Introduction of Logic
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Lecture - 19
Semantic Tableaux Method for Propositional Logic

Welcome back, in the last lecture, we discussed in the detail about a one of the important methods which is considered as a decision procedure method which is call as a truth table method. Truth table method is considered, to be the most simplistic method especially, in this course introduction to logic. It is simplistic in the sense that as long as the number of propositional variables is less in number; that means, to are 3 for example, if you have 2 a propositional variables he have 4 entries in your truth table. And if your 3 a propositional variables like p q r etcetera representing some kind of propositions.

We have 8 entries in the truth table, but the problem is that it is very difficult for as to manage a for example, when you have more than 5. Or 6 variables propositional variables if you have 6 variables; that means, 2 to the power of n entries will be there in the truth table; that means, 2 to the power of 6 may be 64 entries he need to inspect of find out whether, group of statement are consistence are were there a particular kind of a conclusion follows on the premises; that means, the validity etcetera.

For that you know you need to check all the 64 entries; that means, we need to inspect each and row each and every row of your truth table meticulously; that means, 64 rows are there and all the rows he to inspect a, I mean; a those rows in which, where there are not you have thru premises in a false conclusion, if you have true premise and a false conclusion then the argument is; obviously, considered to be invalid. So, instead of inspecting the 64 rows which will difficult for us and there are some other better methods a 1 such method which we will discussing, in this class that is semantic tableaux method. So, this is also called as analytic Tableaux method are it is also called as tree method there 1 other same.

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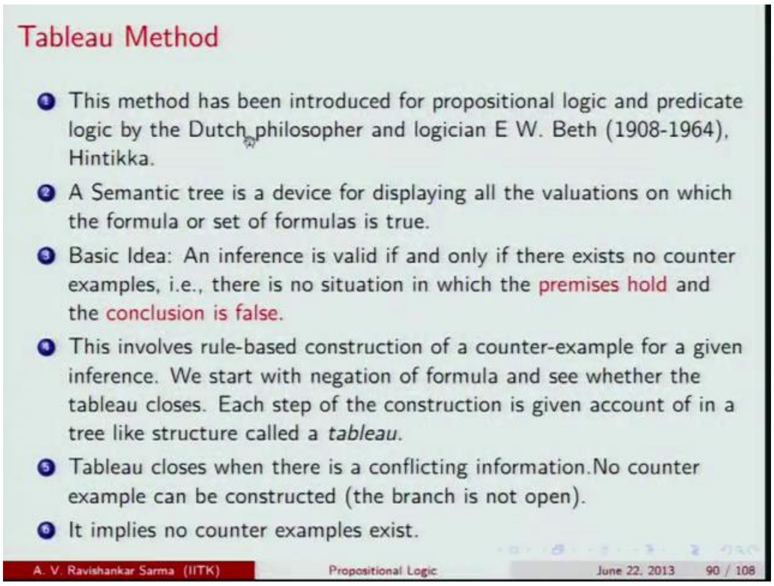

So, this a tree method is very useful find of method, which is originated in the works of a famous logician Beth B e T h in the year 1955 Beth live from 1982 to 1908 to 1964 is considered to be a Dutch philosopher. And in the history of logic it if seams that is method has originated in the works of Beth laters Raymond Smullyan has formulated his book first order logic, is own trees and all is which is a little bit simpler than what Beth has proposed in his analytic tableaux method.

So, then simultaneously around the same year a Intica also develop independently, the same kind of a method it seems that somehow Intica a seems to a propose this method, around the same year, but there is no evidence that whether or not Raymond Smullyan has borrowed something from Intica. So, there is a controversy are debate in who has actually, formulated this method first. So, that is not of interest towards, but; what is of interest towards is to this particular kind of method.

Especially in understanding in understanding the validity of a given formulas are when 2 groups of statements are consistent to each other are 1 can even show, there it givens well form formalize a tautology he or a contradiction or a contingent statements using this tableau method. And you can also show when 2 logical formula 2 well form formulas are logically equal into each other using the same kind of method that is semantic tableaux method. So, it as is an application in automat automated theorem proving and it is also applications in the logic of programs etcetera. And all which will not going to the details of this things.

But, we will try to introduce what, we what exactly this method is all about. We will introduce this method and then we will try to show a which some examples that a given group of statements are consistently whether or not their consistent to each them or when a conclusion follows from the premises; that means, the that is the validity of a given argument etcetera. So, a what is a semantic tree before, we begin its also kind of constructive method.

So, will we doing is given a well form formula will we constructing a corresponding tree diagram for this particular kind thing. Usually trees will have trunks and you have leaves etcetera and a branches etcetera. So, the same way we have, we usually here in this case we have upside down kind of tree usually trees will be down and up wheel have branches etcetera and all, but here we have some kind Offside down kind of tree which you will find it here. For each every formula you will be in the corresponding tree and then we will try to ever evaluate whether, the fall following formula is a tautology etcetera.

So, in the a semantic tree is considered to be device for displaying all the valuations on which, the formula are set of formulas are going to be true. So, 1 of the basic and important the essence of this method is that you know will be constructing some kind of counter example. So, the essence this fun is like this, it consists of finding some kind of counter examples; what is considered to be a counter example? Suppose, if an argument is considered to be invalid especially when your premises at true and the conclusion is false.

So, if you can construct 1 kind of 1 particular kind of counter example, in which your premises such true in the conclusion is false then; obviously, the argument is invalid so; that means, he have we are said to have constructed counter example. So, the basic idea of this method is that and inference is considered to be valid, if an only if their exists the exists a counter example, otherwise the inference is good to valid if an only if there are no counter examples. There is no situation in which the premises hold and the conclusion is false.

So, this also involve some kind of a rule base construction which, we are going talk about in a while from now. And using those rules will construct trees and then they going to show that, if there are no counter examples there are; obviously, the formula is going to the be valid otherwise, if there are a need a counter example we could construct a counter example then; obviously, the argument is considered to be invalid. So, each step of the construction is give in a count of some kind of trellis structure which is also called as a tableau.

So, usually this tableau closes us when there is a conflicting information conflicting information as a. Suppose, if you have formula X and not X then; obviously, the branch close because your a conflicting information. Suppose, if you have some information like it is raining and simultaneously you say that, it is not a raining then that is it conflicting kind of information which 1 to believe you will be in some kind of dilemma. So, it say you incomplete.

So, in that case the branch closes. A literal and its negation appears in a branch then; obviously, the tree closes are that particular branch closes. So, no counter examples, can be constructed for if the branch is not open. So, if the or the branch is close; that means, it implies that there know counter examples exists. So, now this tree method is based on some kind of for rules. So, now, what will be doing in another favorite 20-25 minutes is this that, will become about this rules and then we will construct this tree diagrams first various kinds of well form formulas.

Then they going to show that when a given well form formula is a tautology contradiction are contingence statement. And second we will talk about when a given well form formula is a tautology and third where, we will talk about when to groups of statements are inconsistent to each other and fourth, we will talk about a when 2 groups of statements are or 2 well form formulas are considered to be logically equivalent to each other. So, all these things which, will be trying to talk about in terms of this particular kind of semantic tableaux method and then in the process will also be trying to talk about. So, the important strategies that, will be following while adopting this particular kind of method.

