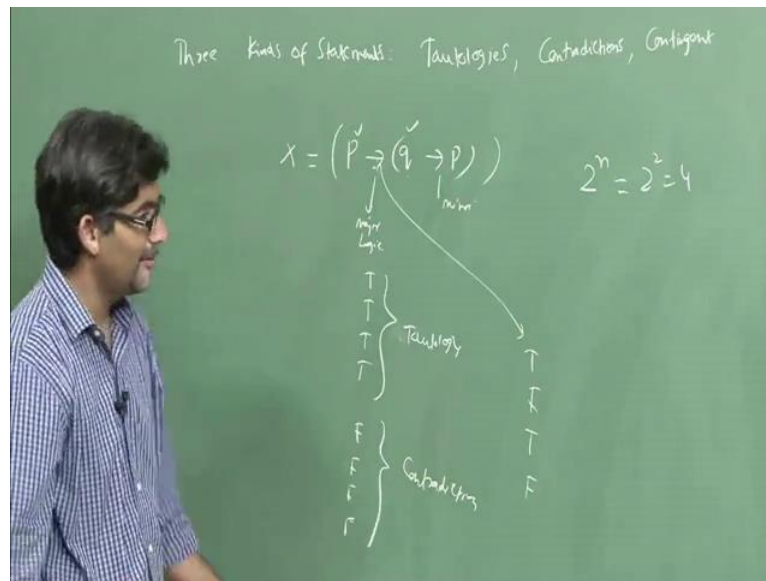
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So, what are we going to do with these truth tables. So, in what way are they going to be useful to us? So, truth tables are used especially to determine first whether the proposition is a logical truth or a logical falsehood that means, it's either a tautology or a contradiction. So, we will look to some example and show that when, I give a formula which is simple example, show that a given formula is a tautology and a given formula is a contradiction.

It can also be used to determine whether a set of sentences are satisfiable that is: whether the sentences can be simultaneously true or not that also, we can determine with the help of truth table. We can also determine whether 2 propositions are logically equivalent each other for example, if you say $p \wedge q$; it is logically equivalent to $\neg(\neg p \vee \neg q)$; how do you know these two are logically equivalent. So, there is a decision procedure method, with which we can say that these two are logically identical that is what we are trying to do in the next unit yes.

In a truth table, you can find out when the given formulas are logically equivalent to each other. We can also determine whether a proposition follows another after logic is all about what follows from what. So, that is validity is a concept, which takes care of what follows from what. So, it is an inference a conclusion from the. So, that means, we can also determine the validity of a given argument using truth table method. So, we will consider few examples and then see, how we can determine whether to begin with first we will talk about whether a given logical formula well from formula is a tautology contradiction or contingent kind of statement.

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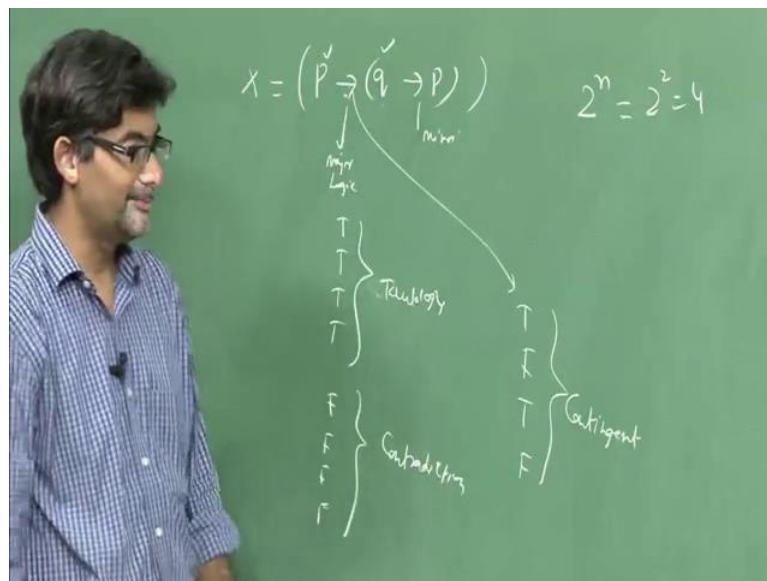
So, let us consider some examples, for improving there are 3 kind of statement which existing propositional logic. So, they are tautologies, 1 formulas which are; obviously, true always true. And there are some other formulas, which are called as contradiction and there are some formulas; which are sometimes true and sometimes false in all, which are consider and the category of contingent statement. The first for most thing which need to do is to identify, what kind of it is: whether it is a tautology and it is a contradiction are is a taken contingent kind of statements.

So, let us consider some example, suppose if you have a formula like this. So, now we want to check whether, this particular kind of formula is a tautology are contradiction are contingent statement is in truth table method So, now there are 2 variables here p and q So, that case 2 to the power of n possibilities increasing in the possibility in truth table that means, 4 entries are possible can I give at truth table So, now what we need to do here is this thing. So, this is considered to be a minor logical connective, because it can at only 2 variables. Now, this is consider to be a major logical connective.

So, under the main logical connective the idea exists that under the once you identify the major logical connective under this major logical connective. If you get on only T is etcetera on it is all T is only then; it is called as tautology the formula which always true

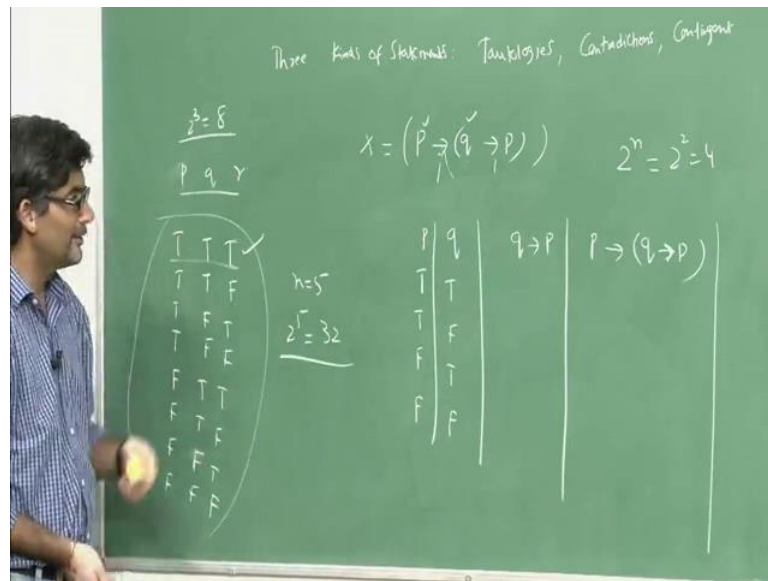
whatever, values you are signed to p and q is always going to be through and all. If there is a case, it is a tautology and if you can only F and all under the main logical connective then; it is a contradiction and in the truth table under the main logical connective here, T is F are T s etcetera 1 T 1 F and 1 T all F and all. Then also, it is called as contingently contingent the formula.

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So, now how to take whether the given formula that, we have taken is a tautology are now so but this we need to construct the truth table.

