Introduction to Logic
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Lecture - 24
Conjunctive and Disjunctive Normal Forms

In the last class, we discussed one of the important decision procedure methods, that is natural deduction method earlier we also discussed 2 table method and the semantic tab locks method.

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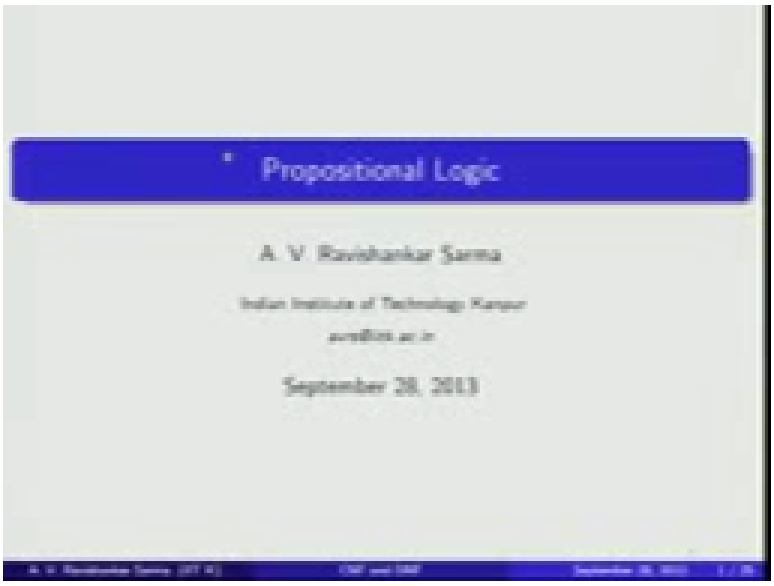

So, in this lecture we will be talking about another different kind of decision procedure method which is called as reducing a given proposition logical formula into its conjunctive and disjunctive normal forms. So, either you can reduce the formula into disjunctions of conjunctions that is DNF or conjunctions of disjunctions CNF. So, what we will achieve with this particular kind of reduction of the formulas into disjunctions and conjunctions of normal forms. It is simple that so whenever you come across conjunctive normal formula.

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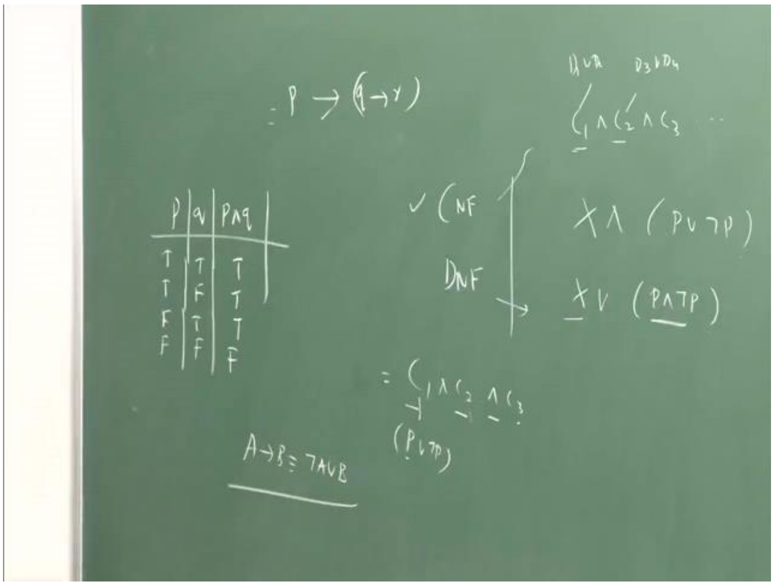

Suppose X and something else p r not p x etcetera and all. We will be talking about 2 different kinds of formulas and all. So, we will be reducing the given proposition logical formulas into either, conjunction of disjunction or disjunctions of conjunctions and all. So, in the conjunctive normal forms it is like c1 c2 c3 etcetera and all whereas, each conjunct is nothing but a disjunction normal d1 d2 etcetera and all d3 d4 etcetera.

So, whereas, in the case of DNF what we have is it is disjunctions of conjunctions and all. So, we have conjunctions of disjunctions and disjunctions of conjunctions. So, we will to the details of it in a while from now so now, observe this first kind of formula. So, in this formula the basic idea of reducing the formulas into conjunctive and disjunctive normal forms is this. So we know that, in the case of conjunctive normal forms suppose in the case of this thing if a conjunction c1 c2 c3 etcetera.

So, this is going to be true only when, each conjunctives true in all otherwise it is going to be false. So, that is the semantics of p and q. So, this reduction of well form formulas into conjunctive and disjunctive normal forms also follows this particular kind of idea. So, that is T T F and F T F T F and then this formula is going to be false only in this case; in all other case it will become T.

So that means, if you have conjunctions of disjunctions and all, then if you come across in each conjunction you come across a literal and its negation and all. Then obviously, the formula is going to be true. So, this formula is going to be true, this is true, this is true, every formula is going to be true provided when you have a formula and its negation is already there. In the same way in the case of disjunctions, in the case of DNF particular whenever you have a formula a literal and its negation is there. Then, it makes the whole formula false; that means, while formula unsatisfied.

So, this is the main idea that we will be using it. So, what essentially we will be doing in reducing the given well form formula into conjunctive and disjunctive normal forms. And talking about the satisfied ability and of course, validity what we will be essentially doing is..., so we will try to reduce the given formulas which occur in this particular kind of format. So, all the implications etcetera for example, if you have a formula p implies q implies r.

So, what you will do is you will remove this implication by using the definition, so that is A implies B is nothing but not A or B either, the reduce formula will have only disjunctions and negations or the reduced formula from the given proposition logical formula; which originally includes implication etcetera and all. So that, we will reduce it to only in conjunction and its negations and all, so that is what we will be doing at in CNF and DNF.

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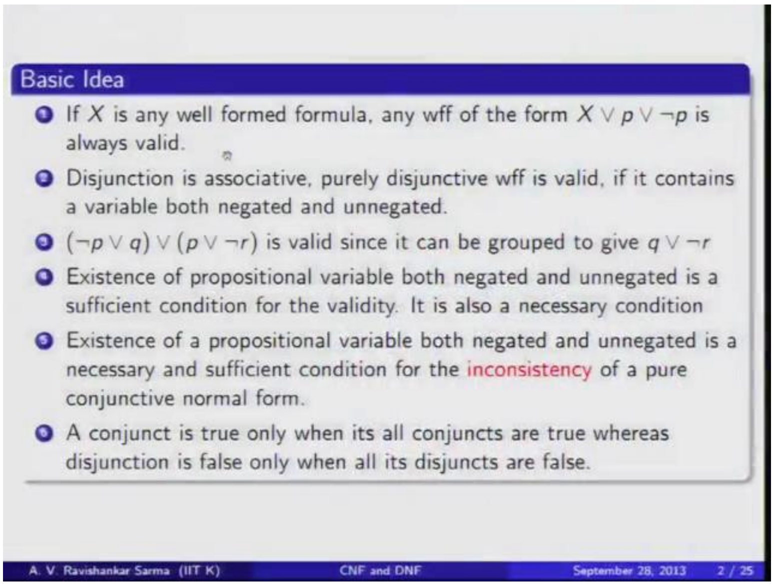

In the process we will be able to talk about whether or not the formula is satisfy able etcetera. So, the basic idea of reducing the given formula into conjunctive and disjunctive normal forms is this. Suppose if x is any well form formula in the propositional logical; which is of this particular kind of format x r p are not we know that p r not is; obviously, tautology and anything which is the dissention of tautology have to be true only. So, because p r not is already true irrespective of whether x is true or x is false that is going to be true only.

So that means, the given formula is obviously, valid and all and other, whatever indentations that you give it for x either it is p or x the variable set exist in x. So, where even thought x becomes true or x becomes false since p r not p is already true. So, that it makes the whole formula true. So, now the other idea is that disjunction is also considered to be associative because, purely disjunctive well form formulas id valid; if it contains a variable and its negation.