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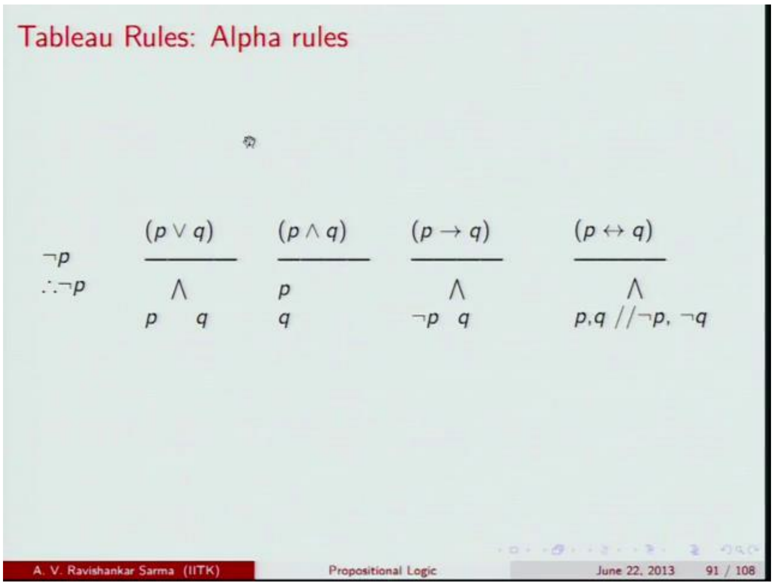

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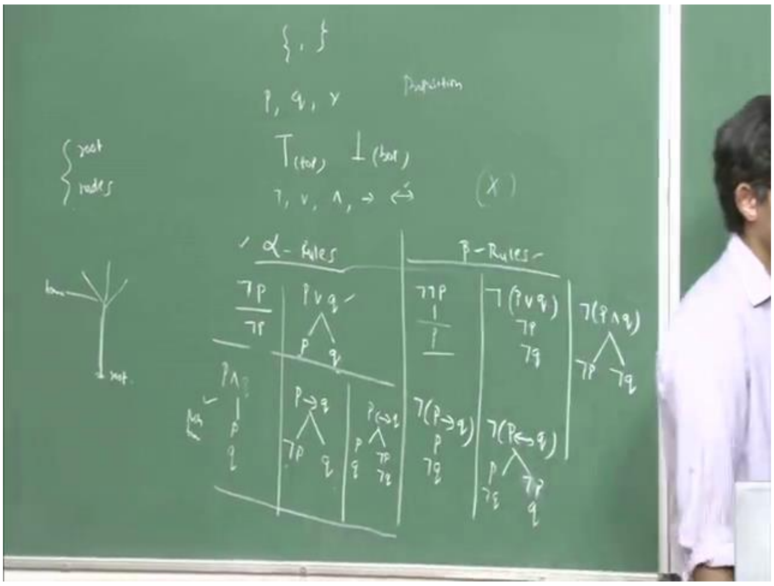

So, the rules are like this: first to begin with we have propositional variables etcetera p, q, r is etcetera. So, this is are all propositional variables and then we have these symbols 1 which is always true is represented, in this way; is usually if the symbol T and then this is represented has bot 1 is always something, which is always false is represented in this. And then we have some other things like parentheses etcetera and all. And apart from that he have this logical connectives negation arc and implies and then the other symbol, which will be using is this 1 at means; the 1 the branch closes will put this mark around cross mark so; that means, the branch closes.

So, now what we are trying to do simply is this that a. So, will be assigning some kind of truth values true these prepositional variables; that means, where interpreting these formulas. So, now, there are some kind of for rules which will to understand before, applying this semantic tableaux method. So, to begin with; so, there are something called a root and then nodes. So, any turn the stand this thing little bit latter, will talk about this thing little bit latter. So, first will talk about some kind of for rules with, which in a you can say whether a given formula is valid are not.

This is the construction tree construction truths. So, suppose if you come across a is simple formula like this: negation of p then you simply like this all the construction of this not p is same as this 1. So, now, if he have formula like this p are q the construction tree for this 1 is p are q; so, now if have a compound formula p and q then; the construction tree there will be like this. So, usually a tree will be like this. So, these are all branches and this is considered to be the root and all. So, this is the formula that, we had trying to begin with.

Now, these formula is reduced a reduced into some kind of atomic preposition and all as you will see in all these rules the things, which have there at the notes are considered to be only atomic sentences; as a p are q are may be negation of that 1 except. So, whenever you have formula p and q you just right it like this. It is a trunk a, it is an upset down kind of tree usually the tree will be like this. Now, you have to reverse it little bit and then you will see the these things. So, now p implies q. So, that definition of p implies q is not p are q so; that means, it is not p or q. So, the branch suggests that there is a dissection.

So, that is that not p are q. So, this is exactly in alliance with the semantic that, we have talked about in the last view classes. So, that means, p are q is going to the falls only when both p are q are false in all other case, it is going to be true in the same way p and q is going to be true only when p q are true in or other case it is going to be false. So, that is a semantic of prepositional logic in the same way p implies q is going to be false only when p is T and q is false, in all other cases going it is going to be true.

So, based on that kind of information, we are just try to come up it some kind of constructive method and then what we are trying to do is for simple formulas like this: we are trying to construct trees tree diagrams are easily a picture says 1000 words. So, a given formula we are trying to construct trees like this. So, now, this is called as a branch and this is called as a trunk of a tree etcetera. So, now, the only logical connective which, is left now here is this 1. So, we will right it here p if an only if q.

So, this is either p q is the case are not p not q you can right not p not q here itself, a p and q you can shift it to the other side, it does not make any big difference it is 1 other thing. So, these are the rules which they have foe each and every logical connective for not this is the thing for are the tree appears to be like these for p and q it at least to be like this. So, these are considered to be alpha roles. So, now in this alpha rules; so, there are some rules which are considered to be branching rules and is wherever, you find and branch this considered to be a branching rule and where ever you do find the branching kind of thing this called as non branching rules now enough.

So, why we a taking about branching and non branching rules because. So, while adapting this particular kind of technique or method. So, there are some kind of strategic is at 1 will be following. So, the one of the important strategies is this that always apply non branch in rules first. So, once you exhaust with non branching rules, you are entering to branching rules. So, now, So, these have this is non branching rule, it is not leading a into any branch; so, non branching rule and all. So, now, all these things are branching kind of rules.

So, given suppose, if you are suppose to apply this method you have to ensure that first you apply the non branching rule and then apply all this rules. So, now these list called as alpha rules usually it is a considered to be positive kind of rule and all. So now, we will be writing beta rules in and all. So, beta rules are exactly negations of these things. So now, if you come across a formula like this at this negation of negation of p it is not the case that it is not the case that, it is raining; that means, it is raining. So, now if you have formula like this you simply substituted with this particular kind of formula p.

