Introduction to Logic
Prof. A. V. Ravishankar Sarma
Department of Humanities and Social Sciences
Indian Institute of Technology, Kanpur

Lecture - 25
CNF, DNF and satisfiability and Validity

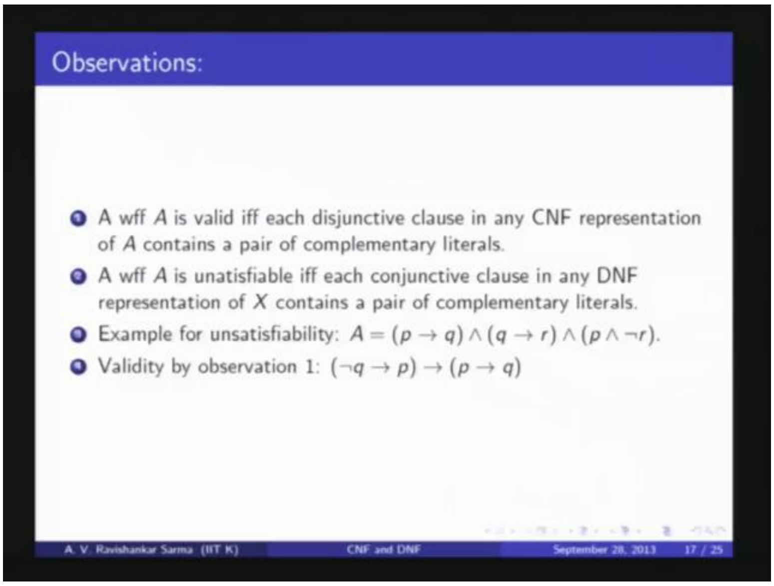
Welcome back, in continuation to the last lecture, where we discussed reducing the given well formed formula in a preposition logic to conjunctive and disjunctive normal forms, where conjunctive normal form is considered to be conjunctions of disjunctions. And disjunctive normal form is disjunctions of conjunctive normal forms and all. So, given a well formed formula, which is in the form of implication and negation etcetera an all, so will try to reduce it to a given normal form.

So, what is a use of reducing it to conjunctive and disjunctive normal forms this is also considered to be one of the important decision procedure method as usually, in any decision procedure in decision procedure method in prepositional logic; for example, if you have taken into consideration 2 table or semantic tab locks method or some other methods which we have considered so far. In all this methods what we did is simply is that given a well formed formula, we are able to check whether a given well formed formula is a tautology.

As you all of us we know all tautologies are considered to be valid formulas and given a set of well formed formulas, we also came to know with these decision procedure methods as a truth table or semantic tab locks method are this method conjunctive normal forms reduced in the given formula into conjunctive and disjunctive normal forms. We can say that a given formula is satisfiable or not. So, under what conditions a given formula is going to be true not only that thing we can also say that when a given formula is considered to be contingent etcetera an all.

So, what we essentially what we are essentially doing is that a given any complex formula we are trying to reduce it to its corresponding DNF, that is disjunctions of conjunctions or conjunctions of disjunctions that is CNF. So, 1 of the important observations that; we can make out by reducing the given well formed formula into a given CNF or DNF.

(Refer Slide Time: 02:25)



So, you have to note that any given formula can be reduced to either CNF that is a conjunctive normal form or DNF that is disjunctive normal form. So, these are some of the important observations which we can make out, but before that. So, what I am essentially trying to do in this lecture is simply is that, I will be talking about some examples of how to reduce given formulas into conjunctive and disjunctive normal forms. And then I will talk about some of the important properties of logic that is satisfiability consistency whether or not a given formula is valid etcetera and all.

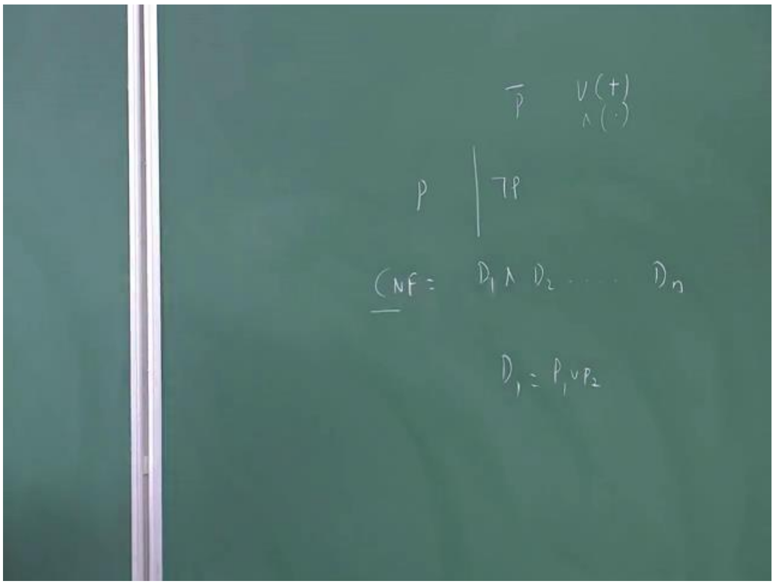
In the 2nd part what, I will be doing is as an application of reducing the given preposition logical formula into CNF and DNF. So, we will try to see its application in analyzing some simple digital switching circuits. So, what we will do in that thing is that a given a complex circuit. So, we will transform it using the principles of preposition logic into essentially, conjunctive and disjunctive either conjunctive or disjunctive normal form. And then we reduce that given formula into a simple formula and that formula corresponds to a simple digital switching circuit.

So, again we reconstruct the circuit based on whatever, simplified formula that we arrived it. And from that you will reconstruct the digital circuit and then that will constitute to be a simple simplified form of a complex digital circuit and then we will also see with some examples. So, one of the important uses of is logic is that it, can also be applied in solving some kind of puzzles. So, we will also see with the help of CNF and DNF reducing the formula into CNF and DNF.

So, we will see that some of the important problem such as: knights and naves etcetera which we have which, we solved it already with the help of semantic tab locks method those things can also be solved by using this reducing the given formula into CNF and DNF. So, essentially CNF reducing the given formula into CNF and DNF will survive some kind of decision procedure method for knowing whether, a given formula is valid or invalid or when 2 groups of when group of statements are consistent to each other. So, these are the following observations in continuation to the last lecture.

So, these are some of the important observations that we can arrive it. So, a well formed formula a after reducing it to conjunctive normal form or disjunctive normal form is valid if and only if, it is the case of conjunctive normal form if and only if each disjunctive clause in any conjunctive normal form representation of a contains a pair of complementary literals. So, what do you mean by a complementary literals.