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So, for these in a the variables as to be need write p and q. This is the only 2 set variables So, we have and then the first 1 have which into take into consideration is this 1 because, it is a sub formula of name formula q in p plus 1 which take into consideration. And then you write truth table for this 1 so for. So, now p is used can take only these values T T F F T F T F; so there is a way of writing this particular way of thing suppose if you have 3 variables.

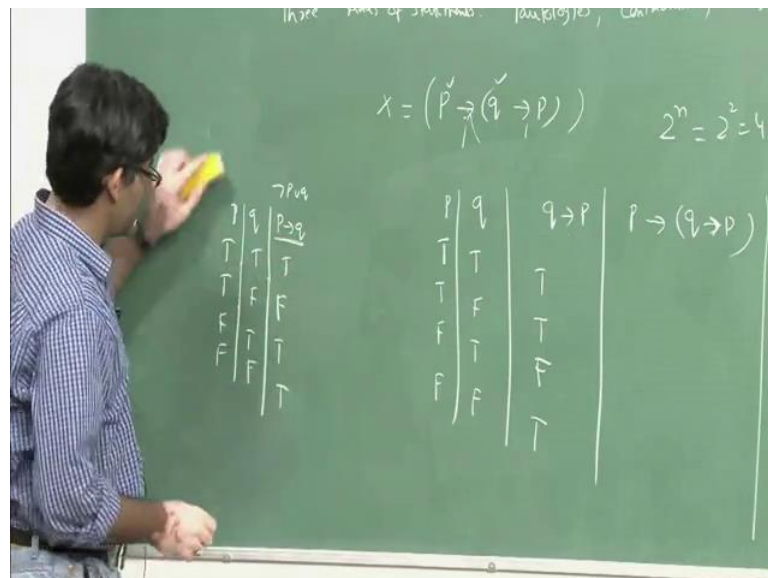
So, what you will do yeah is T T T, because 3 variables are there 2 for a 3 entries are possible and rows which are possible in a given to table, let be 8 entries are possible in first you write all T's 4T's and all followed by that 4 F's. So, this is what you do and then second 1 what you do is T T F F 2T's and 2 F's 2T's and 2F's and all and then 1 second 2T's F F. So, this enough 4 variables are there. So, now once you write first 4T's 4F's and 2T's 2F's and then you write alternative T F T F T F T F and all So, this is going to take care of all the possibilities that, you can talk about in a given formula these are the only values that p s q s r s can take about.

For example, in the first T is p q r etcetera takes only T is and all in the second case p takes value T, q takes value T and r takes value F. So, like these can fill the truth table and all, but the problem is here is that once a number of variables increases for example,

if you have n is equal to 5 then you have to 2 to the power of 5 entries, which have possible that is 32 entries which needs to inspect to find out a given formula is a tautology contradictions and contingent here.

So, it is little bit difficult and all that is may be find out some other kinds of methods which, we can called has the indirect tool method of then will be some other methods. There are some other methods which we will talk about in the next 5 classes. So, there all decision procedure methods about conduct this problem. So, first we need to consider q plus p this is a formula.

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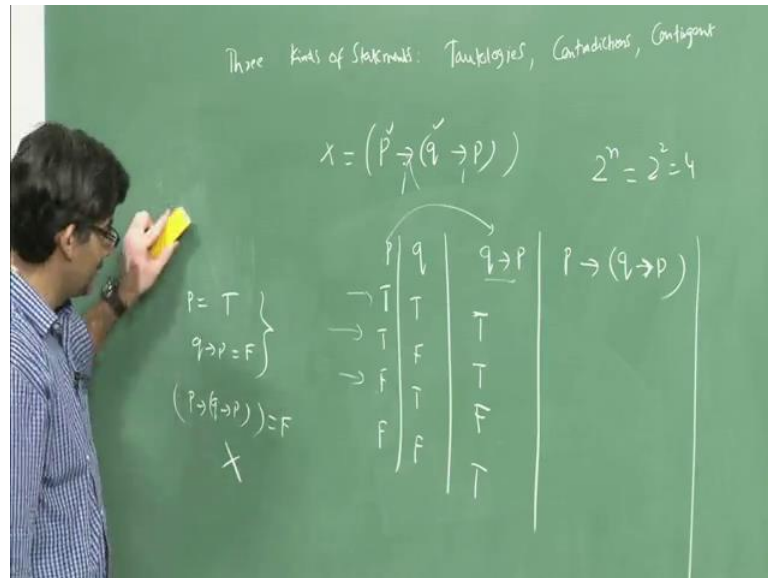


So, now this formula is going to be false only in this case, in all other case is becomes T C all So, Y is because for this particular kind of sequence this is p and q p in plus q is defined as p r q. T T F F T F T F. So, this formula is going to false in lean this case, in all other cases it is going to be true that is the way we defined semantics of p in plus. So, this is the truth table of this 1 q in press p. So, what you are trying to do simply exists that given a well form formula that trying to constructed truth table method. Truth table method is a very good constructive kind of methods.

It is very easy to use especially when the number of variables is less than 4 and it is

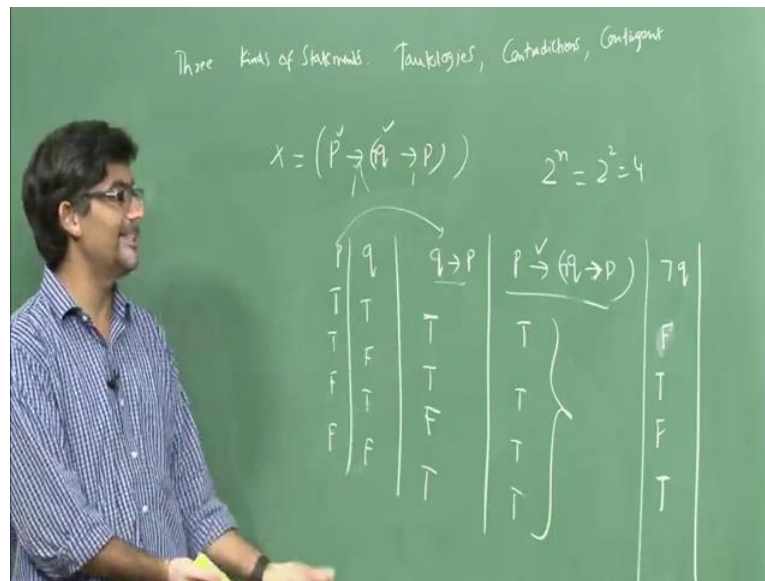
more than 4, it will things will become very difficult it scarifies the entire board and all. So, you will follow some other methods.

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So, now p implies q implies p now you need look a these 2 rows and all here to inspect row in which p is T and q in plus p is for false row. So, there is no row these are all rows and all; there is no row in which p is T and the q plus p is false and all of a this is p is false q impress p false; obviously, it is false in all So, there is no row in which p is T and q in plus q false and all, because that is make a whole conditional p in plus p false and all. So, we want have this particular kind of row. So, that means, all these things true.

