Introduction to Logic
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Lecture - 20
Knight and Knaves Puzzles

Welcome back, in the last class we presented semantic method, which is due to 3 logicians it is originated in the works of with and then later it was simplified by Raymonds William and will find the same kind of work and the work of critical and his work model sets. So, there it seems to 1 of the same. So, semantic is the very interesting important method, which we will come to know whether, are not a given well form formula is valid when 2 performs statement is consistent to each other are when 2 statements are well form formula are logically equivalent to each other etcetera; in continuation to the last class.

So, we discussed about some rules with, which the there are alpha rules and beta rules and then we are also said that the some kind of strategy, which will be adopting a in the process of the using this particular method that is this that. So, when every you come across the non branching formula first you have to utilize this then first and then using branching formulas.

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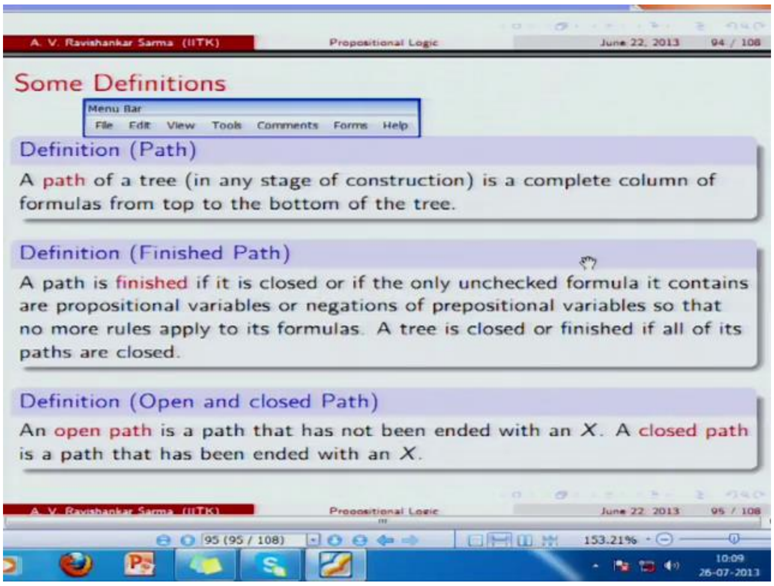

Now in continuation to the last class, you will be talking about some definitions in the context of the semantic method. So, in the semantic method; so, these are some of the definitions we have to will be following. Any way indirectly we discussed all these things, but, in the more formal way will be defining this thoughts the first and for most important thing is what we contest top affective, what we simply trying to do is given a well form formula we are trying to constructed upside down kind of tree.

Then, we are trying to see whether or not a given by wellform formula as valid or consistent and when 2 good formulas are consistent etcetera. so a path of a tree is a complete column a formulas from to bottom of the tree. So, then that is consider to be the path of the tree.

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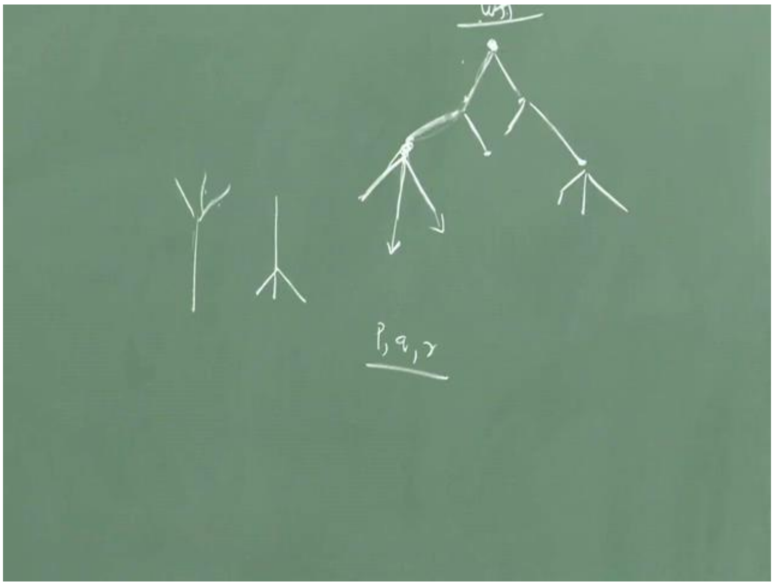

For example, you have this particular kind of thing this is the route this upside down kind of tree a tree will be looking like this these are all branches and this is a trunk, but, in our case it will be like this. So, it is the upside down kind if tree. So, now, suppose if we have formula like this and then as and then this leads to this branch leads to 3 more branches like this for example, this leads to this.

So, now, we have a given wellform formula here. And then this is considered to be the route thats why the welform formula will be sitting and then it is reduced reduced into some kind of atomics variables P q r this, which cannot be further reduced. So, that is why these are called as atomic prepositions at the end of this a bran this path you will end up it only atomic sentences. So, in this tree diagram, this is considered to be 1 path and there is 1 more path going like this and there is 1 more path this is the third path like that you know this is considered to be a path of this particular kind of tree.

Once we construct tree for a given wellform formula that is going to be the path. This is the complete column of all the formulas from top to bottom. Top is the root and the bottom is usually ends of with some kind of atomic sentences because atomic sentences we cannot apply any we cannot further apply any rules. So, that is why it remains at the end of the note as atomic sentences. Once, we come across atomic sentences you will stop constructing the tree. So, the other definition is the finish path that is the closed path a path which is set to be closed especially, if it is closed and a propositional variables are is vacation of the variables exist in the branch. So, if X and not X like this the branch closes or the other way the branch here remains it cannot be extended.

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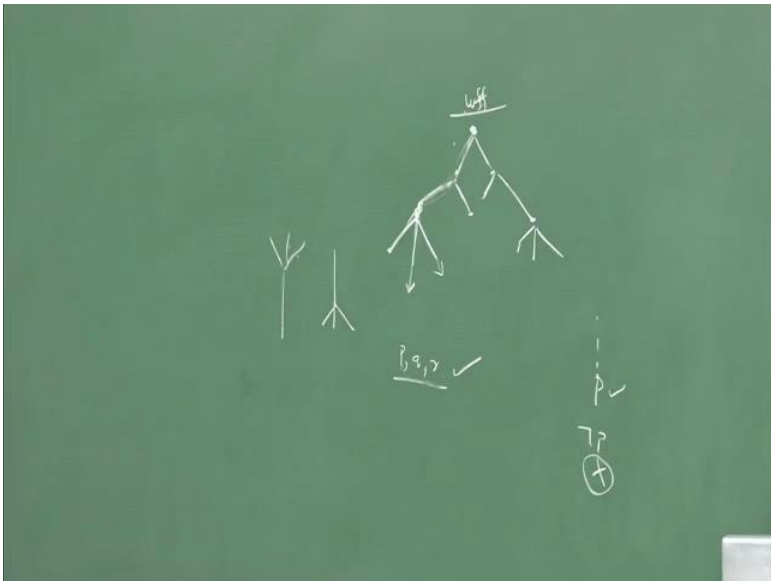

This is that, if it ends with some kind of atomic variables. Atomic sentences P q r etcetera then no further rules can be applied on this P s q s r s etcetera. So, that is why the path ends there itself. So, you will be working, you will be using alpha and beta rules till to such an extent that you will end of with only atomic prepositions either that is a case are you may come across some kind of conflictic information that is, it is considered to be a literal and negation exists then usually put a mark like this; that means, the branch closes here.

So, when the conflicting information exists the branch process are when you are ending off with atomic propositions the no further rules which you can apply; that means, the tree cannot be extended for the it stop there itself. So, the means a trees at to be closed or finished all its paths are closed; that means, you will be checking all the formulas list out all the wellform formulas 1 2 3 etcetera and all to keep checking this formulas using alpha beta rules and constructing tree, these all the formulas are checked and then you will end of with only atomic propositions are then there is no nothing else you can do nothing you cannot extend the tree for the stop were itself.

Because you are existed all the rules and you ended off with atomic sentences and that is considered to be finished path are the other way of saying which is a finished path is that when you have a conflicting information once, we check all the wellform formulas and also it is considered to be finished path. So, now, an open path is the path that has not been ended with that mark X are closed path is 1 path that has been ended with an x. So, this means its particular kind of.

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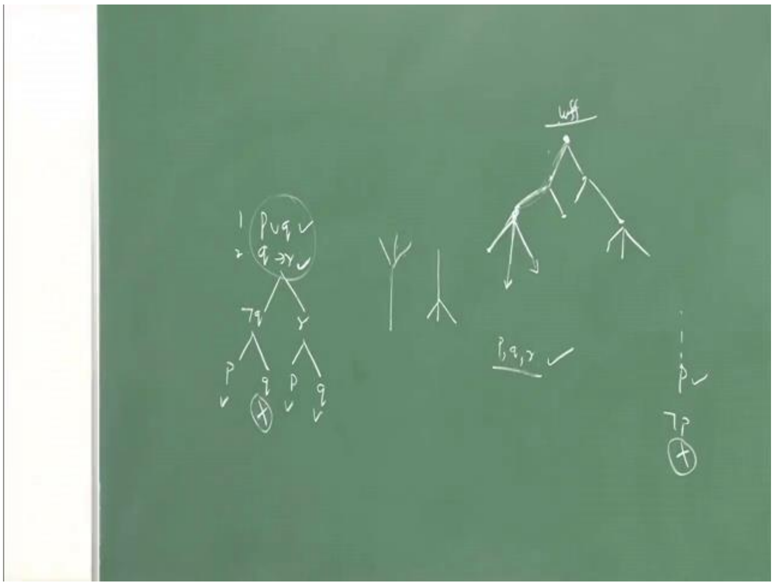

So, suppose if you have some formula P r q and q implies r for example. So, now, you start constructing applying alpha and beta rule first you apply on this 1. So, this is q r and then you apply since, this is check. So, you put tick mark here and then. So, what you will be doing checking this formula and a this is P r q and now each branch we need to write this information. So, now, So all this branches are open. So, there is no conflicting information of. So, this whenever you have a conflicting information q and not q you put this X mark; that means, this branch cannot further extended it closes.

So, this is considered to be a closed path. So, now, we have existed all the rules and all so; that means, the final formulas that existing your tree are going to be only atomic prepositions. So, this is 1 thing which will be know, we are depending what we mean by closed path and the open path etcetera and all. So, whenever you do not come across the mark X is considered to be open path whenever, it is whenever you come across a mark x; that means, there is a conflicting information you market with. So, this are some of the definitions. So, these are the some of the definitions. So, that we will be is in.