So, now using Demorgan's laws, it is a quite simple. So, now negation of p and p are q if you push this negation inside than it will become negation of p and the negation of disjunction will become conjunction. So, that is by be need to right it in this format. So, now as you clearly see this is the formula and than once you apply this rules and or at the end you will find only atomic prepositions; what is an atomic preposition? And atomic preposition is a 1 it, which cannot be further reduce into a any other kind of preposition p r q is can be reduce into p q etcetera and all, but p q is r etcetera.

They are preposition variables that is the most simplistic kind of sentences, which cannot be further reduce into other thing. So, that is they are called as atomic sentences. So, now this is the rule for this negation of p in the same way in negation of p and q using the Demorgan's law at least to a branch, it is negation if push it inside becomes negation of p and negation of conjunction level become disjunction; that is by disjunction will always have a branch. So, this is the form that we have.

So, now, negation of p implies q is simply p and not q we why, because p been plus q is not p or q and negation of not p or q is not; not that is p and not q. So, now, negation of p implies q is a branch again p and not q and not; not p and q. So, these are the only things at we have; this is the rules which will be applying for a judging whether a given well form formalize a tautology where there, it is a contradiction are contingent statement on the 1 hand are when 2 groups of statements are consistent to each other etcetera.

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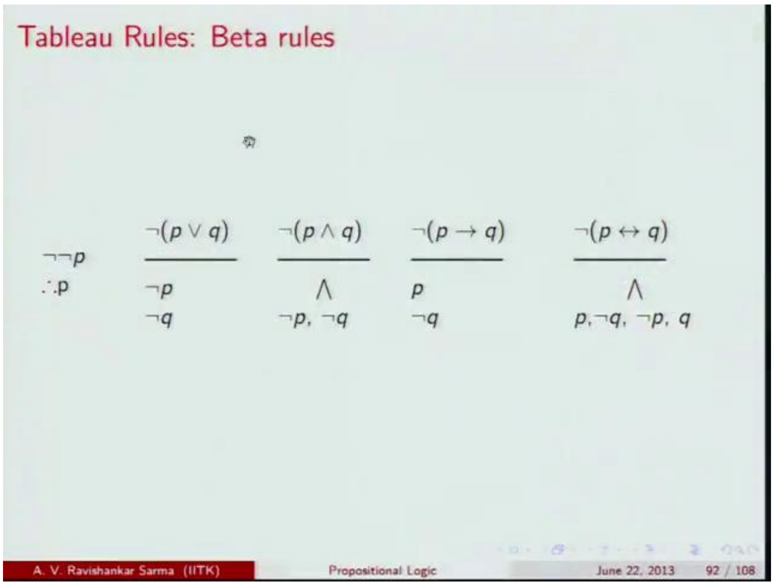

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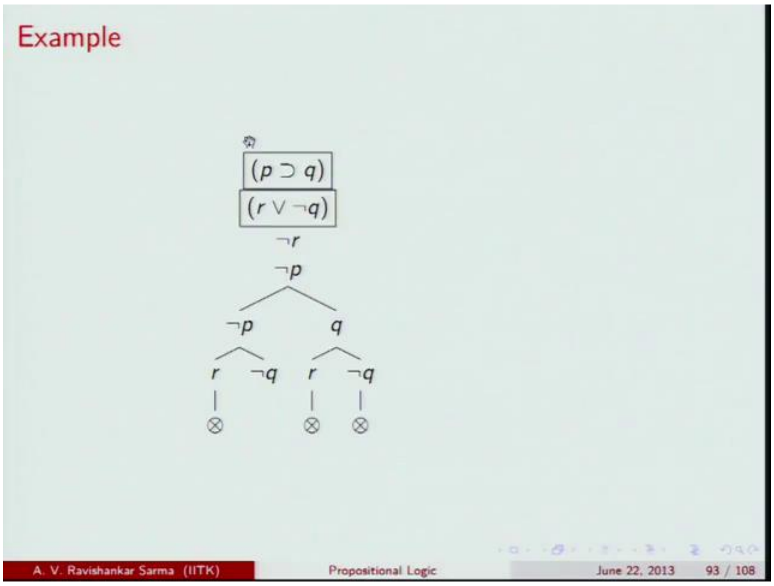

So, now using these rules will be time to talk about a where there are not given formula is a valid or invalid etcetera. So, now to start with the we start with some simple examples. So, we want to see, where there this particular kind of a argument is valid are not. We start with some simple examples in the beginning and then we will move on to some other things.

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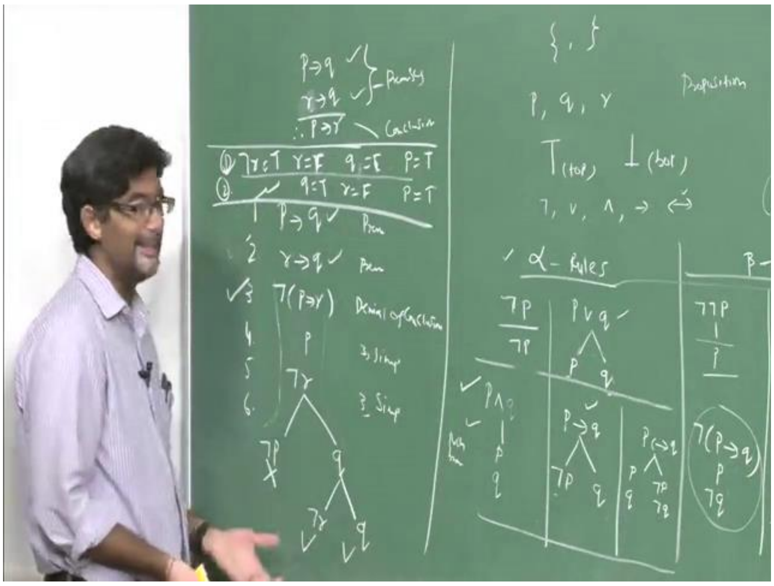

Usually when you right p implies q; q implies r p implies r. And usually is say that it is a valid argument transitive property; obviously, p implies q; q implies r and; obviously, p implies r. So, now, let us do not talk about the valid argument; obviously, valid argument which we know. So, instead of this; what we do is ristraightly change this thing p implies q are q instruct this thing r implies q and then p implies r it does the assume that this is the weather are not this p implies r follows from these 2 things are. So, now these 2 are considered to be usually premises and this is usually called as conclusion.

So, now, how to the now that p implies r follows from these 2 statements p implies q and r implies q where, there it is a it follows. Than it is valid otherwise it is invalid. So, how do which check that; this particular kind of a formula that is p implies r follows from these 2 things. There are various methods 1 method which we have already discussed, that is the: truth able method. And since, there are 3 variables 8 increases will be there in the truth table. So, that is also little bit easy to, but so, what we are trying to do is, we are trying to a trying to see where there p implies are follows these are not.