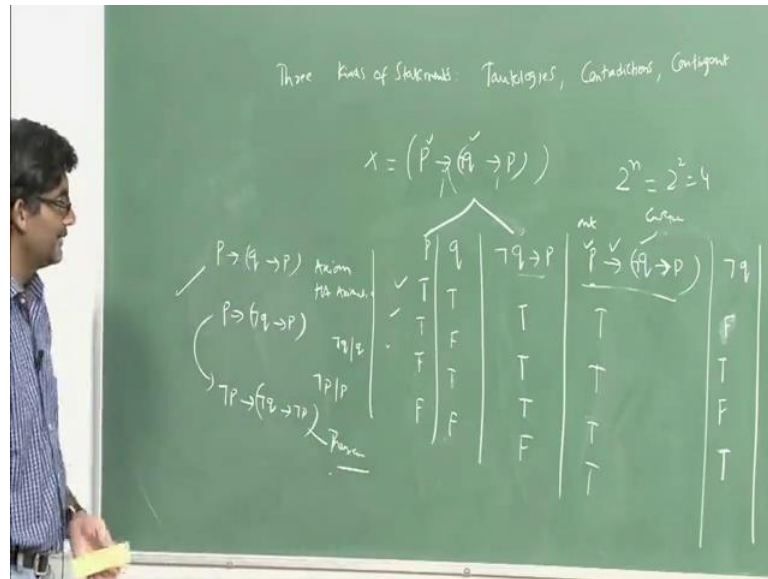
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So, now what we got here is simply this thing under the main logical connective; this is the main logical connective we got all T is; so that makes it particular kind of formula it out all So, what happens if you put some kind of if you change it into some other kind of thing. Let us say this is p implies some what you need to do we need to draw some extra column and all that is mark. So, now observe this q whenever, it is T it becomes F whenever it becomes T whenever, it is T becomes F and there are it becomes F it becomes T that is a semantics of negation.

So, what we have to done you can change in formula into some other thing and all thing than a trying to see a whether; it is tautologies contradiction and or contingency formula there are very simple things to do once he contingent truth table what you mean to do is another main logical can I do we need to inspect whether; a given formula is I mean, it you always get T is in all the rows of your truth table.

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So, now we need to consider not q implies p. So, now truth table changes are that is Y complete So, now not q implies p you have to take into considerate this 1 you are to move from this. So, this becomes false only when not q T and then these false in all in all other cases it becomes T. So, that is a way we defined the semantics So, now we need to consider p implies not q implies so; that means, we need to consider this particular kind of rows, these 2 rows which we need to consider. Is an any row in which you have T is and not to implies p T and all not q must be false and all again in this case there is no row.

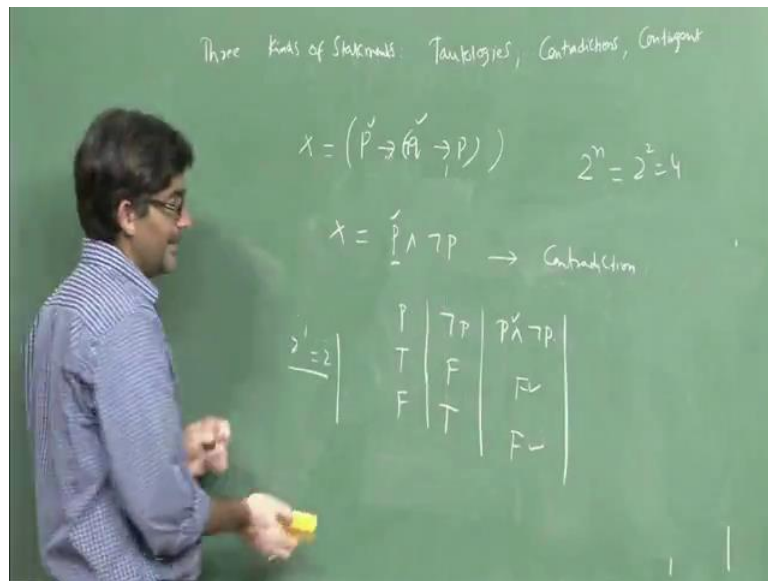
There are all the rows and all we are inspect each and every row here. Is there any row in which is you have antecedent, this is the antecedent and this is whole thing is a consequent is there any row in which this particular thing is T and the right hand side q this is not implies that p false. And this row this is not the case both are T is sold out even this case also it is T and this case also go to be T and in all the cases it becomes T So, p implies not q is also going to be a tautology involve, it is simply straight forward and all.

So, this is usually when axiom and hilbert academic system, which were going to talk about little bit later suppose it is treat this particular thing is as an axiom whatever is

substitute into this 1 that is uniformly substitute this 1 is also going to be a going to be true it. So, example here in this case not q is substituted for q this is also going true well suppose, it substitute not p for p n all then this formula becomes not p implies not q implies not q this is also going to theorem is a 1 it is; obviously, 2 and all. So, uniform substitution you can retain the tautology and all. It is in this sense we just replace q by not q uniformly then it retained by tautologies.

So, this is talk about little bit later when study maxi metric systems. So, this is also an example of a tautologies. So, let us consider some more examples and see this given formula is a tautologies or contradiction and contingent kind of state.

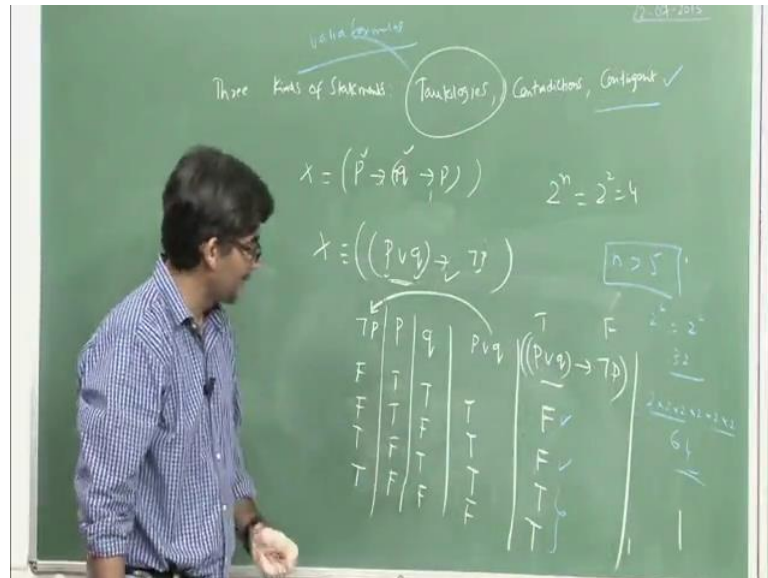
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Suppose if you have a formula p and not p obviously, this is a contradiction your saying that, it is raining and it is not raining; obviously, this is a contradiction. Let us constructed 2 table there is only 1 variable here preposition variable that is p. So, that it has been only 2 entries in the truth table. So, once it is the case p; p and are p the only value such it can take it is p can take values T p can take values F and all and the corresponding variables is p is t; obviously, it is F and if it is F and it is T some have you take p and not p.