And its un negated form and all like, p r not p is it at that is; obviously, tautology and all that makes the other formula also true. So, for example, in this case not p r q it should be end particular it should be r and not p r q and p are not r. So, this is considered to be valid because, negation is un negated forms are there here, so that makes the whole formula true. The first formula true and the second formula is also is true, so that makes whole formula true; this is nothing but p implies q.

So, this is considered to be valid since it can be grouped to give simply q r not r if you simplify this formula you will get q r not r. So, the existences of the propositional variable both negate and un negated is a sufficient condition for validity. So, it is also considered to be necessary condition for validity and all. So, what you mean by validity any formula which is considered to be true a tautology is consider to be a valid formula and all. So, now observe this particular kind of thing and all for example,

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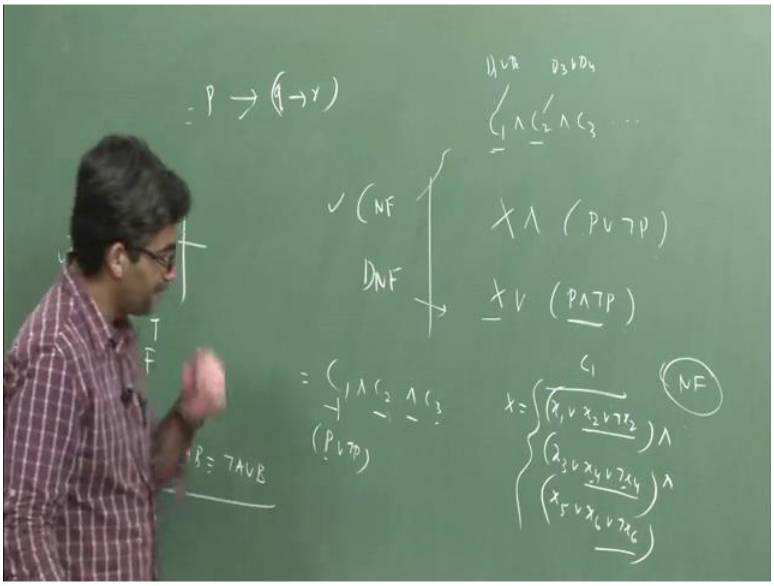

If you have a formula like x1 r x2 r not x2 and another formula x3 or x4 r not x4 and x5 r or x6 r not x6 something like that. So, now observe this particular kind of thing is the conjunctions of all these formulas and all. So, it is conjunctions of distance on, so that is why it is CNF. So, now each 1 each conjunct consist of set of the distance and all. So now, in this case this is already true x2 or x not x true is 2.

So, now in this fact what whatever is there hear is make whole formula true. Now in the same way, a little and its negation is already existing here. So, that makes into true that makes a whole decent 2; that means, this is true, this is true and even this is also. So, case that x6 and not x6 also true, so all the conjunctions the true that is the case here. So, all the conjunctions are true that why, that is result conjunct s is also true c1 c2 and c3 is going to be true; that means, a given formula is tautologies.

So, once again show that its tautology in a of sea can that is same that the given formula is valued formula. So, that is reason why in reduce the given well formula which is complex found which is includes implication, double in implication, etcetera in to normal forms which includes only; the conjunction and negation as its result or disjunction or negation as the only thinks which you see in those normal forms.

So, now existence of a variable both negated and un negated is a necessary and sufficient condition for the inconsistency of a pure conjunctive normal forms. So, for examples: if you come across p and not p in the disjunction particular. So, this formula is going to be like this. So, in case this formula for example: if you come across in the disjunction in the particular, you come across lateral and the negation in all. So, that makes is to the whole conditions false the sense that p and not p is false.

So, it decadent will become falls and disjunction is going to be falls only; when both disjunctive are falls in all other case it's become T. So, in that will be to know that when a given formula is also 1 unsatisfied and all, so provided in literates in negation existing that formula and all. So, conjunct is true when it's all in conjunct are true that is what is case explain of broad and left and side whereas, disjunctive false and both the disjunctive are false.

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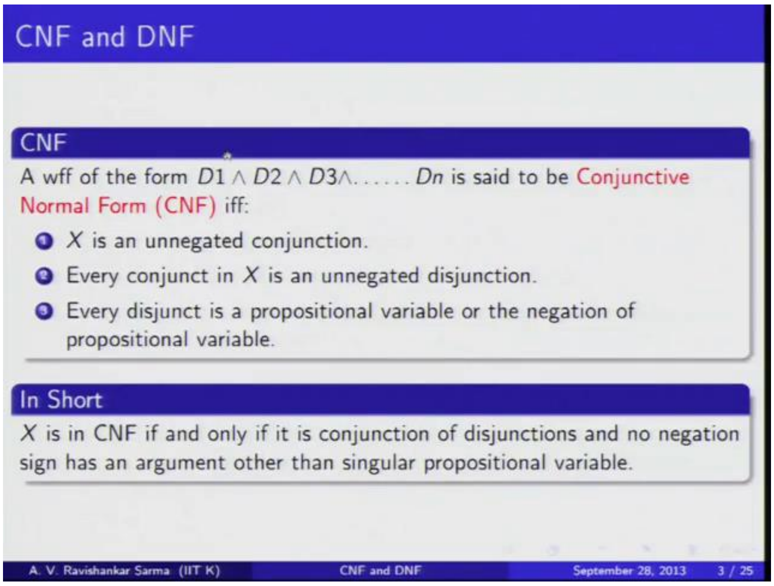

So, now what is the CNF? So, CNF stands of Conjunctive Normal Forms that is given by well form formula you reduced in to conjunction of disjunctions. So, now so it is given and D1 and D2 and D3 where D1 D2 D3 Dn are disjunctions and all, so disjunctions are was represented by r and conjunctions are represented n. So, if it is conjunct of a being there are a and b disjunction means a or b.

So, now well form formula in propositional logic D1 and D2 and D3 and Dn is said to be Conjunctive to Normal Forms. Especially when X is concert to an un negated conjunctions; that means, it is un negated conjunction his p and q extra and all; if it is negated conjunction little lead disjunction all. So, that is why you have role rot is possibility that x has to be an un negated conjunction and every conjunction is disjunction D1, D2, D3 etcetera and all. In x is un negated disjunction and all.

The Third condition is a every disjunctive is a concerted propositional variable is a consorted propositional variable or the negation of propositional variable it should be form in the p r, it should be from in the not p r. So, in short X is conjunctive normal form c n f if and only if it is conjunctions of disjunctions. And no negation sign has an argument other than the singular propositional variable. So, the idea here is a particular kind of thing is trying to talk about Conjunctive Normal Forms.

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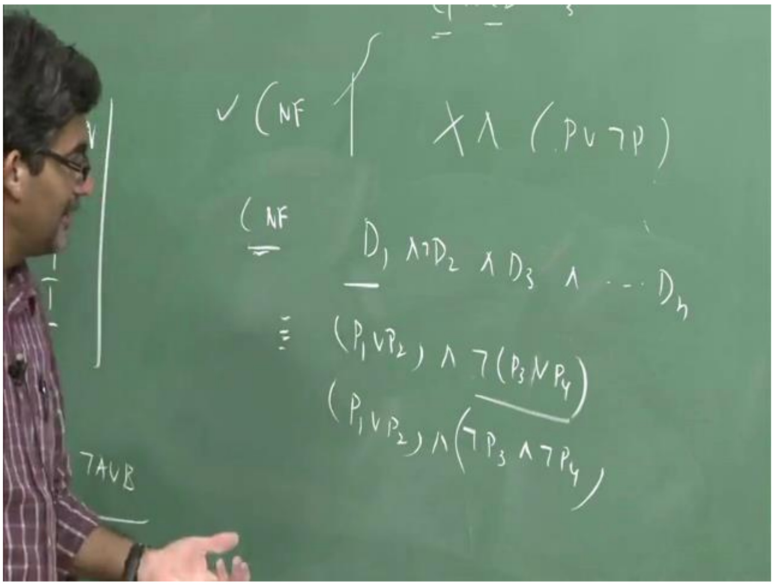

So, what is the Conjunctive Normal Forms? It is the conjunction of distinction D2 D3 etcetera Dn. So, D1 D2 D3 is Dn etcetera are consider to be conjunctions of disjunctions. So, now the idea is that a this conjunctive normal forms should have these things that formula X is an un negative kind of conjunction. For example, if you have something like not D2.