So, now the very essence of semantic tableaux method is this that we are trying to construct a counter example. If he fail to construct counter example: let this the original conclusion is going to be valid. So, what will be doing is we will begin with the same thing will list out this things p implies q and r implies q. So, these are premises and we have a conclusion p implies r. So, the what will be doing is will be negating the conclusion. So, the idea here is this that negation of the conclusion the closer of branch and unsatisfiable then; obviously, your negation of conclusion is going to be false; that means, all the branch closes its going to be false; that means, X has to be.

If X is a tautology; obviously, the formula is going to be valid, because all tautologies are considered to valid formulas. So, now we are trying to a check were there p implies are follows from these 2 or not. It is of conclusion; that means, be denied the conclusion in all and we are trying to construct a counter example, we fail in the closes then; obviously, this is the actual conclusion which be that follows. So, now, these are the compound formulas and all. So, now we will be applying alpha beta rules etcetera.

So, now one of the important strategies is this that we need to apply non branching rules first. So, which formula you taking to consideration is come under non branching rules non branching rules are this is the 1 and then of course, a this is. So, these are the 2 non branching rules at you are fining, because there no branch here, it is the only trunk; trunk of a tree. So, now these looks like this 1 not of p implies q. So, instead of q you have r. So, now, this reduces true p and not r. So, this is 3 nitrification, 3 is this formula and we simplify then its least to this 1. So, we applied a beta rule here, beta rule is talking about the negation of this formula. So, now this is the 1.

So, now, once you apply this particular kind of rule you need to see where, there is any conflict mean information in your a branch in your tree. So, right now, we do not have since we have take this formula then we put this stick mark. So, that you no will not use it again and again otherwise you will confuse and we will use it again. So, once is caught exhausted then you which put a check mark here, let me out use is 1 again. So, now, these are the formulas which are left.

So, now, we use is particularly, kind of thing p implies q; that means, you apply this thing you will constructing it tree for this 1 means. So, it is not p r q. So, you draw a diagram like this under this you put not p and q. We need to pause a second and we need to see whether that is any conflicting information in your in your branch. So, now this branch is going like this, all the way it here it is a trunk and this is a branch. Now, this is going like this. So, 1 is going like this another 1 another branch is like this. So, now he have p here and you have not p here so; that means, there is a concreting information.

So, the branch closes here itself; that means, it staff there itself there is no question of any construction of any counter example possible here, in this case. Since, you know canflitic information that is a information it closes. So, now this branch is still open are. So, now since; this is a formula that we check that already. So, that is where you we are put tick mark here. So, now what is the formula which is left here is this 1 r implies q. So, now, again apply same rule alpha rule and it becomes not are and q. So, now this is also over now we check those check the all of formulas and then; obviously, at the end of all the branches and all only atomic prepositions not a 5.

So, now we have to inspect this particular kind of branch. So, now this branch is open and even this branch is also open; that means, even after the denial of the conclusion, we could still construct a counter example that; satisfied is formula is enough at means; it is possible for the premises to true and a conclusion to be false that is the reason by we have at least 1 or 2 open branches. So, now, we can study this open branches and a we can talk about particular kind of.

So, now, not are means r is false a means; not are is true; that means, r has to be false. And q is true a q is T and then of course, not we have taking to consideration and p T so; that means, assigning these values are false q T and p t. This satisfied this particular a kind of formula; that means, a p implies q r implies q not p implies r. This is the an assignment which is at which means; this formula true open branch means satisfiability. So, this going to make these formulas true. So, this is the 1 a which we have; that means, whenever you give r f q T and p T that satisfy this particular kind of formula what is satisfying this formula at means you are true premises. And the false conclusion; that means, this is a counter example.

For this 1 and another counter example is there here when ever q takes value t; that means, q T are a false not are is true means r is false and then p it t. So, this is another a counter example and are. So, what is that we have achieve a with this particular kind of thing denied of the conclusion does not lead to branch closer; that means, you already set to know construct a counter example; whenever, your set you have construct a counter example then; obviously, the argument is in valid. So, it is possible that your premises are true, but at your conclusion is false; how the conclusion is going to be false.

It is going to be false in 2 different ways these 2 different ways are considered to be the 2 different open branches. So, especially when r is f q is T p is T that is going to make this thing prove and when q is T r is equal to f p is equal to t; then also your premises are true and the conclusion is false even, if at least 1 branch is open by denying the conclusion and; obviously, the argument is considered to be in valid. So, this is based on some kind of falsifiability kind of method looking for thing, which had true you will be looking for things which have false; that means, you are looking for always say or looking for a counter example.

So, it is like have 100 a for example, in a bag which consists of 100 tomatoes a even a 5 have 1 rotten tomato and all. You will say that not all tomatoes are considered to be include in this basket; 1 tomato will is for that thing. So, this is based on some kind of falsifiability a kind of method; as long as it is not falsifiable the formalize going to be true. It is like as long as it is not find a white crow it is going to except, it as a statement at you know you going to aspect this statement that all crows are going to are going to be black. So, now this is the way to show that this argument is in valid. So, what about saw the argument, which are a valid will take a some kind of argument at which are; obviously, valid.

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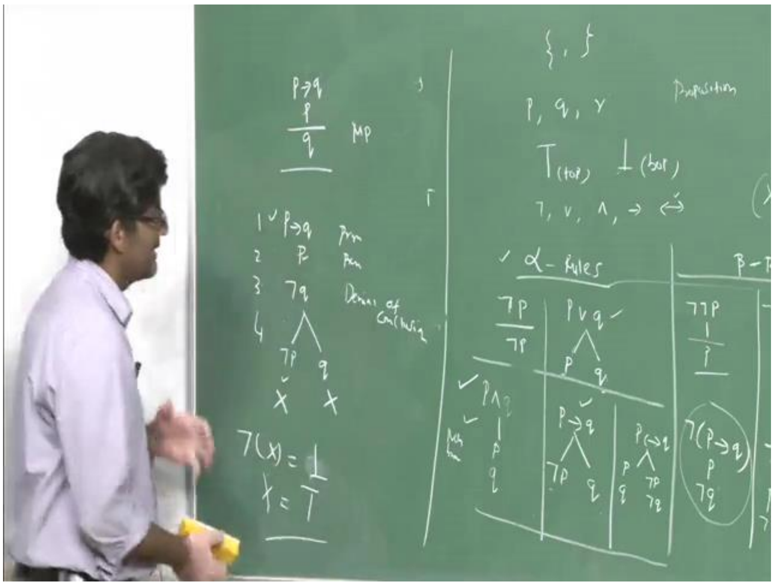

So, this is the 1 p implies q and q. This is the; obviously, the rule which is considered to be; obviously, valid. So, how we show that this q follows from these 2 things p implies q and p accept you list out this premises as it is premises and then what you do here is you negate the conclusion. So, now, we to right here deny of conclusion. So, now once you deny the conclusion we need to check where there it leads to the closer of a branch are not. So, now we need to apply are alpha beta rules depending upon this thing there are no branching rule non branching rule which we can apply here.