So, it is like 1 into 0 that is 0 it means; F only even in these case also it becomes F. So, under the main logical connective p got only F s let means; this p is consider to be a contradiction. Now, let us consider some examples per contingent kind of statements.

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You can take other examples also p r q implies, it is a not some formula as come to mind have retained here p r q implies not. So, now what you need to do here, if first you write p and q there are a 2 set of variables set exists this formula then; you write p or q and here the main logical corrective is particular kind of thing while studying the syntax we found out we found what is going to the major logical connective.

And what is going to the minor logical connective in major logical connective is 1 which connects as many preposition variables is possible and all; that means, this is a major logical connective, because it connects 1 2 3 variables and all. So, that is why it is called as the major logical connective and whatever, logical connective that occurs in the sub formula is consider to be the minor logical connective. So, you are p r q and the next 1 here going to write is this 1 here to consider not be also.

So, this is the 1 which needs to brackets needs to be p F how we are try to find out whether this is a tautologies contradiction and contingency statements. So, now first you

write this thing we have 2 variables you write just like this p T F F alternative T's and alternative F's 2T's and 2F's in here alternative T n alternative F. And T r q to going to be false only this case in all other case it becomes true and then we need to cell this not p and all not the. So, if it becomes T it becomes false as becomes false T and T.

So, now, many to looking to this particular kind of thing and arrow is important in we should not p in press p r q. This is totally different formula p r q implies not is different from not p implies p r q its never the 1 is 1 the same logically identical each other. If the by implication is there then you can go you can move in both directions. So, p or q implies not be now you are find out row in which you have this as this p and then this F then the whole condition will become follows.

So, now, this is 3 and this is F it becomes false again p r q and not p again this is a this false. Now in this case both are thing that mean, it is T. So, now, p r q is F and then this is T and by the semantics of condition whenever, the false the consequent is true the whole condition is; obviously, going to a T n what is that we got is simply listed. Now, in the final column of your truth table you have 2F and then 2T is if it have been in the case that have if you are not all Fs in what then it be a concern to a contradiction, which you we did not get it in the same way if I got all T is and all main the character implication and it is consider to be a tautologies which is not the case here.

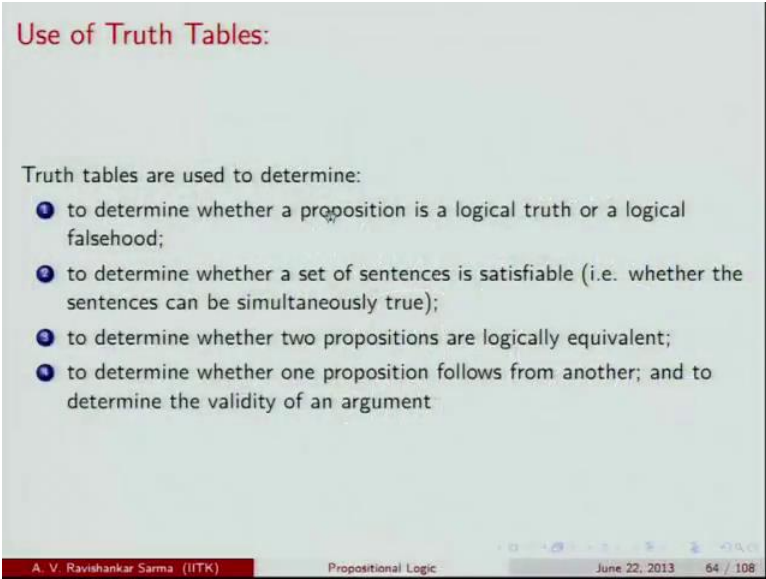
So, then; that means, it is considered to be a contingent statement. So, using truth table you can classify the statement of preposition logic in to tautology contradiction and contingency his truth table method was perfectly find especially, when the number of variables are limited to 3 and 4 all off course, it works have any number of variables. But, it is very difficult for us to constructs the truth table especially, when n is greater than 5 because n is greater than 5 of example 6 variables.

There then we have 2 to the power of 6 and 3 termite 32 entries; which we need to inspect may be 64 2 into 2 into 2 into 2 into 2 into 2 into 2 into this is 8 into 8 64 entries will be there it becomes a difficult for a human being to see all these things and all; I have it is easy but, a computer can easily to this particular kind of thing. So, whatever it is very difficult for us may be the computer can easily. So, truth table still may used as 1

of the provided method for judging method a given formula is a tautologies contradiction and contingency, were we doing all these things simply because a time to extract tautology and all from a given well form formulas.

So, we stated with well form formulas and after this well form formulas some are constraint to be valid formulas which is; obviously, tautologies some are contradiction; obviously, which are invalid in some are concert to be contingent kind of formulas. If you can extract tautologies and all, then what we can have is all valid formulas because all tautologies are; obviously, a valid formula that is a reason why we are in sting on tautologies in particularly. So, easing truth table 1 can talk about whether a given well form formula is a tautologies contradiction or a contingent state.

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Use of Truth Tables:

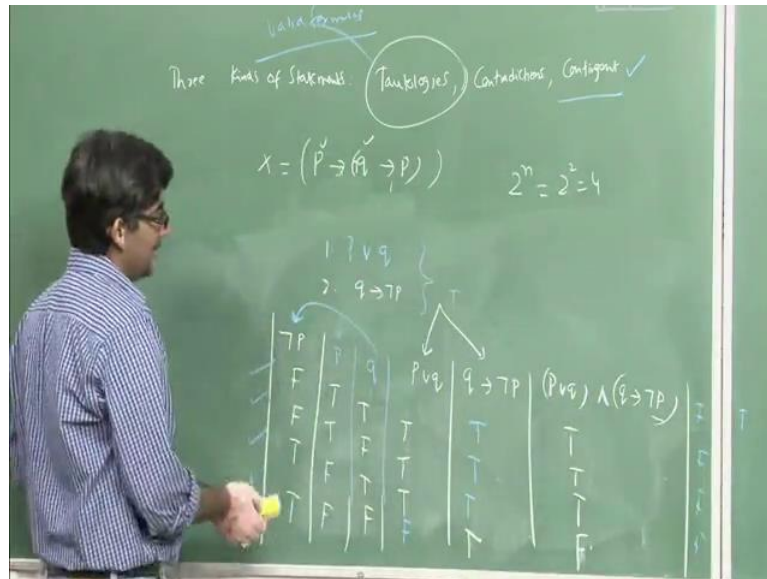
Truth tables are used to determine:

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So, now the second thing which you can do is the particular you can also determined whether a given a set of sentences are satisfiable or not. So, let means it is consider some simple example and will see whether, it is considered to be satisfiable or not.

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Now, we are come across you are translated the English language sentences and then you came across this particular kind of thing p r q is the first sentence a q implies something like not p n of. So, now satisfy ability means; once you assign truth values to p s an q s ultimately these 2 should be simultaneously true and all. This true becomes false allot false unsatisfiable there off. So, how do you using truth table method this is some of the translations are English language sentence may be, it is can be raining or it is not q stands for grass is net if stands for if it grass is net then implies it is not raining here.