For example, let us is p1 p2 and let us see p3 p4 etcetera and all let us assumed, that these are the 2 formula. So, in the sense this is not consider to be a CNF; conjunction of disjunction, because it consist of negation of some kind of formula negation of disjunction and all. So, that means formula as to be reduced to not p3 and not p4 and all. So, it consists of negation of some kind of reverse not consider to be a not conjunction normal form.

So, all that you ensure that all these thing negation are satisfied and all X is an un negated conjunction and all every connect in x is an un negated disjunction that is the thing which need to be followed. Every disjunction in the proposition variable or the negation of should have it is either a preposition variable or it should be negation of this particular kind of variable.

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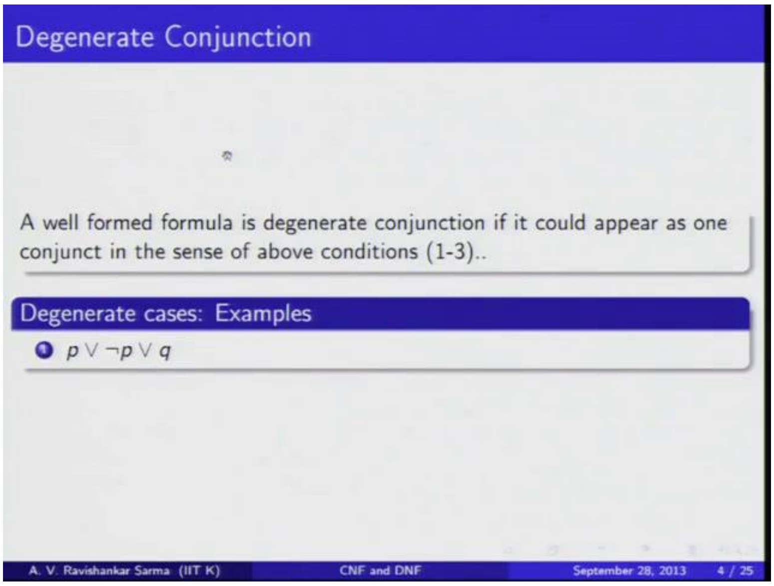

Let us consider, some example of before that the well form formula is consider to be a degenerate conjunction it appears like we have use this 3 rules and all like X is an un negated conjunction. So, that is in this particular kind of formula p or not p or q actually is a disjunction, but it looks like it can also be consider as a conjunction also I mean conjunct c1

So, x is an un negated conjunction there is no negation of formula which is not of x and y extra in all which occurs there. In this formula p or q and every conjunct in x is an un negated disjunction here; we do not have any negative of disjunction kind of thing accrues here is not a p or p extra will not figure out here. The second rule is also followed and every distinct is, a position variable are the negation of a preposition variable; either p in the case first 1 is p and the second case you have not p.

So, literal and negation is there in and all, so every disjunction is a propositional variable are the negation of the propositional variable and it satisfies this particular kind of conjunct which is taken in the form of a dis conjunct satisfy all the thing and all. It is in this sense which is called as degenerated kind of conjunction and all.

Usually, it degenerate and all, but in the whole term p or p are not q we still treated as 1 such kind of formula it exist that here; it can be called as degenerate case of conjunction. So why because, we can still we can write like this is off course, this is a p conjunct p r not p r q may be it wants to extent it write it as p and p r not p and p which is does not make any sense in all.

But still you can write it in that way p and q etcetera q and q same as etcetera or you can even write it as p r in varies way can write it in all, but this is an example of d generate case. So, what essentially we were trying to talk about is particular kind of things. So, what is the CNF?

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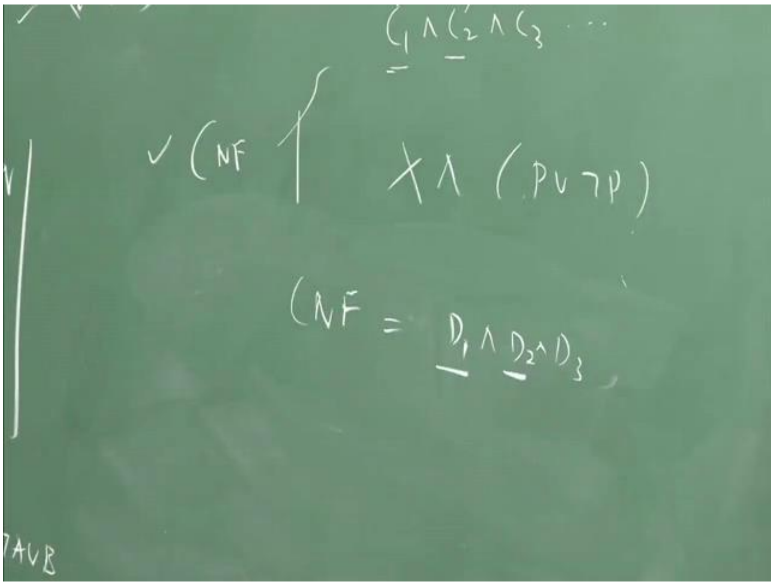

It is a conjunction of distinction in first of all this is conjunction of disjunction in all. So, D1 D2 D3 etcetera, so each 1 is called as the degenerated case of this 1 is p r not p r q. Because, it follows all the 3 rules of the 1 which you have talk about.

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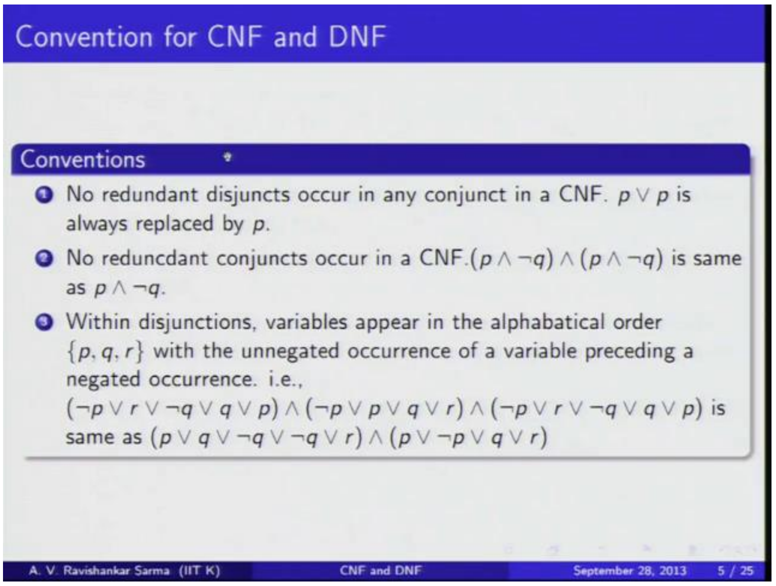

So, now some kind of conjunction which we follows for coming of with CNF and DNF of a given well form formula;, so the convention are like this: No redundant disjuncts occur in any conjunct in a CNF Conjunctive Normal Form so; that means, if you come across a formula like that p r p r p r p something like that. It is simply represented as p. However, number of times you it accrues in the formula 3 times, 4 times it does not matter, but it treated as p, but it's more than p1.

So, redundant disjuncts will vanish in all so now, will just write it as p r p it's like in the a Boolean logic which you have 1 that is 1 plus 1 that is 1 only. So, the same thing which followed here also;, so the only thing we talking about here is the instead of disjunctions instead of plus operation we are talking about disjunction that is the first thing. Convention which you follow and the second thing is, no redundant disjoints occur in the normal forms just like p and not q like.