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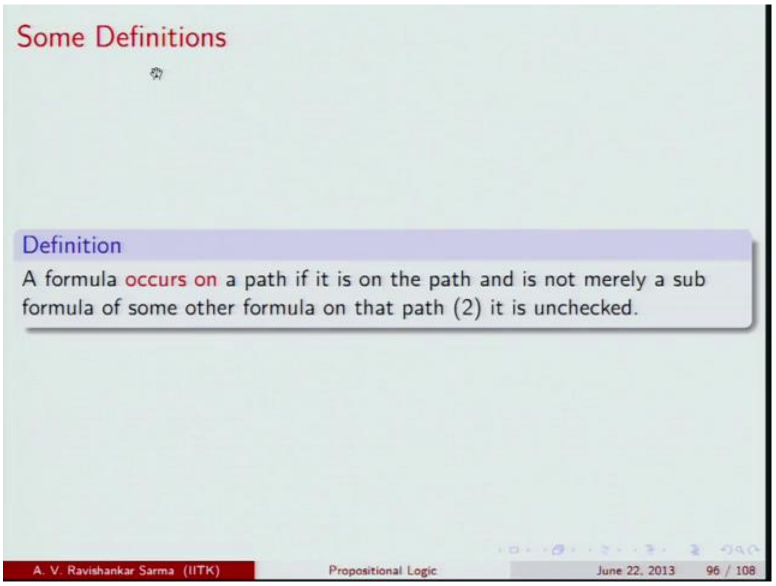

So, another definition is this that the formula occurs on a path, if it is on the path and this various sub formulas of some other formula in that path are second it is unchecked. So, if you once a check the formulas in all it goes it gets exhausted. So, if you once a check the formulas in all it goes it get exhausted. So, it cannot be further used so; that means, either the formulas should be unchecked are if you if it is checked instead of end of with only atomic positions.

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Here as an important strategy which were 1 will be using that strategy is this that first you apply non branching rules before, the branching rules 1 example could be.

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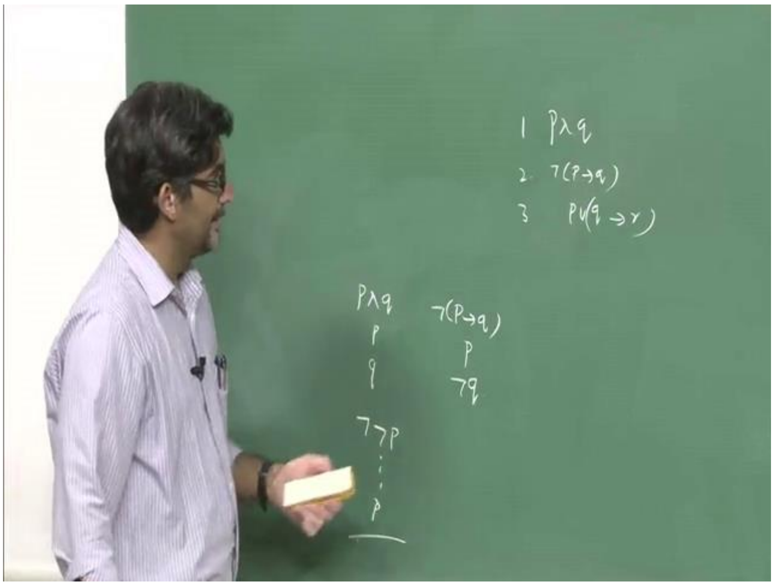

For example, if you have a formula p. And q and we have formula P implies q. Let us say P r implies are for example. So, now, first we need to branching rule. So, non branching rule can be like this. So, these are some of the rules alpha and beta rules the branch will non branching rules are like this; that means, the formulas does not need to some kind of branch. So, this is a non branching rule r this is also another kind of non branching rule suppose, if not of P implies q is simply P and not q. So, usually these 2 are considered to be non branching as another 1 is which you have negation and negation of P and you will get. So, now, clear the strategy here is to use this non branching rules first.

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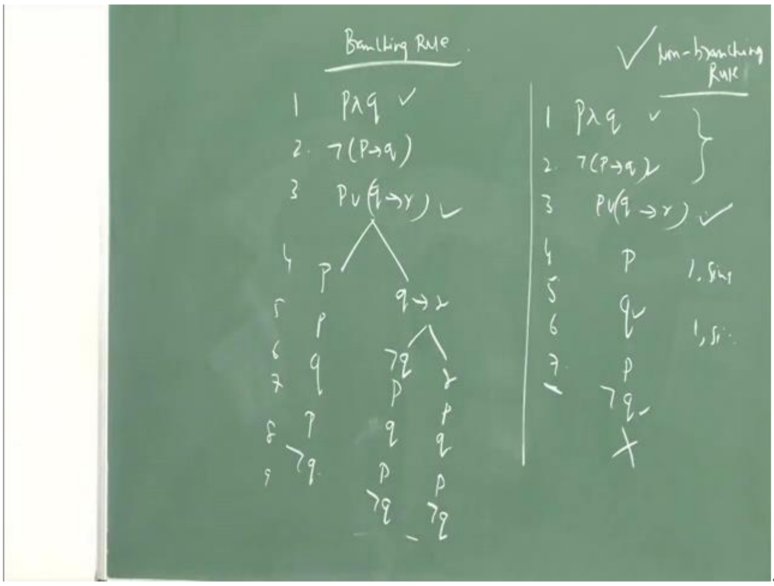

Either you can use this or that suppose, if you open it open this first it leads to so, many branches then it gives an exertive information. Now, so better to use non branching rules first to expand this one. So, P and q is simplifies to P q. So, this is 1 simplification etcetera 1 simplification and then again we are not supposed to use this 1 because it leads to branch it is P r q implies r. So, better exhaust this particular kind of thing. now this is P and not q.

So, this is the strategy 1 adopts in this particular kind of technique first to use non branching rules, I mean; those formulas does not lead to branch other 1s which need to be taken to consideration. So, now as you clearly see here, so now, you have q and not q and all this process here itself. So, now, it does not matter whether or not you used this particular kind of formula P r q implies r etcetera. So, the branch closes here itself. So, this is considered to be a proof for particular thing, that these 3 statements are inconsistent to each other is inconsistent to each.

So, that is why all the branches closes, it does not matter what formula is there in the third kind of for; however, suppose we have use branching rules first here is the problem which comes to let me now, the same formula which we have taken to consideration 1 of this is the correct way of applying this thing non branching rules first, we have use non branching rule. So, now, instead of that we have use branching rule first then here is the problem the problem is like this. Now, instead of expanding these 2 formulas instead of checking these formulas first you are trying to check this particular kind of formula

So, now this leads to P q implies r. So, now, you can use any other formula r P q now, this is the 1 which we are used both are checked. So, now, this leads to be expanded. So, now, P and q leads to be added on both sides. So, now, we check these 2 formulas. Now, you yourself will see that when you use that branching rules first although you are you can use a non branching rules here. This leads to the problem the problem is this that first of all the whatever, you have to trying to show will have more number of steps are if you are suppose. If you are showing the validity of a given formula it involves more steps.

So, now this ended in 7 steps itself the 7 steps we could say that these 3 statements are inconsistent to each other but, here already 4 5 6 and then you 7 we have and then we some more. So, now, we need to write this 1 else. So, P and not. So, now, here it is P and not q P not q. So, now, here we have 8 9 or something else. So, instead of 7 steps, we have 9 steps and all that is the reason while. So, we will always be using non branching rules first and compare to this branching kind of rules in a semantic tableaux tree this is the very important strategy, which comes through practice there is a convention which follow.

So, always whenever, you have a non branching formula better to open it up rather than branching kind of formula, because it in it involves more number of steps. So, in fact, this sematic method can also to be used as some kind of proof method proof procedure and of method a proof is a 1 which consists of number of steps and which ends in intervals of interval of time no proof can be considered to be an affective proof if never ends and it goes on and all and all. So, a proof has to end in finest steps off course, it should take a fined number of a fined amount of time.

Then it construct to be an effective kind of proof, but, you know often once, you start to with non branching rule off course, you will get the answer and all by the number of steps will be more information and economy cannot be maintain. If you branching rules first over non branching rules that is the reason why we follow this particular kind of strategy that is always applying non branching rules before the branching rules.

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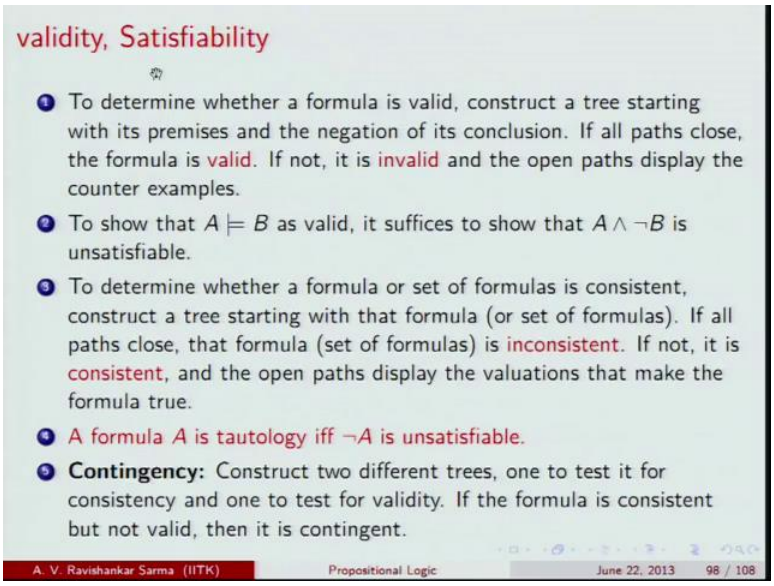

So, now we defined all these things in the context of table method for example, formula is valid especially when you 2 premises and you do not find a false conclusion as long as you do not find a true premises and a false conclusion then; obviously, the formula is going to be valid. So, you have to inspect a in a truth table all the rows and if the any row in which you have premises are true and the conclusion is false then the argument is invalid.

In the same way the formula is set to be satisfy able especially when you need to inspect at least 1 row in which the premises this 1 of the under the main logical connective at least rho 1 T if there all T s and all inset to be unsatisfied the group of statements are unsatisfied whether, in consistent. So, now, same things which will defined in the context of semantic method. So, that is first we want to determine whether a formula is valid 1 is to constructed tree using all the alpha beta rules it, we have been discussing so, far. First what you will be doing is you take the premises into consideration and elevation of the conclusion.

If all the path closes in the formula is considered to be valid, if it is not then it is going to be invalid some examples, which we taken to consideration and then we will see when a given formula are when the conclusions follows from the premises ; that means, the valid.