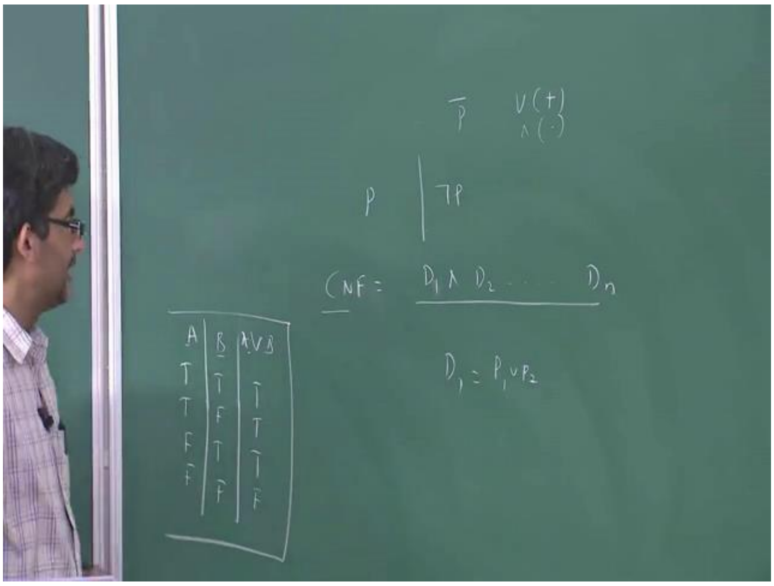
(Refer Slide Time: 05:21)



Suppose, if you have a literal P it is considered to be positive literal and then the complementary of this 1 is not P usually, we represent complementary as this particular kind of thing, but instead of that we are using negation of P in boolean logic, we use complementary for this 1 and then for or we use in boolean logic we use plus and for and it is a multiplication sign is used. So, these are the only differences between the boolean logic and the preposition logic that we are trying to talk about. So, essentially they are more or less the same.

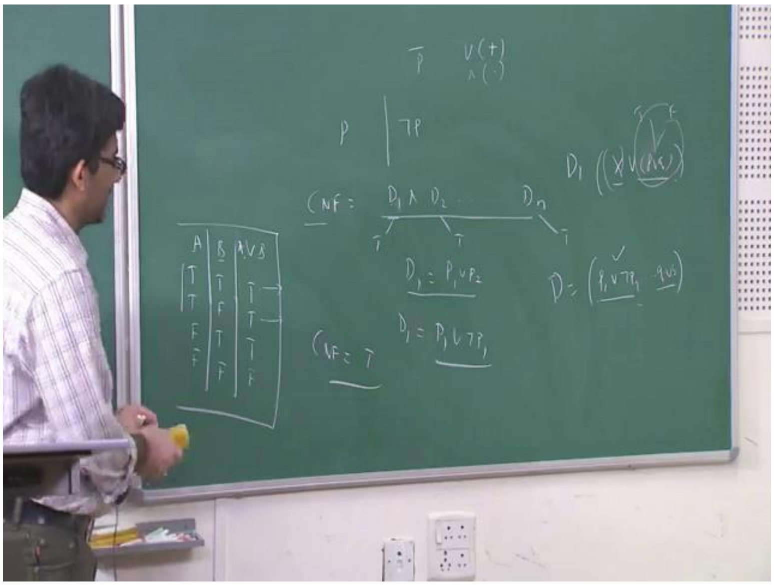
So, what is a CNF. CNF is nothing,, but a conjunction of disjunctions norm. So, first you write these thing conjunctions, of what disjunctions. So, there are several disjunction and all till d n. So, what do you mean by a disjunct, it can be in this form p1 or p2 etcetera and all. So, each term in the conjunctive normal form is a disjunction. So, in this sense a well formed formula is considered to be valid if and only if each disjunctive clause, in the conjunctive normal form contains a literal and its negation and all. So, now, observe this particular kind of formula.

(Refer Slide Time: 06:45)



So, we have semantics of or... So, that is like this you have a and you have b and these are the only values that it takes a takes this values and then alternative t f and all a or b the semantics of a or b that is truth meaning of a formula a or b is nothing,, but truth conditions of a or b and all. So, that is going to be false only when both disjuncts are false in all other cases it is going to be true. So now, this given formula D1 and D2 to dn.

(Refer Slide Time: 07:27)



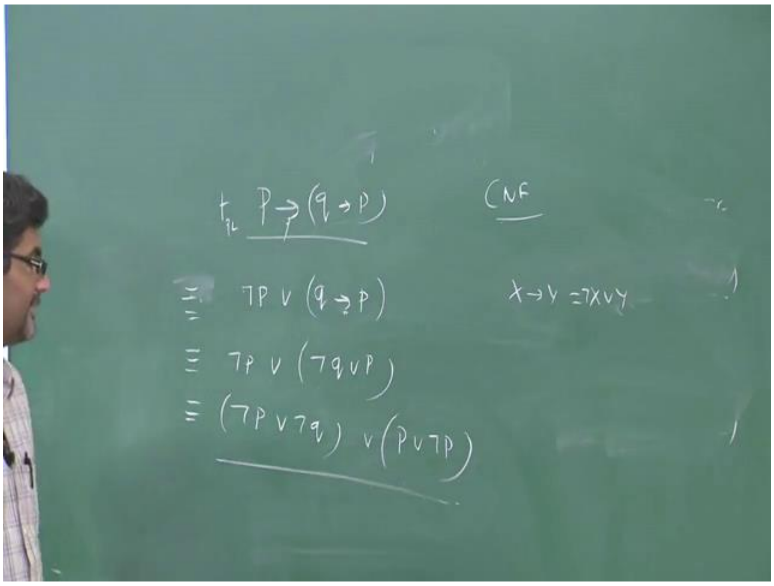
So, this is going to become true if and only if all the disjuncts are true, this has to be true, this has to be true and this has to be true then only your CNF is going to be true, then you have established that a given conjunctive normal form is considered to be a tautology. So, end as usual you know all tautologies are also considered to be valid formulas. So, now, 1 observation important observation is that why we are reducing a given formula into conjunctive normal form is that if it. So, happen that this disjunct has a literal and its negation something like p1 etcetera and all. Then this is always considered to be true only.

So, this is nothing but a tautology and all. So, now, suppose in your disjunction d you already have p1 or not p1 and there are some other letters let exist here, it can be q or s whatever it is. So, now since this is already true. So, now, it is like x or t x or some other formula like P something like that P or q something like that. So, now, since in this disjunct in every disjunct this first formula is already true and all, because of a literal and its negation is there that is always going to be tautology only.

So, now, irrespective of whether whatever follows after this thing is either true or false the whole thing is going to be true only because of this particular kind of thing. So, the first disjunct is already true observe these 2 cases and irrespective of this whatever, disjunct that follows after that 1 a literal and its negation suppose that happens to be true it is also true and the next literal that occurs after the negation and its literal needs negation that is considered to be false then also it is going to be true only.

So, in that sense if each disjunct have to be true there should be at least a literal and its negation has to be there, if a literal and its negation is there; then that disjunction is automatically turning out to be true only. So, in that sense, if each disjunct it. So, happen that each disjunct has a literal needs negation then all the formulas all the disjunct that occurs in the conjunctive normal form are going to be true in that sense this your given CNF is going to be true. So, in that sense a given formula is considered to be a tautology. So, now let us consider some simple examples with which, we will establish this particular kind of observation.

(Refer Slide Time: 10:24)



So, we know that this particular kind of formula P implies q implies p. So, that is a theorem or valid kind of thing this also called as paradox of material implication which will not going to the details of this 1,, but it is a theorem in prepositional logic suppose if you write like this. So, theorem in general prepositional logic. So, now, as a first step in CNF in conjunctive normal form which is a conjunctions of disjunctions and all. So, what you will do is you will start eliminating this implication.