Suppose it have another not q it is reduced p and not q. Even if this occurs 100 times, but it still treated as p and not q. The third thing is without disjuncts that is D1 D2 D3 etcetera and all. Variable appears in the alphabetical order if it does not appears in the alphabetical order we need to use Distribution or Associative Law and make it in some particular kind of order.

So, now the particular order the unengaged occurrence of a variable preceding a negated occurrence of a variable. It's like so that means, un negated 1s to come first whereas, the negated come later. So, example of observe of that kind of thing not p r r not q and p etcetera and all. So, here negated points came first and negated form came later etcetera and all. We need to reorder this particular kind of thing suppose it follows it that kind of convention like a b c etcetera and all; some order will follow.

So, now this will change to so the first disjoints it's not p r r r not q or q r p changes to p r q and not q are r. So, we are reshipping that kind of thing and then you are putting in that some kind of order; after p q follows are follows. These are only the convenience and all. So, these are the 3 things which we follows we avoid lot of redundancy etcetera and all.

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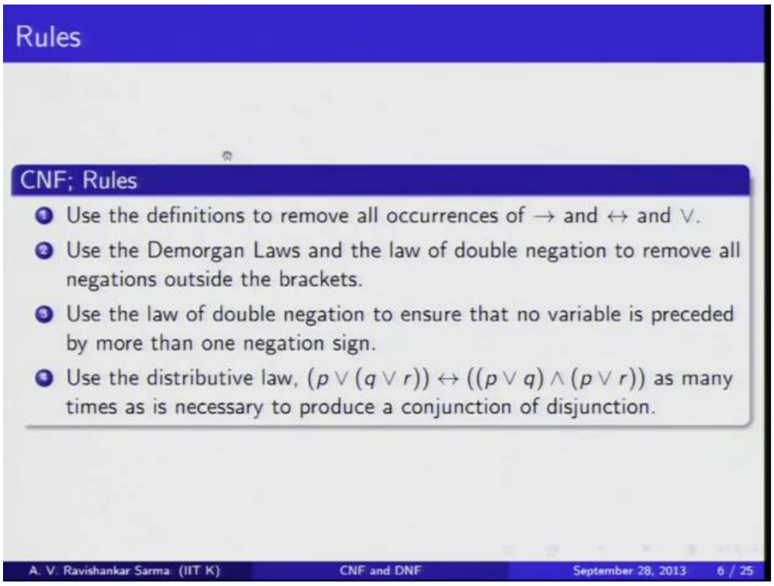

So, now how to convert a given well form formula in to its corresponding conjunctive normal forms; in the same we apply the same procedure it has to different given well form formula is disjunctive normal forms. Where, disjunctive normal form is disjunctions of conjunctions. So, first what we need to know we need to use some definitions to remove all occurrences of implication, double implication and r.

So, that I all converted into conjunction you know to use De Morgan Laws and the law of double negation. That are the last p is equivalent to p to remove all the negation to ensure outside the brackets and all. For example: if you have lot off x and y it is become not x and not y; the same are not x and y become not x and not y. So, then you have the double negation to ensure that no variable preceded by not more than negations sign and all.

That we need to ensure, and then you need to off course some other kind of rules of Distributed Law p r q or r is nothing but p or q and p r q are p r r. Actually, there should be p r q and r or you can use p and q if it is p and r, it will become q p r and p r r; if it is p and q r it become p and q p r p and r. So, these are the Distribution Laws which we apply to this 1.

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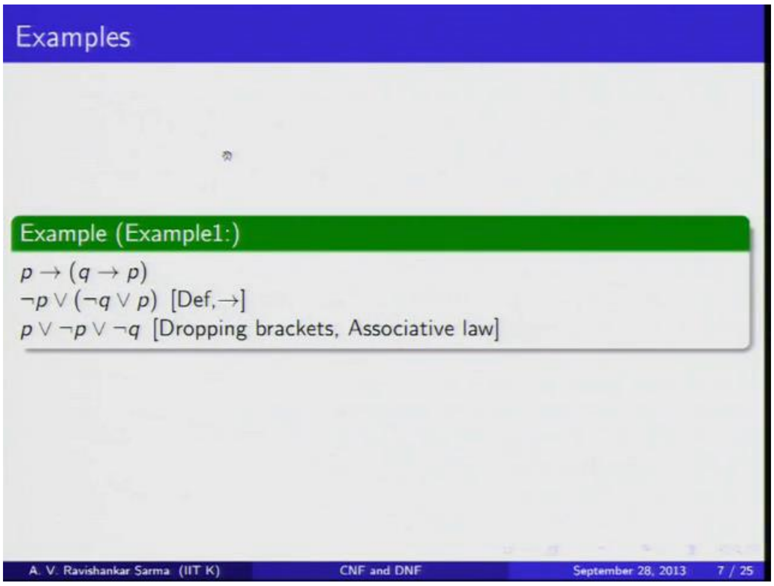

So, that in given well form formula is converted into associative normal form. So, now let us try to convert this particular kind of formula into the conjunctive normal form. So, p plus p what is given so now first thing we need to do is, to eliminate this implication; how do eliminate this implication by using simple definition of material that is p implies q implies is nothing but not p r q.

So, that is the reason why the first conditional p implies p will become not p r again q implies p is written as not p r p; that is the definition that you are used in the first step. So, now further simplifies to you can use Associative Law, and then you can regroup re order it in such a way that it will become p r not p r not q. So, this is kind of case which we have talk about earlier that is the degenerated case of the special case of conjunctive normal forms.

The degenerate cases where we have all the rules 3 rules are applied on this particular kind of thing. So, this a p or not q is finally, consider to be in the particular kind of normal form. So, this as such it appears to not to be in the conjunction, but we can write it in that way that p or not p or not p r and the s same thing write it p r not q. So now, this is nothing but this is like c1 and c2; where each c1 is nothing but a disjoint.

So, in this case p r not are not q because, already true in all because you know the tautology. So, anything which are q is to going to be true only; it is in that sense it is consider to be valid formula also. Off course, in this is a valid formula in the preposition logic because, it sense of paradox of material implications. In the last class, we showed that using natural deduction we showed that p implies q implies p is theorem or construct to be valid formal.

So, what essentially we should here this is given a formula we reduce this we eliminated application by using the De Morgan's Laws the definition of material implication extra and all. And we reduce the formula into a formula which consists of only disjunction or negation and all. So, this is the only thing in the final formula.

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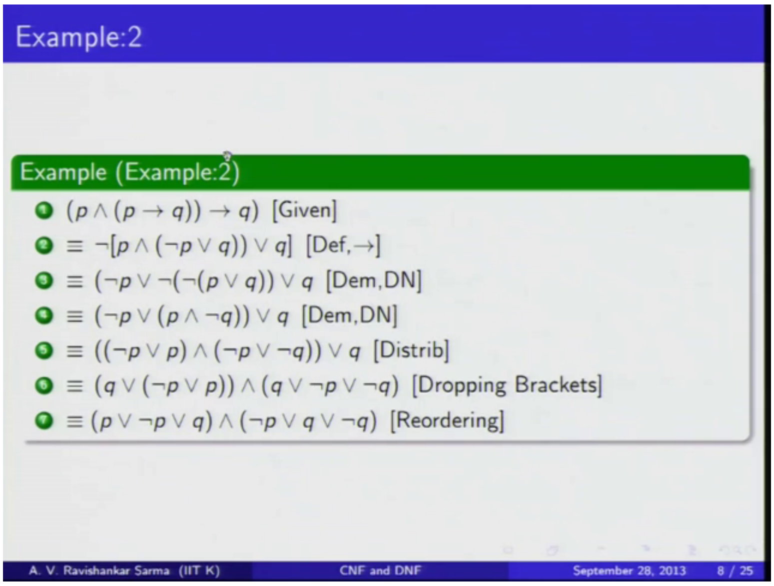

So, more examples which we taking to consideration how to reduce given formula into conductive normal formula;, so conjunct in normal formula is conjunction of disjunction; each conjunction combination of different formula which are in the form of disjunction. So, what is given to us is p and p implies q at implies q is what it is given to us. So, now what essentially 1 is to do is in this formula we have implication and conjunction is there.