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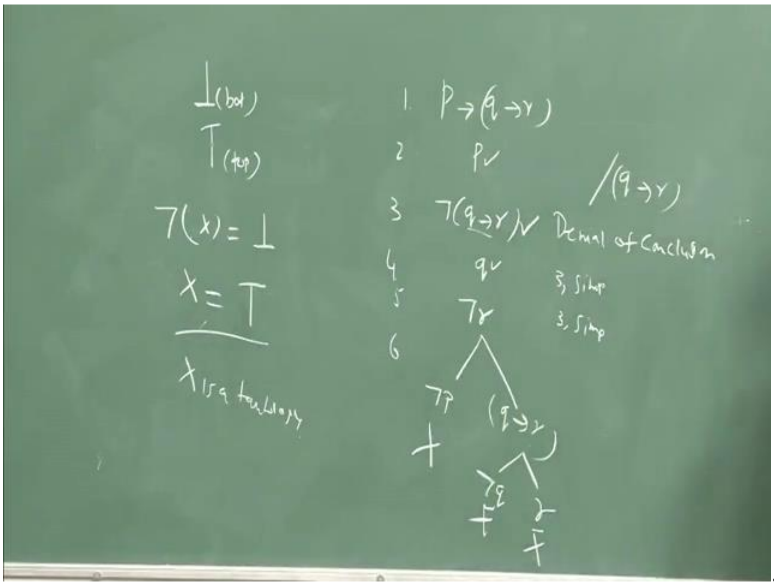

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Some simple examples can be like this: P implies q implies r. Let us considered, this 1 q and from that q implies r. So, these are considered to be premises and this separated by an hyphen this is considered to be the conclusion. So, now, in the sematic tableaux method we added to show that q plus r follows from P plus q plus r and P, what you will be doing is first you deny the completion so; that means, you need to write the denial of conclusion, once you denial the conclusion.

Now, you will be constructing the tree diagram for this thing and then you need to see whether all the branches closes up. So, now as usual we follow we applying non branching rule first so; that means, you need to open up this 1. So, this is q and not r. So, this is 3 can simplifies it got it get simplifies then, you will get q and not r. So, now we check this formula. So, that is why we put this mark. So, now you open up this thing is an atomic sentence we not have to do much. So, now we need to open up this 1 P implies q implies r.

Now, this a branch, because you have a formula X implies y X implies y is a construction tree for this 1 is not X and y. So, now, this is not P and q implies r is as it is. So, now we have P here and not P here which branch closes somehow, we need to expand this thing little bit for them. So, this q plus r will become again this rule used X implies y not X and y. So, now this become q implies r become this 1 not q r somehow we have q and not q is a conflicting information this branch closes and you have r here and you have not r here this also process.

So, what is that we got simply waste that negletion of the conclusion leads to branch conclusion; that means, it is unsatisfied; that means, leads to a contradiction, I mean; avail a branches close its then negletion of X is this 1 then X has to be T were, this stand for both it always falls the formulas which are always falls and this T this is totally different from the formula that we use t. So, this stands for truth off course, this is also considered to be true, but, always prove a.

So, this is the symbol is depends as top so; that means, X is a that is what we have said; that means, actual conclusion is the 1 which stands, because we neglected the conclusion and led to contradiction. So, that is why you have to retain the original conclusion that is q implies r. So, negletion of q plus r is to contradiction; that means, q plus r has to true q plus r is a true conclusion of this 2 premises. So, it is in this way 1 can show a given form then a when a conclusion follows from the premises; that means, the validity.

For validity what we need to do simply is that to take the negation of the conclusion and see whether, all the branches closed all the branches closes then the negletion of the conclusion and see whether, all the branches closed and all the branches closes and the neglection of the conclusion is false; that means, actual conclusions stands as it is. So, its negletion of the conclusion does not lead to a branch closure then; that means, there are at least some kind of a interpretations, which satisfies your truth premises are true and conclusions are false; that means, you already constructed a counter amount.

So, that is the reason why sematic method 1 of the 1 of the essence semantic tableaux method look for some kind of counter modules; here we could not come off with any counter module less of the argument is valid. So, the another way of showing particular kind of argument is valid or invalid this simply this things. Suppose, if there are 2 formulas A and B and B is a logical consequence of A are this A implies in a logically implies B as valid it sufficient to show that A and not B is unsatisfiable. So, if you want to say that this is the on which, we want to C.

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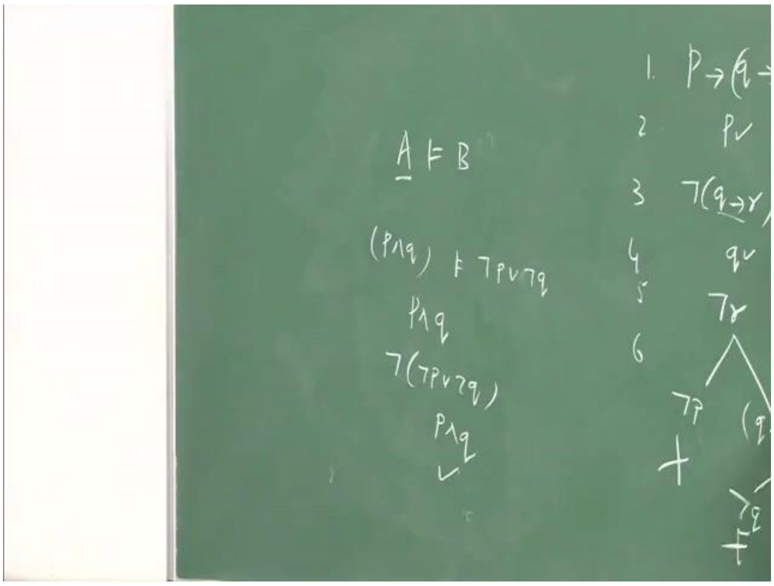

So, now let us take formula A as B and q and then B as something like not B r not v not q something like there is 2 formulas and 1. And you have this and your somehow want to show that this is a logical consequence of this 1 what, we need to do is first we need to write like this not P r not q and then we constructed a tree for this 1 and then this becomes not P and not q in all the branches it becomes, it the branch processing, but, that is not we are trying to say.

So, what we are trying to show is this particular kind of. So, P and q and not of not P are not q, if this remains unsatisfyable. So, then this is the logical consequence of this 1, but, actually that is not the case. So, now you expand this 1 it becomes P and q not P is in not not q is q negation of this junction is conjunction. So, now, this branch is open satisfy this particular kind of thing; that means, this is not a P and not q has to be unsatisfyable showing that, this particular kind of argument is other way this is considered to be in worth. So, that is another way of showing that whether or not a given formula is valid or invalid.

So, what we are trying to do is a logically implies; that means, b is logical consequence of a. So, A is consider to be premises and B is considered to be conclusion suppose, if we can come across at least true premises and A false conclusion and obvious the argument is invalid. So, another thing which you can do with the help of semantic tableaux method is you can show whether, 2 sets of formulas that to given formulas are consistent to each other.

But for that what we need to do is list out all the formulas and then constructed tree and at least 1 branch is open that is construct to be consistent with all the branches process, there it is considered to be inconsistent. And there is another way of saying that the given formula is topology a formula is considered to be topology means; always true if an only if not A is unsatisfied. So, that is this particular kind of thing, what we are trying to say here is.

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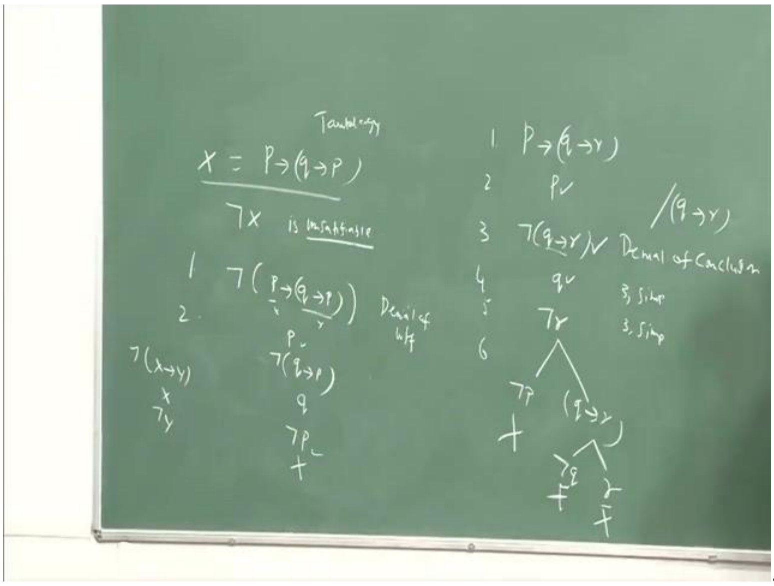

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So, you take A any formula P implies, q implies this is what is the formula, which is given to us. So, now according to the definition, which want to show that this particular formula is a tropology, what we need to show is this is not of X is unsatisfied unsatisfiable, in the sense that if take a neglection of this 1 all branches should closed. Somehow, that is what we are trying to C not of X P. So, this is denial of original welform formula given welform formula.

Now, this has to be unsatisfiable; that means, all the branches should closed after applying alpha beta tools. So, now this is a 1 form this is another form X and y. So, not of X plus y is X and not. So, now, you constructed tree for this 1 become q implies P some of this further this is to q and not p. So, now you have P here and not P here conflicting information the branch process here; that means, neglection of the formula reach to the branch pressure all the branches process here, there is only 1 branch here is a path this.

So, this branch process so; that means, it is considered to be unsatisfiable at least 1 branch is open in the construction of your given the formula then it said to be satisfied. But, in all the branches alone even 1 branch here, that process here so; that means, that is not of X is said to be unsatisfied is not is X is considered to be unsatisfiable then;

obviously, the given formula is considered to be topology and which note that all topologies are; obviously, valid formulas. So, this is a relation between satifiability and validity; validity against topology. So, something is considered to be a topology only 1 the negation of formula is considered to be unsatisfied.

So, the negation of X is considered to be satisfiable then this is not considered to be a topology first of all made in continuant statement are it can be may be the. So, we cannot say that given formula is considered to be valid. So based on the information that: negation of the given formula is satisfied. So, we have to ensure that not is unsatisfied then only we can say that a is a kind of valid formula or it is all formulas; obviously, topologies the other thing, which we can do with the help of semantic tableaux method is this continges you are defined statements into statements of proportion logic into 2 categories topologies, which are always true contradiction, which are always false and continuant statement, which can be sometimes true sometimes false.

So, for this we need to construct 2 different trees 1 is suites whether, a test for the consistency another 1 is test further the validity. So, if the formula is consistent but, not valid there it is said to be contingent.

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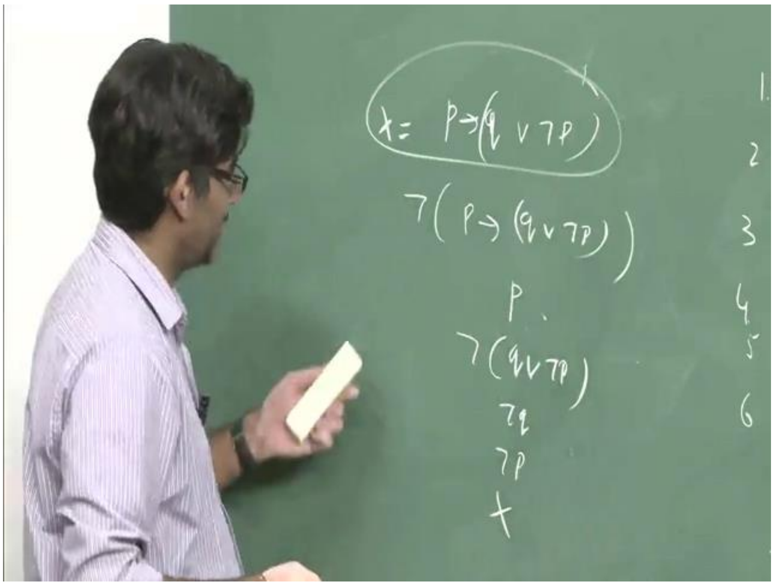

For example, if you have a formula like this formula like this for example, P implies q r not P etc some are this is the formula that we have. So, now, what we do is you nugget this formula and then see whether, all the branches goes; that means, not of X is unsatisfied that is not you are trying to show. So, now, this is extend this is use alpha beta rules or beta rule actually and this becomes true or not true. Now, this is not q or not p. So, this closes; that means, it is a considered to be a topology. So, now let us take another example where the branch does not loss.