So, any rule which you can use; so, now we to apply this 1 this is already and atomic sentence nothing needs to be done. So, now, p implies q is simply not p or q. So, now you have p here and you are not p here is branch closes and not q and q here, it is a concrete information this branch also closes. So, what did we get negation of the conclusion leads to the closer of branch; that means, it is false usually the present it has this 1 bot this is the symbol that let me use it so that means, X has to be; obviously, true since exists a X is true means; X is a tautology, X is a tautology means; it is a valid formula all tautologies in propositional logics are; obviously, all valid formulas. That is these and by logisticians, will be insisting on tautology there especial kinds of statements which are; obviously, are always considered to be true.

They are like not giving kind of truths always true and there are some other groups of statements group of formulas which are; obviously, false it is like 2 plus is equal to 5 is obviously, false and 2 plus is equal to 5 it is; obviously, false and 2 plus is equal to 4 is always true. So, now this is the way which you can show the validity of a given formula for validity what you need to do is denial the conclusion and then see where, there the branch closes are not all the branch closes are not all the branch is closes are not. If at least 1 branch is open; that means, the open branch is considered to it has to be analyze in detail.

So, I means; open branches indicates as that there is a counter example; what is the counter example: we have true miss false conclusion. So, that is going to be that case, if you construct 1 counter example that means; obviously, going to be in valid argument. So, what else 1 can do with help of is particular climatic tableau method. So, now you are training to see whether a given group of statement are consistent to each other are not. So, what that you will be doing is the existing.

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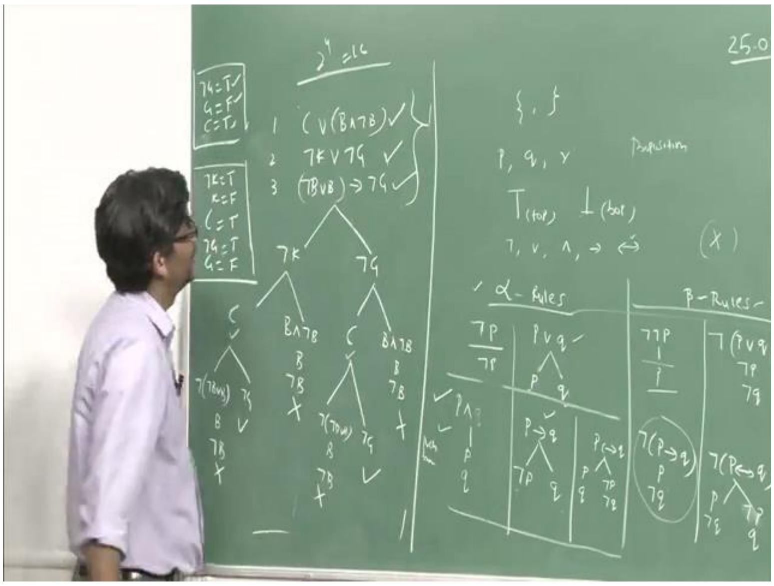

Let us consider an example and will tried to see whether; this particular kind o p f group of statement are consistent to each other or not B at B the first statement and the second statement is not case some statement, which does not matter whatever we are take consideration and 3 not B are B implies not G for get over what this C is B is etcetera an all. Correspond to some kind of statement, it can be may be something like it some kind it some kind of statement which the express in all. So, now given English language sentence, we have converted in to the language of proposition logic.

Now, we are going to see whether these are consistent to each or not; that means, simultaneously they can be true or not that is what we are training to. So, now we are there is the; obviously, there is no conclusion in had to look in to negation of conclusion and C the branch closes etcetera an all. So, will keep it has it T is in all. So, now we are traing to find out model in which training to in trip it this formula this proposition is variables some kind of values to it. So, that all this things will turn over to be true in all; that means any to inspect the open branches here.

So, now so, as usual we need to apply in on branch rules in all here, there is no scope for any non branch rule and all. So, you can open up any formula and then constrict tree for this things. So, now let us considered the second 1; we can choose the first 1 also. So, this is not k are g. So, that is why we applied this particular kind of p are q means; either p is case or q is case. So, now this is over tick marks this 1, now second formula we can take any formula in to consideration. So, now we will open up the first 1; it is either C are B and not B. And the same kind of information he need to passing to the other branch also. So, C and then B and not B.

So, now, this formula is also. So, now, each same is apply this alpha beta rules etcetera; we need to see whether any branch true or not. So, this branch remains open. So, now this we can write it in this wave B and not B, which can be write that form of but it is B and not B. So, this is also B and not B. So, now we have a compacting kind of information B and not B this branches closes and here, also this branch close. Now, own the branches that are open are this 2. So, now whatever has remain the formula need to apply to this need to return just below this open branches. So, now what is left now this is the 1.

So, now this can be written has this is X and this is Y and now X implies Y is not X or Y; that means, not of not B are B and not G remains the same. And now, this branch is this 2 branch already close then to very much. So, now this is not B r B same information you put it in the all the open branches that, you will see here. So, now this not g; now you further simply it is using Demorgan's law etcetera are maybe we apply any 1 of this rules there, then not on this 1 you put negation in side this becomes B and then not of negation of dissention will become conjunction that is why we are writing 1 below the other.

So, this like a this formula and then this 1 and not G remains same and then this becomes B and not B not, not B is B and not of dissection is cizenction that is we are writing just below this 1 and this is not B. So, now all the things are exhausted. So, now we need to see this particular kind of thing here B here and not B here is branch closes and C not k all this branch remain sop hen and this; obviously, closes because of conflicting information of B are not B. And then this branch also remains open branches are the 1 which need to inspect.

So, these are the branches are make this formulas true; that means, each open branch will serve has a model some kind of in each branch is consider to be satisfiable in all. For example, when you give when you assign value not K T, that means; K is false. And C; C as T and then not G has T means; H is false under this particular kind of assignment of truth values. These 3 statement are going to be satisfable; that means, that is going to be true together. In the same way another open branches that, you will find it here in this tree diagram is this when you give valuation this is 1 kind of interpretation under which this formulas are going to be true together are satisfiables.

The other 1 is the 1 which is need to see not G is equal to t; that means, G is false the first 1 and then C is true we assign some kind of true valued T to C the only value that, we can say to C T are f an all and G not G not G we already have this information and all. So, you have this particular kind of information that also satisfied this particular kind of an all. So, these are the things say in to interpretation, which satisfied this formula; that means, this formulas going to be consistence especially, we are interpreted in this particular kind of wave. So, list out all the formulas 1 after another consisted tree diagram and you find any open branch and open branch correspond to the satisfiablity consistence.

So, I means; formula can be true together especially when you have when assign some kind of values like this so; that means, these 3 formula are simentenacily set to be consistence to each other. So, this is another wave of showing that; these formulas are going to be consistence an all suppose if I used truth table method. So, now the number of variables are 1 2 3 4 4 and 4 variables. So, there; that means, there are 16 need to inspect. So, I mean; each row we realized true an all the rows in the final column what whatever value; that means, that value you going to get still those rows in to inspect.

So, instred of doing all this thing most imprestik kind of method, it essay to used on some simple kind of rules Demorgan's rules will helps. So, with help of which we can easily see that you now here, particular kind of things satisfies this formulas; that means, under this particular kind of in interpretation is going to be true all the formula is going to be true. So, now, we can also. So, that we given formula are consider to be in consistence to each other so; obviously, these 3 formulas are let to be consistence.