So, each eliminate this implication how would elimination this implication using definition that is a imply b is nothing but not a r p. So, this is p and p implies q that is taken as x. That means, not of x or y; y is same as q here, so that is happen in the second case so now, 1s you use the definition. So, now there is a negation outside the bracket in all, you push it inside by using De Morgan’s laws.

So now, in the second stead the negation is outside the bracket in all then pushing it inside. The negation of conjunction will come disjunction negation of a formula p become not p not p are not p q that is the first 1 are q is a case. This is not still in conjunction normal form; that means not the conjunction and disjunction. But you have to reduce the using further apply De Morgan’s rules and whenever, you have not not p etcetera and all and use the notion of double negation.

So, the first formula not p are not p are q become now p and not q and q is remains the same. So, it is not p now use Distributed Law x r y a and z will become a x or y and x are z. That is what we have done here not p r p and not p are not q or q is remains same here. So, now, 1s your one done this thing what you need to know this is that is re order the formula in such a way that there is some order maintain conjunction we need to followed p presided by q etcetera.

So, now is q is not p r p and not p r and not q and the q it is use Distributive Law becomes q r no p r p and then q r not p r p not q, so now drop the brackets and all. So, now this again you can use some kind of re ordering, it will become the Associate Law here; it will become p r not t and q. So, you have ensure that un negated term comes first followed by that negation of its term and then followed by anything off course and all.

In the same way in this case also not p r q are not q; that means, first un negated term came first and then the negated comes later. So, this is what we reduce given formula p and p implies q implies q reduce to this 1. So, what great about reduce this particular kind of formula and all. So now, observe this particular kind of formula this is like a x and y kind of thing.

So x and y when it is good we true, when both conjuncts are true. So, now in this case if you observe the first conjunct that is that involves p and p are q p and not p is obviously tautology. So anything whether q false are q is true that is going to be true only. That means, the first conjunct will be automatically; first distinct is automatically true. So, coming back to the second 1 here also we have the literal 2 and its negation is in there so that means, it always true.

Now irrespective of the not p occurs in that formula whether it as true or false that is going to be true only. So that means, the entire conjunct C1 D1 and D2 both are true that is why the given formula is true; that means, given formula is a tautology. So, how this happen in all especially become each disjoint has literal and its negation occurs in particular kind of distinct in a conjunctive normal form. So, that makes the particular kind of formula true; that means, tautology and hence the given formula is valid kind of formula.

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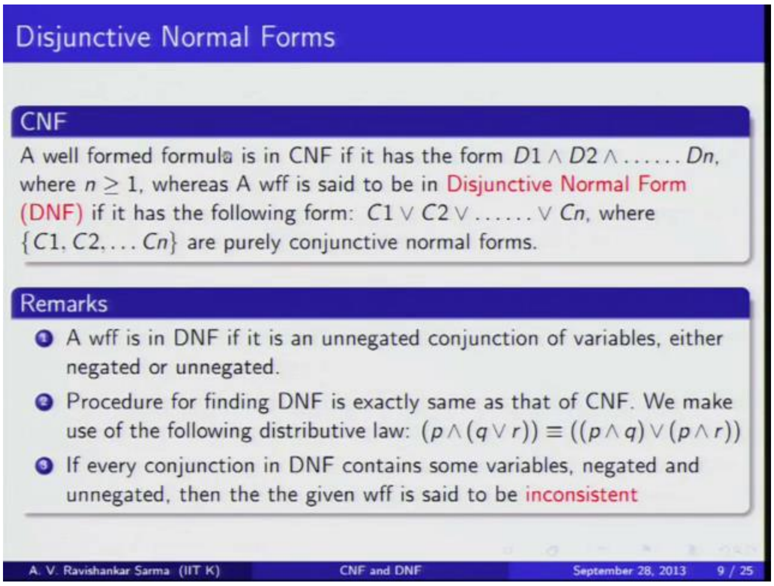

So, now a well form formula is in CNF especially when it has form is a conjunctions of distinctions. First you need to talk about the connective that occurs there that is a conjunction of what disjunction even D1, D2 to Dn, where n is greater than 1, whereas A is consider to be a well form formula which is distinct in normal formula the other way d and this 1. If it is disjunctive of conjunction and all, so how we need to this particular kind of formula.

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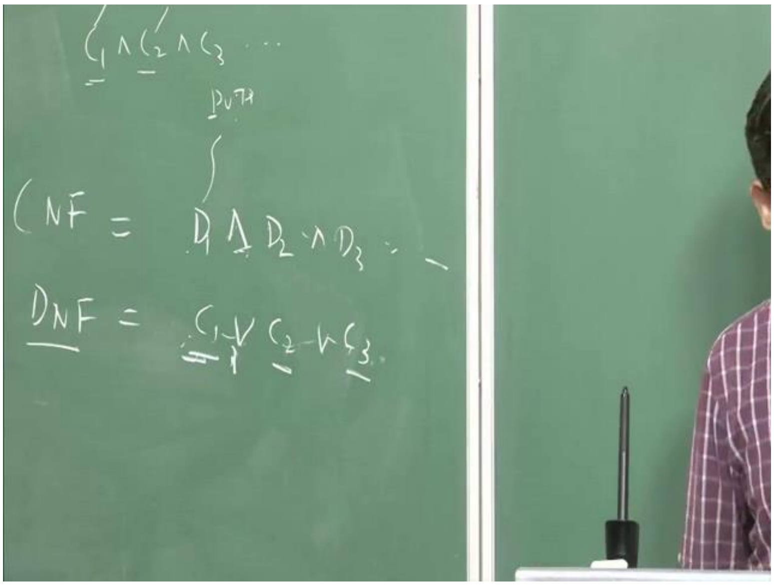

Some time it will be confusing for us in the case of conjunctive normal form. So, what appears here is it is a conjunction of disjunction and all D3 etcetera. So, now DNF is disjunction of conjunctions c3; the conjunctive normal from is going to be true when all the distinct in which that occurs in the normal conjunctive form is going to be true. So when this happens, especially when you have a formula like literal negation occurs in this particular kind of formula and then each distinct will be true.

Then, they make the conjunctive formula like from true hence valid. In the same way DNF in the case of DNF each conjunct has to be suppose each conjunct is false, then that makes the whole formula unsatisfiable and all. So for showing the validity use CNF showing the un stability we will be using DNF. So, DNF distinct to normal forms are nothing but distinction of conjunction first you need to sign here the distinction of the conjunction and all.

In the same in the conjunctive normal form first you right the conjunction here conjunction of what distinction D1 D2 D3 etcetera. So, now there are some remark which needs to be discuss here. So, a well form formula is a distinctive normal form if it is an un negated conjunction of variables; that means, the no formula occurs like this it is not of p r q and etcetera and all whereas, individual let us have negation and you can have p and we can have not p also; so either negated or un negated.

The procedure for finding DNF is same as CNF; that means, remove the implications sign first using the definition and you push the negation inside and use De Morgan Law and Distributed Laws etcetera. And I ultimately we will have in the case of DNF you will have distinction of conjunction and all. If every conjunction in DNF contains some variable and negatives negated and un negated then the well form formula is to be inconsistent.