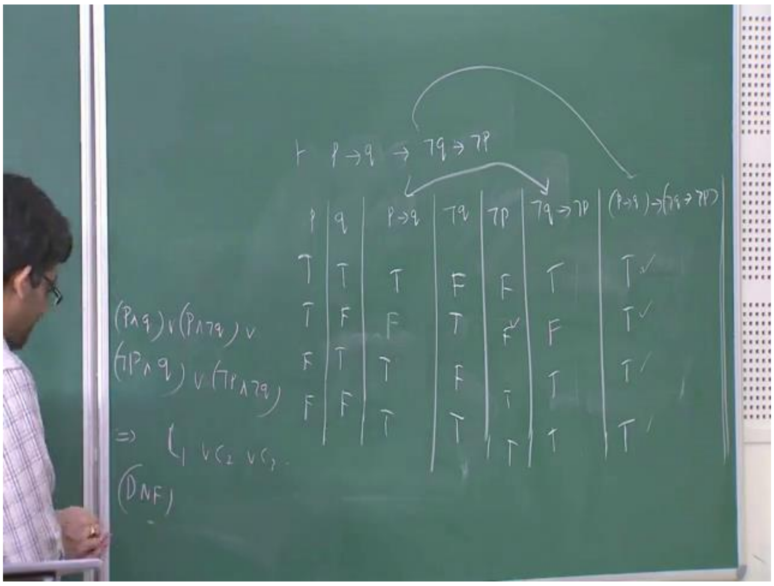
So, you eliminate this implication by using this particular kind of rule, x implies y is same as not x not x or y what essentially, you are doing is you are reducing this implication to simple disjunction its negation and all in the final formula what you will find is you will not find implication and all this time you will not get it, but only signs that you will come across is negation or end conjunction and all. So, in that sense you are reducing the given formula into CNF. So, now as a first step, what you will do is. So, now, the whole thing is taken as x.

So, now, x implies y means not of P or q implies p. So, this is the first step that we are trying to see. So, now, this will bracket should be there. So, now, you reduce the you eliminate this implication, then you will get not q or p. So, this is not P or whatever is there here is the 1 which we have written and the second step this will become this. So, now what you can do is you can use distribution law and all or associative law this will become not P or not q or P or not P not P or not q and not P or p. So, now, observe this particular kind of disjunction this is.

So, this particular kind of disjunction observe this particular kind of thing it is in this form x or y. So, now, since this is already true P or not P is also always considered to be true only. So, now, irrespective of whether this formula is going to be t or f this is always going to be true only, because of the semantics of disjunction. So, 1 particular disjunct is true then irrespective of whether this whole term is false or true this going to make the whole disjunct true it is in that sense. So, this formula is going to be a tautology. So, this is a 1 way of representing 1 particular observation is that in a given well formed formula suppose if it.

So, happen that a literal needs negation occurs then that is considered to be a tautology and it is also considered to be a valid formula. So, similar kind of thing which 1 can do with the help of a truth table also.

(Refer Slide Time: 14:00)



Example, suppose you know that this formula is always considered to be valid only, that is a law of contraposition P implies q is nothing,, but not q implies not p. So, now, from the truth table also 1 can draw the CNF DNF. So, now, there are 2 variables here P and q that is why there are 4 entries that are possible. So, now, then you need to take into consideration P implies q then the next 1 is not q and not P and then not q implies not P and then the final 1 is this 1 the whole thing. So, that is P implies q implies not q implies not p.

So now just quickly, will construct the truth table and all. So, since there are only 2 variables in the truth table, there are only 2 to the power of n entries will be there 2 to the power of n rows is going to be there. So, there are 4 rows which are possible. So, now, in this case t t f and ft f alternative t and alternative f is the 1 which you write. So, now, P implies q is going to be true is going to be false only in this particular kind of case in all other situations. So, that is going to be true only.

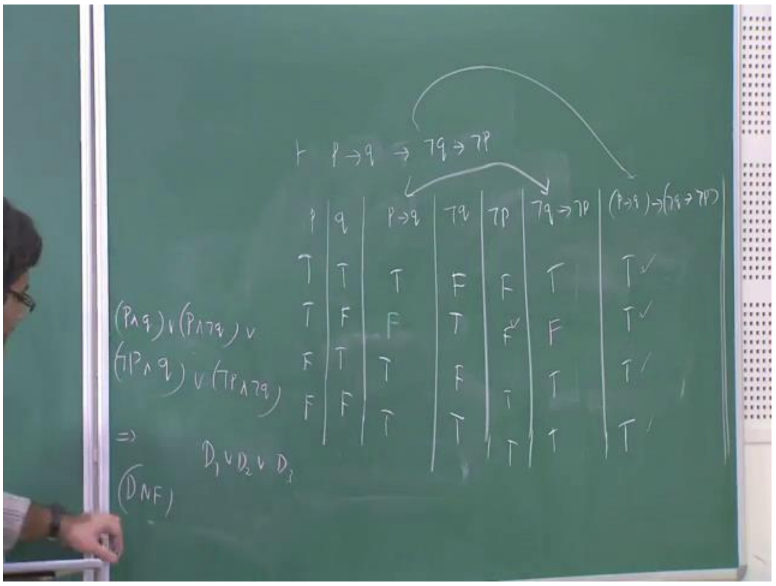
So, in all other cases is going to be t. So, this is all we know about material implication that is P implies q is nothing,, but not P or q. So, now, not q is exactly the opposite of these things. So, now whenever it becomes t it becomes false whenever, it is f it is t and f and t and not P this 1 this is f f and t t exactly the opposite of this values and all when P takes value t not P takes value f, so now not q implies not p. So, now, this formula is going to be false only in this case. So, that is when the antecedent true the consequent is false it is going to be false in all other cases it becomes t.

So, now, you need to observe the implication of these 2 things. So, that is going to be your final formula. So, why we are what is that we are essentially, trying to do is that from the truth table also you can make out you can write disjunctions DNF's and CNF's and all. So, now, this 1; now we need to check whether there is any formula in which P implies q is t and not q implies not P is false. So, now, this is the antecedent in which you 3 t's are there. So, now, we need to check whether there are there is any formula in which your antecedent is true and the consequent is false.

So, now, you will observe that there is no way no row in which you have P implies q t and not q implies not P false and all that is in that sense all are going to be t only. So, all this things are turning out to be true. So, now what we can do is this thing. So, from the truth table 1 can construct 1 can construct a DNF or CNF. Let us see, what how we do it. So, now, for the DNF's you take into consideration all the things, which are true all the rows are true only. So, now, we start constructing this thing conjunctive normal form for this 1.

So, now, what are the values for this 1 what are the rows in which it is true in all the rows it is true only. So, now it is like this whether P and q P and q or. So, now next formula is or P and not q or. So, now this 1; so that is not P whenever, it is P is false; it is represented as not P and then not P and q or this 1 both are false not P and not q. So, what essentially, we did is this thing that. So, it is like d1, c1 or c2 or c3 etcetera. So, now what is this c1 it is in a disjunctive normal form. So, where each disjunct is a conjunction.