Let us functions some example, write this then it will see whether disstatistable or not at means simultaneously; it as to be true and all then only we can true especially then that case we can say that theorem these are satisfiable satisfiable or the other way of saying that thing is consistent. Now, what do you need again in we draw the truth table and then we need to see whether; at least some particular kind of row in particular kind of row we generate T is T is. So, now you have p q now this is a formula, which we have here and then q implies not p; now e to study this particular kind of thing p r q and q implies not.

So, under the main logical connective; that means, end if at least there is 1 of 1 case 1 row in which you generate T is then that is consider to be satisfaction; let it all F it is

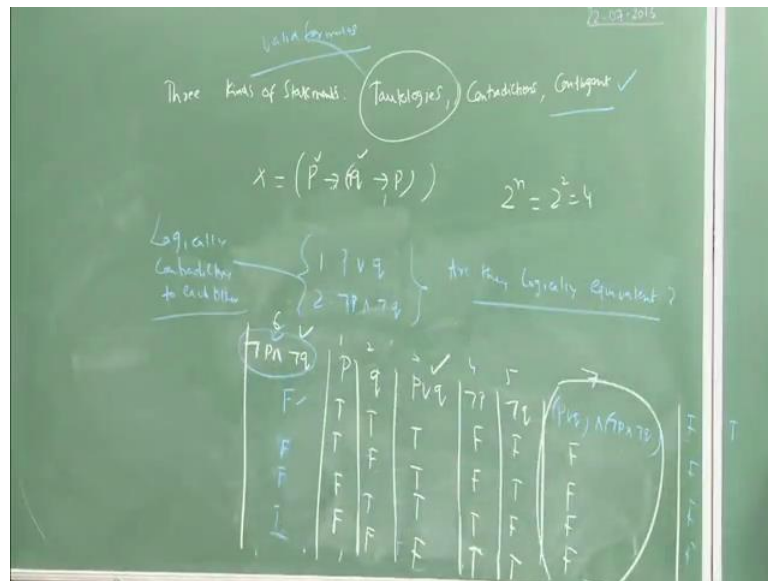
consider to be unsatisfiable. So, as usual we have two variables and you can take T T F and then alternative T alternative F; these are the only values that you can take. So, now, you have not T full write it for not t. So, exact even p becomes T then it will become false then it is false T n T. So, negation of T is takes a values F F T T. So, now you feel these things p or q is going to be false only in this case and both are false and both constituents are false in all other cases it is T.

So, now, q implies not p; that means, q p be to take this 1 q implies not p directions are important. So, this is going to be false when consequence true and the consequence false and then in this case becomes T; now, this also T, now this is also T. Now, what we need to consider is these 2 things; now, only under this particular kind of thing it is false in all other cases it becomes the component formula is going to be T. So, now what is that we have achieved with this particular kind of construction of the truth table, we are trying to see whether these two statements that we are returning in the board are jointly consistent jointly becomes true.

There are 3 different cases in which it becomes true. So, these are the 3 rows which needs to inspect only in this row it becomes false a does not matter and at least 3 different situations the formula p r q and q or not q implies not p is simultaneously, true that p or q implies not p are consistent to each other or at least one T is there in this 1 and then the major logical connective, it is also considered to be satisfiable. So, will talk about formal definition of satisfiability little bit later but, as per has 2 table is concerned when you take 2 formulas p r q. And the combine formula; under this major logical connective if you come across at least 1 T then it is considered to be satisfiable.

Suppose, if come across all F s and all F's etcetera, then it is considered to be unsatisfiable; some example we can take in to consideration we can understand it little bit. So, this formula considerable satisfy able and you can even call it has simultaneously you can call it as p or q is consistent with q implies not p.

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So, now you take some other formulas and you can see whether these are consistent to each other or not not p and not q. So, the way of return itself show that these you are contradictory to each other off course, using truth table again you can find out whether or not these are unsatisfiable satisfiable etcetera. So, now, this is p r q and not p and not q. So, you will quickly going to regions of this 1 T T F and F is a 1 which you write it first and then alternative T's and F's it is boring retails of all these things

So, now, quickly p r q this falls only in this case in all other cases it becomes T not p is exactly the opposite of this particular kind of column. So, now, this is F F T T. Now, not q is exactly opposite of this 1 F T F T. So, not this is of the end know end. So, we need to take the conjunction of this particular kind of thing. So, we need to write not p and not q as well. So, this first second third fourth fifth and sixth and this is seventh 1. So, this a way to need write. So, we need to thing conjunction of these 2 things.

So, this is going to become true only in this case in all other cases, it becomes false because at least in 1 of the conjunction in false in this cases. So, now this is s F F etcetera. So, all this cases it becomes now, we need to consider this and this now in all these cases at least 1 of the conjunction is false and all. So, that makes the whole conjunct false some So, now, what is that we got we simply this things under no

particular kind of interpretation; interpretation assigning some kind of values oh what are you doing assign values p and q.

So, under no particular kind of assignment of the truth values to a given propositional variables we got T's and all T's is particular in the final column of your truth table you got all F's. So, it is in this sense these 2 formulas are in suggestion to each other are it is also considerable should be unsatisfiable. So, this is a way we can determine whether a given formula is satisfiable and consistency using the truth table method. We can also determine whether to formulas are logically equivalent to each other. So, now, again the going to the details of this 1 we take the same example into consideration. So, 2 formulas are set to be logical identical especially, when the truth table matches.

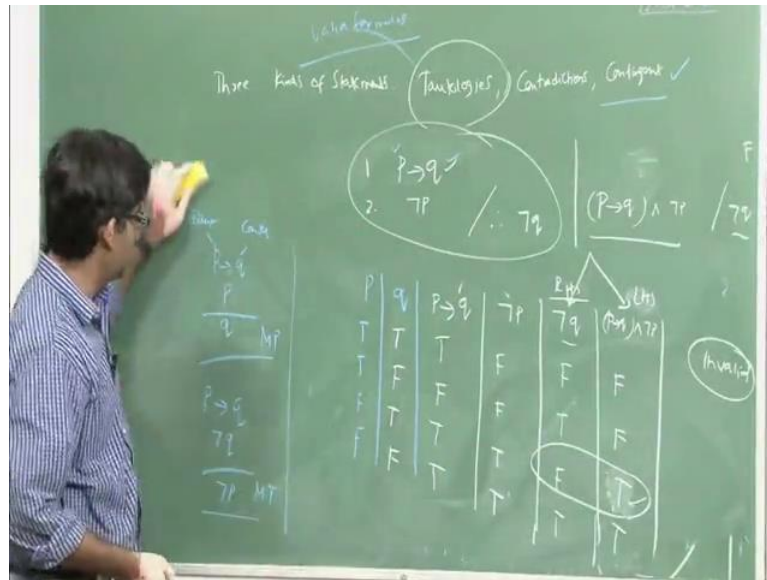
So, now what we need to see here is need to observe the truth values of this 1 are there logically, equivalent that is what is the question we are trying to answer; so what we saying is when that a truth table matches exactly then they are consider to be logically equivalent. So, now to each observe p and q and this 1; now if you have value F here we got p's and we have F here we got T when we have F here you have T g r. And then whenever we are have of we have F here that is a truth table does not match but, what else you can say about it then the truth values of 2 particular given formulas are exactly opposite to each other and are set to be contradiction to each other.