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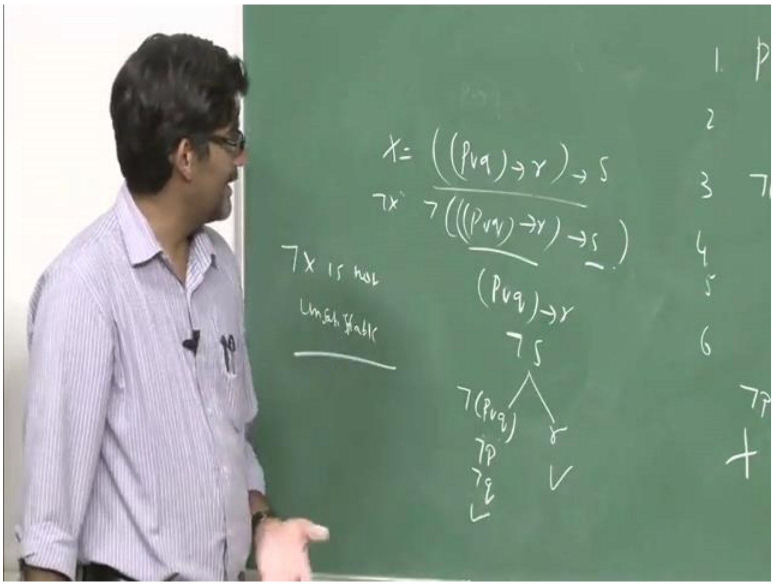

So, now this example could be P r q implies r implies something like s some formula, we take into concession whatever consist movement just I am writing it on this. So, now, you want to whether, it is a contingent or topology or contradiction. So for this again the original formula and start constructing a. So, now, you write the same thing here bracket since to be return clearly. So, this is 1 formula this is another formula. So, now, this can be written as P implies q implies r not s it was not of X impulse y, it is a not y. So, now, you further expand these things than this becomes P or q and r.

So, this is 1 mission this further simplifies to not P not q and here, observed that all the branches remains open; that means, negation of the given formula does not lead to branch process; that means, not X is not unsatisfiable definitely, it is not a topology is particular formula is not topology. So, since if it is 1 branch is remains open. So, when it is considered to be contingent kind of a contingent kind of state. So, this is to ensure that this is not unsatisfiable. Once it is not unsatisfiable then, we can really say that this can be called as a contingent kind of state.

So, for the contingency what we need to do is you have to construct 2 different trees 1 is to test for the consistency and the other 1 is to test the validity. So, the 1 which we have mentioned it here is left hand side in the board is we check for the validity of a given formula. So, that is 1 test which, we are trying to do definitely, it is consider to be an invalid kind of formula some of your contingency very another thing, which we need to follow.

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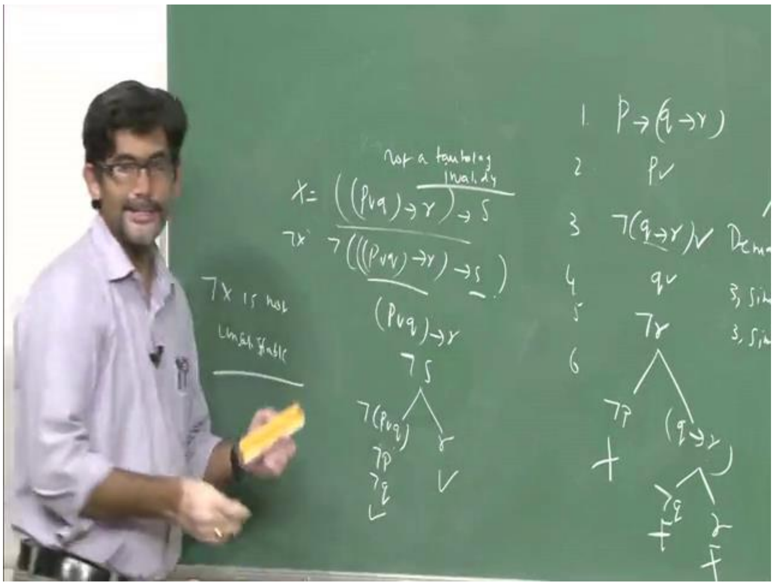

So, that is now here we showed that it is not X is not unsatisfied; that means, is this formula is not at topology not at topology but, and; obviously, it is invalid group. So, this is not enough because, there are it can be a contradiction can be even contingent statement. Now, what we need to do here is instead of negating this formula you need to check this formula in the consistency. For consistency what you will be doing is you do not take negation into consideration just you leave the formula as it is and.

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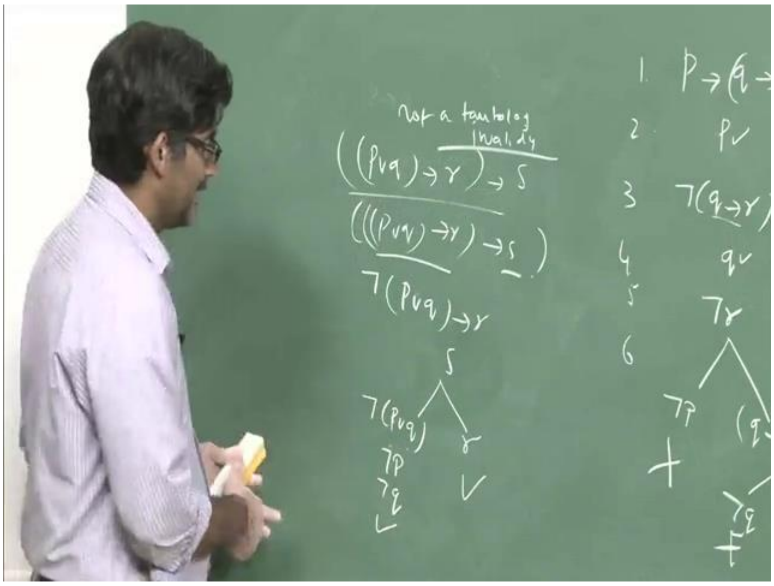

You start constructing a tree for this 1 some of this becomes like this. This is X this is y. So, not X and y. So, now changes somehow this change to this is not P into now this further simplifies to this 1.

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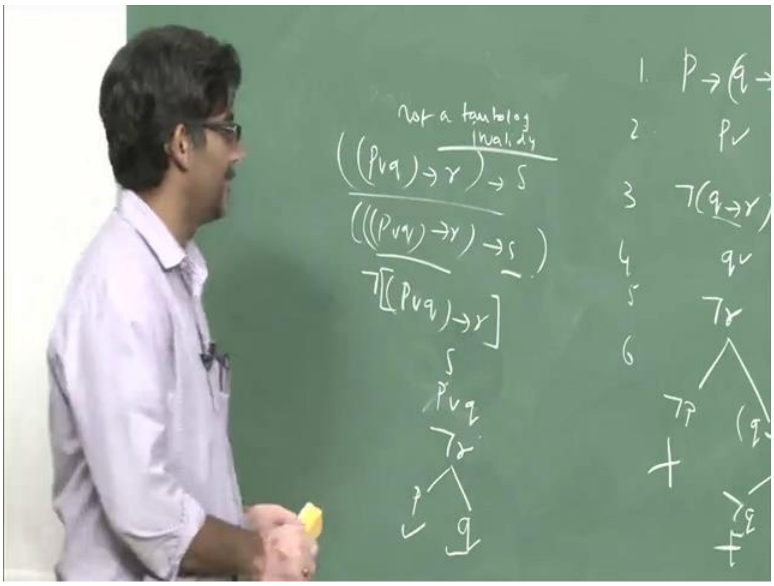

This is the end this is the whole thing which we need to take into consideration negation of this 1 and s. So, now, negation of P or q implies are is this thing P r q and not r is a now P r q multiplies to this 1. So, now, all the formulas are open I mean all the branches are open; that means, it is considered to be consistent kind of formula. So, this formula is definitely not valid it is not a tautology, but, definitely, it is not a contradiction also because at least all the branches of open and all.

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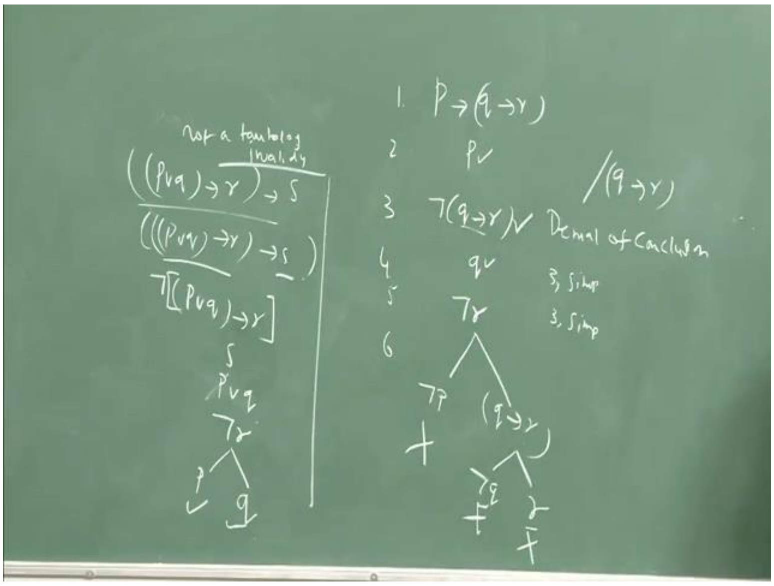

So, now, for example, in the process of checking the consistency you came across all the close branches and on then this formulas going to be a contradiction. So, the idea here is this that for contingency, we need to construct 2 different trees first we need to check the validity of a given formula that tells as whether, given a formula is a tautology or not, but, it does not tell us about whether, it is a contradiction are contingent kind of statement for contingency and contradiction you need to go for another test that is the test for the consistence.

For consistency we do not denial the conclusion, we just leave the formula as it is any construct the tree as in the case of there is explain on the left hand side of the board. So, suppose imagine a situation where in which all the branches closes; that means, a formulas it to be inconsistent. So, if it is inconsistent; obviously, that formula is going to be a contradiction, but we did not come across the particular kind of situation at least 1 branch is open that is considered to be satisfied.

So, this formula and the 1 which we have written on the board P r q implies r implies s is considered to be a contingent kind of formula is not a contradiction because, in the process of checking in the consistency the branch does not closes that all the branches does not close. So, it is not a contradiction and for the tautology, we already checked it in the beginning that you know negation of the formula leads to the closure of branch that all the branches that is means not access unsatisfiable, but, also we did not get is.