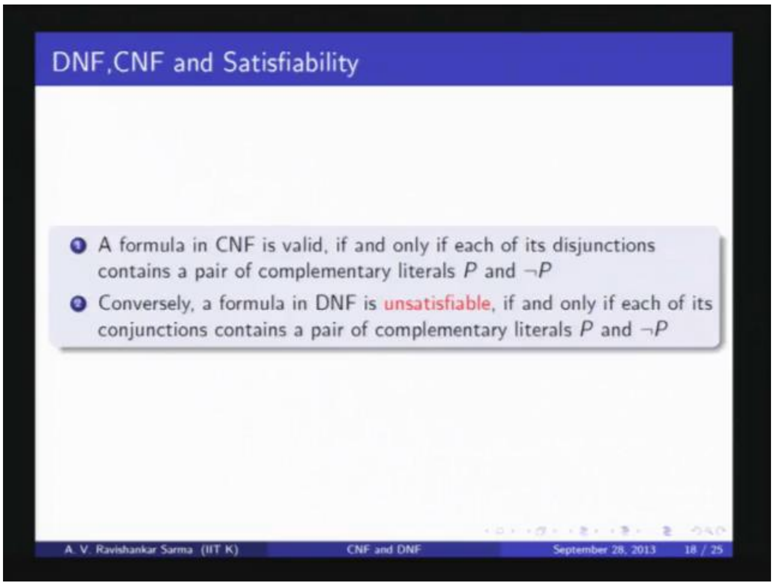
(Refer Slide Time: 19:14)



So, this is like should be written in this way D1 or D2 or d3. So, where each disjuncts is a is a conjunct and all. So, how did we essentially write this 1. So, under what condition this whole formula is going to be true. So, now when P takes value t and q takes value t that is 1 particular kind of condition under which this formula is going to be true. And then you have P and not q in that case it is going to be t and you have to list out all the rows in which the this formula is going to be t.

So, this is what we have at the end. So, that is in the form of disjunctive normal form D1 or D2 or D3 where each d 1 is a conjunct. So, this is the disjunctive normal form. So, now from the truth table 1 can make out you can write down the corresponding DNF or CNF. So, that is 1 way of doing it that is what is the important observation that, we can make out here.

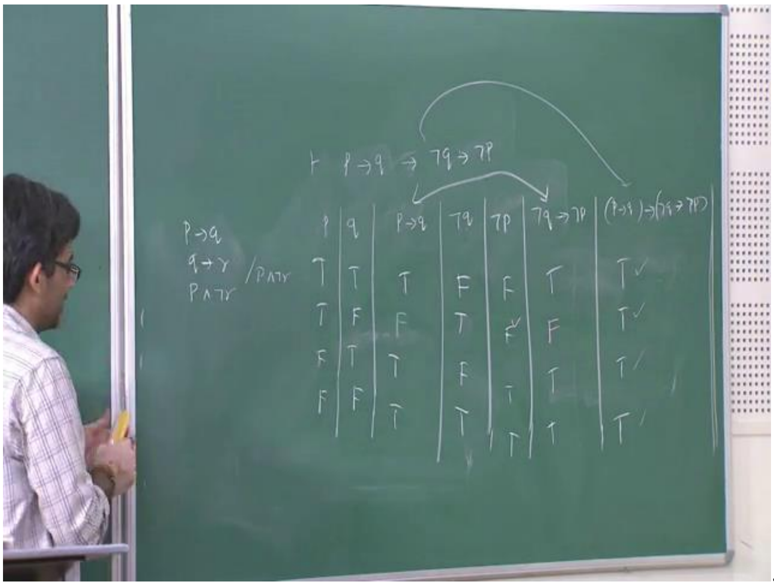
(Refer Slide Time: 20:20)



So, from the truth table also 1 can construct a corresponding DNF or CNF. So, now the second observation is that a given well formed formula is considered to be unsatisfiable, if and only if each conjunctive clause in any DNF representation of x contain a pair of complementary literals. So, it is so happened that in a DNF disjunctive normal form. It is. So, happen that you have both P and not P is there in any 1 of these formulas in each and every disjunct then all disjuncts will turn out to be false, in that case a given DNF is going to be false then it is said to be unsatisfiable.

So, for example, in this case P implies q and q implies r and P and not r. So, that is considered to be unsatisfiable, because in the last term in particular P and not r. So, that is having problem. So, that is why that that term is going to be false that makes the that makes the conjuncts false and all. So, when a formula x contains a pair of literal and its complementary that is not P, then in the given form the whole formula is going to be false and all. So, that makes this form unsatisfiable and all.

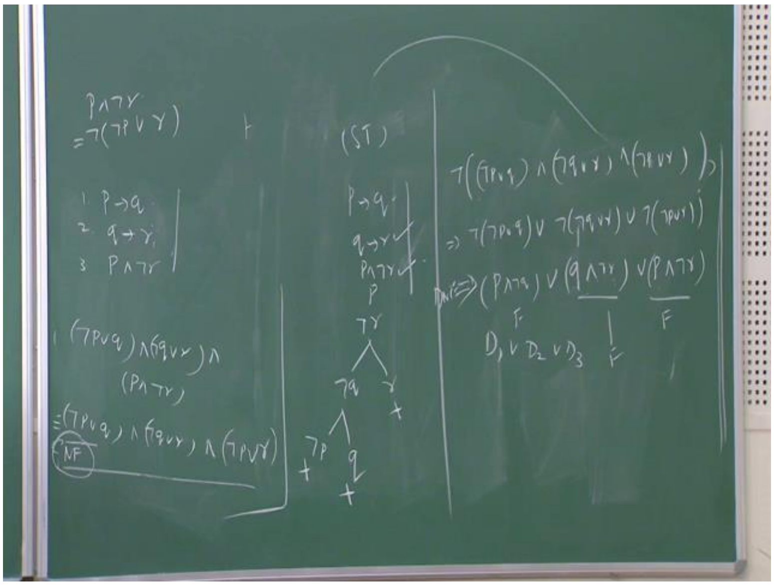
(Refer Slide Time: 22:01)



.

So, let us see why it is a case that P implies q and P implies q and q implies r and P implies P and not r. So, you add to this 1 why it is considered to be unsatisfiable and all.

(Refer Slide Time: 22:23)



So, now it is like this thing first 1 can be written as this 1. Now, we are trying to say why it is considered to be unsatisfiable and all. Let us see whether, it is unsatisfiable or not. So, now, the first statement can be written as this 1 P or not P or q and the second 1 is not q or r this the second 1 and then the third 1 is P and not r. So, this needs to be transformed into this not P or q. Now, the second write it in this way not q or r and. So, this can be written as this 1 use de Morgans law and then conversation to that its corresponding disjunctive form.

So, it is P and not r will become not P or r. So, why it is become like this P and not r is nothing but if it take the negation into consideration you have to take the negation of P and r. So, now if you see this 1 not of not P is P only not of disjunction is conjunction and this is not. So, this is same as this 1; so now why this formula is considered to be unsatisfiable. So, now we need to further reduce this particular kind of formula and then you will see why it is a case that it is going to be unsatisfiable and all.

So, now there are several ways of showing that whether or not this particular kind of formula is unsatisfiable or not. So, first there is a method which we have we came to know that is a semantic tab locks method using that you can see whether, it is unsatisfiable or not. So, now you can write it in this way P and not r is represented in this way. So, now, this formula is checked and all. So, now we are using semantic tab locks method just to see whether it is unsatisfiable or not.

So, when these 3 formulas are going to be unsatisfiable when you construct a tree for this formulas and all if all the branches closes then that is considered to be unsatisfiable if at least 1 branch is open. So, that is considered to be that is considered to be satisfiable and all. So, now we will come back to this particular kind of format little bit later. So, now we are checking it with semantic tab locks method whether it is considered to be satisfiable or not. So, now this is not q and r. So, this in this way you can write it. So, now you have r here and not r here.