So, that means, here is F we will find T here and whenever we find T here we will find F here. So, it is in this sense these 2 formulas are logically contradictory to each other contradictory to each other because they are exactly, opposite truth values of in the truth tables whenever, p r q is T. Now, we will see here F whenever we have F here we have T here; that means, exactly opposite of truth values it has is 2 formulas are exactly opposite truth values when also these to are set to be logically contradictory to each other.

So, we can also talk about logically equivalence with the help of truth table method that means, especially, when the truth values of 2 particular well form formulas matches then they are considered to be logically equivalent. So, now the 4 thing we can determined with the help of truth table; is the very important thing which is considered to be validity

of a given argument. So, let us consider some simple examples and will see whether the particular formula follows from this thing or not. So, randomly we take some formula into consideration and then we will see whether particular things follows a that or not.

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Let us see, we have formula p plus q and then you have formula not p and then this is a conclusion; which separated and all and then you will write this you write not. So, now using truth table whether or not this particular thing follows and all but, usually speaking. So, we have some kind of rules; which is p implies q is a case p is a case then q follows. So, this is called as a true it is; obviously, truth preserving kind of rule so; obviously, always valid in the same way there is other kind of role which we commonly used that is this q . So, here this is antecedent and this is consequent.

So, now this is antecedent have consequent, if you delay the antecedent in you are to delay the consequent and you delay the antecedent. So, this is called as true. So, now you can clearly see here inside of denied consistent here we denied the anticipant part and then they denied the consequent is might working the day to day discourse, but, in the first in the classical logic other proposition logic that we are trying to talk about this is considering to be a in valid arguments. So, we want to see Y it is an invalid argument.

So, we want to see why it is an invalid argument using a particular decision method procedure method, which we have been talking about. So, that is truth table. So, now what are the variables existing here p is and q means 4 entries are possible that is there are 2 variables here. So, you write down the same thing $T T F F$ and alternatives $T F$ etcetera and the formulas that, we have are here this p implies q and you are not p . So, p implies q it becomes false only in this case in all other cases it becomes T . So, this is a semantics of implication that we have p is not exactly a opposite of this $F F T T$.

So, now we write q as well not q is $F T F T$. Now, what we need to do here is now, we can write like this p implies q and not p and then separated by you have not q . Now, hope to the conjunction of p is 2 which the 1 we needs to write here that is p implies q and not. So, that is this becomes false this becomes false this becomes T and F this becomes false this is T . So, now we need to observe rows in such a way as is a any row in which the left and side here left and side is this particular kind of thing p implies q and not p . Do have any row in which this whole thing is T and a conclusion is false.

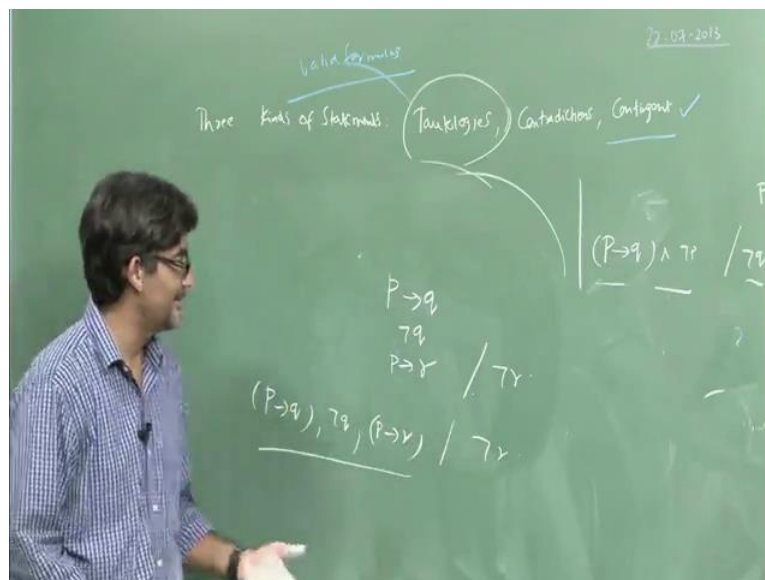
So, now e to inspect the rows what are the rows are certainty to inspect these rows these are the 2 rows which we need to inspect. So, this is the first 1. So, this is on the left hand side and this is on the right hand side. So, now easier any row, in which we have T 's on the left hand side and T on the right hand side and F on the left hand side and all. So, now so there are different ways to this thing. So, now here is an instance where 1 second p implies q and not p it becomes $F F$ and $F F$ p and $T T T$ and T ; this becomes T . So, this is the 1 which we need to look and T . So, when is argument is considered to be and invalid argument.

So, invalid argument is an argument. So, which we have true promises and false conclusion that is what we have been telling write from the beginning of this course under basic concepts we was that the argument is invalid especially, when it is impossible argument is valid particular especially; when it is impossible to the promises to be true and the conclusion to be false. So, now these are promises to be conclusion and this is considered to be conclusion here in this case is it possible that you are a promises are true and the conclusion is false.

So, now observe this particular kind of row. So, your promises are true and miss both taken together are true and your conclusion is false not true is false to be conclusion here. So, that is false; that means, we can have a counter example in which your promises are true and the conclusion is false that makes this arguments invalid. So, that is say this argument is invalid solid we know that: a invalid is construct a truth table method and then be or inspecting left hand side in the right hand side and right hand side is usually consider as a promises. If you are true promises you can only have a true conclusion you cannot have false conclusion.

If you can come across to promises and a false conclusion then the argument is invalid So, it is in that sense this particular kind of argument is consider to be an invalid kind of argument. If an consider some other examples of and you can show whether, given arguments is valid or invalid it has consider some more examples and will see whether with the help of truth table that particular kind of arguments is valid or invalid p implies q not q that is say p implies r then from that n you are derived not r.