So, it is not a tautology and not a contradiction. So, it is considered to be a contingent kind of formula. So, here that is the way in which, you contest can check the contingency of the given formula you need to construct 2 different trees.

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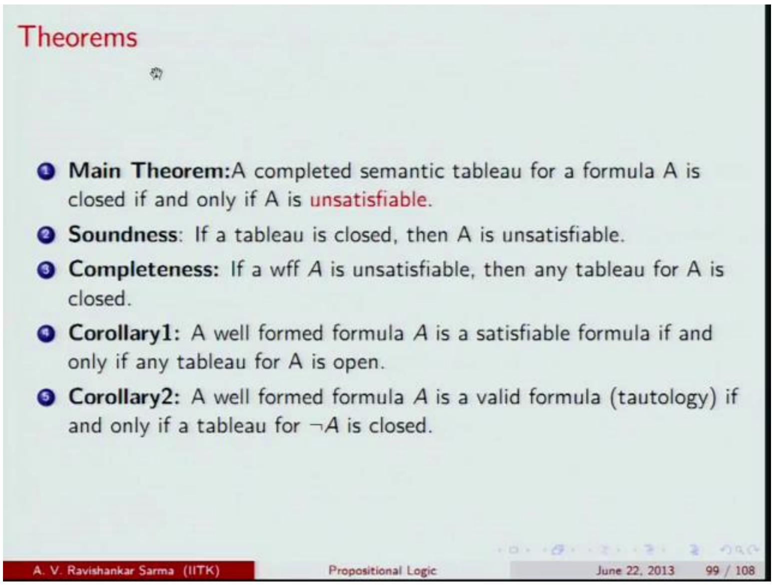

So, know here some of the interesting and important which is using it for the when we talk about something on a metrology; that means, some important theorems later. So, 1 of the interesting theorems is: we are not going using this theorems you just highlighting 1 of the important theorem which will be making use of it later in another context, I will be explaining theorems in later detail. So, a completed semantic tableau for a given formula A is at to be closed if an only if is set to be unsatisfied.

So, we take the neglection of the formula not X and then it leads to the closure of all the branches then; obviously, not access considerably unsatisfied is not X is going to be unsatisfiable and; obviously, X has to a the tautology that is 1 thing we have observed it already. And the second most important thing is a the sound has the in the context of semantic tableaus method. So, in the context of semantic tableaus method is like this. If the tableau is closed then; obviously, A is said to be unsatisfied; that means, our formula X and you construct the tableau for that and if all the branches closes that aims at to be unsatisfied.

So, usually is a case that, know soundness relates there are 2 things, which is important prepositional logic that is the co we are not discussed in detail about something about talking about in the next few classes. So, if something is considered to be true, then it is also provable are something is provable is also true and it is something is provable and it is true called as soundness. And an if something is already true then, it has to find it proof then that is considered to be completeness; what is provable is true and whatever is true has to be provable.

Interestingly in prepositional logic and both the cases it happens. All the true formulas are provable and all the provable formulas are has to be true it proves lots of things, but, it end of the day it is false and does not make any sense to us; that means, all provable formulas are true and all true formulas are provable and propositional logic in this sense is considered to be complete. Completeness says that: if a welform formula is unsatisfiable then any is; obviously, closed I can show that all the branches close; 1 of the important corollarys of this 2 theorems which will be explaining it little bit later just we are just highlighting what we mean by this theorems and all.

A well form formula A is considered to be satisfiable formula, if an only any tableau for is open at least 1 branches open it is considered to be satisfiable and in the same way corollary to is that a welform formula a is a to a valid formula, if an only if the tableau for not a considered tableau for not a on it is. So, happens in all the branches closes that is the case than it is considered to be a valid formula.

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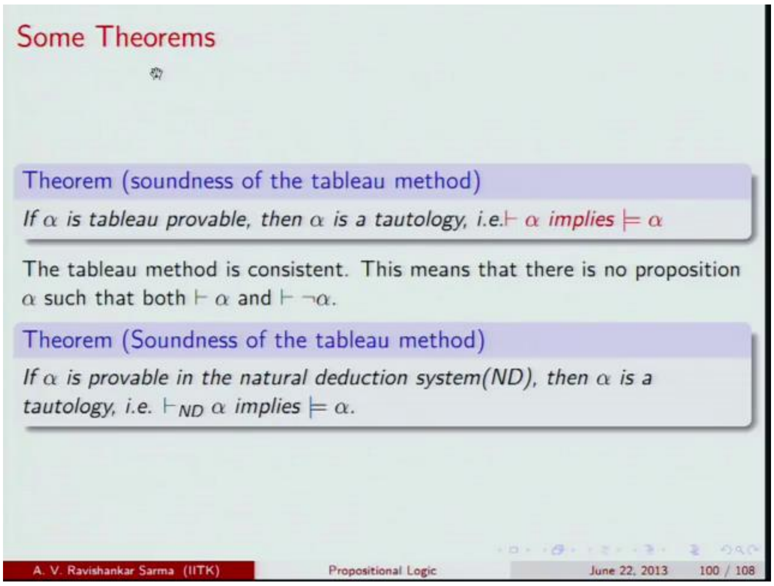

So, soundness in the contest of tableau method is simply like this. If alpha is tableau provable then; obviously, alpha is considered to be at tautology what is tableau provable. So, you have a given formula X and neglect the formula and it leads to all close it off all the branches; that means, not X is falls; that means, X has to be true. So, if alpha is tableau provable then; obviously, alpha has to be given formula has to be tautology something is provable and it is true then it is called as a sound.

So, the tableau method is also considered to be consistent in the sense that by proving certain theorems, it is in tableaux method which never happens that you come across a you prove both alpha and not alpha and all is either, it has the case that you have to prove only alpha are it has to be the case that that prove only not alpha. So, now, the soundness of tableau method is like this if alpha is provable in the match system, which will be talking about little bit later then alpha is also considered to be tautology natural reduction method is a 1 which and which you will find this proofs of some given formula.

Same thing is you can prove it with the help of sematic tableaux method as well. So, something is provable it has to be true and something is true it has to be provable then the system is consider to be complete.

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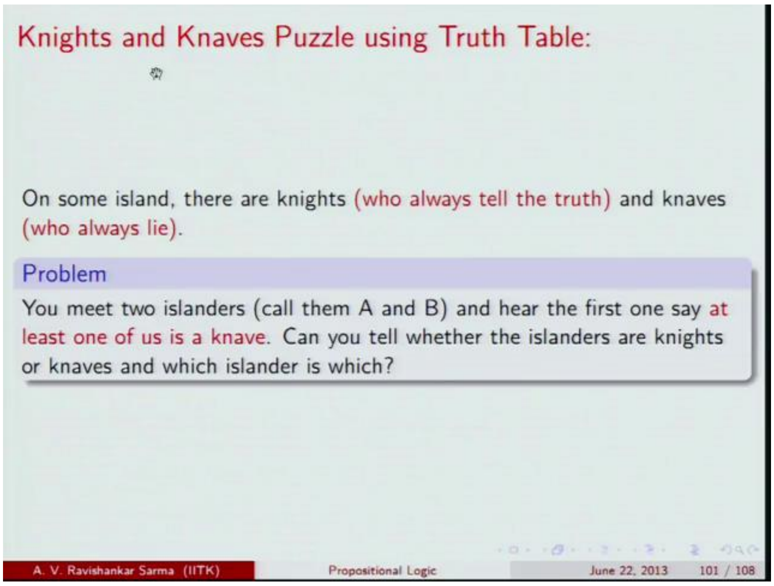

So, so far we discussed about semantic tableaux method and then we talked about when formula is valid consistent satisfiable and all these things. So, the will apply these semantic tableaux method. So, 1 of the important things, which we will be making use of this semantic tableaux method is all in some kind of puzzles. So, here some of the interesting puzzles, which are could up by Raymond’s William. And Raymond’s William come off with a various books all these books are quite interesting it includes lots of puzzles 1 of the interesting books are like this the title of the book is what is the name of the book that is that is to be considered be the title of the book.

The other book is lady or tiger and lots of other books, where we discussed all these puzzles and tries to solve this things using the principles basic principles of logic a few puzzles which, you will take up in this class. And then, you will end this lecture and all. So, here is an interesting puzzle which is called as Knights and Knaves puzzles this puzzles can be solved by a using semantic tableaux method, it can be solved by using tool table method. So, the puzzle is puzzle goes like this a story behind the puzzle is like this.

So, and some island there are too many 1 always speaks truths they are considered to be Knights and some other kind of inhabitants there always lies for example, if he ask particular kind of inhabitant is 2 plus 2 is equal to 4 then it is a Knight he will tell the answer is yes, it is a Knave he will tell that is 2 plus 2 is equal to 4, if the Knave will answer no; that means, he always lies whatever trues are there he always say it not the case.

So, now, that is a particular kind of island; that means, cleverly designing such a way that this true and false everything is a clear black and white either something sentence another true or false. So that is an island were you will be going. So, now, you went to that particular island is all an imaginary kind of situation. So, all stories, but, lot of things can be done with the help of this particular kind of things. So, now, you meet to islanders that is call them A and B; now, you here the first 1 saying that at least 1 of us is a need. So, now, can you tell whether the islanders are Knights or Knaves based on whatever, information that a person is trying to you.

So, what is happening here is that you are a stranger to visited that particular kind of island and then, you are trying to question this we have trying to ask some questions. So, that you will get definitely yes or no kind of answers and with the help of those answers which, we are trying to judge whether their Knights are needs. So, for solving this kind of Knights and Knaves problems the first thing which, we need to know is some kind of notion.

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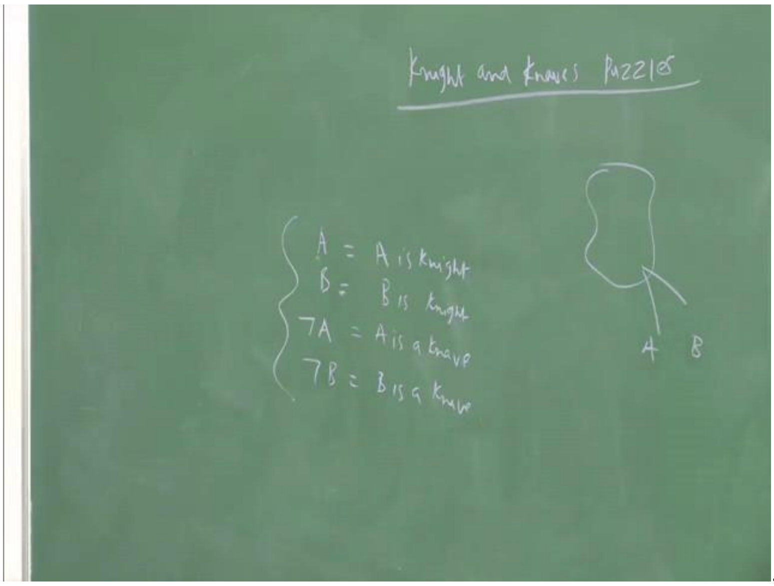

So, what we have discussing is Knights and Knaves for these using either truth table are semantic tableaux method some simple problems, which you will be considering and then we will move on to some kind of difficult kind of problems. So, now so, this is an island. So, there are only 2 kinds of inhabitants A and B. So, now suppose A is a Knight and you represented like this only. So, A is Knight in the same way suppose, if A same thing letter B; that means, B is Knight suppose, if you write like this not A; that means, A is a Knave and not B, B is a Knave.