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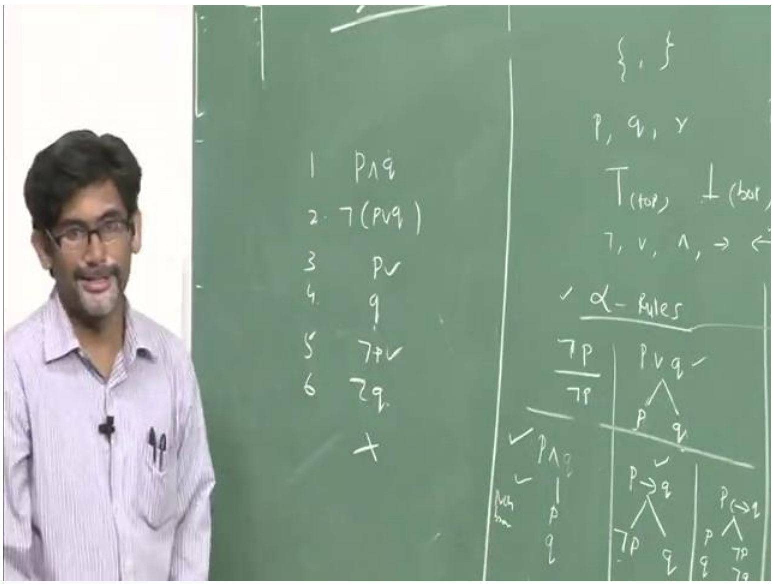

For example, if you have formula p n q and p are q; I have written deliberately choose on this example just shows that this 2 formulas are in consistence each other. So, now first you state all this things resort this things with some numbers and then applying non branching rules first p n q is non branching rules; that is why you apply is first and then you apply branching rules in all of course, both of branching rules on the we have to very much. So, any rule which you can apply; so, this is non not thing and not of conjunction become not q. So, now you will see here p and if there.

So, list out the premises list out this formulas and then if all branches close then that is set to be unsatisfied it is also called as inconsistence. So, p and q is inconsistence with not p and q that is; obviously, the case because p and q exactly not p are q not of p are q p and q all; obviously, 2 statement which each other always inconsistence an all un satisfied all. So, this the way in which you can show that given formula formulas are consistence and in consistence to each other or satisfable an all. So, that other things which you can do with help of this semantic tableaux method.

So, that is you can show whether, given formulas are logically identical to each other and later we will see whether this is going to be statement is going to be tautology are not that also 1 kind do with the help of semantic tableaux method.

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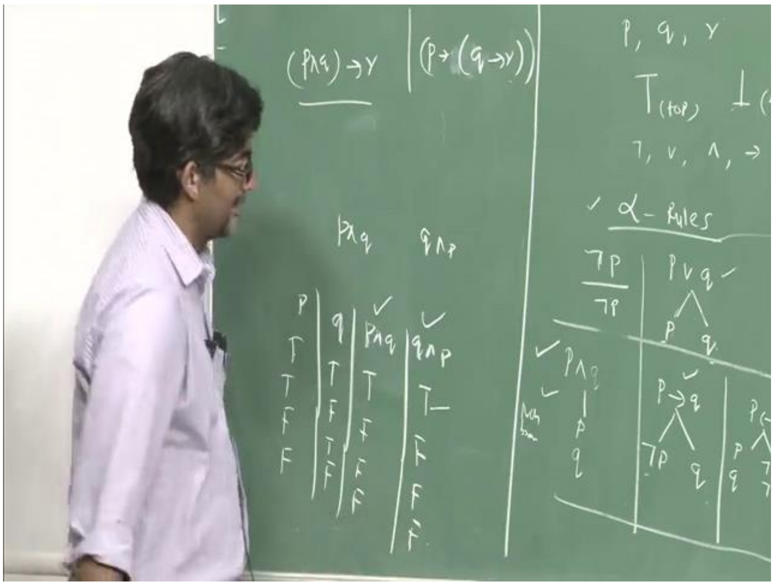

So, now we are trying to see whether. So, these are the 2 formula. Now, we are trying to see whether they are logically identical true each other or not p implies q implies r bracket here. So, now this 1 can do it several waves in all again using truth table method. If the truth value this 1 exactly matches with truth table are truth values of this on under the under the main logical collective then these to are set to be logically identically each other for example, p n q and q n p here p q and p n q q n p. So, you have 2 variable that is why write 2T and 2f alternative T and alternative F.

This formula is going to be false I am going to true in this case in all other case is become false in all exactly q n p also the same thing. So, this formula q n p is going to true when both p q are true in all other case is; obviously, it becomes falls. So, now according to the truth table, I mean; this exactly matches with other on an all for example, when p n q takes values T even q n p also take T. So, this matches with this matches with this and this matches with this in that sense p n q and q n p are logically identical to each other. So, that is 1 wave of for showing a the other wave of showing it is using from other kind of thing. So, that is this 1.

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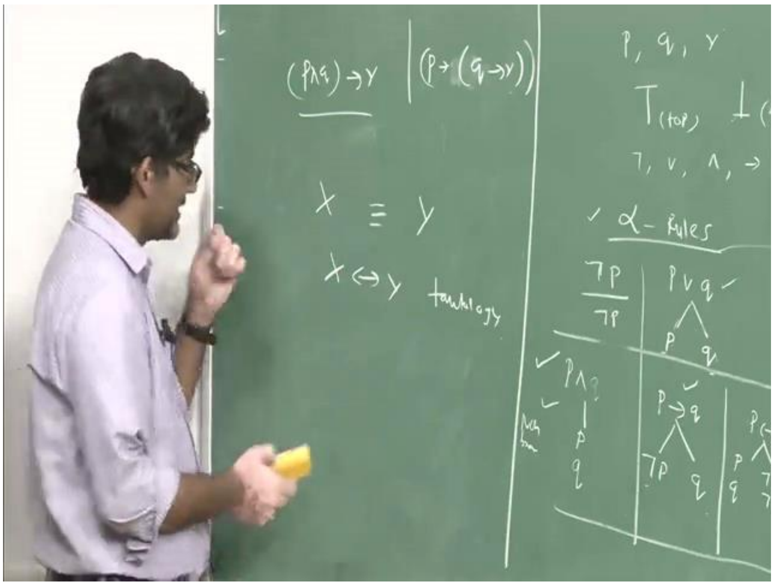

Suppose, X is formula and Y is another formula. So, when do we say that these 2 are logically identical to each other these 2 are logically identical to each other especially, when X if an only if Y is consider to be a tautology. If you can show that X if an any if Y is a tautology then X and Y are set to be identical to each other that is logically equal in to each other. So, now what we are trying to see here is this thing. So, now we are trying to see whether this formula is logically equalent to this or not for that what will doing is putting this particular kind sign and then we are trying to see whether this formula is going to be tautology or not. So, no this is the formula which is there now.