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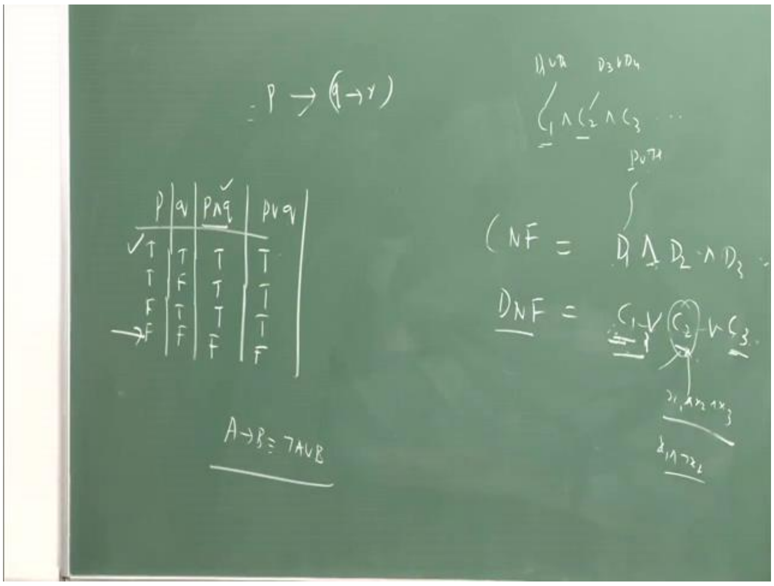

So that means, in formula like this they are all conjunction and all. So, like x1 x2 x3 so there all conjuncts, so now it’s so happens that you have x1 and not x2 in any 1 of this in all this conjuncts and all. So that means this makes this formula this make this c2 false; in the same way c1 having the particular kind of thing a conjunction and a literal and its negation occurs in each conjuncts and all.

That means every conjunct false and since all the p r q false when both are false; that means, every conjunct is false and this is going to be inconsistent or unsatisfable kind of formula and all.

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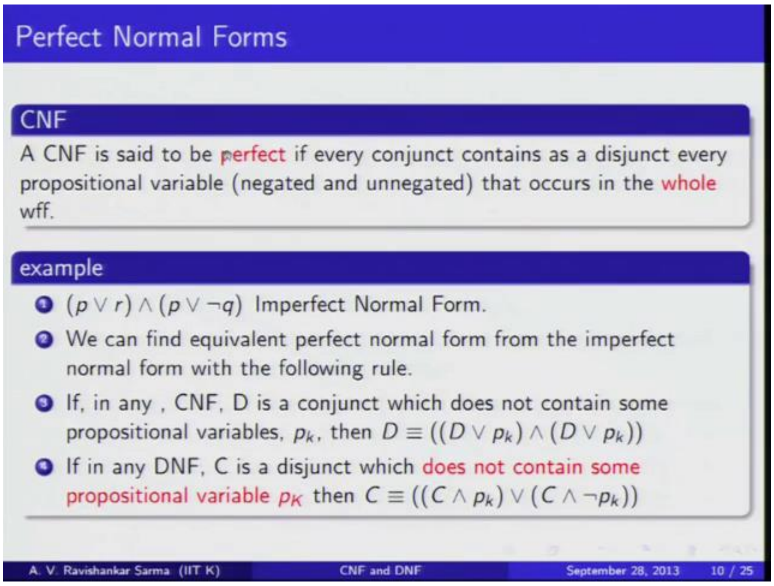

So, there is the question comes to this there may be some perfect normal forms CNF a conjunctive normal form is to be a perfect normal form especially if a every conduct contains as a distinct; every prepositional variable negated and un negated that occurs in the whole well form formula. For example, if you take example serve a purpose, so here p r r kind p r not q. So, this is an imperfect kind of normal form.

So, what 1 is to do here we such a need to find a perfect normal form correspond to this imperfect normal form. So, how do we find it this is a procedure for find it. So, if in any conjunctive normal form D is a conjunct; that means, D is a conjunct does not contain from some prepositional variables pk, then D is represented as D r pk. If any distinctive normal form c is a decadent which does not contain some preposition variables pk.

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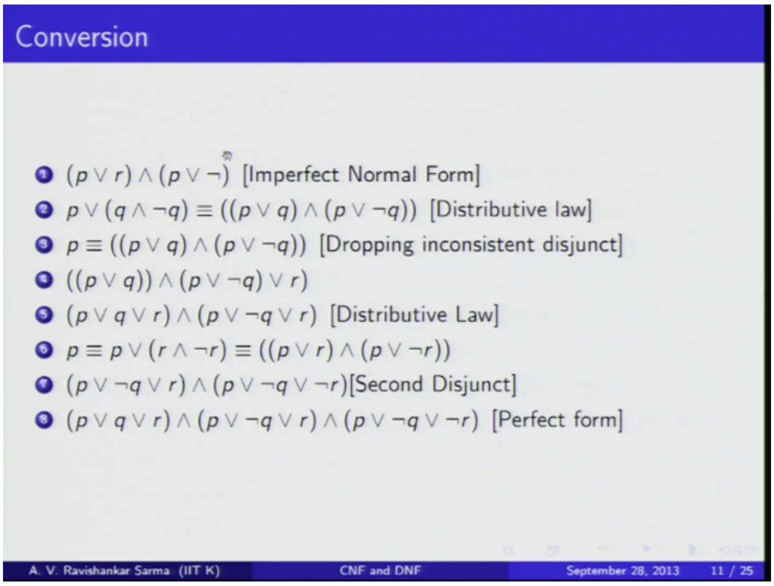

So how do we reduce an imperfect normal form into a perfect normal form? So, the first 1 is this thing p r r and p r not q for example that is taking into consideration. So, now a formula p r q and not q is written in this sense p r q are p r not q. So, now this is p is same as dropping the in consist disjunction and all. What is the in consist disjuncts? q and not q if you drop that particular thing q and q not q is obviously false in all and p r false in p1.

So, p leads to p is nothing but p r q and p r not q drooping the in consist distinct and all. So, now the p r q and p r not q there is the first 1 and then the second 1 p r q and p r not q are all. So, this is another kind of thing imperfect form that is translated into this 1 p r q or r is into Distributive Law a and we can write it in p r not q or r. So, now again this reduces to p p r q and pr not q or r. So, p is equal into p r r and r. So, it is r and r is reduce to f, so p r f is p only and p r r and p r not r.

So, now pr not q r r and pr not q or r, so now, we have doing all this things, we have something called a perfect normal form. So, what is that we have done here is this that, we have reduce the given formula into corresponding kind of a perfect forms, imperfect forms have translated into perfect line of forms. So, something is an imperfect conditional imperfect well form formula can be reduced to perfect kind of formula.

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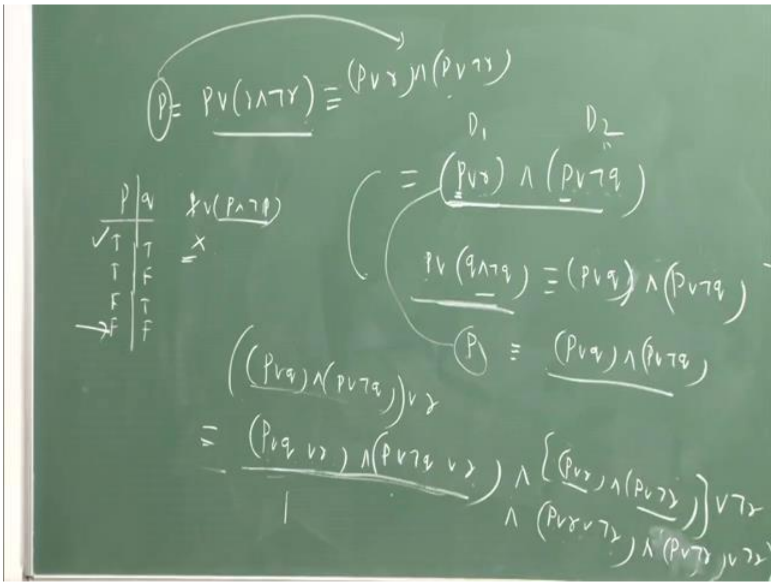

p r r and p r not. So, now this can be written as this 1. So, the first thing P it can be written as this 1 distinctive normal form this is nothing but p r q and this we are try to translate into perfect kind of formula. So, now p r we are adding this particular kind of formula q and not q this is nothing but p r q and p r not q. So, this is using distributive property pr not q and p r not q this is first is p r q and p r not q.