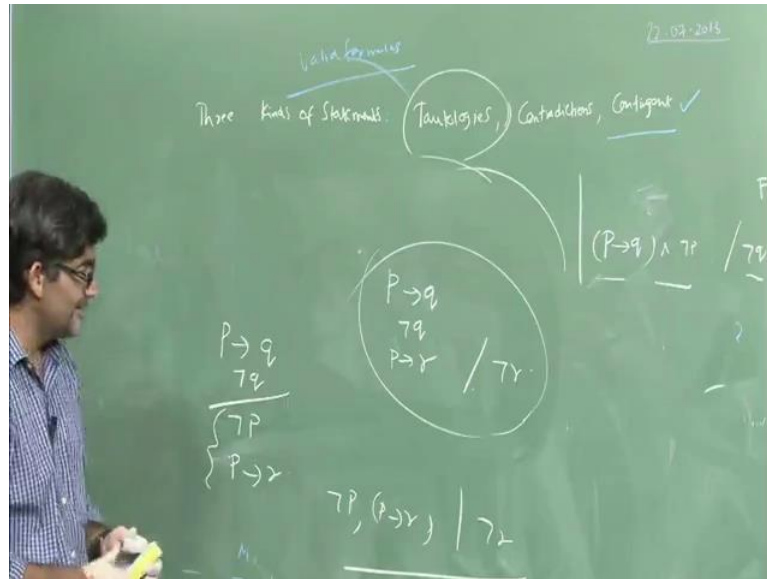
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So, now since we have 3 variables we is the truth table will be relatively little big. So, now p implies q not q and p. So, now we need to inspect a row in which suppose, if you can come across all these things true and not r is false then we leads to a particular kind

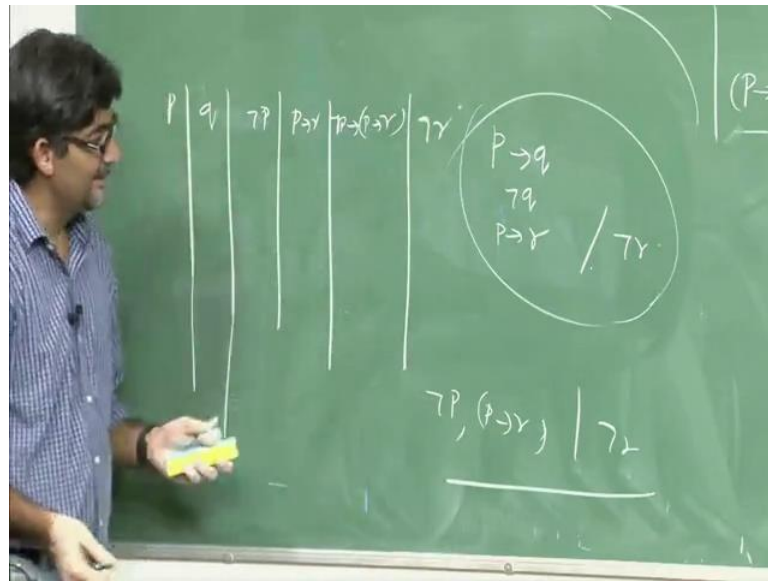
of thing that this formula is considered to be invalid. So, if involves 3 variables. So, the truth table will be relatively bigger. So, inside of that 1 can try to solve this particular kind of problem using some different kinds of methods, which we are will be talking about in the next class.

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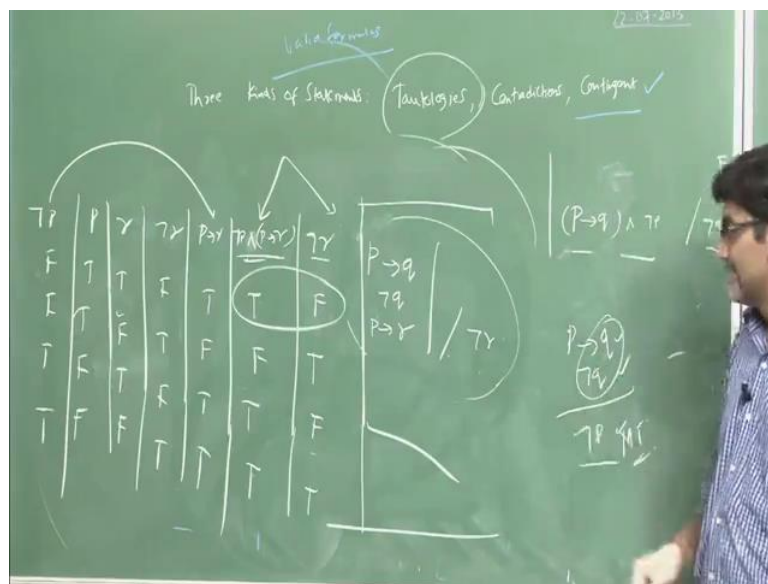
So, now we trying to see whether not are follows from these 2 or not; so simple thing is here p implies q and not q from this we will get not and not p and p implies r. So, which we once you have inspect some not p p implies are whether or not not. So, this religious to 2 variables now; these are talking about q talking about missing not 3 p implies r not r. So, if you can say that this is a concerned to be valid, if you can say that promises are true and as the conclusion are false conclusion are false then it is invalid and other it is going to be valid and all So, now, the radios is to this particular kind of thing.

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So, now, quickly you can draw truth table or this 1 not p and p plus r not p implies. R T implies r etcetera and then not r; there are 3 variables here and again the 2 variables in fact.

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So, p r and not r, so r is T T F F this is T F T F and then not r is F T F T is what is not a p

implies r this becomes false only in this case in all other cases it becomes T. So, now not T not p and not p implies r . So, these are the things which needed to take into consideration not p is this 1 F F T T not p implies p implies r . So, now this is false and this is also false T and T true and T and is called with true. So, now what over so now we are trying to see whether not or follows from these things are not. So, now you are reduce a formula into 2 variables and all including p answer. So, now not r is this 1 whenever you have T you have F here T F F is T.

So, now what we need to do here is need to observe these 2 rows is there any row in which you are 2 promises and false conclusion. So, now clearly we have this particular kind of row in which a promises are true that means not the p implies r r true and the left right hand side is false and all. So, this row is sufficient enough to show that this particular kind of argument is invalid is in that have to inspect any other row, because invalidity requires that at least 1 counter example. If you can come across with 1 counter example can which your promises are true and the conclusion is false and then; obviously, the argument is invalid.

So, what is that we have done here first we have reduced p implies q and not q into this particular kind of thing. So, this is a logical consequence of this 1 from this you can derived not p is the a logical consequence of these 2 by using now you remove the 1 particular kind of variable q is not required here constructed to table F these things we have seen that a premises of true and conclusion is false and hence the argument is obviously, an invalid argument. So, what we have seen here is that we determine whether; the given formula is a tautologies contingent statements or contradiction.

Or we can even say when 2 groups of statements are satisfiable each other; or we can even say when 2 groups of statements well form formulas are logically equivalent to each other. There equivalent to each other especially, when the truth values matches and we can also talk about whether or not a given well form formula given conclusion follows from the premises again you can constructed to the truth table method.

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Strategy for constructing truth table:

- 1 Across the top of the left-hand side of the table, list each primitive proposition that occurs in p .
- 2 Beneath this, fill in each combination of T's and F's, beginning with an 'F' beneath each primitive proposition and ending each column with a 'T'.
- 3 Write out the proposition p across the top of the right hand side of the table. Leave some space between each symbol.
- 4 Starting with the smallest subformulas of p (i.e. those nearest the top of p 's construction tree), fill in the column under the main connective of those subformulas with 'F's and 'T's according to the truth table for the connective in question.
- 5 Repeat the previous step until there is a column of 's and 'T's under each connective. Now **highlight the column under 'p's main connective**, as this is the information that we are looking for.

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Construct Truth Table for the formulas:

- 1 $p \wedge q \vee (\neg p \vee \neg q)$
- 2 $(p \wedge q) \wedge (\neg p \vee \neg q)$.