So, this is to each other. So, this closes and then. So, whatever is left is this 1 not P and q. So, now P and not P closes and you have q and not q this also closes. So, now, from this method we can make out that these 3 statements are inconsistent to each other. So, now, according to our this thing. So, this is in CNF it is a conjunctions of disjunctions and all. So, this formula if it has to be false and all if this is going to be false if all these at least 1 of these disjunction is false and all then, it is going to make the whole conjunct false when you say that P and q is false at least 1 conjunct is false then you can say that the whole thing is false.

So, there are ways to say that. So, now what we observation tells us is that a well formed formula is said to be unsatisfiable, if each conjunctive clause in any DNF representation of x contains a pair of complementary literals and all, but here the formula is in CNF. So, now we need to convert this formula into corresponding DNF and all. So, if you take the negation of this whole formula then it will be converted into your corresponding DNF and all. So, that is not P or q and not q or r and not P not P or r. So, now the negation of this 1 is this.

So, now, this will become not of not P or q now negation of conjunction will become disjunction and then each term will be like this and negation of conjunction will become disjunction here and again negation of not P and r. So, now so what essentially we are trying to do is the formula is in CNF, but we are trying to converted into disjunctive normal form. So, once you converted into disjunctive normal form then, we use we make use of the observation that is a term which consist of both a literal and its negation then that conjunct has that term will be false and all.

So, now this translates into you have to use de Morgans laws then it, will become P and not q not of not P is P not of q is not q. So, now, this is or now this will become q and not r now or this is P and not r. So, now this is in the disjunctive normal form that is D1 or D2 or d3, where each disjunct is a conjunct. So, like in this first case d 1 is nothing but P and not q and D2 is q and not r and P D3 is P and not r. So, now, 1 of the important observation is this that whenever, your disjunct that is which is nothing but a conjunct here P and not q a literal and negation and all P and not q in each 1 of this terms a literally is there and unnegated form and the negated form is there and all.

If, you have q and it is not r and then P not r etcetera and all. So, that makes all this things false in any given DNF this is DNF of your formula this case. So, in your DNF each conjunct is having a literal negation or sometimes it need not have to be its own negation,, but negated formula and unnegated formula unnegated literal then obviously, that disjunct D1 has to be false and hence and since each disjunct is false that makes the whole formula false and all.

So, that is what is the important observation that we can make out a well formed formula a is considered to be unsatisfiable; that means, the formula is going to be false if and only if each conjunctive crossing any DNF. So, that is here in this case P and not q q and not r P and not r we will see negation and unnegated term and all unnegated term is followed by a negated term. So, in that case all d 1 d 2 d 3 are going to be false and hence is formula is going to be false that is why it is considered to be unsatisfiable. So, we can also show that a given formula is valid or not by again reducing it into conjunctive disjunctive normal form.

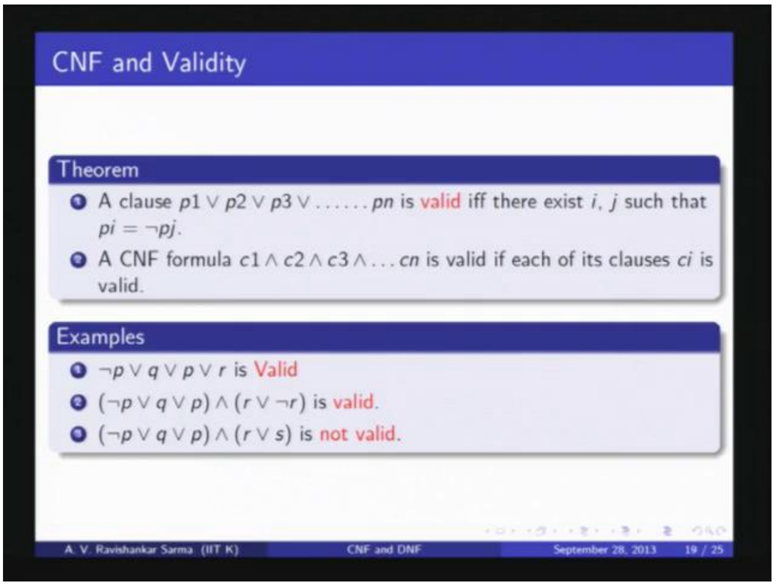
So, these are some of the important observations which 1 can make out. So, these are the 2 important things which are directly related to satisfiability of a given formula. A formula in conjunctive normal form is considered to be valid if and only if each of it is disjunctions because in conjunctive normal forms is a conjunctions of disjunctions and all. So, in each of such kind of disjunctions it contains a pair of complementary literals like P not P and all.

So, then each conjunct is going to be true so; that means, all the I mean; the whole formula is going to be true; that means, tautology and hence it is a valid formula and conversely a formula in DNF like this the 1 which we have explained it on the board is considered to be unsatisfiable; that means, is going to become the formula is going to become false, I mean; it takes a value 0 if and only if each of it conjuncts contain a pair of complementary literals like P or not P or P and not q etcetera and all.

So, the idea here is that 1 formula is 1 literally is true and the other literally is consider to be false its makes the whole formula whole d 1 false only since all disjuncts are false and the given disjunctive normal form whatever formula that exists in the disjunctive normal form D1 or D2 D3 all are false. So, hence D1 D2 D3 the DNF is going to be false; that means, is going to be unsatisfiable. So, this we make use of it in solving puzzles also while solving the knights and naves puzzles, we translate the statements into the appropriate language of prepositional logic.

Then we say that either we write it in the CNF or DNF usually, we write it in the CNF conjunction of disjunctions then if whether or not we will check whether or not a given formula is satisfiable and all. So, usually knights and naves puzzles are translated into those problems in which usually, satisfiable and all that makes some of this sentences true and all. So, then we will look for the solution whether or not a given person who is talking about something is a knight nave and all we will come to it little little while from now.

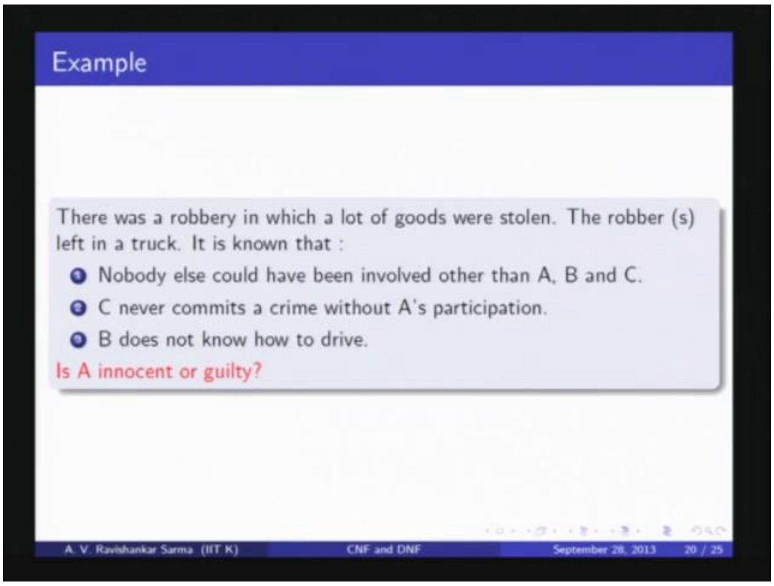
(Refer Slide Time: 32:49)