So, now a this reduces to just p now what you mean by p is p r q and p r not q. So, what exactly we are doing is that since it is not in perfect normal form all; conjunctive normal form where this is D1 and D2. So, we are trying to reduce to into perfect kind of normal form. So, now, using this particular kind of equaling this substitute in this particular kind of thing. So, now this p substitute here.

So now what will happen, p r q and p r not q r r, so this is the first distinct and all. So, this will become this 1 this is p r q r r and this is and p r not q r r. So, this is the second distinct you know. So, how did you get this particular thing? First we included this particular kind of thing pr q and not q this is same as p only; where you do anything new only this is nothing but p r q and p r not q. So, now the same thing whatever we generated here this we substituted in this 1.

So, that we are written here are distinction r, so this is what we get. So, the same thing we substitute into the second 1 you will get, you will generate the other 1 p r. So, now what we will do here is,, so now p r q and p r not q in the same way you can write p so now, p as p r r n not r. So, this is same as p only.

So, now written as p r r and p r not r so now, you substitute this p value whatever we got it here into the second 1. So, now this becomes p you substitute this 1 p r r and p r not r this is the first thing, we got it for p p is nothing but this 1 the whole thing r not r. So, now this is as it is in all and this will become p r r r not r and p r r and p r r p r not r. So, p r r not r this is the first thing, then you have to use p r not r r not r.

So, now this seems to be in the perfect kind of normal form. So, initially we have just p r r and p r not q. So, now if you are try to write in normal form we need to ensure that, all the distinct occur here will have all this letters and all; whereas in this case the q is missing here where as in this case r is missing. So, this not in a perfect form and all.

A perfect form includes all the distinction of what distinct occurs in the given formula. So for that reason, what we have done is simply is that we have just taken 1 formula into consideration; we this is an equaling of this particular kind of thing p for example, if you have x r p and not p is same as x only. Because, p and not p is; obviously, false in all x are something which this false in x it is like x plus 0 is x.

So, now essentially what we have done here this is that, this is an in perfect normal form and in the sense that that distinct D1 does not consist of q the variable q here. In the same way, in the case of decision 2 we do not have r here. So, we need to transform this in this such a way that if want you have it in forfeit normal form. Then, we need to make use of this particular kinds of transformation an ultimately you we see here.

This is the final translated version of the same formula p r r and p r not q that is like this p r q r r and p r not q r r and followed by that we have p r r not r and p r not r and not r and all. But now here, so can be say that particular kind of formula is a valued are not. So, for validity what needs to be the case is that all congest have to be true. So that means, each decent have to be through to make this formula true.

So, we can talk about such kind of think for only this 2 formulas p r not q r r at least little lets negation is there not its negation; same negated formula un negated formula is there. But when it comes to this particular thinks p r not p r q are r if this any decent to will become false, is going to make the hole content false in all. Had it been case that? It as at least 1 not r here and then if we have some p and not p a some think lets here.

Then, it makes all the congests all the design through and it makes the contented to normal formula true. So, this is the way in which planking convert a and imperfect normal form that is p r r and p r not q into this particular kind of format and all. Where, we have not change the inter structure of the formula and all, but we made use of same definitions all like p is nothing but p r q and not q.

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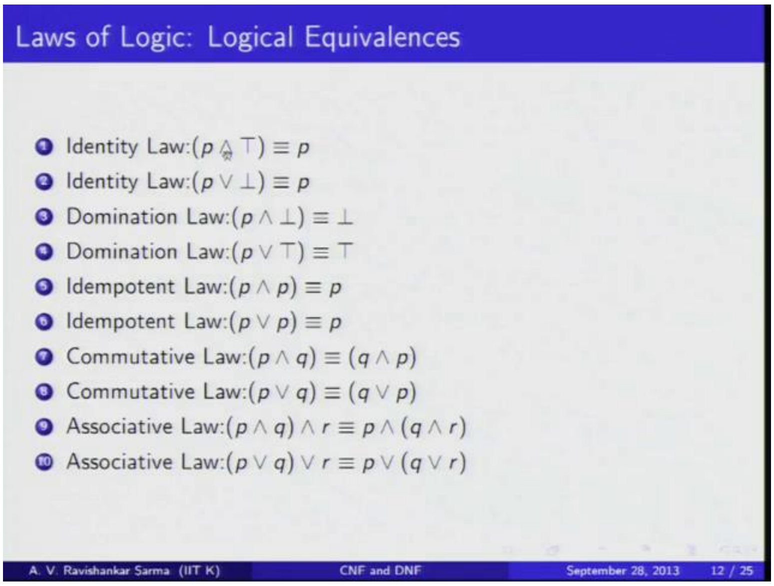

In addition to this particular kind of think definition and pushing the implication in side etcetera and all. So, what we trying to do is we are making use of some loses of logic also. So, 1 is the first 1 is law of indentify T and something which it is the tautology is obviously, p only. Or Identify Law is not a indentify law, but p r contradiction is; obviously, p only it's like in the Boolean logic p r0 p pulse 0 is p only.

The Domination Law if a add any think any continuation to a given formula contradiction dominates and all, so the is given statement is go on to a contradiction. The Domination Law with respect to disjunction if add any tautology to a disjunction disjoints, that is going to be true only. The tautology dominates all over here in the first case contradiction dominates there. And the Idempotent Law p and p nothing but p p r r also p only.

The Commutative Law as been no p and q is nothing but q and p which happens which acres for even disjunction also p r q is nothing but q r p only. Associative Law p and q r are p r q r r is same as pr now brackets q r r next. So, this are thinks which we already know.

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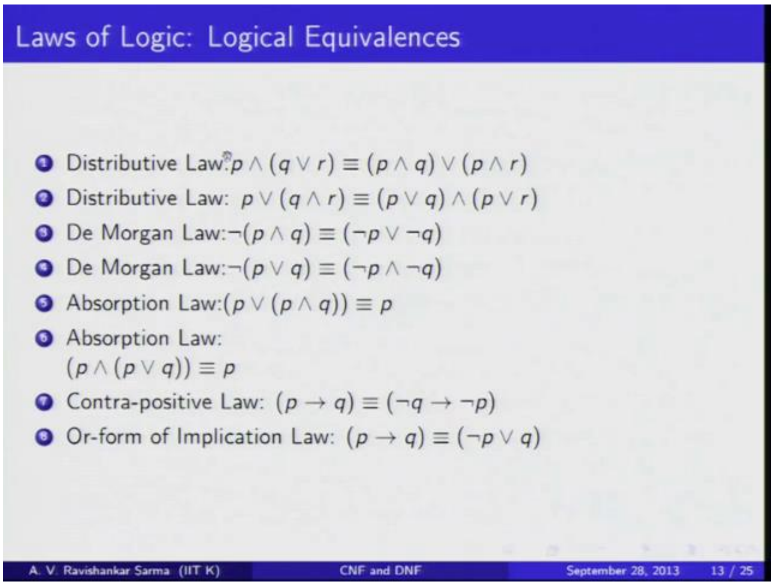

The De Morgan Law's are as usual not of p in q will become not p and not q and Absorption Law is simply is in to be, little bit surprising to this something we is p r p an q will become p.