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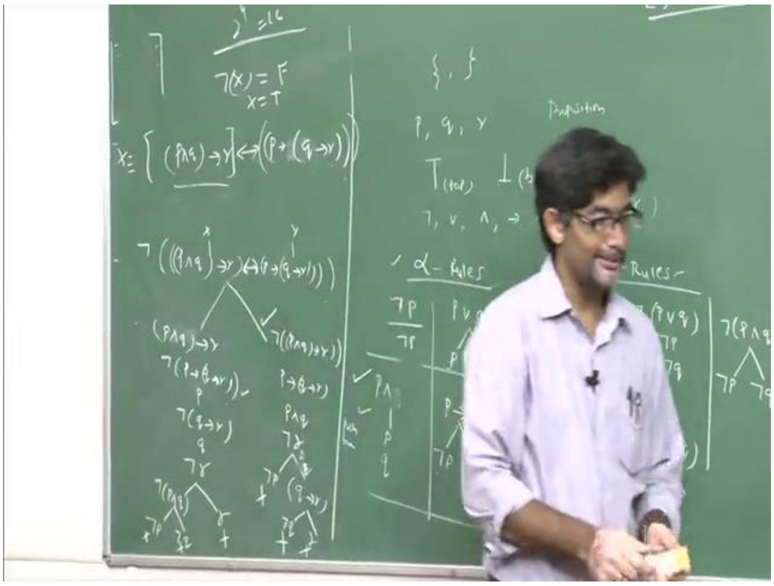

So, now, the idea of semantic tableaux method is simple that we now you start with counter example first that means; you negative the conclusion negation of X is this 1 not of the enter thing which need to right. So, p implies r this is the first formula and then the second formula is 1 p implies r if the negation of these formula leads to closer of branch in all. That means negation of X is false; that means, X are to be T if X is T then; obviously, this is consider to be valued in all.

So, now if X if an only if Y is tautology then; obviously, this too are set to be logically identical to each other. So, now this we treated has X this as y. So, now we need to apply negation of p implies q. So, now we need to apply this particular kind of rule. So, this is this is X, I mean; p yen q implies r the first 1 that is the X part and then not of p implies q implies r little bit big an on here that is the first 1 extend not Y and then not of p and q implies r that is not X and Y, Y is same as this much backed need to be properly in all.

So, now, we need to farther simplify this thing then it will be like this we need to expand this thing then it will be like this we need to expand this branch an all. So, once you apply branching rule here. So, that is this becomes not p n q and then this become r. Now, this is over now this is thing. So, now you farther expand this thing it become not p not q and this is as it is. So, now you apply a rule again here not of p implies and q implies r. So, this is.

So, here what we need to here is better to used non branching rules first. So, here we apply first rule for this 1. So, this is p and not q implies r.

So, now we apply rule for this 1 first. So, always strategy is that first you open up a branch a formula which leads to non branching kind of formula an all. So, first you expand this 1 other this 1. So, now this become this. So, now this farther reduce to q and not r because not of q implies r is q not r. So, now you are apply this 1 here. So, this is not of p n q and then r. So, now see your r and not r is branch closes. So, now this farther expand to not p n not q. So, now you have p here all the wave down here you have not p this branch closes and you have not q here and q here this branch also close.

So, the left hand side all the branches closes an all. So, now we have to see the right and side of this 1. So, the now first you open formula which leads to some kind of non branching non branching rule is 1; which need to applied first. So, this is X and this is y. So, that is why not of this 1 leads to nonbranching kind of rule that is p and q and then not r. So, this is the formula which we have use it here this is the 1 not of p implies q p n not q. So, not q instead of not q you have r here. So, now this is over. So, now you have apply branching rule it does not apply.

So, this is not p and q implies r. So, now p n q this p n q is written as p and not q p n q 1 after another this is the rule which we used p n q is we can write p q has a trunk this reflects trunk an all. So, now you have p and now not p this closes. Now, it expanded farther it becomes not q and r; now not r here r this is closes and q here this is here this also close. So, now, non other branches remains open. So, that lead to negation of the given well form formula is to the closer of the branch; that means, what we are showed is this particular kind of not of X is false; that means, X is tautology.

So, that means these to p n q implies r and p implies q implies r are set to be logically identical to each other. So, we established that actually this is the thing which we. So, the mistake here by in p likes. So, in this wave you can show that true given logicals formulas well forms formulas are set to be logically equalent to each other are other wave showing it is that how do you shows that a given well form formulas is a tautology especially, even you derived the well form formulas this negate well form formula. If all the branch is closes then; that means, negation of by given formula X is false that is; that means, X has to be tautology. So, these are the sum of things 1 can do with help of semantic tableaux method.

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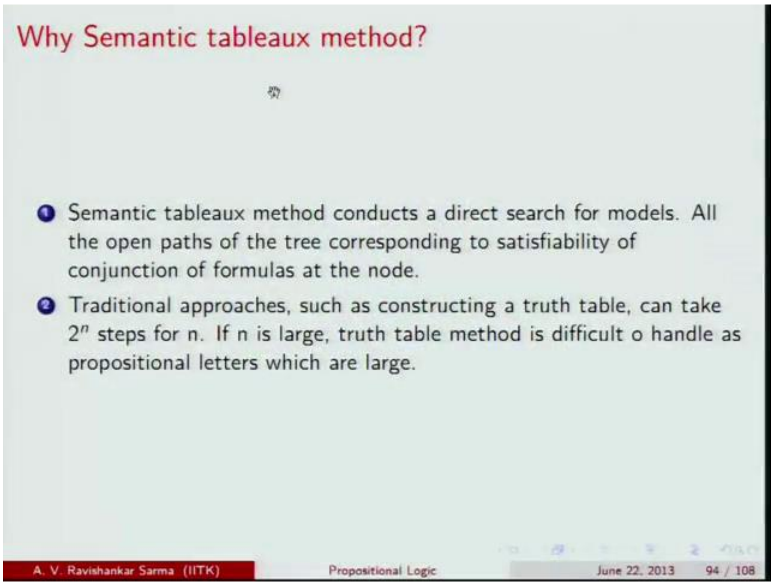

So, you can show that a formula tautology you can show that: 2 group of statement consistent to each other are satisfable are you can even show that: 2 given logical formulas are consist equalent to each other etcetera. All these things are consider to be some kind of decision procedure methods. So, the advantages of this semantic tableaux method conducts a direct search for models; models in sense that whenever, you find a open branch that is consider to be a model then is under these particular kind of assignments they give a formula is going to be true.

For example, p and q is going to be true especially; when both are both p and q are going to be true in all other cases is going to be false so; that means, when you assigned truth values p T q T and that will serve as a come some kind of mode. All the oven pass of the tree that you are seeing in this all this example corresponding to satisfiability of conjunction of formulas at the node. Suppose, if all the branches close; that means, unsatisfiability. So, now, traditional approaches such as constructing a truth table etcetera is can take 2 power of n step for any given n.

For example, n stand for number of variables set existing given formula. If any is too big an all; that means, large truth table method is difficult o handle, because number propositional variables are large. If it is more than 6 it become 64 increase are the we need to inspect to we find out whether a whether a given well form formula is valued or not.