So, there are some particular kind of problems the problems here is these thing. So, now you are a stranger you visited this island. Now, you are trying to know what they are. So, you ask some question and all this is what you here the first 1 say his at least 1 of us is a knave the first 1 is A.

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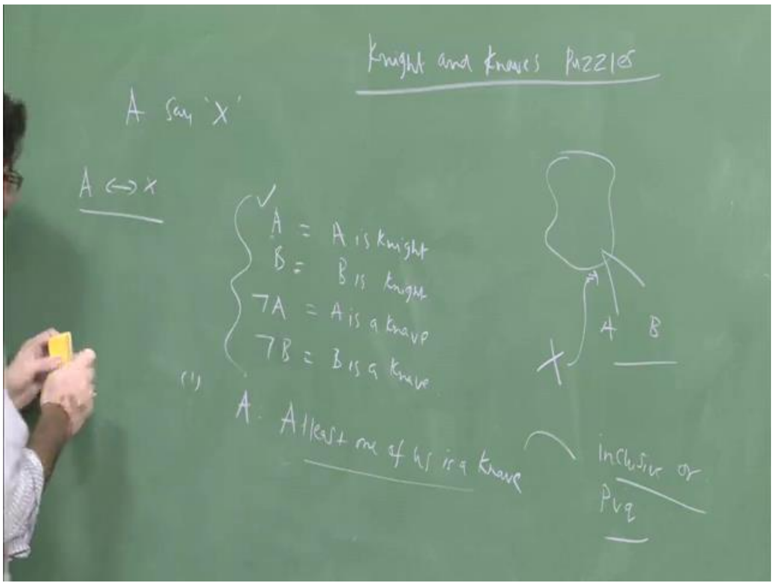

Now, what is the thing A says at least 1 of us is a Knave. This is the information that a gives. So, now, you are a stranger to went to this particular kind of island now, you are trying to decide whatever answers that they give you are trying to decide what type they are? Do not know whether he is a Knight he is a Knight and all. But, based on the information that they give you will be drawing some kind of conclusion.

So, these are some kind of reasoning problems which you can solve it various number of numerous methods and all but, since we have studied semantic tableaux method and truth table method in greater detail. So, we will be talking about this particular kind of method. So, now a says X some x; that means, he will be saying 1 of us his a Knave or some other kind of thing. So, this can be represented, as a backend bi implication shift and only if X.

So, now, the first problem that, we will be solving is this particular kind of thing you went to island and then you came across to inhabitants instead of both are talking the first person a is saying that at least 1 of us is a Knave. So, from that, what you can judge about A and D. So, now, this at least 1 of us is a knave it can be translated as that is usually, A consider as inclusive or. So, that is P r q suppose if we have they have said exactly 1 of us is a knave then it is considered to be a an exclusive r that is either a has to be the case b has to be the case, but, not both of them that cases information about exactly 1 of us is a Knave, but, usually 1 of us is a knave is usually translated as inclusive all that is P r said now this formula reveals us to particular kind of thing. So, now, what it says at least 1 of us is a knave.

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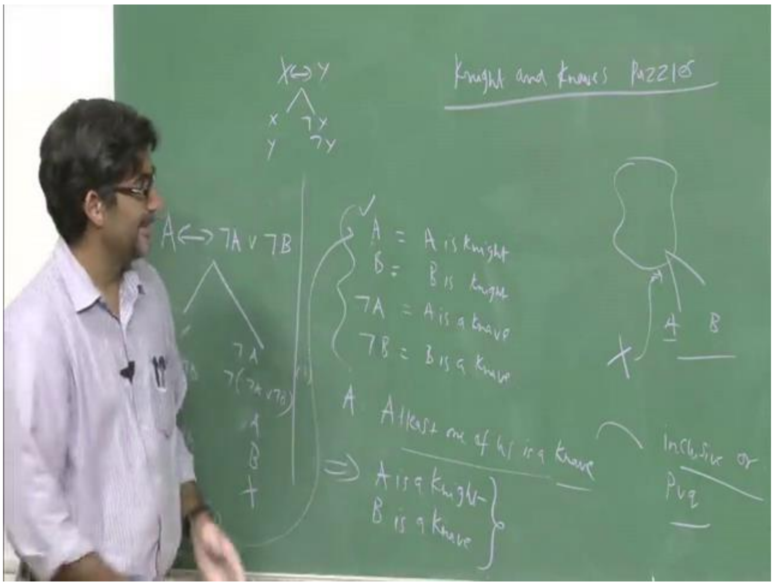

So, that means A is saying; that means, a you cannot if an says that at least 1 of us is knave; that means, either A has to be knave are B has to be a Knave. So, now, we need to construct A semantic tableaux per this particular kind of thing then we can come to know what is A and what is B. So, what is what is happening here, you went to a strange island you ask them there all tell about anything, but, there is a this particular kind of thing at least 1 of us is a Knave from that we need to judge what they are. So, now, this is the information that A is trying to give at least 1 of us is a Knave.

So, now, we need to see when this formula is going to be satisfiable that is going to give us answer for this particular kind of thing by there was A is knave or B is a Knight etc all this information. So, now we construct A tableaux for this 1 for example, if we have a formula X implies y. So, this is either X y is the case X and y is a case or not X and not y. So, this is the tree for this particular kind of thing. So, now, you draw semantic tableaux for this 1 this is A and not A and B and not B and then the second 1 is not A not of not B or not B.

So, now, you further expand it then this becomes not A and not B and here is a not of not is a not of neglection of this junction is conjunction that is why you write it, just below this 1. And now, this becomes the B. So, now, we need to see whether this any conflicting information in the branch. So, now, you have a here not a present branch closes and this branch in its open now, you have a here not a this batch close. So, now, when this formula is going be satisfiable and especially even we need to inspect the open branch the open branch is only this 1 in this open branch the information that we have a and not B.

So, you have a here and not B here that satisfy this particular kind of formula when a is T and B is false and this makes the whole formula true and this is the 1 which are looking for I come to your original interpretation, if you right only A and all; that means, A is a Knight and if you write not B and B has to be knave. So, the solution of this 1 is that A is a Knight and B is a knave. So, this is the A solution for this particular kind of problem. So, when somebody tells 1 of us is an Knave then this has to be the solution.

Suppose, if you have said that at lea exactly 1 of us is a knave and all; that means, 1 ruling out the other possibility and all. So, then we need to write in a different way. So, this will become like this.

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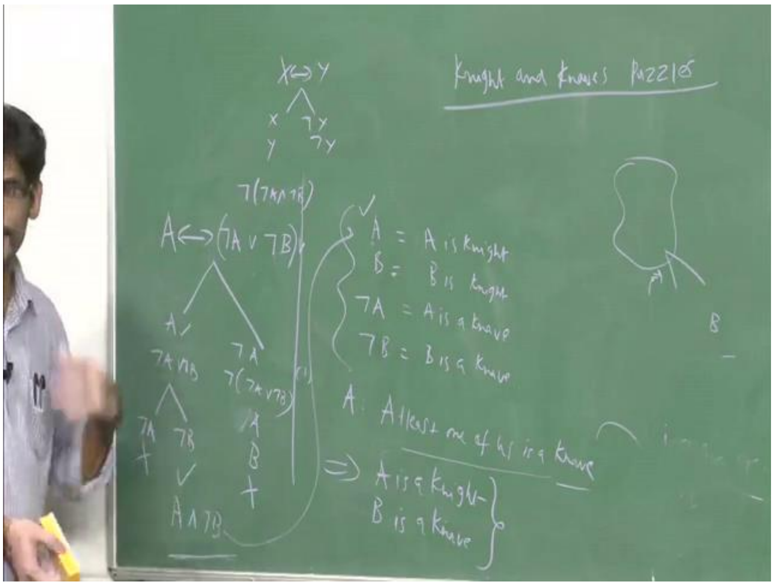

So, not only this is the case and on exactly 1 of the sernights or it is the not the case that both are knaves and all not A and not B. So, this also we need to take into consideration; that means, 1 explores the other possibility then we need to draw the semantic tableaux for this 1. And then you can see whether or not the open branch is the 1, which you need to inspect then you can see the corresponding answer per this particular kind of problem. So, now let us consider some more examples of this and we will see what can be done of this particular kind of thing.

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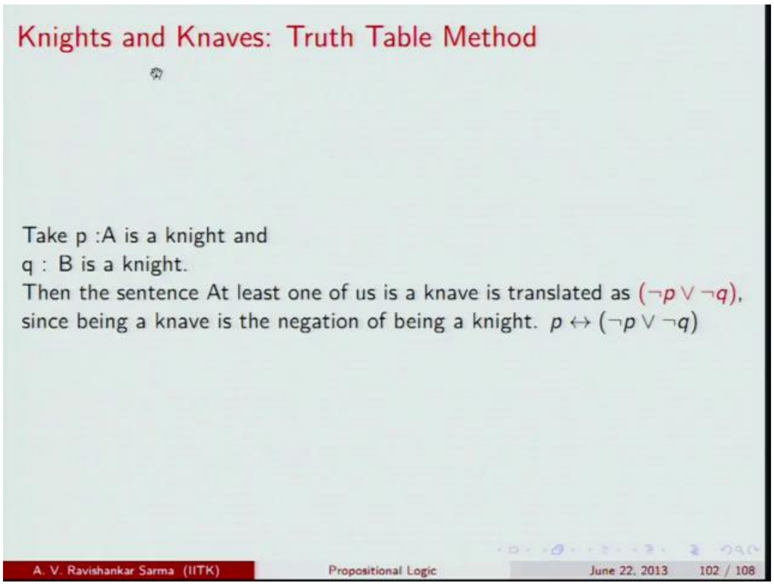

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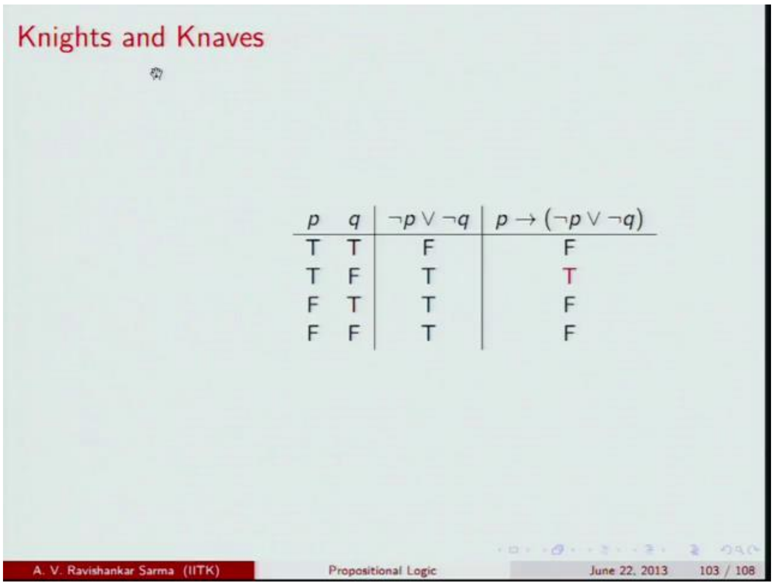

So, now, let us considered that this is: the 1 which, will be considering. So, this is the 1 which can be solved with the help of truth table method also. So, this is like this we have P q and not P or q is stands for at least 1 of us is a Knave and then P implies not P or not q is the 1 which, we need to sick because P is saying this particular kind of statement. So, now, at least it should be bi implication and that is the correct 1, but, you have a inspect a row in which this formula satisfiable; that means, second row is the 1 which satisfies particular kind of formula; that means, P has to be T and q has to be f that satisfies particular kind of formula. So, using truth table method is also you can solve this particular kind of problem.