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Tautologies, contradictions, contingent statements

- 1 Tautologies have only 'T's in the main column of their truth table.
- 2 A statement is a **contradiction** if and only if it is false on every assignment of truth values to its atomic components.
- 3 A statement is **contingent** if and only if it is true on some assignments of truth values to its atomic components and false on others.

Truth table method works for a works better for this thing but the number of variables increases, if present some kind of difficulties see this have this will be a set sofa 1 can have tautologies especially when under the main logical connective only T is. And statements is consider to be contradiction especially, under the main logical connective you have F and the statements is contingent. If only if it is true on some assignment that means, a sign d some kind of values p q is an T a l F.

Ultimately under the main logical connective you have to T s to F and may be 1 F 1 T and 3 F etcetera and all etcetera at least 1 T should be the in under the major logical connective.

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Examples:

Using truth tables, determine which of the following are tautologies. For any that are not, give a valuation which does not satisfy the sentence.

- 1 $(p \rightarrow \neg q) \rightarrow \neg(p \rightarrow q)$
- 2 $(p \rightarrow \neg q) \rightarrow \neg p$
- 3 $(p \rightarrow q) \rightarrow \neg(q \rightarrow p)$
- 4 $\neg(q \rightarrow p) \rightarrow (p \rightarrow q)$
- 5 $(p \rightarrow (q \rightarrow r)) \rightarrow (p \wedge q \rightarrow r)$
- 6 $(p \wedge q \rightarrow r) \rightarrow (p \rightarrow (q \rightarrow r))$.

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It is that case, it is consider to be contingent. So, 1 kind determine the formula are is a tautologies are not using this particular kind of thing.

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Procedure for consistency, inconsistency

- 1 Symbolize all the propositions of the argument in question.
- 2 Put the premises in conjunction- if there is more than one- by pairs, associating to the left.
- 3 Construct a truth table for this conjunction.
- 4 If the conjunction is **tautological or contingent**, the premises are **consistent**. If the conjunction is contradictory, the premises are **inconsistent**. In other words, premises of an argument are consistent if there is atleast one interpretation making them all **true**.

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So, there are some particular kind of statements, which occur in the natural language and which are presented in the natural language; there is English there is what first to do is

we need to translate the given english language statements is to appropriate language of proposition logic. Then we can talk about a particular formula is Contingent or tautology or it is consider to be contradiction and all.

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Procedure for finding Logical Equivalence:

- 1 Symbolize all the propositions of the argument in question.
- 2 From these two symbolized statements, construct a third statement of the form $p \leftrightarrow q$, where p is one of the original symbolized statements and q the other.
- 3 Construct the truth table analysis of this **biconditional**.
- 4 If the **biconditional** is either **contingent or contradictory**, the original two statements are **not** logically equivalent. But, if the biconditional is **tautological**, the original two statements are logically equivalent.

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Logical Equivalence: Example

- 1 If the Neuron is alive(A) and fires(F), then it has a given minimum number of excitatory fibres(N). $(A \wedge F) \rightarrow N$.
- 2 If the Neuron is alive, it has a given number of excitatory fibers whenever it fires. $A \rightarrow (FN)$.

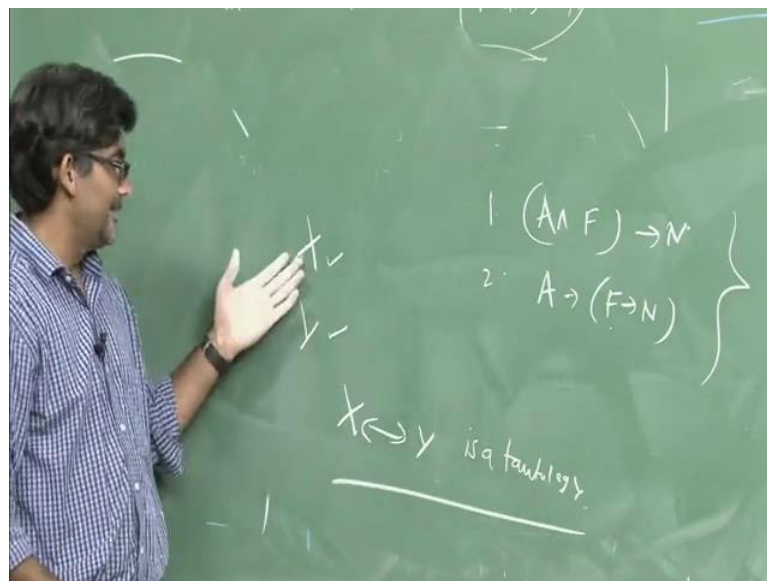
$[A \wedge F] \rightarrow N \leftrightarrow [A \rightarrow (F \rightarrow N)]$ is a **Tautology**.

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So, examples if have this particular kind of thing if the neuron is alive and fires them a

given minimum of excitatory fibers e and F implies N . And if you want to say neuron is liability as given number of excitatory fibers wherever, it is fires A implies F N . So, now if want to say fibers that these 1 and 2 are logically equivalent to each other. So, then what you need to do here is this. So, the first formula in this 1 is like this before that I will talk about what you mean by saying that b is to a logically equivalent to each other.

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The first formula is A and F implies N and the second formula is A implies F implies N . So, now these are the 2 formulas; which are which be got by translating these to statements. So, now if want to show that this is going to a tautologies and then; that means, if 1 the first statement is considered to be X and second statements considered to be Y and it; obviously, becomes F if an only if Y if Y . So, if you want to say that this to logical equivalent to each other what you need to do here, if this here a formula X and you have a formula Y .

So, X and Y considered logically equivalent especially, when if an only if by is A tautologies. If you can say that X if an only if Y you can show that X ; if an Y is a tautologies then; obviously, then X and Y are set to be logically, equivalent these 2 statements whether or not logically each other for that: we want to need check is first you need translate the English language sentence into appropriately into the language of

propositional logic. And then let us assume that the first formula is X that is $A \wedge F$ implies N and the second formula is $A \wedge F \wedge N$ and then X if and only if Y if you can show that that is a tautology than already shown that X and Y are set to be logically identical to each other.

So, in this case what we have discussed is simply based that we started with truth table method, which is consider simplistic method which works fine for number when the number of variables proposition variables are less. So, with the help of truth table method, I can talk about consistency I can talk about whether or not a particular formula whether or not a conclusion follows from the premises that is a logical validity. Or you can also talk about whether and not to given well form formulas are logically equivalent to each other by showing that X we can only if Y concerned to tautologies.

So, in the next class, we will be talking about a particular different kind of method, which works perfectly alright even if the number of variables is more than 4. So, that particular method is called a simultaneously, tubules method see we have to note that we have covering semantic methods so on. So, then we will move on to this simultaneous tubules method and we will talk about the essential feature of simultaneous tubules method.

We can talk about same thing like whether and not given well form formula is consistent to each other contingent etcetera all these things; whether a given formula is valid etcetera. All these things well can you know the help of simultaneous tubules method. And then next class, you are going to talk about simultaneous tubules method.