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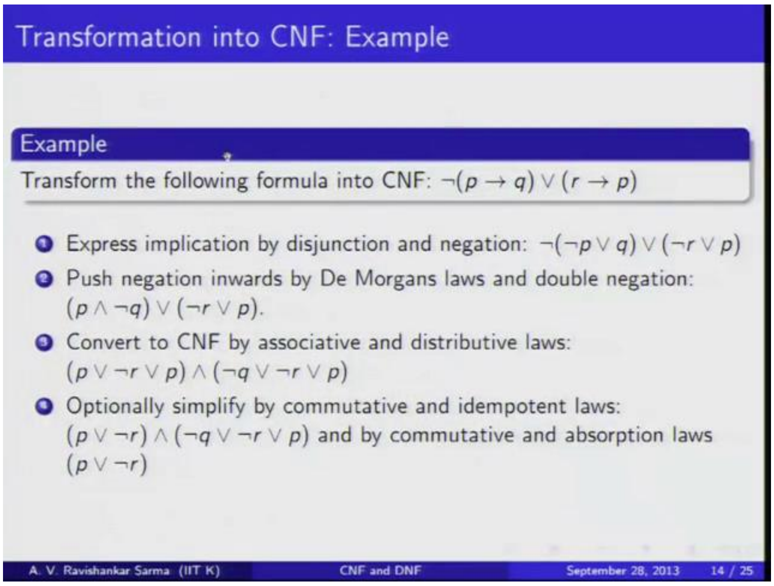

So, now same more examples how to convert not of p implies q r r implies p in to which corresponding congestive disjoint normal forms. The first step what will do is your write we remove the implication. So, then p in pulse q will become not p r q that is for way negation of that particular king of formula or r implies p is return as not r r p.

So, now in the first decent first we are push in this negation inside then it will become p and not q an r not r are p remains same. So, just let me what and the board and all, so that he will get no this think. But is the just 1 formula we I work for connect in the rest of the formula 1 can transforming to the appropriate DNf's and all. So, the formula that we are trying to reduce is this think that of p implies q implies r r p.

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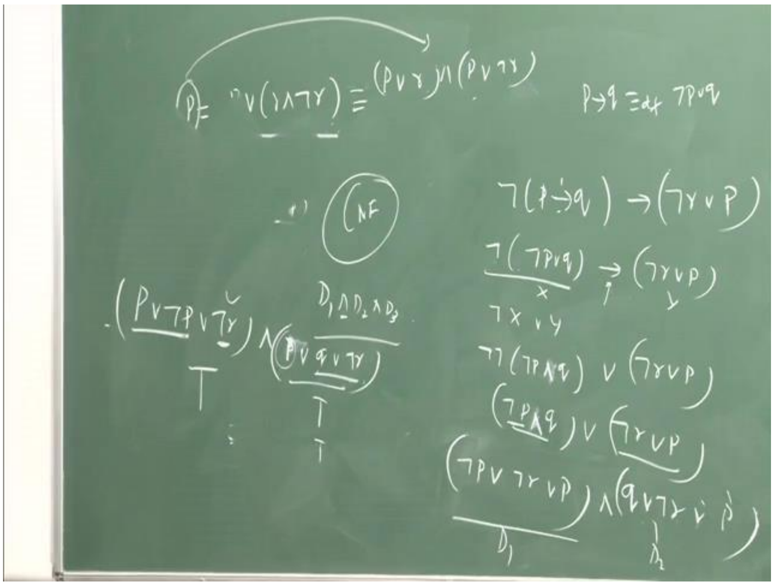

So, now to reduce which in to CNF; CNF is like this, conjunctions of disjunctions D3 etcetera. So now the first think which we do is, you have to eliminate to this implication and all. So, now how we eliminate this think. So, we have a definition p implies q pulse by definition a nothing but not p are q. So, this is what you write it here and then you remove in the next step and all.

So, now this is not r r p, so now in second step we will remove this particular kind of implication. So, now this will become the whole think is treated as x and this as y x implies y is nothing but not x are y. So, now we substituted for x this is not of not of not p q or not are p. So, now this is now not of for this 1 will become this 1 not p or q or not r or p.

So now, we can use Distributive Law and all, then we will get not p or not p r q or not r or not p or q not of p implies q. Then r p not of not of not p implies q yes here think is that, not of not p are q it will become n negation of disjunction will become contention of this is that as it is. So, now we can use Distributive Law and all then you will get not p or.

So, not p are q are not r are not p r q r p not of p implies q, and then r p not of not of not p implies q here think is that not of not p are q which will become n negation of disjunction will become congestion of this is the as it is. So, now this needs to change like not p and q are not r p. So, now this step will become. So, now you apply this think not p and q aha not p and q and not r r not p and q and p and q and p.

So, this does happening. So, now it is not p and q are no now this will become not p r. So, now this think we will taking to consideration and you remain keep it like this only not r or p and q and is 1 which is coming here. So, now, q r not r or p so now this is in aha that digestive form D1 and this is D2 now it is a conjunction of a and d 2 that w 8 is called as conjunction of disjunction that why CNF conjunctive normal form.

So, now whether are not this formula is satisfy able are not. So, not in not in a perfect form and all now, we need to reorder it and then we can write it like this. So, now not p r p now this comes first un negated term comes first forward way, that we are negated term and then not r and in the same way q r not r you write it in this way p or q r not r. So, now in this case you have p r not be; which already through it is a tautology.

Now here is put it whether not r is true or false is going to be T. So, now even in this case of also p or q r not r and negated term an un negated term is there here. So that means, it ensure us that weather are not p is false and p is true this hole formula is going to be t only. So that means, the whole formula is going to be true; that means, is formula is going to be valid formula.

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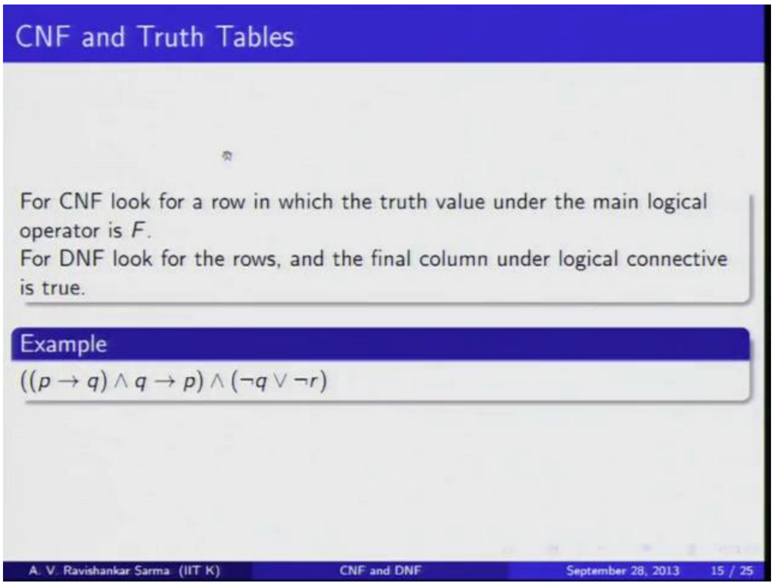

So, in that sense we can talk about validity of like even formula expressly in the form in this case of CNF. So, in this class what we have seen is that a give well form formula; we reduce is it in to its corresponding conjunctive, a disjunctive normal formula. Conjunctive Normal Form is nothing but is a conjunction of disjunction and disjunctive normal form is disjunction of conjunction in all.

So, the basic idea of reducing this conjunctive given formula into conjunctive and disjunctive normal forms is this, that the movement you see the structure of this formulas. We can immediately come to know, that using the semantic of disjunction, semantic of conjunction you will come to know expressly in the case of CNF. If each D1 is true then conjunct to normal formula is called to be true in the case of DNF its disjunction of conjunction.

If each conjunction is false when we can show that a formula is then and all. So, this is the advantage of reducing the given formula in to even well normal formula in to corresponding deceptive and conjunction normal forms. And sometimes it may appear that, this formulas are will not appear in a perfect form like, all the variable will not acre in a given formula.

So, we need to so little bit of transformation without distributing the truth value of fit just like in for example, we can replace p with p r q q and not q. So, that is p r q and p and not q r and all. So, like this 1 we can transform in perfect normal forms in to perfect normal forms 1 is some example we are seen in this particular kind of class. In the next class what we are going to see is, this that we will making use of his conjunctive and digestive normal forms.

Especially, some solving some kind of puzzle as well as you will be making use of this conjunctive normal forms and disjunctive normal forms. In analyzing some simple stitching digital circuits. So, we will talk about an a analyses of as an application of this conjunctive and disjunctive normal forms will be talking about, the analyses of simple digital stitching circuits.