So, now this is an important theorem which is a worthwhile to mention here a clause P 1 or P 2 or P 3 etcetera is considered to be valid. If there exists some i and j such that that P i is equal into not P j and all; that means, A literal and its negation exist in a given formula then; obviously, it is going to be valid a conjunctive normal form c1 c2 c1 and c2 c3 where each c1 is a disjunct is going to be valid if each of its clauses ci that is D1 or D2 D3 are true. If all these things are true c1 is true and c2 c3 all are true then; obviously, CNF is going to be true that is, it is a tautology and hence it is a valid formula, so now in the example that are there here not P or q P or r. So, that is considered to be valid because, if you rearrange it in a certain way then it will become P or not P q or r. So, at least if P or not P is already true; we know that it is a tautology. So, in the whole formula is; obviously, going to be true only irrespective of whether P the other preposition variables whether it takes true truth or false then, it is going to be true only. So, that makes the whole formula true; that means, all true prepositions are considered to be tautologies.

In the second case also we have a literal needs negation in the in c1 and c2. So, a literal needs negations is there that is why that makes c1 true for example, c1 is nothing but not P or q or p. So, in that P or not P is already there irrespective of whether, q is true or false the first c1 the first term c1 is going to be; obviously, true in the same way in the seocnd term r and not r a literal needs negation is a that is always going to be true hence both the terms are true it is going to be a tautology, and obviously all tautologies are considered to be valid formulas.

(Refer Slide Time: 34:49)

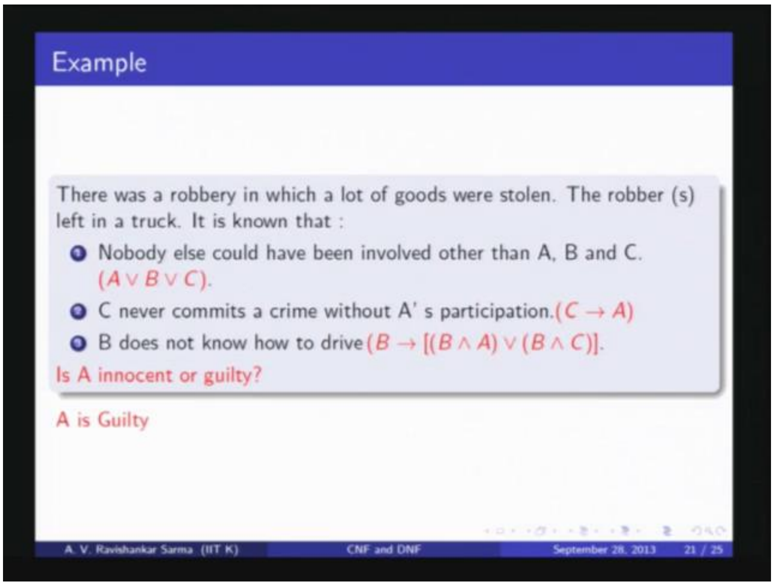


So, now let us try to solve some simple examples, with while making use of this particular kind of idea that is a literal needs negation exist in a given CNF then that formula is going to be a tautology. And hence it is a valid formula. So, now in these kinds of puzzles we are trying to look for the satisfiability or unsatisfiability of a given formulas and all. So, what we essentially we do is like this.

Let us consider simple example which we already discussed it, in the context of semantic tab locks method, but again we try to do the same thing, with the help of a converting this particular kind of formulas into conjunctive and disjunctive normal forms. So, here is a story which goes like this there was a robbery in which, lots of goods were stolen and it. So, happen that the robbers left in a truck and it is also know to us that this is a some of things which are known to us nobody else could have been involved other than only these 3 persons that is A B C.

So, no d is involved in this particular that information, we are sure of and the seocnd case is this that; C never commits a crime without a’s participation. So, wherever C is there you can assure we can surely say that A is already there. So, now the third thing which we already know is this particular kind of B does not know how to drive; that means, if B comes al1 out of all this things stolen goods and all he cannot free. So, he cannot run away because he do not know how to drive the truck and all; that means, he needs help of either A and C. So, now, with this information; we have to whether or not we need we need to show whether A is innocent or a is guilty. So, now, these problem require some kind of representation.

(Refer Slide Time: 36:44)

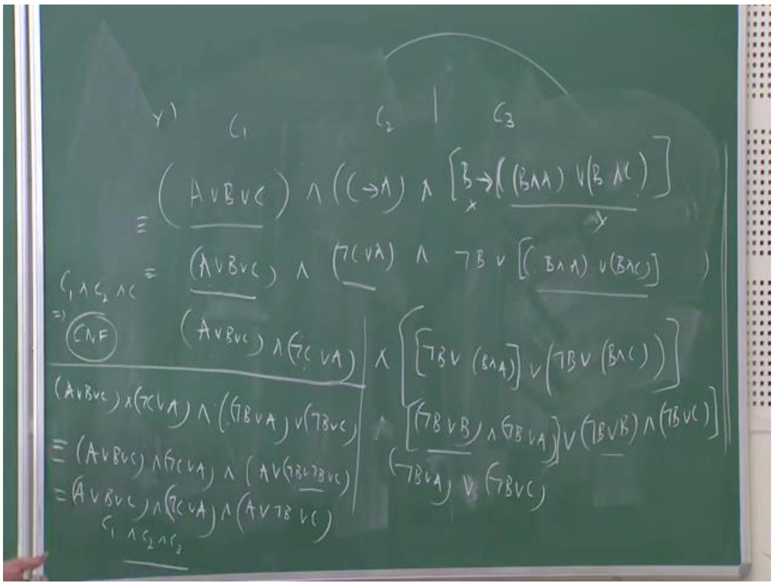


So, that is when, I say that A B C etcetera and all; A is guilty, B is guilty, C is guilty and all if I say not A not B not C then A is innocent not B means; B is innocent not C means; C is innocent. So, that particular kind of representation is what is needed in the beginning of solving this particular kind of problem. So, what essentially we are trying to do is we translate the given english language sentence into appropriately into the language of prepositional logic and then we try to convert it into CNF and DNF and from that, we can make out when it is going to be satisfiable and all is the which, we are trying to look for.

So, now the first statement can be represent in this way nobody else could have been involved means either A is involved or b is guilty or C is involved either 1 of them is guilty and all C never commits A crime without A's participation; that means, C implies A. So, now third statement B does not know how to drive. So, B if B has to be accompanied with that is A first sentence or B has to be accompanied by C that is why B and C. So, now we have set of formulas and all. The first 1 is obviously, in particular kind of format A or B or C.

Let us consider it as term c1 and the next 1 he need to change it into appropriately into a corresponding form and all. So, now, given this formulas, we are trying to check whether by reducing this formulas into conjunctive or disjunctive normal forms we are trying to check whether, we are trying to find out whether a is guilty or not. So, now, let us try to convert this formula into corresponding normal form and all. So, now, this is the thing which we have. So, these are the 3 formulas that we have.