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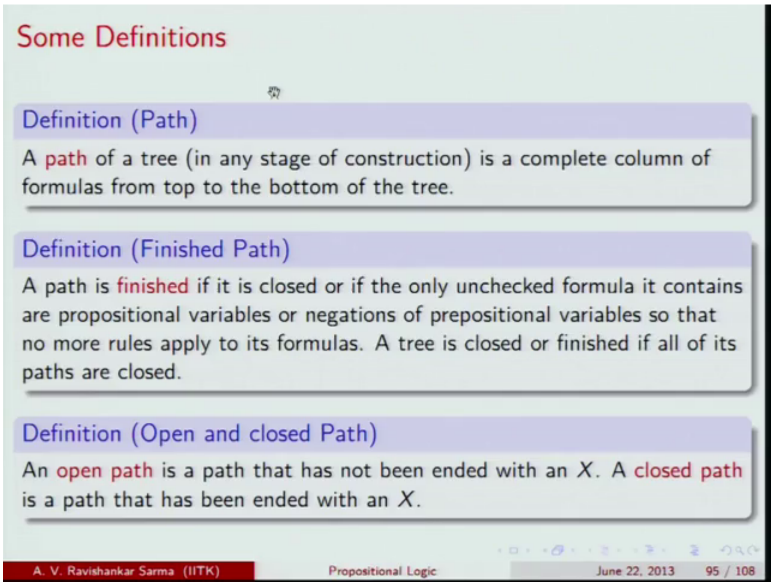

So, these are some of the definitions that will just discuss in very informal wave about this particular kind of method. There are some definition, which will be following will constructing the truth this method semantic tableaux method. So, the first definition is about the path; a path of a tree in any stage of construction wherever, the left hand side you will see tee diagram for this particular kind of thing. So, a path of a tree is complete column of formulas from top to the bottom of the tree an all for example, in this case.

So, this is consider to be 1 path and this is 1 path and is going like this and ending like this. This is path no 1 and this is 1 more path exactly the same thing and this going like this and it end here, another path true; that means, is start all the way from the route and ends with some kind of atomic proposition. So, that is to be the path is already we have discuss in some there other, but we have discussing in what we mean by path of a tree. So, this is what is consider to be path definition of a finished path; that means, path is no wave in which you can progress farther; that means, in there is a information there is no wave in which you can go farther why you are not able to go farther?

Because whenever, you have in consistence information you can derived anything. So, that is a reason why we want to avoid the this kind of information. So, this is the wave in which 1 can shows that.

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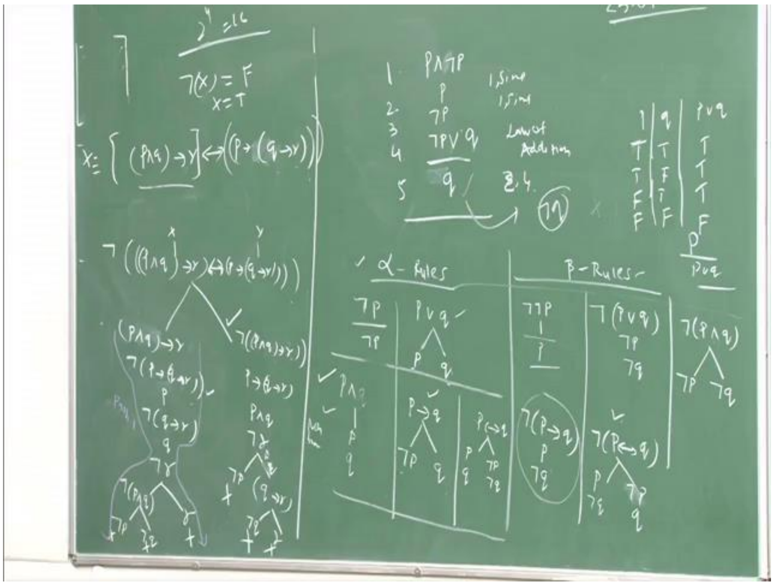

Suppose, if have complicit information like this p and not p to begin with you have a information like this it is not. So, now, he just state this things like this: already p and not p. So, now fourth 1 not p are this is 1 simplication and 1 simplication, you will get this 1. So, now, we can safely add another kind of proposition without disturbing truth value and this 1. If suppose, your assume that not p is already true then whatever, you add after this 1 q is is all always going to true an all.

Because, of the semantic is like this that when this is; obviously, true he respect you whether it is q is T q is false this is going to be true only means p are q and p are q so; obviously. So, this become false only in this case in al other case is becomes T F T F in all other case is going to be true. So, whatever you add after this 1 this formula you are written the same thing an all it is also going to be true. So, that is why you can safety are any kind of strong kind of proposition. Suppose, if he p stands for it is we can safely I other kind of proposition that fix plaice an all that is the things about particular kind of thing.

So, now what trying to shows is: that whenever information you can derived anything. So, this is what is called as law of addition this is truth resaving kind of law, which you commonly see in logic or an important loss of logic. So, that means, you have p and you can easily safely are p are q without disturbing the truth value this 1. So, now quickly what you can see is 3 and 4 sorry 2 and 4 q. So, that is the reason why we are not going farther and then whenever, you have conflicted information you stop that in the same wave. So, you have proved q is case q is any kind of proposition, which comes has consequence of kind of information.

Exactly in the same way you follows some other kind of steps, you can even prove not q also are may be some kind of proposition so; that means, is you start with inconsistence etcetera we can derived anything. So, that is the reason whenever, you have come across a close branch in you will stop there itself. So, a path is consider to be finished if it is; if the only unchecked formula it contains are only propositional variables and; that means, no wave which go beyond are the negations of prepositional variables. So, that no more rule can apply on this 1 no more alpha beta rules can apply on that 1 so; that means, tree is set to a close or finish if all of its path closed.

Open path is path that has no that has been marked with X and closed path in the mark with tick mark and the close mark with some kind of crossed. In this class, we just introduced semantic tableaux method for propositional logic has 1 of the important decision procedure method. And we are seeing with some examples, that: how the semantic tableaux method can be used to decide whether, a given well form formula with tautology suppose if you can.

So, that that given well form formula is tautology; obviously, is the formula is going to be valued are you can even show that when 2 groups of the statement are consistence to each other that is what you can show. And then you can shows with help of semantic tableaux method means; constructive method that when 2 given logical formulas are proposition well form formula are going to be logically, equalent to each other. So, what will be doing in the next class is that, will be applying in this semantic tableaux method; particularly in solving some of the important logical as well as once we translate the English languages sentences in to the language of propositional logic.

Then we can see whether conclusion follows from the premises or not again by using semantic tableaux method. A semantic tableaux method edges over the truth table method especially, in the sense that: when the number of variables are more then 4 or 5 semantic tableaux method is essay to used. So, that is why you can edges over the truth table method. So, depends upon our convenience, which method will be using in all. In the next class you will be seeing some of the important logical puzzles interesting logical puzzles that, can be solved with the help of this semantic tableaux method.