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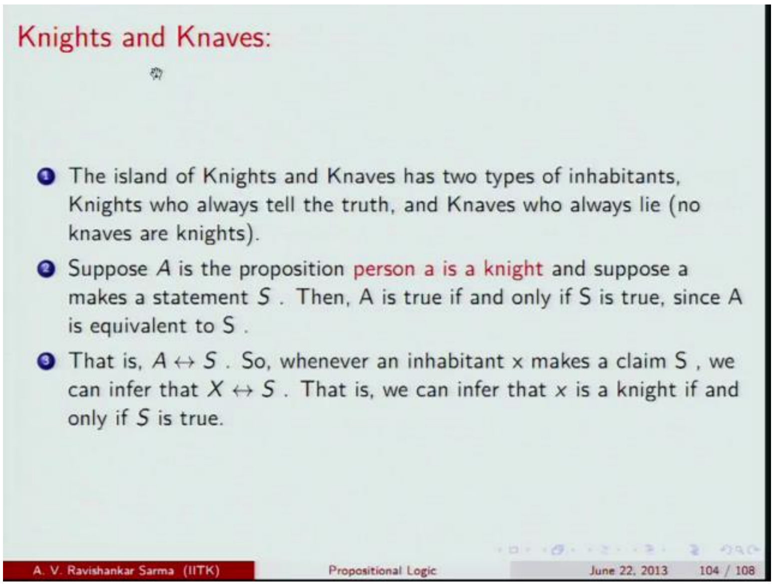

So, let us consider some more examples then will be see, what is the situation.

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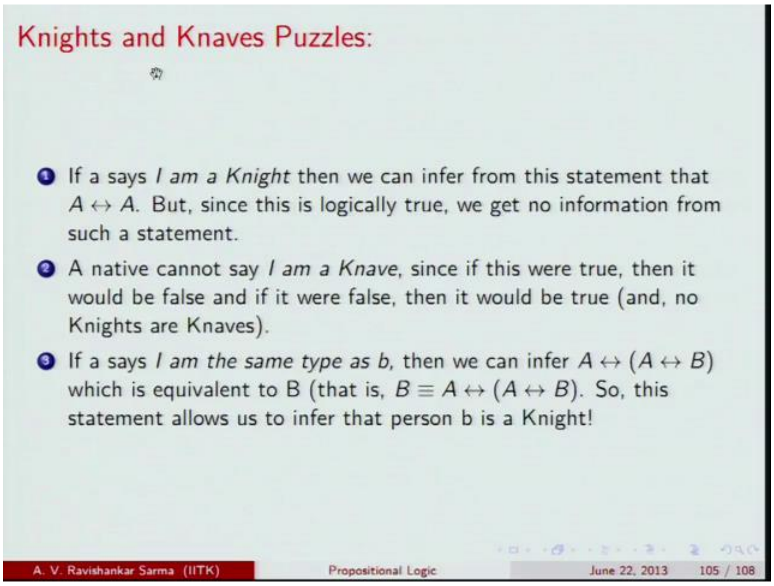

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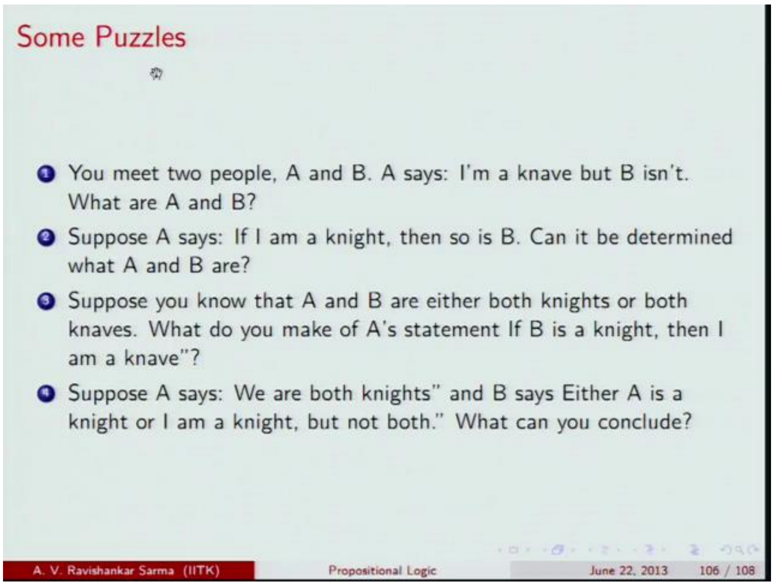

So, this are some of the notations that, we will be following we can solving this particular kind of puzzles. So, you meted 2 kinds of people A and B suppose, if a says, I am a knave that b is it. Let us considered that particular kind of thing.

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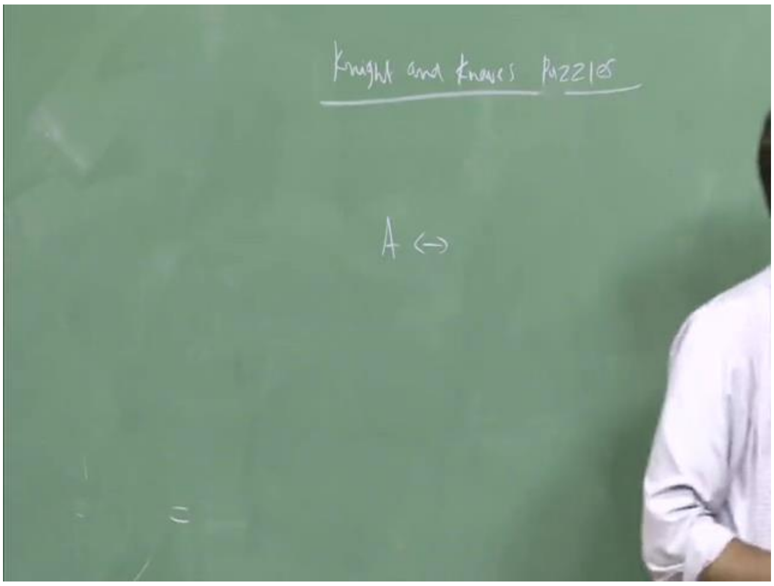

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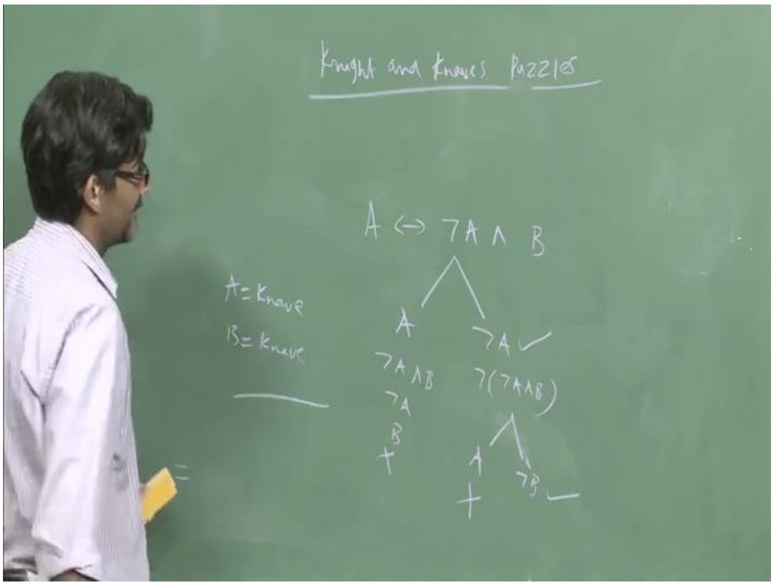

So, A is saying he is saying this particular kind of thing, I am a knave, but, b is not and knave, but, B is not that means, B is a Knight. So, now, from this information what is the 1 which you are trying to get no again you construct a semantic tableaux method using semantic tableaux method you construct a tree for this 1 first is A not B and then this is not of not A and B. So, now, this is this branches clear itself, because not A and B because, A and not it process here itself and this can be expanded to this is a not A is a and then this is not B some of this branches also closes again this is the information that we have.

So, A is a knave and B is also considered to be A knave and all suppose, if you says this particular kind of thing that, I am a knave but, b is not the case. So, then it has to be the things that both of them are Knaves suppose in the second problem suppose, if A says I am A Knight. So, is B then can we determined; what are this thing again the same thing which will be using. So, we this can this class.

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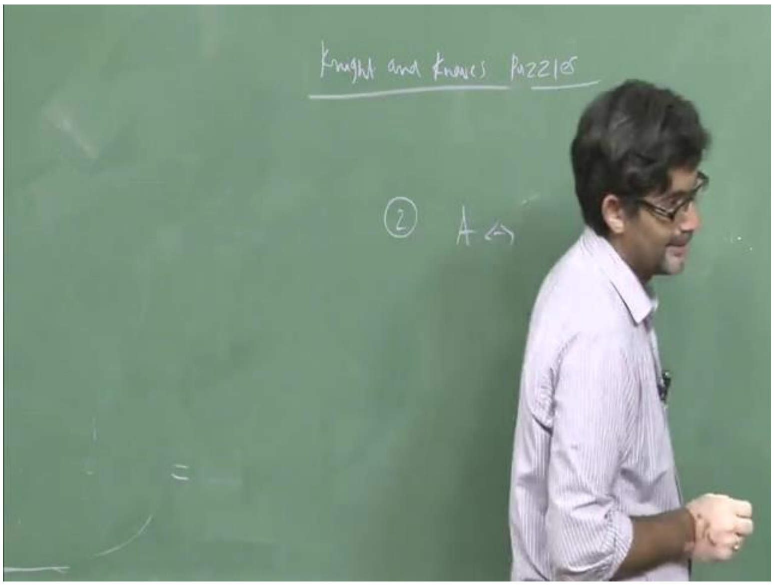

So, A says the second problem if it like A says means A if an only if this thing if I am a knight; that means, A is a Knight then the. So, is B; that means, B is also Knight.