(Refer Slide Time: 38:47)



So, that is first 1 is A or B or C conjunction the next statement is, what is that C implies a and the third 1 is B implies B and A this bracket should be there here particular or B and C. So, now it looks like that it is like in this particular kind of formats c1 c2 c3 c1 and c2 and c3. So, that is why it is called as CNF; I mean, you cannot call it as CNF this stage, but we need to convert this things each and every formula into conjunctions of disjunctions and all. So this already all disjuncts only.

So, now, this is as it is A or B or C. So, this is disjunct only and we need to write this thing as not C or A and we need to convert this thing into corresponding normal form. So, that is. So, now this whole thing is taken as x, I am sorry y and this as x. So, then B implies x implies y is not x or y. So, that is not B or the whole thing B and A or B and C. So, what we are essentially doing is we are reducing this formula into its corresponding normal form. So, these things are already transformed into corresponding disjunctions and all, so D1 and D2 etcetera and all.

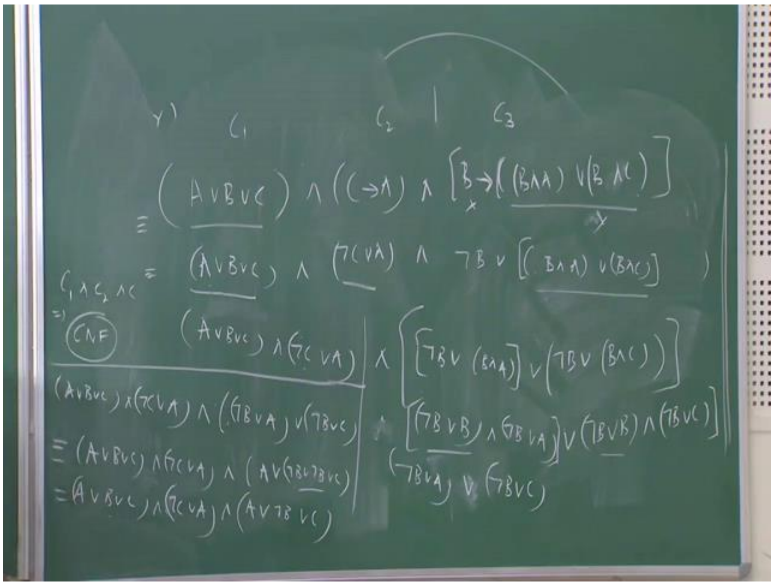
So, now this is as it is A or B or C and not C or A and no this is the 1 you have to use associative law and all. So, then this will become not B or B and A or this is the first 1 or not B or B and C. So, this is the second kind of thing. So, now we have to further reduce it into this particular kind of format. So, now you have to distribution laws here. So, this you need have to do anything here this will remain as it is. So, now, we need to convert this thing into particular kind of thing not B or B the first 1 and not B or A. So, this is what it reduces to or now this 1 is not B or B and not B or C; I mean this formula reduces into this 1.

So, now, we know that B or not B is; obviously, essentially, it is going to be t only. So, you may not to worry much about it. So, this always going to be t usually, represent is as this letter t. So, in the same way here not B or B is also going to be t. So, these terms will vanish and all here. So, now, what will remain here is not B or A or the other 1 here is this 1. Now, this is particular kind of connection and not B or C. So, now, this formula is reduces to obvious tautology and all this does not make any sense. Now, this is not B or A or whatever is there here and whatever is left is not B or C.

So, now so what are the formulas that we have now here. So, now, we need to see this box and all we have A or B or C and not C or A and. So, not C A and these are the formulas not B or A or not B or C. So, now this further reduces to this thing. So, why what is what essentially, we are trying to do is that a given well given formulas c1 c2 what c3 and all we are trying to reduce it into its corresponding normal forms and all and once you reduce it into some kind of format either CNF or DNF then, we can talk about whether this formula is going to be satisfiable or not.

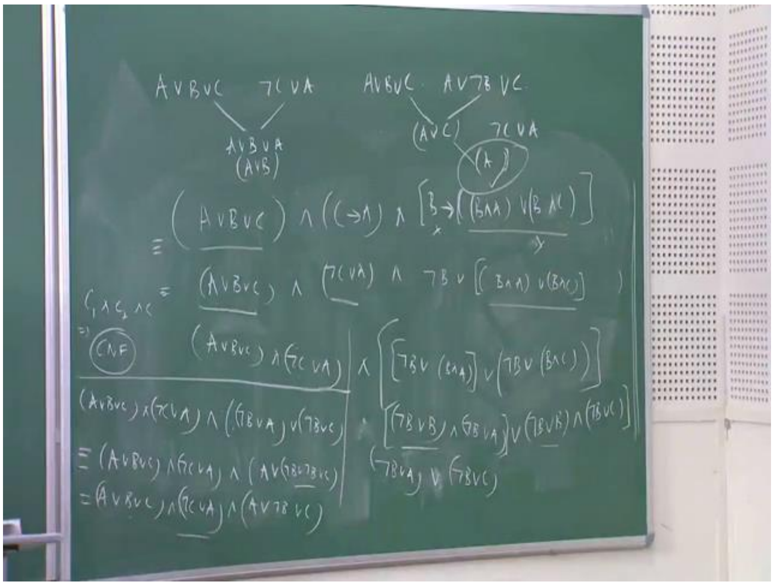
So, not B or A or not B or C. So, now this reduces to A or B or C and not A or A is as it is now here, we can use some kind of associative law and all then we can say that A or this not B goes not B or not B or C. So, now, not B are not be same as 1 not b only. So, this is A or B or C and not C or A and then A or not B or C. So, now we have everything is in the form conjunctive normal form each c1 c2 and c3. So, now we need to inspect that literal needs negation is there here it does not matter even here also negation and unnegated form is there that makes this 2 formulas true and then. So, now it its further reduces to this 1.

(Refer Slide Time: 45:50)



So, there is a way to talk about this thing once you convert this formulas into a given conjunctive normal forms and all.

(Refer Slide Time: 46:04)



Here, is a rule which we use A or B or C and not c or a. So, these 2 resolves into simple formula that is A or B and C are not C is going to be true only. So, then this reduces to this 1. So, this is nothing,, but A or B. So, now in the same way A or B or C A or B see the first term and the last term that is A or not B or C. So, now this resolves into A and B or not B is B and not B cancels and all then whatever, remains is C A or A or C yeah. So, this is another term. So, now, A or C and this term not C or A will end it in a while from now not C or A. So, this translates to a why because A or C and not C or A because this translates into I mean; this reduces to simply a resolves into A.

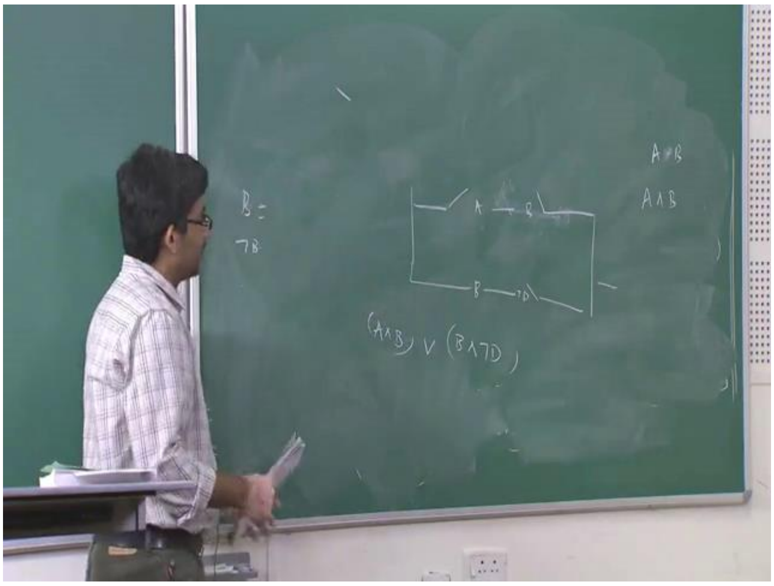
So, now we got this final thing that we got A is the case and all; that means, A has to be guilty and all. So, what essentially we did is clearly is that first you translate it the english language sentence into appropriately into the prepositional logic and then we reduce the non normal forms into normal forms and then we came across this particular kind of formula and all A or B or C not c or a. If you further simplify it and all. So, then ultimately we have this 2 literals A or B or C and not c or a these resolves into A or B or A and B or A is nothing but A only it becomes B.

So, this particular kind of method is what is called as resolution reputation method, which we will talk about in the next class. So how whenever, you 1 of the advantages of reducing the given formula into conjunctive normal formula, is this particular kind of thing. So, once you come across any conjunctive normal form then, you can use this resolution and reputation method to further reduce it into its corresponding formulas. So, these 2 resolves into A or C and A or C and not C or A.

So, this becomes true only in the case that A has to be true and all these conjuncts have to be true only when A has to be true otherwise it is going to be false. So, ultimately what we got is this particular kind of solution that observes this 1 A is considered to be the case; that means, A is considered to be guilty. So, now, this is the way in which 1 can solve some of the important puzzles and all. So, now we will move on to 1 simple example were this particular kind of thing is used and all.

So, this can also be used in analyzing some simple digital circuits and all. So, I will talk essentially about the basic idea of that 1 and I will try to solve 1 example based on particular kind how a given complex digital switching circuit can be translated into simple kind of formula and all.

(Refer Slide Time: 50:08)

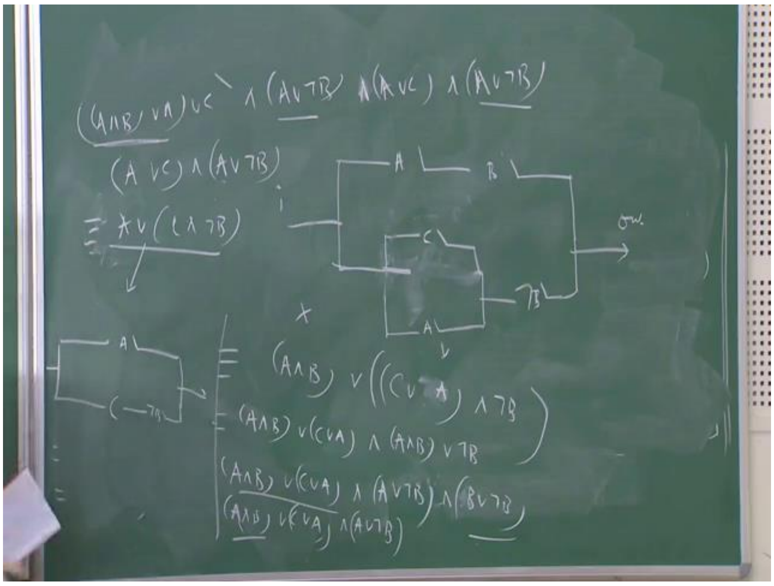


So, now usually, in the digital switching circuits whenever, you have this particular kind of thing A this A and then B suppose, if these 2 switches are in a series then usually you write it as A and B usually in the case of boolean interpretation, it is multiplication of B. So, whenever 2 switches are in they are arranged in a parallel kind of thing. Let us say there are 2 more switches like this D and all then you write it write this 1 as the first 1 is in the series that is why you write it as A and B and this 2 are in parallel.

So, that is why you use this particular kind of symbol or in the simple digital switching circuits you can call it as plus operation. So, now for example, if not D is there. So, here the notation is this that you usually take it in this way the moment are write B; that means, b is closed you switched it on or if I write not B; that means, B is open the switch is off. So, that is only difference here. So, this 1 can be written as B and not D because B and not D are in a series. So, you can write it in this way.

So, what you will essentially do is that: given a simple digital switching circuit, we translate it appropriately into a given prepositional logic and then we trying to simplify this formula and then this simplified formula is corresponds to simplified kind of circuit. Let us consider, 1 simple example with this we will end this lecture. So, here is this example.

(Refer Slide Time: 52:01)



So, now you have A switching circuit a like this and you have B and then this connected in this sense C. So, these 2 are connected parallelly A and this 1 and not B is the 1 which is there here. Now, you will generate some kind of output and all. So, this is the input and you will get some kind of output and all. So, this diagram essentially, says that this c and A are connected parallelly and of course, A and B are in series C and A are connected parallelly and then not B is also connected in a series.

So, now, this is written in terms of prepositional logic like this. So, the first formula is written as this thing since A and B are in series; that means, both switches have to be. So, that current passes through it. Now second 1 or it should be like this C. So, this is going to be the right kind of diagram. So, now this is in parallel. So, that is why you write it in this way C or A and now, this is in series that is why it is not B. So, now, here are the 2 formulas and now, these formulas are further reduces to this particular kind of thing this is as it is A and B or C or A and B and B or not B.

So, now we use distributive law and all suppose, if you take this as x and this as y. So, now the first term a and b or c or not a that is a first term and now, A and B or not B. So, now, this is going to be your second term. So, now you need to rearrange it a little bit and then this will become like this A or B or C or a as it is and this will become A or B A or not B. So, the first 1 and A or sorry B or not B. So, now again used a distributive law and all. So, this will become A or not B and A B or not B. So, now so this changes it to this particular kind of thing A and B or C or A and A or not B. So, because B or B is; obviously, true and all you can ignore this particular kind of thing.

So, now you have A and B C or A and A or not B. So, with this I try to end. So, now observe this 2 terms A and B A and B or C or A and A or not B. So, now again we need to use some kind of associative law and all. So, now, this will become A or B C or A and you are be this thing. So, now use distributive law on this 1 it will become A and B or A or C and A or not B A or not B then A and A or C and A or not B. So, ultimately this reduces to this particular kind of thing since A or B and A or B is same. So, now so this will become A or not B will become a only, because of law of observation.

So, this is A or C and what else is the case what is left and A or not B. So, now, this can be written as A or C and not B. So, this is what we have reduced into. So, now, a complex final remark is that a complex digital circuit, when it is transformed in by using the principles of prepositional logic this transformed into this particular kind of thing. So, now, you start worrying about your circuit and all. So, now this says that you have a switching circuit A and then it moves to goes to like this and then not B. So, now this is going to be your simplified circuit corresponding to this 1.

So, what is given here is this that a given complex circuit can be reduced into a simple switching circuit and all. So, in this lecture, what we did is simply this that we given a well formed formula, we tried to reduce it into conjunctive and disjunctive normal forms and then we talked about we solved dome kind of puzzles by using this reducing it into conjunctive and disjunctive normal forms. And then we tried to see whether, we can simplify the complex digital circuit into a simple switching digital circuit. So, in the next class, we will be talking about a resolution reputation method; which is considered to be an outcome of reducing the given formula into a conjunctive normal form.