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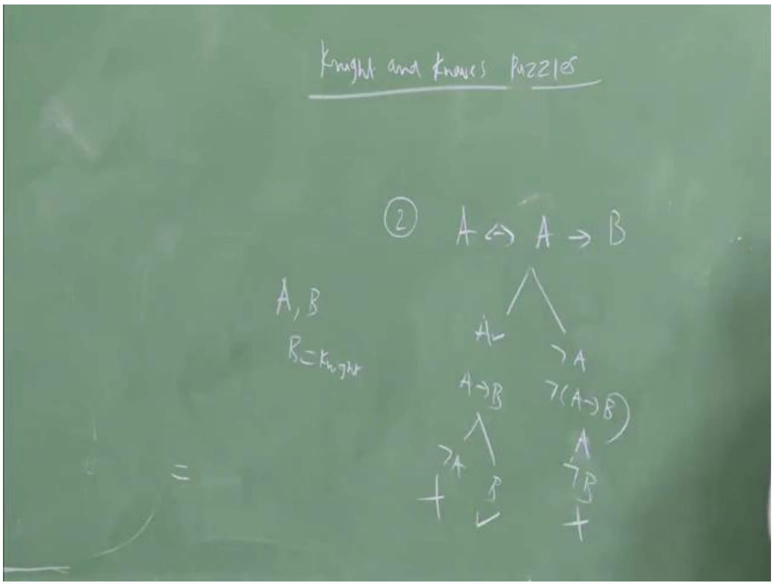

So, this is the formula, we can translate that statement into this particular kind of formula a implies a implies single d. So, now, again you construct semantic tableaux method using semantic tableaux constructed tree for this 1 and this become a implies b then not A exactly in the neglection of this 1. So, now, this further extends to not A and B because, A and not A this branch process and this branch remains open. Now, this is A and not B this branch process.

Now, open branch this are the 1s; which we need to instruct; that means, we have a here; that means, A is A Knight and B is a Knight; that means, suppose, if you went to island and you asked them a particular kind of inhabitant replies the saying that if I am a Knight then. So, is B from that information if that has to be he translated into appropriate language of propositional logic.

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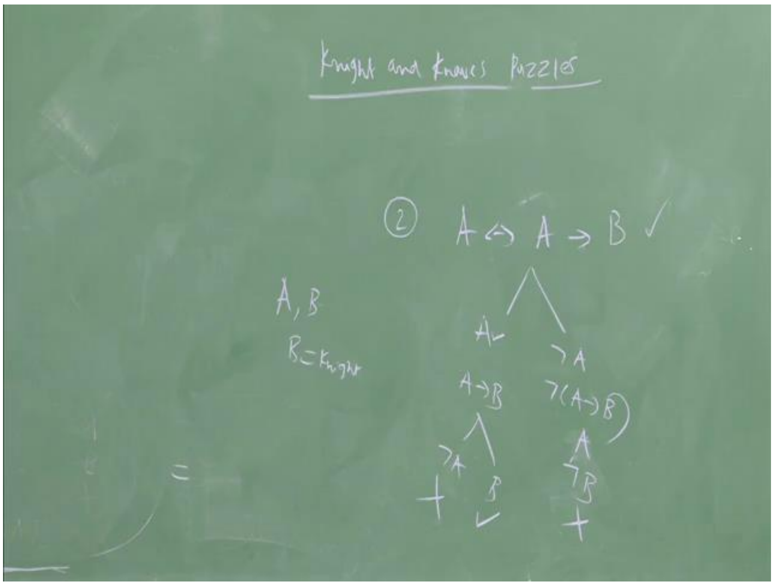

Then this is the formula with which you can be translated into and then you constructed a tree and then you are observing the open branches and then the open branches corresponds to at the answer that is a has to be Knight and B has to be Knight to satisfy this particular kind of state is state. So, in this way suppose of we are making some kind of sample statistical survey that how many number of people are Knights how many number of people are Knaves etc.

Then, we need to translate given formula in appropriately into the language of propositional logic and then you constructed tableaux method and we can see whether or not whether they are knights or Knaves.

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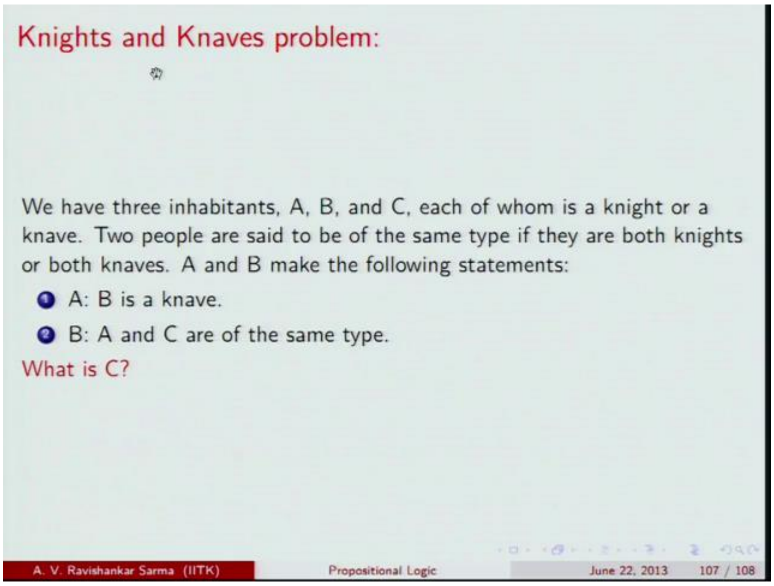

So, let us considered 1 last example and then here we have 3 A, B, C 1 of them each of whom can be Knight or can be a Knave, it cannot be both and all and other important thing which, we need to not is that Allier cannot tell truths and all if the that is the case then it leads to a big problem; which is called as liars paradox. So, that thing which we will try to avoid the Allier cannot tell truths a Allier always lies. So, here is the information that we have asked A and B and the knaves talking about particular kind of thing A is saying B is a Knave.

So, this particular kind of thing, I will describe it and then maybe you can solve it in your free time. So, now A says that: B is a knave that is the first sentence that we have in this problem.

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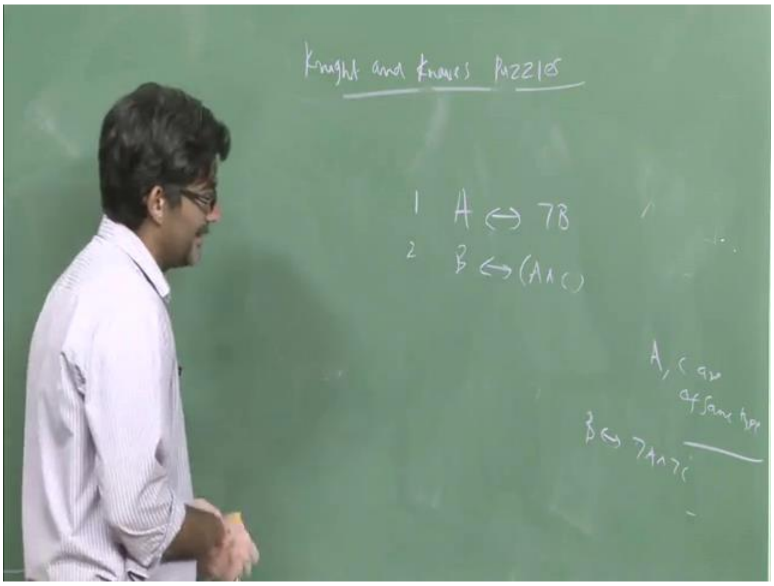

Now B says A and C are of same type A and C are of same type whose saying this thing d is saying that A and C are of same type either, it should be this particular kind of thing are you can even taken to consideration d as not A and not C. So, this satisfies that both are Knights; that means, they are at the same time and the other 1 satisfies this thing not A and not C we can take that also into consideration, but not both of them. So, they are all same of. So, this satisfies this particular kind of formula.

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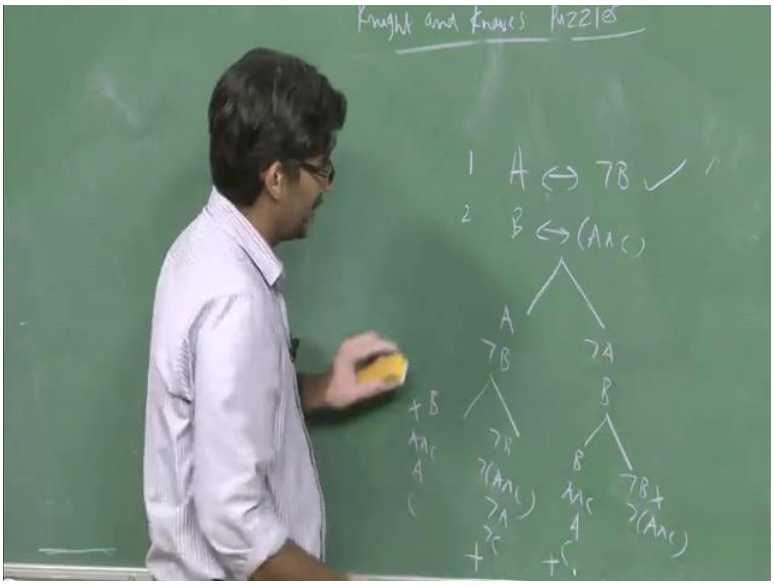

Now, you are trying to determine what is C n quickly, we can draw a semantic tableaux method for this 1 A and not B then this is not A and B not of not B is P in the first formula that, we checked it, So, now the second formula is this thing B and A and C not A not of B and C same information, which you write it here that is B A and C and not B and not of A and C sometimes A branch process even before itself, it in have to go do anything wrong.

So, this B and not B here is branch process here itself and now this is not A and not C since A and not A is in is that branch also close this. So, now, in this A and the C is simplifies to this 1. Since A and not is that this branch also progress. So, now B and not B. So, it is a problem here 1 second; so, there is some problem with representation of the this 1. So, will talk about it, in the next class; so, there is there is a translation seems does not seems to be correct and all. So, we need to a translated properly.

So, then we will get the answer. So, in this class what we have done is you have talked about some of the definitions of semantic tableaux method, some of the definition, in the context of semantic tableaux method. And then we have applied this sematic tableaux method all in some of the interesting puzzles that is these are the puzzles, which are cooked up a Raymond’s for a these are all called as Knight and Knaves puzzles Knights and Knaves puzzles; what you will be doing is you are a stranger you cu went to the island and then you ask them what type of interment argue.

So, suppose if the reply that based on the reply that, you get from them and you are trying to convert this information into language propositional logic. And then we are judging what kind of type he is suppose if they say both are Knaves both are Knights etc and all and based on the you translated into language of proposition logic. And then you are trying to handle it properly, with the help of proposition logic.

In the next class, we will be seeing some more examples in the context of semantic tableaux method; what will be doing is will be translating some of the English language sentences appropriately, in the language of propositional logic. And then see whether, the argument follows or not.